Results in Physics 35 (2022) 105358

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Abundant new optical soliton solutions related to q-deformed Sinh–Gordon model using two innovative integration architectures

Nauman Raza [a], Saima Arshed [a], H.I. Alrebdi [b], Abdel-Haleem Abdel-Aty [c,d,*], H. Eleuch [e,f,g]

[a] Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore, Pakistan
[b] Department of Physics, College of Science, Princess Nourah bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia
[c] Department of Physics, College of Sciences, University of Bisha, PO Box 344, Bisha 61922, Saudi Arabia
[d] Physics Department, Faculty of Science, Al-Azhar University, Assiut 71524, Egypt
[e] Department of Applied Physics and Astronomy, University of Sharjah, Sharjah, United Arab Emirates
[f] College of Arts and Sciences, Abu Dhabi University, Abu Dhabi 59911, United Arab Emirates
[g] Institute for Quantum Science and Engineering, Texas A&M University, College Station, TX 77843, USA

ARTICLE INFO

Keywords:
Traveling wave transformation
Soliton
q-deformed Sinh–Gordon equation, generalized projective Riccati equations (GPRE) method
$\exp(-\phi(\xi))$-expansion method

ABSTRACT

In this article, the q-deformed Sinh–Gordon equation has been investigated using $\exp(-\phi(\xi))$-expansion and the generalized projective Riccati equations (GPRE) methods. These methods successfully extract trigonometric and hyperbolic solutions along with the suitable constraint conditions, imposed on parameters for the existence of solutions. Besides that, the numerical results are displayed through various graphics to show the diverse propagation patterns of solitons. The proposed equation has opened new horizons for modeling those physical system where symmetry is violated.

Introduction

Dynamical models form the foundation of many scientific domains, yet they are little documented in the literature. They are a major topics of study in mathematical and applied sciences. Such models are based on nonlinear ordinary or partial differential equations. Nonlinearity has significant impacts on describing the dynamics not just in macroscopical systems, but also in microscopical systems governed by quantum physics laws [1,2]. We can provide many examples like bifurcation, chaos, Rogue waves, stability, lasing, phase transition, correlations, squeezing, super radiance, and solitons [3–12]. Solitons are of particular interest due to their potential for novel applications in a variety of sectors of sciences. Soliton is a traveling wave solution with particle-like characteristics. This propriety can be described in general by a balance of dispersion and nonlinearity. The essential feature of integrable models is the soliton solution [13–15]. Solitons are generated by a number of well-known nonlinear PDEs such as Kadomtsev–Petviashvili equation, nonlinear Schrödinger equation, Sin-Gordon, Korteweg–de Vries equation, Kundu–Mukherjee–Naskar equation, and Sinh–Gordon equations [16–23].

Nonlinear phenomena can be seen in many fields, like hydrodynamics, fluid mechanics, plasma physics, quantum electronics, and optics [24–26]. NLPDEs are commonly used to gain a better understanding of the tendency of these areas, specifically some attractive models derived from the Boussinesq equation. Multiple schemes for extracting exact as well as the numerical solutions to nonlinear evolution equations (NLEEs) have been devised in order to give sufficient information for comprehending physical events occurring in a number of science and engineering fields such as the unified strategy, to discover the exact solitary wave solutions, Painleve approach, Wronskian formulation, linear superposition principle, Hirota bilinear method, inverse scattering method, invariant subspaces, novel auxiliary equation strategy, and symmetry reduction strategy. The fractal vibrational principle always leads to approximate solution instead of exact solutions are discussed in [27–29]. This significance has led in the creation and implementation of a variety of strategies in this respect [30–36].

More and more attention is given to these nonlinear challenges, and much nonlinear identification research will eventually be characterized as NLEEs. As a result, determining how to find their exact solutions are critical for associated nonlinear scientific analysis, and this has always been a crucial topic in mathematics and physics research. The extraction of solitons to NPDEs, in particular, has become one of the most fascinating and engaging areas of study. Significant progress has been made in recent years, and numerous robust and efficient strategies for obtaining accurate NLEE solutions have been established. The GPRE and the $\exp(-\phi(\xi))$-expansion schemes are regarded as the most appropriate strategies for generating exact solutions for NLPDEs.

* Corresponding author at: Department of Physics, College of Sciences, University of Bisha, PO Box 344, Bisha 61922, Saudi Arabia.
E-mail address: amabdelaty@ub.edu.sa (A.-H. Abdel-Aty).

https://doi.org/10.1016/j.rinp.2022.105358
Received 24 January 2022; Received in revised form 6 February 2022; Accepted 17 February 2022
Available online 26 February 2022

These techniques are straightforward, easy, and successful, and they may be applied to a wide range of NLPDEs to identify solitary wave solutions. These approaches are useful because they are applicable to equations with a large balancing number. In other analytical solution methods, the computational cost rises as the balancing number rises.

This article deals with the generalized q-deformed Sinh–Gordon equation (Eleuch equation) [37]. It describes

$$\frac{\partial^2 m}{\partial x^2}-\frac{\partial^2 m}{\partial t^2}=\left[\sinh_q(m^\gamma)\right]^p-\delta, \tag{1}$$

where $\sinh_q$ is the function of Arai q-deformed defined by

$$\sinh_q(x)=\frac{e^x-qe^{-x}}{2},\quad 0<q\le 1. \tag{2}$$

For $q=1$ gives standard sinh functions. The $\cosh_q(x)$, $\tanh_q(x)$ and their reciprocal along with their useful properties are well described in [37,38]. It is not restricted to mathematical, theoretical physics and applied mathematics, but also provides applications in physics when the symmetry of the explored system is missing or broken. We want to use the proposed approaches to examine the optical soliton solutions of Eq. (1). On the contrary, this work proposes unique optical solitons and various types of solutions to this nonlinear model. This article describes a significant advancement and gradual enhancements to past work.

The remainder of the article is structured as following. Section "The mathematical analysis" covers the fundamentals of mathematical analysis. Section "Outline of the analytical techniques" presents an overview of the applicable techniques. Section "Construction of solutions using proposed methods" describes how to generate soliton solutions using the proposed method. Section "Graphical illustrations" contains a graphical representation of the obtained solutions to the problems. Finally, in Section "Conclusion", the work's conclusion is presented.

The mathematical analysis

The following transformation is used to determine the traveling wave solution of Eq. (1) [37],

$$\zeta=\frac{1}{\sqrt{1-\alpha^2}}(x-\alpha t), \tag{3}$$

where α denotes the speed of traveling wave. Using Eq. (3), Eq. (1) has been converted into following ODE [37],

$$\frac{d^2m(\zeta)}{d\zeta^2}=\left[\sinh_q(m^\gamma(\zeta))\right]^p-\delta. \tag{4}$$

In this paper, Eq. (4) has been studied for the case $p=1$, $\gamma=1$ and $\delta=0$ which represent the q-deformed-Sinh–Gordon equation. The traveling wave solutions for Eq. (4) will be obtained in the subsequent sections using proposed analytical methods by applying the transformation

$$n(\zeta)=e^{m(\zeta)}. \tag{5}$$

After utilizing Eq. (5), Eq. (4) becomes

$$-2n'^2+2nn''-n^3+qn=0. \tag{6}$$

Outline of the analytical techniques

This section goes over the analytical techniques in detail.

Method-I: exp(−ϕ(ζ)) method

According to this technique [39,40], the solution has the form,

$$n(\zeta)=\sum_{p=0}^{N}A_p\left(\exp(-\Phi(\zeta))\right)^p, \tag{7}$$

A_p represents arbitrary constants and $\Phi(\zeta)$ obeys the following ODE

$$\Phi'(\zeta)=\exp(-\Phi(\zeta))+m_1\exp(\Phi(\zeta))+l. \tag{8}$$

Eq. (8) possesses the following hyperbolic, trigonometric and rational solutions.

When $l^2-4m_1>0$ and $m_1\neq 0$, then

$$\Phi_1(\zeta)=\ln\left(\frac{-\sqrt{l^2-4m_1}\tanh\left(\frac{\sqrt{l^2-4m_1}}{2}(\zeta+C)\right)-l}{2m_1}\right). \tag{9}$$

When $l^2-4m_1<0$ and $m_1\neq 0$, then

$$\Phi_2(\zeta)=\ln\left(\frac{\sqrt{4m_1-l^2}\tan\left(\frac{\sqrt{4m_1-l^2}}{2}(\zeta+C)\right)-l}{2m_1}\right). \tag{10}$$

When $l^2-4m_1>0$ and $m_1=0$ and $l\neq 0$ then

$$\Phi_3(\zeta)=-\ln\left(\frac{l}{\cosh(l(\zeta+C))+\sinh(l(\zeta+C))-1}\right). \tag{11}$$

When $l^2-4m_1=0$ and $m_1\neq 0$ and $l\neq 0$, then

$$\Phi_4(\zeta)=\ln\left(-\frac{2(l(\xi+C))+2}{l^2(\zeta+C)}\right). \tag{12}$$

When $l^2-4m_1=0$ and $m_1=0$ and $l=0$, then

$$\Phi_5(\zeta)=\ln(\zeta+C). \tag{13}$$

The balancing principle determines the value of N. When Eq. (7) is substituted into ODE, an algebraic equation involving powers of $\exp(-\Phi(\zeta))$ is produced. A system of algebraic equations for A_p is obtained by equating the coefficients of each power of $\exp(-\Phi(\zeta))$ to zero.

Method-II: Key points of GPRE technique

The solution takes the form given by GPRE method [38]

$$n(\zeta)=A_0+\sum_{p=1}^{N}\Omega^{p-1}(\zeta)\left[A_p\Omega(\zeta)+B_pY(\zeta)\right], \tag{14}$$

where A_0, A_p and B_p are the unknown constants. The functions $\Omega(\zeta)$ and $Y(\zeta)$ obey the following ODEs

$$\Omega'(\zeta)=e_1\Omega(\zeta)Y(\zeta), \tag{15}$$

$$Y'(\zeta)=R_1+e_1Y^2(\zeta)-M\Omega(\zeta),\qquad e_1=\pm 1, \tag{16}$$

where

$$Y^2(\zeta)=-e_1\left(R_1-2M\Omega(\zeta)+\frac{M^2+r_1}{R_1}\Omega^2(\zeta)\right). \tag{17}$$

Eqs. (15)–(16) possess the following solutions

Family 1 $e_1=-1,\ r_1=-1,\ R_1>0,\ M\neq 0$

$$\Omega_1(\zeta)=\frac{R_1\,\mathrm{sech}(\sqrt{R_1}\zeta)}{M\,\mathrm{sech}(\sqrt{R_1}\zeta)+1},\qquad Y_1(\zeta)=\frac{\sqrt{R_1}\tanh(\sqrt{R_1}\zeta)}{M\,\mathrm{sech}(\sqrt{R_1}\zeta)+1}.$$

Family 2 $e_1=-1,\ r_1=1,\ R_1>0,\ M\neq 0$

$$\Omega_2(\zeta)=\frac{R\,\mathrm{csch}(\sqrt{R_1}\zeta)}{M\,\mathrm{csch}(\sqrt{R_1}\zeta)+1},\qquad Y_2(\zeta)=\frac{\sqrt{R_1}\coth(\sqrt{R_1}\zeta)}{M\,\mathrm{csch}(\sqrt{R_1}\zeta)+1}.$$

Family 3 $e_1=1,\ r_1=-1,\ R_1>0,\ M\neq 0$

$$\Omega_3(\zeta)=\frac{R\sec(\sqrt{R_1}\zeta)}{M\sec(\sqrt{R_1}\zeta)+1},\qquad Y_3(\zeta)=\frac{\sqrt{R_1}\tan(\sqrt{R_1}\zeta)}{M\sec(\sqrt{R_1}\zeta)+1}.$$

Family 1 $e_1 = 1,\ r_1 = 1,\ R_1 > 0,\ M \neq 0$

$$\Omega_4(\zeta) = \frac{R_1 \csc(\sqrt{R_1}\zeta)}{M\csc(\sqrt{R_1}\zeta)+1}, \qquad Y_4(\zeta) = \frac{\sqrt{R_1}\cot(\sqrt{R_1}\zeta)}{M\csc(\sqrt{R_1}\zeta)+1}.$$

Plugging Eq. (14) into ODE and collecting the coefficients of same powers of $\Omega^p(\zeta)Y^j(\zeta)$ $(j = 0, 1, \dots; p = 0, 1, \dots)$ (or $\Omega^p(\zeta)$, $p = 0, 1, \dots$) and equating them to zero. The constants A_0, A_p, B_p, m and R, $p = 1, 2, \dots, N$ are extracted upon solving the system of algebraic equations.

Construction of solutions using proposed methods

This section gives the extraction of soliton solutions for the proposed model Eq. (1) for $p = 1$, $\gamma = 1$ and $\delta = 0$ by employing one of the most efficient analytical methods namely $\exp(-\phi(\zeta))$-expansion method and the generalized projective Riccati equations (GPRE) method.

Method-I

In this part, Eq. (6) is solved using the $\exp(-\phi(\zeta))$ approach to produce soliton solutions.
In Eq. (6), balancing n'^2 and n^3 produces $N = 2$. The suggested solution is as following:

$$n(\zeta) = \sum_{p=0}^{2} A_n \left(\exp(-\Phi(\zeta))\right)^p, \tag{18}$$

Using the approach described in Section "Outline of the analytical techniques". The values of the unknowns A_0, A_1, A_2 are calculated as:
SET 1:

$$A_0 = l^2, \quad A_1 = 4l, \quad A_2 = 4, \quad q = (l^2 - 4m_1^2).$$

The hyperbolic function solutions corresponding to **SET 1** are
When $l^2 - 4m_1 > 0$ and $m \neq 0$, then

$$m(\zeta) = \ln\left[\frac{\left(l^2 - 4m_1^2 + l\sqrt{l^2 - 4m_1^2}\tanh[\frac{1}{2}\sqrt{l^2 - 4m_1}(C+\zeta)]\right)^2}{\left(l + \sqrt{l^2 - 4m_1}\tanh[\frac{1}{2}\sqrt{l^2 - 4m_1}(C+\zeta)]\right)^2}\right]. \tag{19}$$

The trigonometric function solutions are
When $l^2 - 4m_1 < 0$ and $m_1 \neq 0$, then

$$m(\zeta) = \ln\left[\frac{\left(l^2 - 4m_1^2 - l\sqrt{-l^2 + 4m_1^2}\tan[\frac{1}{2}\sqrt{-l^2 + 4m_1}(C+\zeta)]\right)^2}{\left(l - \sqrt{-l^2 + 4m_1}\tan[\frac{1}{2}\sqrt{-l^2 + 4m_1}(C+\zeta)]\right)^2}\right]. \tag{20}$$

The hyperbolic function solutions corresponding to **SET 1** are
When $l^2 - 4m_1 > 0$ and $m_1 = 0$ and $l \neq 0$ then

$$m(\zeta) = \ln\left[l^2\coth[\frac{1}{2}l(C+\zeta)]\right], \tag{21}$$

where C is the constant of integration.

Method-II

In this article, Eq. (6) is solved using the GPRE method to generate soliton solutions. Eq. (14) has the following form for $N = 2$.

$$n(\zeta) = A_0 + \sum_{p=1}^{2} \Omega^{p-1}(\zeta)\left[A_p\Omega(\zeta) + B_pY(\zeta)\right], \tag{22}$$

where A_0, A_1, A_2 and B_1, B_2 are the unknown constants to be found by inserting Eq. (22) in the Eq. (6) and adopting the proposed method as mentioned. The values of unknowns A_0, A_1, A_2 and B_1, B_2 are summarized as **SET 1**.

$$A_0 = \sqrt{q}, \quad A_1 = -2M, \quad A_2 = \frac{2(M^2+r_1)}{\sqrt{q}}, \quad B_1 = 0, \tag{23}$$

$$B_2 = \pm\frac{2\sqrt{M^2+r_1}}{q^{\frac{1}{4}}}, \quad R_1 = \sqrt{q}.$$

The hyperbolic solutions have been calculated corresponding to **SET 1**, for **Family 1**

$$m(\zeta) = \ln\left[\sqrt{q} - 2M\left(\frac{\sqrt{q}\,\mathrm{sech}(\sqrt{\sqrt{q}}\zeta)}{M\,\mathrm{sech}(\sqrt{\sqrt{q}}\zeta)+1}\right) + \frac{2(M^2-1)}{\sqrt{q}}\left(\frac{\sqrt{q}\,\mathrm{sech}(\sqrt{\sqrt{q}}\zeta)}{M\,\mathrm{sech}(\sqrt{\sqrt{q}}\zeta)+1}\right)^2 \pm \frac{2\sqrt{M^2-1}}{q^{\frac{1}{4}}}\left(\frac{\sqrt{q}\,\mathrm{sech}(\sqrt{\sqrt{q}}\zeta)}{M\,\mathrm{sech}(\sqrt{\sqrt{q}}\zeta)+1}\right)\left(\frac{\sqrt{\sqrt{q}}\tanh(\sqrt{\sqrt{q}}\zeta)}{M\,\mathrm{sech}(\sqrt{\sqrt{q}}\zeta)+1}\right)\right]. \tag{24}$$

The hyperbolic solutions have been calculated corresponding to **SET 1**, for **Family 2**

$$m(\zeta) = \ln\left[\sqrt{q} - 2M\left(\frac{\sqrt{q}\,\mathrm{csch}(\sqrt{\sqrt{q}}\zeta)}{M\,\mathrm{csch}(\sqrt{\sqrt{q}}\zeta)+1}\right) + \frac{2(M^2+1)}{\sqrt{q}}\left(\frac{\sqrt{q}\,\mathrm{csch}(\sqrt{\sqrt{q}}\zeta)}{M\,\mathrm{csch}(\sqrt{\sqrt{q}}\zeta)+1}\right)^2 \pm \frac{2\sqrt{M^2+1}}{q^{\frac{1}{4}}}\left(\frac{\sqrt{q}\,\mathrm{csch}(\sqrt{\sqrt{q}}\zeta)}{M\,\mathrm{csch}(\sqrt{\sqrt{q}}\zeta)+1}\right)\left(\frac{\sqrt{\sqrt{q}}\coth(\sqrt{\sqrt{q}}\zeta)}{M\,\mathrm{csch}(\sqrt{\sqrt{q}}\zeta)+1}\right)\right]. \tag{25}$$

Applying GPRE method, the values of constants are determined for $e_1 = 1$ as
SET 2

$$A_0 = -\sqrt{q}, \quad A_1 = 2M, \quad A_2 = -\frac{2(M^2+r_1)}{\sqrt{q}}, \quad B_1 = 0, \tag{26}$$

$$B_2 = \pm\frac{2\sqrt{-M^2-r_1}}{q^{\frac{1}{4}}}, \quad R_1 = \sqrt{q}.$$

Following trigonometric solutions have been calculated corresponding to **SET 2**, for **Family 3**

$$m(\zeta) = \ln\left[-\sqrt{q} + 2M\left(\frac{\sqrt{q}\sec(\sqrt{\sqrt{q}}\zeta)}{M\sec(\sqrt{\sqrt{q}}\zeta)+1}\right) - \frac{2(M^2-1)}{\sqrt{q}}\left(\frac{\sqrt{q}\sec(\sqrt{\sqrt{q}}\zeta)}{M\sec(\sqrt{\sqrt{q}}\zeta)+1}\right)^2 \pm \frac{2\sqrt{-M^2+1}}{q^{\frac{1}{4}}}\left(\frac{\sqrt{q}\sec(\sqrt{\sqrt{q}}\zeta)}{M\sec(\sqrt{\sqrt{q}}\zeta)+1}\right)\left(\frac{\sqrt{\sqrt{q}}\tan(\sqrt{\sqrt{q}}\zeta)}{M\sec(\sqrt{\sqrt{q}}\zeta)+1}\right)\right]. \tag{27}$$

Following trigonometric solutions are calculated corresponding to **SET 2**, for **Family 4**

$$m(\zeta) = \ln\left[-\sqrt{q} + 2M\left(\frac{\sqrt{q}\csc(\sqrt{\sqrt{q}}\zeta)}{M\csc(\sqrt{\sqrt{q}}\zeta)+1}\right) - \frac{2(M^2+1)}{\sqrt{q}}\left(\frac{\sqrt{q}\csc(\sqrt{\sqrt{q}}\zeta)}{M\csc(\sqrt{\sqrt{q}}\zeta)+1}\right)^2 \right. \tag{28}$$

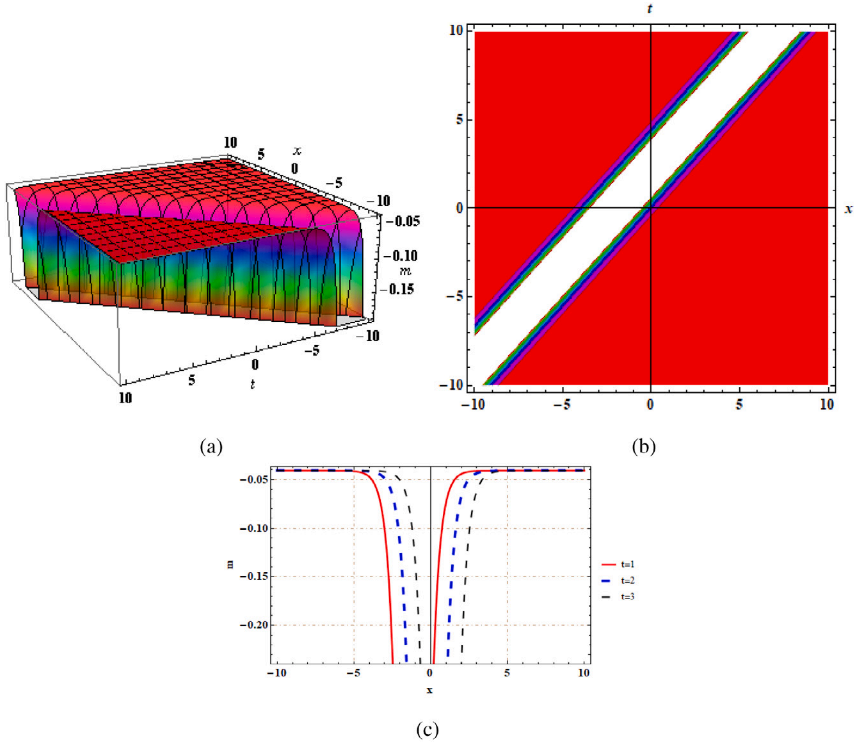

Fig. 1. (a) is **3D plot** of Eq. (19) by taking $l = 1$, $m_1 = 0.01$ and $C = 0$, (b) Contour plot and (c) is **2D line plots** of Eq. (19) against x.

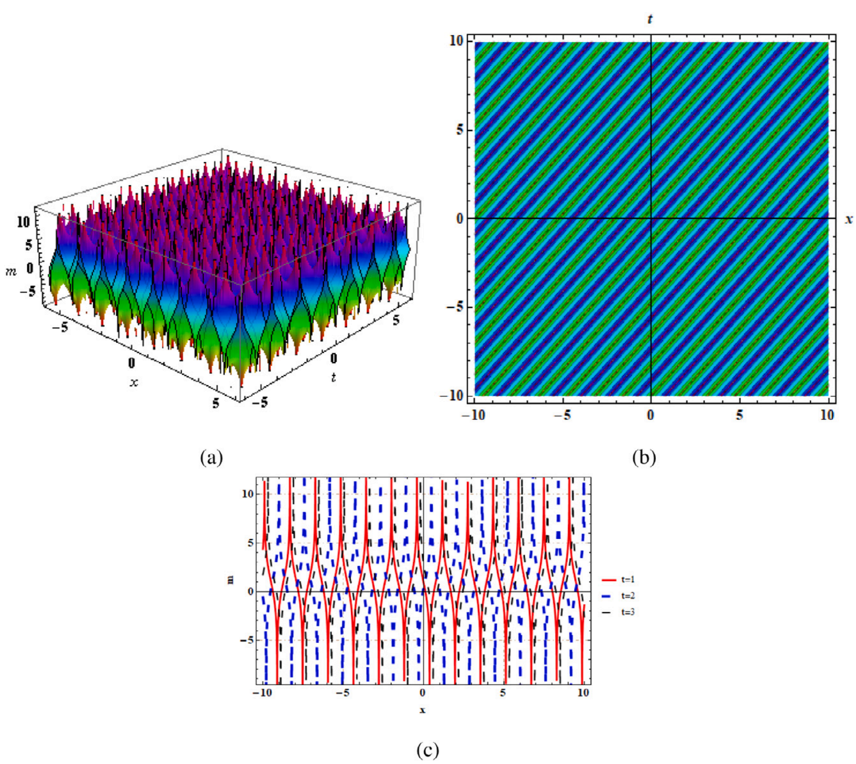

Fig. 2. (a) is **3D plot** of Eq. (20) by taking $l = 1$, $m_1 = 1$ and $C = 0$, (b) Contour plot and (c) is **2D line plots** of Eq. (20) against x.

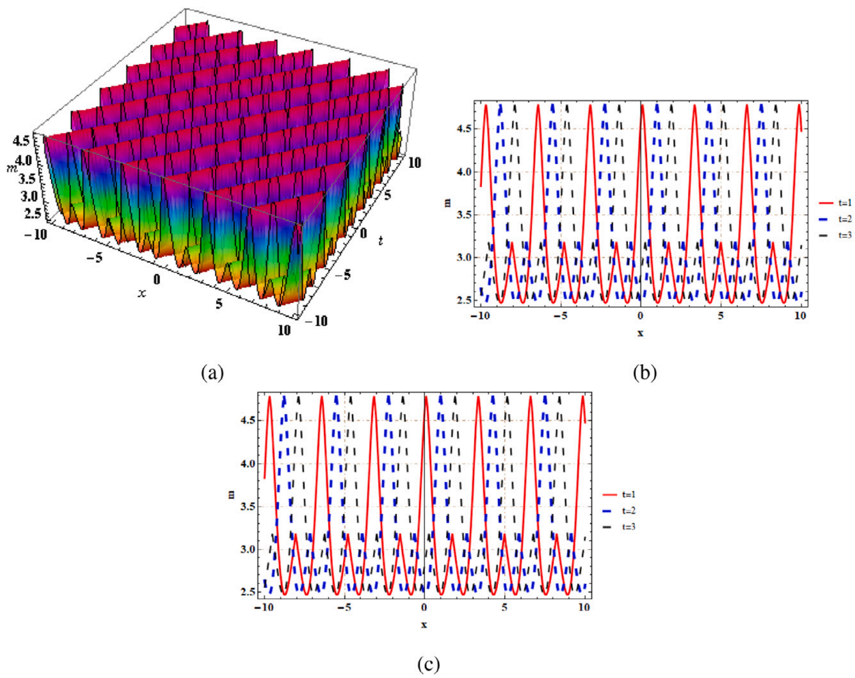

Fig. 3. (a) is **3D plot** of Eq. (28) by taking $M = 1.5$, (b) Contour plot and (c) is **2D line plots** of Eq. (28) against x.

$$\pm\frac{2\sqrt{-M^2-1}}{q^{\frac{1}{4}}}\left(\frac{\sqrt{q}\csc(\sqrt{\sqrt{q}}\zeta)}{M\csc(\sqrt{\sqrt{q}}\zeta)+1}\right)\left(\frac{\sqrt{\sqrt{q}}\cot(\sqrt{\sqrt{q}}\zeta)}{M\csc(\sqrt{\sqrt{q}}\zeta)+1}\right)\Bigg].$$

Graphical illustrations

In this part, we provide graphical illustrations of few of the determined results. It is important to mention here that explicit and consistent wave solutions are extracted by applying two different reliable schemes. The graphical illustrations of few of the determined solutions are depicted here. It is important to mention here that explicit and consistent wave solutions are extracted by applying two different reliable schemes. Figs. 1–3 represent, the 3D, 2D plots and contour plots of Eqs. (19), (20) and (28) respectively by taking $q = 0.5$ and $\alpha = 0.9$.

Conclusion

Our work provides a detailed presentation of the sine–Gordon models as well as the presence of many soliton solutions. The $\exp(-\phi(\zeta))$-expansion and the GPRE methods are effectively employed in this research to obtain solutions to an interesting q-deformed Sinh–Gordon equation. The investigation of analytical solutions and numerous properties of the traveling wave solutions for the q-deformed Sinh–Gordon equation are certainly pertinent. These approaches have successfully retrieved trigonometric and hyperbolic solutions, as well as the appropriate constraint conditions imposed on parameters to ensure solution existence. Such current results provide a solid framework for proceeding forward in this direction. The q-deformed Sinh–Gordon equation is useful not only in applied mathematics and mathematical physics, but it is also relevant to applications in physics when the system model's symmetry is broken or missing. Using various parameters, several dynamic features of the generated solutions were investigated and illustrated in figures. The Figs. 1–3 show that the model under study has diverse propagation patterns and consists of trigonometric and hyperbolic solitons. It is further highlighted that the proposed solutions are efficient, novel, and can be used to various fields of science, particularly nonlinear optics.

CRediT authorship contribution statement

Nauman Raza: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Visualization, Writing – original draft, Writing – review & editing. **Saima Arshed:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Visualization, Writing – original draft, Writing – review & editing. **H.I. Alrebdi:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Abdel-Haleem Abdel-Aty:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Validation, Visualization, Writing – original draft, Writing – review & editing. **H. Eleuch:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Validation, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This research was funded by Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2022R106), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

References

[1] Yusuf A, Sulaiman TA, Mirzazadeh M, Hosseini K. M-truncated optical solitons to a nonlinear Schrodinger equation describing the pulse propagation through a two-mode optical fiber. Opt Quantum Electron 2021;53(10):1–17.
[2] Kilbas AA, Srivastava HM, Trujillo JJ. Theory and applications of fractional differential equations. Amsterdam: Elsevier; 2006.
[3] Lai PL, Fyfe C. Kernel and nonlinear canonical correlation analysis. Int J Neural Syst 2000;10(5):365, 377.
[4] Berrada K, El Baz M, Eleuch H, Hassouni Y. Acomparative study of negativity and concurrence based on spin coherent states. Internat J Modern Phys C 2010;21(3):291–305.

[5] Mohamed ABA, Eleuch H, Ooi CHR. Non-locality correlation in two driven qubits inside an open coherent cavity: Trace norm distance and maximum bell function. Sci Rep 2019;9(1):1–10.
[6] Joshi DG, Vojta M. Nonlinear bond-operator theory and 1/d expansion for coupled-dimer magnets. II. Antiferromagnetic phase and quantum phase transition. Phys Rev B 2015;91(9):094405.
[7] Jhangeer A, Munawar M, Riaz MB, Baleanu D. Construction of traveling waves patterns of (1+ n)-dimensional modified Zakharov Kuznetsov equation in plasma physics. Results Phys 2020;19:103330.
[8] Arshed S, Raza N, Rahman RU, Butt AR, Huang WH. Sensitive behavior and optical solitons of complex fractional Ginzburg–Landau equation: A comparative paradigm. Results Phys 2021;28:104533.
[9] Darvishi MT, Najafi M, Wazwaz AM. Conformable space–time fractional nonlinear (1+1)-dimensional Schrödinger-type models and their traveling wave solutions. Chaos Solitons Fractals 2021;150(1):111187.
[10] Raza N, Jhangeer A, Rahman RU, Butt AR, Chu YM. Sensitive visualization of the fractional Wazwaz-Benjamin–Bona–Mahony equation with fractional derivatives: A comparative analysis. Results Phys 2021;104171.
[11] Boutabba N, Eleuch H, Bouchriha H. Thermal bath effect on soliton propagation in three-level atomic system. Synth Met 2009;159(13):1239–43.
[12] Eleuch H, Prasad A. Chaos and regularity in semiconductormicrocavities. Phys Lett A 2012;376:1970–7.
[13] Arshed S, Rahman RU, Raza N, Khan AK, Inc M. A variety of fractional soliton solutions for three important coupled models arising in mathematical physics. Internat J Modern Phys B 2021;2250002.
[14] Sassaman R, Heidari A, Biswas A. Topological and non topological solitons of nonlinear Klein–Gordon equations by He's semi-inverse variational principle. J Te Frankl Inst 2010;347(7):1148–57.
[15] Seadawy AR, Yasmeen A, Raza N, Althobaiti S. Novel solitary waves for fractional (2 + 1)-dimensional Heisenberg ferromagnetic model via new extendedgeneralized Kudryashov method. Phys Scr 2021;96(9):125240.
[16] Ding CC, Gao YT, Deng GF, Wang D. Lax pair, conservation laws, Darboux transformation, breathers and rogue waves for the coupled nonautonomous nonlinear Schrödinger system in an inhomogeneous plasma. Chaos Solitons Fractals 2020;133:109580.
[17] Wazwaz AM. Travelling wave solutions for combined and double combined sine-cosine-Gordon equations by the variable separated ODE method. Appl Math Comput 2006;177(2):755–60.
[18] Akram G, Arshed S, Imran Z. Soliton solutions for fractional DNA Peyrard-Bishop equation via the extended (G0/G2)-expansion method. Physica Scripta 2021;96:094009.
[19] Seadawy AR, El-Rashidy K. Dispersive solitary wave solutions of Kadomtsev–Petviashvili and modified Kadomtsev Petviashvili dynamical equations in unmagnetized. Results Phys 2018;8:1216–22.
[20] Liu JG, Zhu WH. Multiple rogue wave, breather wave and interaction solutions of a generalized (3 + 1)-dimensional variable-coefficient nonlinear wave equation. Nonlinear Dynam 2021;103(2):1841–50.
[21] Kumar D, Park C, Tamanna N, Paul GC, Osman MS. Dynamics of two-mode Sawada-Kotera equation: Mathematical and graphical analysis of its dual-wave solutions. Results Phys 2020;19:103581.
[22] Rezazadeh H, Kurt A, Tozar A, Tasbozan O, Mirhosseini-Alizamini SM. Wave behaviors of Kundu-Mukherjee-Naskar model arising in optical fiber communication systems with complex structure. Opt Quantum Electron 2021;53(6):1–11.
[23] Tozar A, Tasbozan O, Kurt A. Optical soliton solutions for the (1+ 1)-dimensional resonant nonlinear Schröndinger's equation arising in optical fibers. Opt Quantum Electron 2021;53(6):1–8.
[24] Kivshara YS, Davies BL. Dark optical solitons: Physics and applications. Phys Rep 1998;298(2–3):81–197.
[25] Hao R. Optical soliton control in inhomogeneous nonlinear media with the parity-time symmetric potentials. Opt Commun 2015;338:265–8.
[26] Eleuch H, Prasad A, Rotter I and. Enhancement of photon intensity in forced coupled quantum wells inside a semiconductor microcavity. Phys Rev E 2013;87(2):022916.
[27] Wang Kang-Le. A study of the fractal foam drainage model in a microgravity space. Math Methods Appl Sci 2021;44(13):10530–40.
[28] Wang KangLe, Wang KangJia. A new analysis for Klein–Gordon model with local fractional derivative. Alex Eng J 2020;59(5):3309–13.
[29] Wang K. New variational theory for coupled nonlinear fractal Schrödinger system. Internat J Numer Methods Heat Fluid Flow 2022;32(2):589–97.
[30] Raza N, Rahman RU, Seadawy A, Jhangeer A. Computational and bright soliton solutions and sensitivity behavior of Camassa–Holm and nonlinear Schrodinger dynamical equation. Internat J Modern Phys B 2021;2150157.
[31] Raza N, Hassan Z, Butt AR, Rahman RU, Abdel-Aty AH, Mahmoud M. New and more dual-mode solitary wave solutions for the Kraenkel-Manna-Merle system incorporating fractal effects. Math Methods Appl Sci 2021. http://dx.doi.org/10.1002/mma.7967.
[32] Bagheri M, Khani A. Analytical method for solving the fractional order generalized KdV equation by a beta-fractional derivative. Adv Math Phys 2020;2020.
[33] Sousa JC, Oliveira EC. A new truncated M-fractional derivative type unifying some fractional derivative types with classical properties. Int J Anal Appl 2018;16(1):83–96.
[34] Abdalla MS, Eleuch H, Barakat T. Exact analytical solutions of the wave function for some q-deformed potentials. Rep Math Phys 2013;71(2):217–29.
[35] Fabian AL, Kohl R, Biswas A. Perturbation of topological solitons due to sine-Gordon equation and its type. Commun Nonlinear Sci Numer Simul 2009;14(4):1227–44.
[36] Biswas A, Rezazadeh H, Mirzazadeh M, Eslami M, Zhou Q, Moshokoa SP, et al. Optical solitons having weak non-local nonlinearity by two integration schemes. Optik 2018;164:380–4.
[37] Eleuch H. Some analytical solitary wave solutions for the generalized q-deformed Sinh–Gordon equation $\frac{\partial^2\theta}{\partial z\partial\zeta} = \alpha[\sinh_q(\beta\theta^\gamma)]^p - \delta$. Hindawi Adv Math Phys 2018;5242757. http://dx.doi.org/10.1155/2018/5242757, 7 pages.
[38] Arshed S, Biswas A, Mallawi F, Belic MR. Optical solitons with complex Ginzburg–Landau equation having three nonlinear forms. Phys Lett A 2019;383(36):126026.
[39] Raza N, Arshed S, Sial S. Optical solitons for coupled Fokas-Lenells equation in birefringence fibers. Modern Phys Lett B 2019;33(26):1950317.
[40] Zayed EME, Alurrfi KAE. The generalized projective Riccati equations method and its applications to nonlinear PDEs describing nonlinear transmission lines. Commun Appl Electron 2015;3(4):1–8.