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Analytical and semi-analytical ample solutions of the higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term

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ABSTRACT

In this paper, we investigate distinct novel analytical and semi-analytical solutions of the higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term by the employment of there different schemes. These schemes are the generalized auxiliary equation method, the generalized exp-$(-\phi(\xi))$ expansion method, and the Adomain decomposition method that are considered as useful tools in this field. The suggested model in this study is used to explore the dynamics of light pulses for sub-10-fs-pulse propagation in the framework of computational simulations. The primary research of our study focuses on the case where the carrier wavelength of the soliton is much shorter than the spatial width. Herewith, the soliton frequency is smaller than the carrier frequency. The shorter femtosecond pulses (< 100 fs) is desired to increase the bit rate of pulse propagation. Their loss for the short-wavelength pulses that are propagating through the wave-guide is negligible. New analytical solutions are obtained, then it used to evaluate the initial and boundary conditions that are used in calculating the semi-analytical. Moreover, the accuracy of obtained solutions is explained by evaluating the absolute value of error between the obtained analytical and semi-analytical.

1. Introduction

Partial differential equations are of fundamental for several applications [1–3]. These equations have been used to formulate many natural, engineering, mechanical, and physical phenomena [4–8]. During the last decade, many aspects have been explored for the partial differential equations [9–12]. Solitary waves have attracted the interested of many researchers [13–17]. Solitary wave is a wave that propagates without any change in shape and size [18–22]. Many analytical and approximate schemes have been used to investigate the physical dynamics of these waves [23–26]. We can cite the simplest equation method, modified tanh-function method, B-spline method, iterative method, and other methods [27–31].

In this paper, we investigate the solitary and numerical solutions of one of the essential models in the optical fiber, which is the sub-10-fs-pulse propagation model [32–36]. Optical models are a vital icon of telecommunication technology [37–39]. In fact there is a particular interest for optical soliton that propagates for a long distance without changing its shape and without being attenuated [40]. This model is also known as a higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term [41–44], and describes the propagation of sub-10-fs-pulse [45].

Many ultra-fast applications been based on sub-10-fs pulse generation that has reached robustness and maturity. These applications depend on the octave-spanning spectral width, the exceptionally high peak intensity of the femto-second lasers, or the coherence properties. The mathematical analysis of such pulse allows the determination of its proprieties with high accuracy [46,47]. This study is also more accurate than simple characterization methods [48]. It allows the measure of the pulse properties by exploring the bandwidth in the single-cycle regime, the electric field of the sub-10-fs pulse, and the shapes of the optical solitons [49,50].

This is only an example

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The paper is organized as follows: In Section 2, an introduction and historical study of higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term as well as analytical and approximate schemes to this model's are presented. In Section 3, a summary of all the steps of our paper is detailed.

2. Application

The higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term can be formulated as: [51–55]:

$$\begin{aligned} &S_x - \iota(a_1 S_{tt} + a_2 |S|^2 S) - a_3 S_{ttt} - a_4 (|S|^2 S)_t - a_5 S(|S|^2)_t - \iota a_6 |S|^4 \\ &\quad S - a_7 (|S|^4 S)_t - a_8 S(|S|^4)_t = 0, \end{aligned} \tag{1}$$

where $\iota = \sqrt{-1}$, $S = S(x, t)$ is the electric field, x, t refer respectively to the normalized distance along the fiber and normalized time with the frame of the reference moving along the fiber at the group velocity, respectively. $(a_1, \ldots, a_8)$ are real parameters that represent velocity dispersion, self-phase modulation, third-order dispersion, self steepening and self-frequency, respectively. Such that,

$$a_1 = \frac{\beta_2}{2},\ a_2 = \gamma_1,\ a_3 = \frac{\beta_3}{6},\ a_4 = \frac{-\gamma_1}{\omega_0},\ a_5 = \gamma_1\, T_R,$$

where

$$\beta = \frac{n(\omega)\omega}{c},\ \beta_j = \left(\frac{d^j\beta}{d\omega^j}\right)_{\omega=\omega_0},$$

$(\gamma_1, \beta_j (j = 1, 2, 3), \omega_0)$represent a coefficient of the cubic nonlinearity, the inverse of group velocity, the group velocity dispersion parameter, the third order dispersion and frequency, respectively. The coefficients (a_6, a_7, a_8)describe a quintic non-Kerr nonlinearities that appear from the expansion of the refractive index. The power of the intensity I of the light pulse can be represented in the following formula:

$$r = r_0 + r_2 I + r_4 I^2 + \cdots, \tag{2}$$

where (r_0, r_2, r_4) represent the linear refraction index, nonlinear refractive indices and originate from (3–5) order susceptibilities, respectively. Scaling the variables of Eq. (1) in the following form:

$$S = \varepsilon_1 \nu,\quad x = \varepsilon_2 \chi,\quad t = \varepsilon_3 \tau.$$

Eq. (1) can be transformed to:

$$\begin{aligned} &\nu_\chi - \iota(\nu_{\tau\tau} + |\nu|^2\nu) - \nu_{\tau\tau\tau} - \alpha_1 (|\nu|^2 \nu)_\tau - \alpha_2 \nu (|\nu|^2)_\tau \\ &- \iota\alpha_3 |\nu|^4 \nu - \alpha_4 (|\nu|^4 \nu)_\tau - \alpha_5 \nu (|\nu|^4)_\tau = 0, \end{aligned} \tag{3}$$

where

$$\alpha_1 = \frac{\varepsilon_1^2 \varepsilon_2 a_4}{\varepsilon_3} = \frac{a_4 a_1}{a_2 a_3},\ \alpha_2 = \frac{\varepsilon_1^2 \varepsilon_2 a_5}{\varepsilon_3} = \frac{a_5 a_1}{a_2 a_3},\ \alpha_3 = \varepsilon_1^4 \varepsilon_2 a_0 = \frac{a_0 a_1^3}{a_2^2 a_3^2},$$

$$\alpha_4 = \frac{\varepsilon_1^4 \varepsilon_2 a_7}{\varepsilon_3} = \frac{a_7 a_1^4}{a_2^2 a_3^3},\ \alpha_5 = \frac{\varepsilon_1^4 \varepsilon_2 a_8}{\varepsilon_3} = \frac{a_8 a_1^4}{a_2^2 a_3^3},\ \varepsilon_1 = \sqrt{\frac{a_1^3}{a_2^2 a_3^2}},\ \varepsilon_2 = \frac{a_3^2}{a_1^3},\ \varepsilon_3 = \frac{a_3}{a_1}.$$

By using the following wave transformation

$$\nu(\chi, \tau) = \nu(\vartheta) e^{\iota\eta},\ \textit{where}\ (\vartheta = s\chi + \tau\ \&\ \eta = k\chi - \Omega\tau),$$

separating the real and imaginary parts of the transformed equation and using the rational relationship between the coefficients of (ν, ν^3, ν^5), that gives

$$\frac{(s - 2\Omega + 3\Omega^2)(1 - 3\Omega)}{k + \Omega^2 - \Omega^3} = \frac{(3\alpha_1 + \alpha_2)(1 - 3\Omega)}{3(1 - \Omega\alpha_1)} = \frac{(5\alpha_4 + 4\alpha_5)(1 - 3\Omega)}{5(\alpha_3 - \Omega\alpha_4)},$$

Eq. (3) becomes

$$\nu'' - a\nu + 2b\nu^3 + 3c\nu^5 = 0, \tag{4}$$

where

$$a = (s - 2\Omega + 3\Omega^2),\ b = \frac{3\alpha_1 + 2\alpha_2}{6},\ c = \frac{5\alpha_4 + 4\alpha_5}{15}.$$

2.1. Analytical solutions via generalized auxiliary equation method

Applying the generalized auxiliary equation method allows putting the general solution of Eq. (4) in the next form:

$$\nu(\vartheta) = \sum_{i=0}^{n} (a_i \phi(\vartheta)^i + d_i \phi(\vartheta)^{-i}),\ \textit{where}\ \phi'(\vartheta) = \sqrt{\sum_{i=0}^{m} h_i \phi^i(\vartheta)}. \tag{5}$$

Identification the values of (n, m) according to balance rule, we obtain $(n = \frac{m-2}{4})$. We assume that $(m = 6,\ n = 1)$. By substituting Eq. (5) and its derivatives into Eq. (4), collecting all coefficients of the same power of $\phi^i(\vartheta)$ where $(i = -5, -4, \ldots, 4, 5)$ and solving the obtained system, we get:

$$b_0 \to -a_0,\ b_1 \to 0,\ c \to -\frac{h_6}{a_1^4},\ a \to h_2,\ b \to -\frac{h_4}{a_1^2}.$$

According to these values, the solitary wave solutions are:

Case I For $h_1 = h_3 = h_5 = 0$, $h_0 = \frac{8h_2^2}{27h_4}$, $h_6 = \frac{h_4^2}{4h_2}$, we get:

$$b_0 \to -a_0,\ b_1 \to 0,\ c \to -\frac{h_4^2}{4a_1^4 h_2},\ a \to h_2,\ b \to -\frac{h_4}{a_1^2}.$$

If $[h_2 < 0,\ h_4 > 0,\ \epsilon = \pm 1]$:

$$S_1(x, t) = 2\sqrt{\frac{2}{3}}\ \epsilon_1 \sqrt{-\frac{h_4}{b}} e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2\left(-\frac{3}{\tan^2\left(\frac{\sqrt{h_2}\ \epsilon\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)}{\sqrt{3}}\right) - 3} - 1\right)}{h_4}}, \tag{6}$$

$$S_2(x, t) = \sqrt{\frac{2}{3}}\ \epsilon_1 \sqrt{-\frac{h_4}{b}} e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2\left(-\frac{3}{2\cos\left(\frac{2\sqrt{h_2}\ \epsilon\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)}{\sqrt{3}}\right) - 1} - 1\right)}{h_4}}. \tag{7}$$

If $[h_2 > 0,\ h_4 < 0,\ \epsilon = \pm 1]$:

$$S_3(x, t) = \frac{2\ \epsilon_1 \sqrt{-\frac{h_4}{b}} e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2\left(\frac{3}{\cos\left(\frac{2\sqrt{h_2}\ \epsilon\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)}{\sqrt{3}}\right) + 2} - 1\right)}{h_4}}}{\sqrt{3}}, \tag{8}$$

$$S_4(x, t) = \frac{2\ \epsilon_1 \sqrt{-\frac{h_4}{b}} e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2\left(-\frac{3}{\cos\left(\frac{2\sqrt{h_2}\ \epsilon\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)}{\sqrt{3}}\right) - 2} - 1\right)}{h_4}}}{\sqrt{3}}. \tag{9}$$

Case II For $h_0 = h_1 = h_3 = h_5 = 0$.
If $[h_2 > 0,\ \epsilon = \pm 1]$:

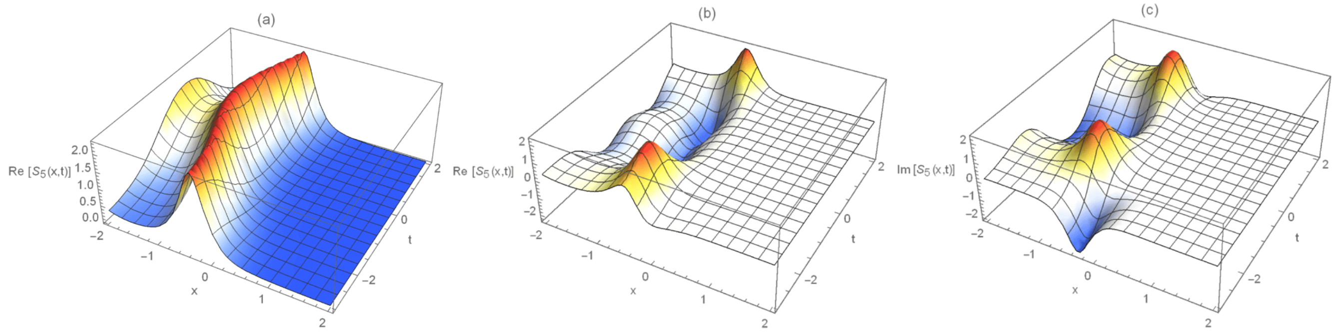

Fig. 1. Solitary wave solution in three-dimensional plots for absolute, real, and imaginary value of Eq. (6) for $[b = 9,\ h_2 = -4,\ h_4 = 4,\ \kappa = 2,\ s = 3,\ \epsilon = 1,\ \epsilon_2 = -2,\ \epsilon_3 = -3]$.

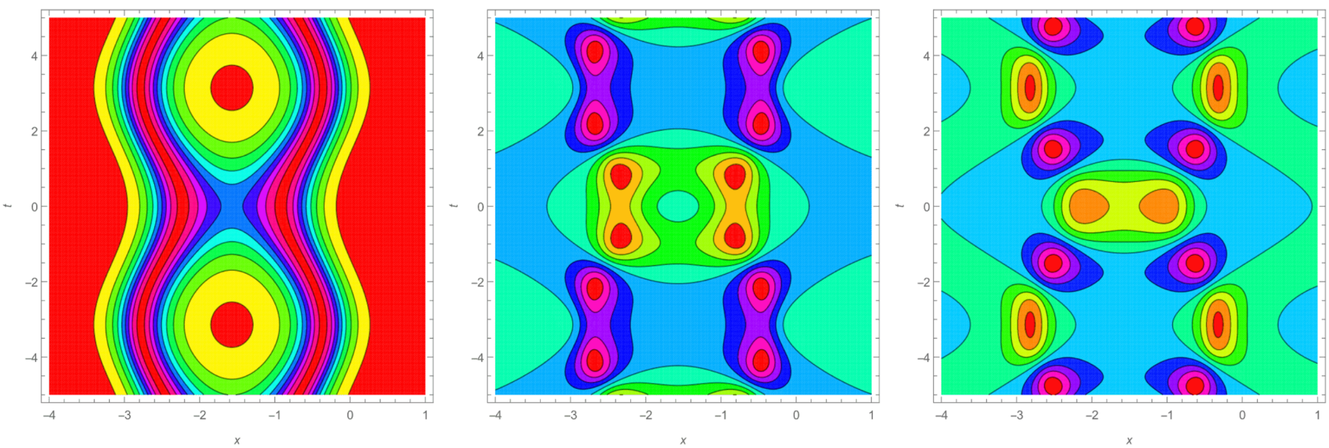

Fig. 2. Solitary wave solution in contour plots for absolute, real, and imaginary value of Eq. (6) for $[b = 9,\ h_2 = -4,\ h_4 = 4,\ \kappa = 2,\ s = 3,\ \epsilon = 1,\ \epsilon_2 = -2,\ \epsilon_3 = -3]$.

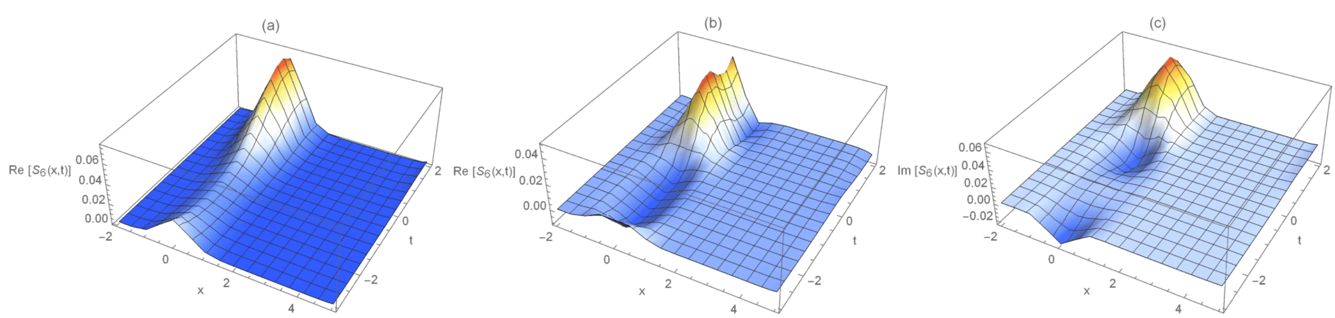

Fig. 3. Solitary wave solution in three-dimensional plots for absolute, real, and imaginary value of Eq. (11) for $[b = 9,\ h_2 = -4,\ h_4 = 4,\ \kappa = 2,\ s = 3,\ \epsilon = 1,\ \epsilon_2 = -2,\ \epsilon_3 = -3]$.

$$S_5(x,t) = \epsilon_1 \sqrt{-\frac{h_4}{b}}\, e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2 h_4 \operatorname{sech}^2\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)\right)}{h_2 h_6 \left(\epsilon \tanh\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)\right) + 1\right)^2 - h_4^2}}, \tag{10}$$

$$S_6(x,t) = \epsilon_1 \sqrt{-\frac{h_4}{b}}\, e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2 h_4 \operatorname{csch}^2\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)\right)}{h_4^2 - h_2 h_6 \left(\epsilon \coth\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)\right) + 1\right)^2}}. \tag{11}$$

If $[h_2 > 0,\ h_6 > 0,\ \epsilon = \pm 1]$:

$$S_7(x,t) = \epsilon_1 \sqrt{-\frac{h_4}{b}}\, e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{-\frac{h_2 \operatorname{sech}^2\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)\right)}{2\sqrt{h_2 h_6}\ \epsilon \tanh\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)\right) + h_4}}, \tag{12}$$

$$S_8(x,t) = \epsilon_1 \sqrt{-\frac{h_4}{b}}\, e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2 \operatorname{csch}^2\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)\right)}{2\sqrt{h_2 h_6}\ \epsilon \coth\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \kappa\right)\right) + h_4}}. \tag{13}$$

If $[h_2 < 0,\ h_6 > 0,\ \epsilon = \pm 1]$:

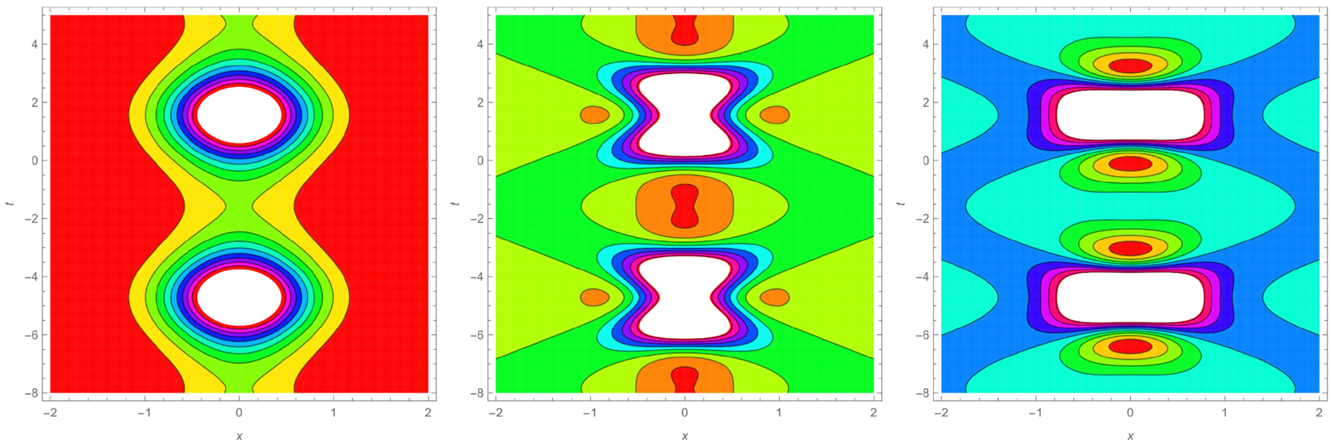

Fig. 4. Solitary wave solution in contour plots for absolute, real, and imaginary value of Eq. (11) for $[b = 9,\ h_2 = -4,\ h_4 = 4,\ \varkappa = 2,\ s = 3,\ \epsilon = 1,\ \epsilon_2 = -2,\ \epsilon_3 = -3]$.

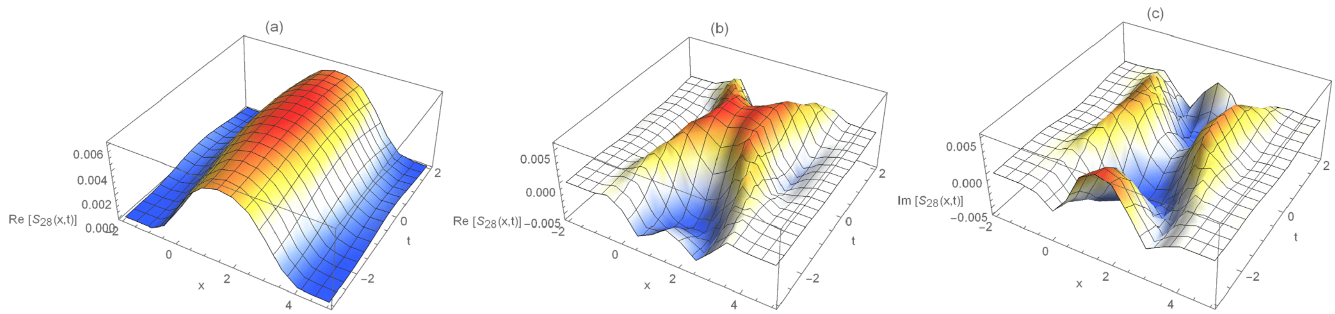

Fig. 5. Solitary wave solution in three-dimensional plots for absolute, real, and imaginary value of Eq. (35) for $[b = -1,\ b_2 = 4,\ b_3 = 5,\ h_4 = 1,\ h_2 = 2,\ s = 3,\ \epsilon = -3,\ \varkappa = -4]$.

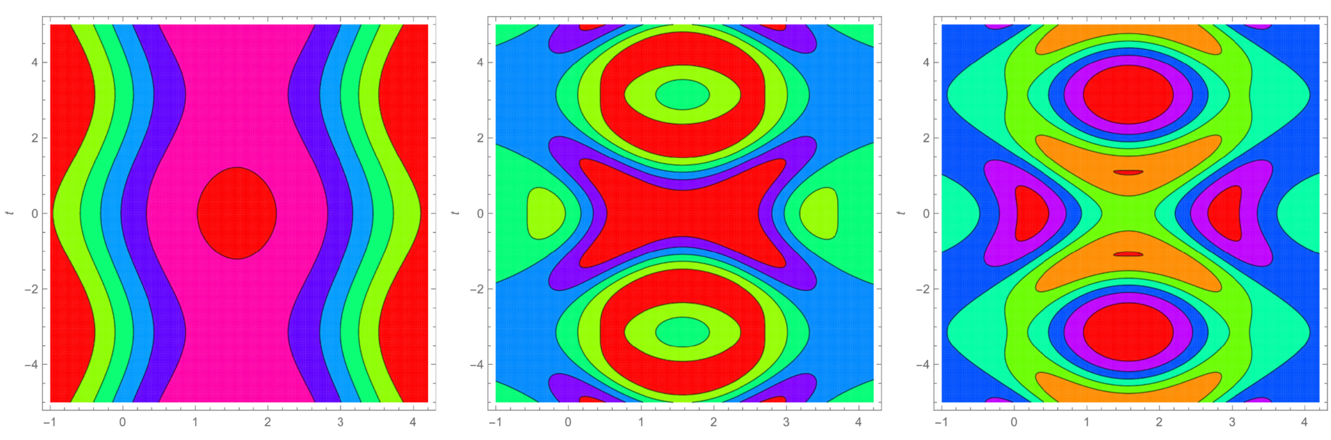

Fig. 6. Solitary wave solution in contour plots for absolute, real, and imaginary value of Eq. (35) for $[b = -1,\ b_2 = 4,\ b_3 = 5,\ h_4 = 1,\ h_2 = 2,\ s = 3,\ \epsilon = -3,\ \varkappa = -4]$.

$$S_9(x,t) = \epsilon_1 \sqrt{-\frac{h_4}{b}}\, e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{-\frac{h_2 \mathrm{sech}^2\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \varkappa\right)\right)}{2\sqrt{-h_2 h_6}\ \epsilon \tan\left(\sqrt{-h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \varkappa\right)\right) + h_4}}, \tag{14}$$

$$S_{10}(x,t) = \epsilon_1 \sqrt{-\frac{h_4}{b}}\, e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2 \mathrm{csch}^2\left(\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \varkappa\right)\right)}{2\sqrt{-h_2 h_6}\ \epsilon \cot\left(\sqrt{-h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \varkappa\right)\right) + h_4}}. \tag{15}$$

Case III For $h_0 = h_1 = h_3 = h_5 = 0,\ h_4^2 - 4h_2 h_6 > 0$.
If $[h_2 > 0,\ \epsilon = \pm 1]$:

$$S_{11}(x,t) = \sqrt{2}\ \epsilon_1 \sqrt{-\frac{h_4}{b}}\, e^{i\left(\frac{kx}{\epsilon_2} - \frac{t\Omega}{\epsilon_3}\right)} \sqrt{\frac{h_2}{\sqrt{h_4^2 - 4h_2 h_6}\ \epsilon \cosh\left(2\sqrt{h_2}\left(\frac{sx}{\epsilon_2} + \frac{t}{\epsilon_3} + \varkappa\right)\right) - h_4}}. \tag{16}$$

If $[h_2 < 0,\ \epsilon = \pm 1]$:

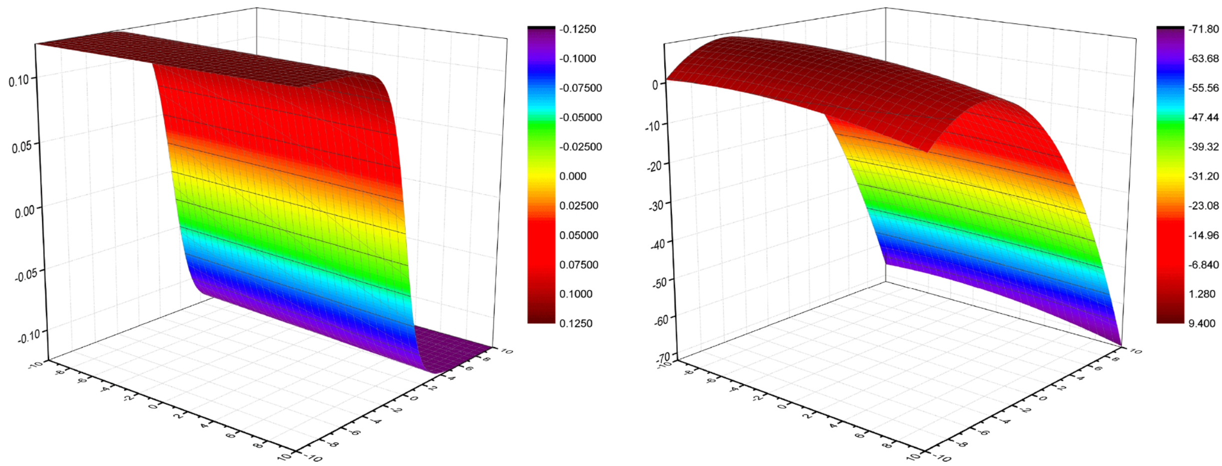

Fig. 7. Three dimensional wave solution of Eqs. (35) and (49) in three dimensional plot for $[b = -1,\ b_2 = 4,\ b_3 = 5,\ h_4 = 1,\ h_2 = 2,\ s = 3,\ \epsilon = -3,\ \kappa = -4]$.

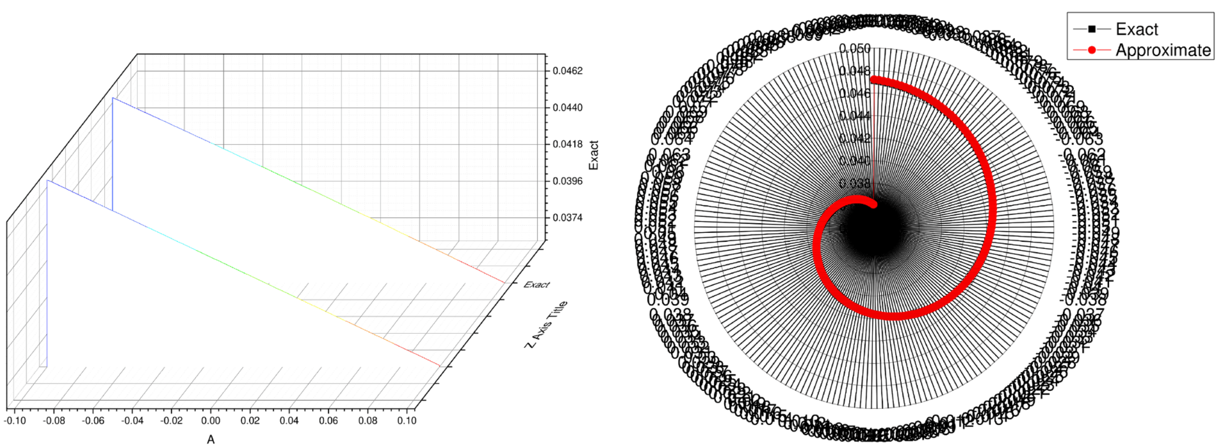

Fig. 8. Waterfall and radar plots of Eqs. (35) and (49) in three, two, and contour plots.

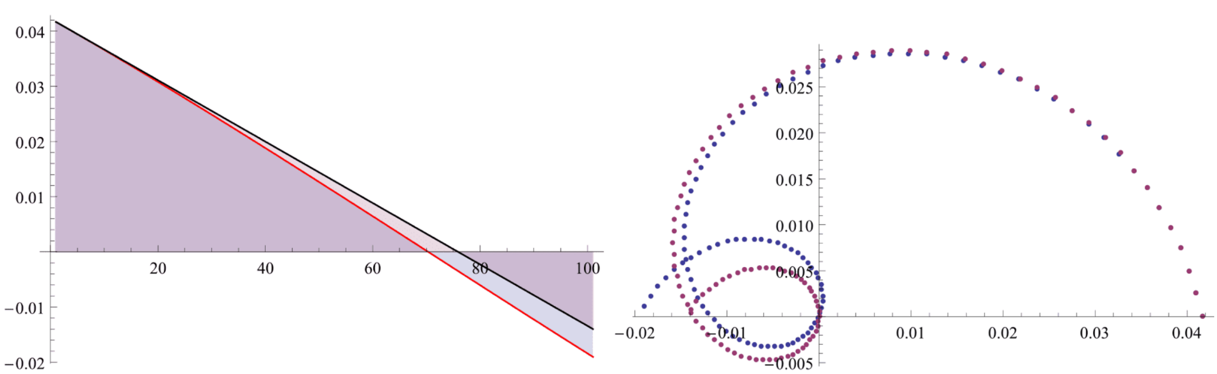

Fig. 9. List line plot and List polar plot of Eqs. (35) and (49).

$$S_{12}(x,t)=\sqrt{2}\,\epsilon_1\sqrt{-\frac{h_4}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{\frac{h_2}{\sqrt{h_4^2-4h_2h_6}\,\epsilon\cosh\left(2\sqrt{h_2}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)-h_4}},\tag{17}$$

$$S_{13}(x,t)=\sqrt{2}\,\epsilon_1\sqrt{-\frac{h_4}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{\frac{h_2}{\sqrt{h_4^2-4h_2h_6}\,\epsilon\sin\left(2\sqrt{-h_2}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)-h_4}}.\tag{18}$$

If $[h_2>0,\ h_4<0,\ h_6<0,\ \epsilon=\pm1]$:

$$S_{14}(x,t)=\sqrt{2}\,\epsilon_1\sqrt{-\frac{h_4}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{\frac{h_2}{\sqrt{h_4^2-4h_2h_6}\cosh\left(2\sqrt{h_2}\,\epsilon\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)-h_4}},\tag{19}$$

$$S_{15}(x,t)=\sqrt{2}\,\epsilon_1\sqrt{-\frac{h_4}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{\frac{h_2}{\sqrt{h_4^2-4h_2h_6}\cosh\left(2\sqrt{h_2}\,\epsilon\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)-h_4}}.\tag{20}$$

If $[h_2<0,\ h_4>0,\ h_6<0,\ \epsilon=\pm1]$:

$$S_{16}(x,t)=\sqrt{2}\,\epsilon_1\sqrt{-\frac{h_4}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{-\frac{h_2\mathrm{sech}^2\left(\sqrt{h_2}\,\epsilon\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)}{(h_4-\sqrt{h_4^2-4h_2h_6})\sec^2\left(\sqrt{h_2}\,\epsilon\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)+2\sqrt{h_4^2-4h_2h_6}}},\tag{21}$$

$$S_{17}(x,t)=\sqrt{2}\,\epsilon_1\sqrt{-\frac{h_4}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{-\frac{h_2\mathrm{csch}^2\left(\sqrt{h_2}\,\epsilon\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)}{2\sqrt{h_4^2-4h_2h_6}-(h_4+\sqrt{h_4^2-4h_2h_6})\csc^2\left(\sqrt{h_2}\,\epsilon\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)}}.\tag{22}$$

Case IV For $h_0=h_1=h_3=h_5=0,\ h_4^2-4h_2h_6<0$.
If $[h_2>0,\ \epsilon=\pm1]$:

$$S_{18}(x,t)=\sqrt{2}\,\epsilon_1\sqrt{-\frac{h_4}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{\frac{h_2}{\sqrt{4h_2h_6-h_4^2}\,\epsilon\sinh\left(2\sqrt{h_2}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)-h_4}}.\tag{23}$$

Case V For $h_0=h_1=h_3=h_5=0,\ h_4^2-4h_2h_6=0$.
If $[h_2>0,\ \epsilon=\pm1]$:

$$S_{19}(x,t)=\epsilon_1\sqrt{-\frac{\sqrt{h_2h_6}}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{-\frac{h_2\left(\epsilon\tanh\left(\sqrt{h_2}\,\epsilon\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)+1\right)}{\sqrt{h_2h_6}}},\tag{24}$$

$$S_{20}(x,t)=\epsilon_1\sqrt{-\frac{\sqrt{h_2h_6}}{b}}e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{-\frac{h_2\left(\epsilon\coth\left(\sqrt{h_2}\,\epsilon\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)+1\right)}{\sqrt{h_2h_6}}}.\tag{25}$$

2.2. Analytical solutions via generalized exp- $(-\phi(\xi))$ expansion method

Using homogeneous balance rule on Eq. (4), yields $n=\frac{1}{2}$. By applying another transformation $\nu=\varphi^{\frac{1}{2}}$ to Eq. (4), leads to

$$-a\varphi^2+2b\varphi^3+3c\varphi^4+\frac{\varphi\varphi''}{2}-\frac{\varphi''}{4}=0.\tag{26}$$

By calculating the value of balance between the terms of Eq. (26), we obtain $N=1$. The general form of solution Eq. (26) is in the form:

$$\varphi(\vartheta)=\sum_{i=0}^{N}a_i\,e^{-i\phi(\vartheta)}=a_0+a_1\,e^{-\phi(\vartheta)}.\tag{27}$$

Substituting Eq. (27) and its derivative into Eq. (3). Collection the coefficient of the $e^{-i\phi(\vartheta)}$, where $(i=0,1,2,3)$ and equating them by zero, we get a system of algebraic equations. Solving this system of equations by any computer software, we obtain:

Family 1:

$$a_1\to\frac{a_0\sqrt{\sigma}}{\sqrt{\mu}},\ a\to0,\ b\to\frac{\mu\sigma}{4a_0^2},\ c\to-\frac{\mu\sigma}{3a_0^2},\ \lambda\to2\sqrt{\mu}\sqrt{\sigma}.$$

Consequently, the solitary traveling wave solutions of Eq. (1) are given by:

Case I. For $\sigma=1$, we obtain:
For $\mu\neq0,\ \lambda\neq0,\ \lambda^2-4\mu>0$:

$$S_{21}(x,t)=\epsilon_1\,e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\left(\frac{\sqrt{\frac{\mu}{b}}}{2}+\frac{\mu}{\sqrt{b}\left(\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)-\lambda\right)}\right)^{0.5},\tag{28}$$

$$S_{22}(x,t)=\epsilon_1\,e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\left(\frac{\sqrt{\frac{\mu}{b}}}{2}+\frac{\mu}{\sqrt{b}\left(\sqrt{\lambda^2-4\mu}\coth\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)-\lambda\right)}\right)^{0.5}.\tag{29}$$

For, $[\mu=0,\ \lambda^2-4\mu>0]$:

$$S_{23}(x,t)=\frac{\lambda^2\,\epsilon_1\,e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}}{4b\left(e^{\lambda\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)}-1\right)^{0.5}}.\tag{30}$$

For, $\mu\neq0,\ \lambda\neq0,\ \lambda^2-4\mu=0$

$$S_{24}(x,t)=\epsilon_1\,e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\left(\frac{\sqrt{\frac{\mu}{b}}}{2}-\frac{\lambda^2\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)}{4\sqrt{b}\left(\lambda\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\vartheta\right)+2\right)}\right)^{0.5}.\tag{31}$$

For, $[\mu=0,\ \lambda=0,\ \lambda^2-4\mu=0]$:

$$S_{25}(x,t)=\frac{\epsilon_1\,e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}}{4b\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)^{0.5}}.\tag{32}$$

For, $[\mu\neq0,\ \lambda\neq0,\ \lambda^2-4\mu<0]$:

$$S_{26}(x,t)=\epsilon_1\,e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\left(\frac{\mu}{\sqrt{b}\left(\lambda-\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)\right)}-\frac{\sqrt{\frac{\mu}{b}}}{2}\right)^{0.5},\tag{33}$$

$$S_{27}(x,\,t) = \epsilon_1\, e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)} \left(\frac{\mu}{\sqrt{b}\left(\lambda-\sqrt{4\mu-\lambda^2}\cot\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)\right)}-\frac{\sqrt{\frac{\mu}{b}}}{2}\right)^{0.5}. \tag{34}$$

Family 2:

$$\sigma \to \frac{a_1\lambda}{2a_0},\, \mu \to \frac{a_0\lambda}{2a_1},\, a \to 0,\, b \to \frac{\lambda^2}{16a_0^2},\, c \to -\frac{\lambda^2}{12a_0^2}.$$

Consequently, the solitary traveling wave solutions of Eq. (1) are given by:

Case I. For $\sigma = 1$, we obtain:

For, $[\mu \neq 0,\ \lambda \neq 0,\ \lambda^2 - 4\mu > 0]$:

$$S_{28}(x,\,t) = \frac{\epsilon_1\, e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\left(\lambda-\frac{4\mu}{\lambda-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)}\right)^{0.5}}{16b}, \tag{35}$$

$$S_{29}(x,\,t) = \frac{\epsilon_1\, e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\left(\lambda-\frac{4\mu}{\lambda-\sqrt{\lambda^2-4\mu}\coth\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)}\right)^{0.5}}{16b}. \tag{36}$$

For, $[\mu = 0,\ \lambda^2 - 4\mu > 0]$:

$$S_{30}(x,\,t) = \frac{\lambda\,\epsilon_1\, e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\sqrt{\coth\left(\frac{1}{2}\lambda\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)}}{16b}. \tag{37}$$

For, $\mu \neq 0,\ \lambda \neq 0,\ \lambda^2 - 4\mu = 0$

$$S_{31}(x,\,t) = \frac{\lambda^2\,\epsilon_1\epsilon_2^2\epsilon_3^2\, e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}}{4b(\lambda sx\,\epsilon_3 + \epsilon_2\,(\lambda t + \epsilon_3(\lambda\kappa+2)))^{0.5}}. \tag{38}$$

For, $[\mu = 0,\ \lambda = 0,\ \lambda^2 - 4\mu = 0]$:

$$S_{32}(x,\,t) = \frac{\epsilon_1\, e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}}{4b\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)^{0.5}}. \tag{39}$$

For, $[\mu \neq 0,\ \lambda \neq 0,\ \lambda^2 - 4\mu < 0]$:

$$S_{33}(x,\,t) = \frac{\epsilon_1\, e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\left(\lambda-\frac{4\mu}{\lambda-\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)}\right)^{0.5}}{16b}, \tag{40}$$

$$S_{34}(x,\,t) = \frac{\epsilon_1\, e^{i\left(\frac{kx}{\epsilon_2}-\frac{t\Omega}{\epsilon_3}\right)}\left(\lambda-\frac{4\mu}{\lambda-\sqrt{4\mu-\lambda^2}\cot\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\left(\frac{sx}{\epsilon_2}+\frac{t}{\epsilon_3}+\kappa\right)\right)}\right)^{0.5}}{16b}. \tag{41}$$

For both families (1, 2);

We find that for the case where $\lambda = 0$ it is impossible to generate solution to the contradictory of obtained parameter values.

2.3. Semi-analytical solutions via Adomain decomposition method

Applying Adomian decomposition method on Eq. (4) enables us to rewrite in the form:

$$L\,\nu(\vartheta) + R\,\nu(\vartheta) + N\,\nu(\vartheta) = 0, \tag{42}$$

where $(L\ \&\ R\ \&\ N)$ represent a differential operator, a linear operator and a nonlinear term, respectively. Using the inverse operator L^{-1} on 42, we get

$$\sum_{i=0}^{\infty}\nu_i(\xi) = \nu(0) + \nu'(0)\vartheta + a\,L^{-1}\left(\sum_{i=0}^{\infty}\nu_i\right) - 2\,b\,L^{-1}\left(\sum_{i=0}^{\infty}A_i\right) - 3\,c\,L^{-1}\left(\sum_{i=0}^{\infty}A_i\right). \tag{43}$$

Under the following condition $[a = 0,\ a_0 = \frac{3}{8},\ a_1 = \frac{1}{4},\ b = 4,\ c = -\frac{16}{3},\ \lambda = 3,\ \mu = 2,\ \sigma = 1,\ \kappa = 0]$ Eq. (4) gives:

$$\nu_{analytical} = \frac{1}{8}\left(3-\frac{8}{3-\tanh\left(\frac{\vartheta}{2}\right)}\right). \tag{44}$$

So that, we obtain:

$$\nu_0 = \frac{1}{24} - \frac{\vartheta}{18}, \tag{45}$$

$$\nu_1 = -\frac{\vartheta^7}{4960116} + \frac{\vartheta^6}{944784} + \frac{139\vartheta^5}{2099520} - \frac{427\vartheta^4}{1679616} + \frac{859\vartheta^3}{2239488} - \frac{287\vartheta^2}{995328} - \frac{\vartheta}{3981312} + \frac{1}{37158912}, \tag{46}$$

$$\nu_2 = -\frac{5\vartheta^{13}}{5076758087856} + \frac{19\vartheta^{12}}{5727624509376} + \frac{3109\vartheta^{11}}{6364027232640} - \frac{10397\vartheta^{10}}{11570958604800} - \frac{52207\vartheta^9}{6171177922560} + \frac{41711\vartheta^8}{914248581120} - \frac{100787\vartheta^7}{914248581120} + \frac{181967\vartheta^6}{1218998108160} - \frac{238739\vartheta^5}{2167107747840} + \frac{186143\vartheta^4}{5201058594816} + \frac{305\vartheta^3}{3467372396544} - \frac{31\vartheta^2}{1541054398464}, \tag{47}$$

$$\nu_3 = -\frac{145\vartheta^{19}}{26580270631909726368} + \frac{21655\vartheta^{18}}{348807761976639917952} + \frac{207041\vartheta^{17}}{51675223996539247104} - \frac{1921043\vartheta^{16}}{43424557980285081600} - \frac{58816267\vartheta^{15}}{101323968620665190400} + \frac{74953019\vartheta^{14}}{9006574988503572480} - \frac{143413339\vartheta^{13}}{25733071395724492800} - \frac{4450621817\vartheta^{12}}{18475025617443225600} + \frac{2332090183\vartheta^{11}}{1492931363025715200} - \frac{19484518861\vartheta^{10}}{3583035271261716480} - \frac{39679799717\vartheta^9}{682482908811755520} + \frac{323220066373\vartheta^8}{707760053582561280} - \frac{4779859417955\vartheta^7}{3397248257196294144} + \frac{8033560188919\vartheta^6}{3235474530663137280} - \frac{24159330543841\vartheta^5}{10065920762063093760} + \frac{8042387879353\vartheta^4}{8052736609650475008} + \frac{6019750139\vartheta^3}{2684245536550158336} - \frac{95551549\vartheta^2}{170428288034930688}, \tag{48}$$

According to Eqs. (45)–(48), we get an approximate solution of Eq. (4) take this out

$$\nu_{approx.}(\vartheta) = -\frac{145\vartheta^{19}}{26580270631909726368} + \frac{21655\vartheta^{18}}{348807761976639917952} + \frac{207041\vartheta^{17}}{51675223996539247104} - \frac{1921043\vartheta^{16}}{43424557980285081600} - \frac{58816267\vartheta^{15}}{101323968620665190400} + \frac{74953019\vartheta^{14}}{9006574988503572480} - \frac{168757339\vartheta^{13}}{25733071395724492800} - \frac{4389335417\vartheta^{12}}{18475025617443225600} + \frac{33675704333\vartheta^{11}}{16422244993282867200} - \frac{113520144641\vartheta^{10}}{17915176356308582400} - \frac{45453476261\vartheta^9}{682482908811755520} + \frac{355510386757\vartheta^8}{707760053582561280} - \frac{29196430051567\vartheta^7}{16986241285981470720} + \frac{341174393885\vartheta^6}{92442129447518208} + \frac{128230434494419\vartheta^5}{2013184152412618752} - \frac{2038874563373639\vartheta^4}{8052736609650475008} + \frac{1029601822304507\vartheta^3}{2684245536550158336} - \frac{49142611464253\vartheta^2}{170428288034930688} - \frac{221185\vartheta}{3981312} + \frac{1548289}{37158912} + \ldots \tag{49}$$

So that, the approximate solution of Eq. (1) is

$$S_{approx.}(x,\,t) = \frac{9}{64}e^{i\left(\frac{7x}{10}-\frac{8t}{11}\right)} \times \left[-\frac{145\varphi^{19}}{26580270631909726368} + \frac{21655\varphi^{18}}{348807761976639917952} + \frac{207041\varphi^{17}}{51675223996539247104} - \frac{1921043\varphi^{16}}{43424557980285081600} - \frac{58816267\varphi^{15}}{101323968620665190400} + \frac{74953019\varphi^{14}}{9006574988503572480} - \frac{168757339\varphi^{13}}{25733071395724492800} - \frac{4389335417\varphi^{12}}{18475025617443225600} + \frac{33675704333\varphi^{11}}{16422244993282867200} - \frac{113520144641\varphi^{10}}{17915176356308582400} - \frac{45453476261\varphi^9}{682482908811755520} + \frac{355510386757\varphi^8}{707760053582561280} - \frac{29196430051567\varphi^7}{16986241285981470720} + \frac{341174393885\varphi^6}{92442129447518208} + \frac{128230434494419\varphi^5}{2013184152412618752} - \frac{2038874563373639\varphi^4}{8052736609650475008} + \frac{1029601822304507\varphi^3}{2684245536550158336} - \frac{49142611464253\varphi^2}{170428288034930688} - \frac{221185\varphi}{3981312} + \frac{1548289}{37158912}\right]^2, \tag{50}$$

where $\left[k = 7,\ s = 12,\ \Omega = 8,\ \epsilon_1 = 9,\ \epsilon_2 = 10,\ \epsilon_3 = 11,\ \varphi = \frac{t}{11} + \frac{6x}{5}\right]$.

Value of (ϑ)	Exact solution	Approximate solution	Error
$\xi = 0.01$	0.0411101882844529	0.04111110705709359	9.187726406970076$\times 10^{-7}$
$\xi = 0.02$	0.04055187674857813	0.040555465132705776	3.5883841276451722$\times 10^{-6}$
$\xi = 0.03$	0.03999175103864777	0.03999977001618954	8.01897754176828$\times 10^{-6}$
$\xi = 0.04$	0.03942983043980941	0.03944402385955532	0.0000141934
$\xi = 0.05$	0.03886613454105958	0.038888228756352845	0.0000220942
$\xi = 0.06$	0.038300683233448596	0.038332386742444345	0.0000317035
$\xi = 0.07$	0.037733496708183545	0.037776499796779904	0.0000430031
$\xi = 0.08$	0.03716459545462936	0.03722056984217522	0.0000559744
$\xi = 0.09$	0.036594000258207915	0.03666459874609141	0.0000705985
$\xi = 0.1$	0.03602173219819493	0.03610858832141702	0.0000868561

See Figures 1–9.

3. Conclusion

In this paper, we studied a sub-10-fs-pulse propagation model uses two analytical techniques and one semi-analytical scheme. This model represents via a higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term that describes the dynamics of light pulses. We implement a generalized auxiliary equation method, generalized exp-$(-\phi(\xi))$ expansion method, and Adomain decomposition method to determine the solitary wave solutions and numerical solutions of this model. We obtained novel and different formulas of solutions. Furthermore, we calculated the value of the error between the analytical and approximate solutions, and we also show the convergence between analytical and semi-analytical solutions. The performance of both analytical techniques could be extended to other forms of nonlinear partial differential equations.

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