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System of six-level atom interacting with a quantized field in the existence of time-varying coupling

Mariam Algarni [a], Kamal Berrada [b,*], Sayed Abdel-Khalek [c,d], Hichem Eleuch [e,f,g]

[a] Department of Mathematical Sciences, College of Science, Princess Nourah Bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia
[b] Department of Physics, College of Science, Imam Mohammad Ibn Saud Islamic University (IMSIU), P.O. Box 5701, Riyadh 11432, Saudi Arabia
[c] Department of Mathematics and Statistics, College of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia
[d] Department of Mathematics, Faculty of Science, Sohag University, Sohag 82524, Egypt
[e] Department of Applied Physics and Astronomy, University of Sharjah, Sharjah 27272, United Arab Emirates
[f] College of Arts and Sciences, Abu Dhabi University, Abu Dhabi 59911, United Arab Emirates
[g] Institute for Quantum Science and Engineering, Texas A&M University, College Station, TX 77843, USA

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ABSTRACT

In the present manuscript, we extend the Jaynes-Cummings model in the framework of six-level atom (LA) system in the presence of a quantized field describes by an ordinary coherent state or a superposition state. We investigate the dynamics of the population, coherence and quantum entanglement in terms of the system parameters. We analyze the time variation of the quantifiers in the presence and absence of time-varying coupling of the six-LA–field system and make comparison among the quantifiers. Furthermore, we show how the quantifiers can be controlled according to the values of the system parameters. The obtained results illustrate that the proposed system provides interesting values of the quantum coherence and entanglement which can be considered as good candidates for executing quantum optics and information schemes.

Introduction

By addressing the incompleteness of quantum mechanics, quantum entanglement (Q-E) was proposed [1–5]. A remarkable feature of composite systems known as Q-E is the inability of the joint quantum state to be expressed as the product of the quantum states of its system components [1,2]. One of the most advantageous phenomena in quantum computation is entanglement, which illustrates the use of non-local correlation [6]. Various topics and phenomena have been better understood in various physical sciences domains thanks to the study of Q-E and the outcomes of its measurements [6,7]. An increasing body of research on the Q-E phenomenon has lately aided in advancing quantum information theory [8–13]. Due to the importance of Q-E in several applications, quantum systems in higher dimensional spaces are studied and investigated, and a novel function for this kind of correlation in multiparticle systems is revealed [14].

Quantum coherence (Q-C) has been considered as an essential features of quantum systems. Coherence theory purpose to examine the fundamental distinction between the quantum and classical realms, leading to a better understanding of classical limits [15–18]. Furthermore, this featured quantum property exposes the mechanisms that ultimately lead to quantum-enhanced devices [19–22]. In quantum systems, several approaches have been proposed to detect the amount of Q-C [15]. Recently, numerous studies have been suggested for characterizing and exanimating the Q-C [23–30]. In comparison to nonclassical correlations, Q-C exhibits its dual characters. Due to the fact that any quantum system is inevitably interacted with its external environment, the Q-C is sensitive to external noise. This demonstrates that the Q-C is not simple to be created, controlled, and preserved in quantum systems. Therefore, it is necessary to establish and protect Q-C in the domain of quantum optics and information. The development of many applications of quantum technology depends on understanding and analyzing the interactions between quantum fields and atomic systems. A simple example is the case of a two-level atom interacting with a quantum electromagnetic field in the context of rotating-wave approximation. Theoretical description of this bipartite system is provided by the Jaynes-Cummings model [31].

Numerous generalizations of this model have recently been

* Corresponding author at: The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34151 Trieste, Italy.
E-mail addresses: mmalgarni@pnu.edu.sa (M. Algarni), berradakamal@ymail.com (K. Berrada), sabotalb@tu.edu.sa (S. Abdel-Khalek), heleuch@sharjah.ac.ae (H. Eleuch).

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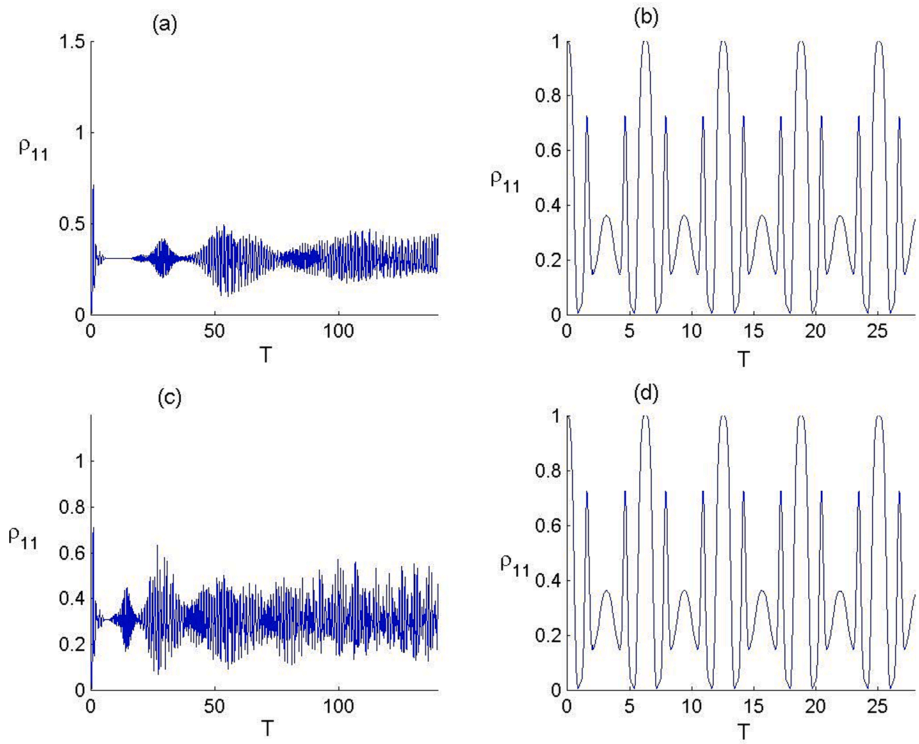

Fig. 1. The time variation of the population $\rho_{11}=\langle 1|\rho_{6LA}|1\rangle$ of a six-LA in the upper state, where the field initially in the coherent state (r $= 0,\ \alpha = 5$) for the panels (a, b) and in a superposition state (r $= 1,\ \alpha = 5$) for the panels (c,d). Panels (a,c) correspond to $f(\tau) = 1$ and (b,d) correspond to.$f(\tau) = \sin(\tau)$.

developed and explored [32–37]. Recently, a control strategy incorporating a fixed carrier frequency constraint on the optimal field is presented by considering the generation of maximum coherence in a six-LA system through solving the equation of Schrödinger [38]. By using the optimal control theory, it is shown that the optimal control theory optimization leads to control the central frequency of the control field and optimize processes of off-resonant in multilevel systems. Based on the aforementioned considerations, we continue in this work the investigation of quantum phenomena and analyze the optimal conditions needed for applications in quantum technology. We investigate the dynamical behavior of the population, coherence and quantum entanglement considering a quantum system that consists a six-LA in the presence of a quantized field describes by an ordinary coherent state or a superposition state. We analyze the time variation of the quantifiers in the presence and absence of time-varying coupling of the six-LA–field system and make comparison among the quantifiers during the dynamics. Furthermore, we show how the quantifiers can be controlled according to the values of the system parameters.

The manuscript is structured as follows. In Section 2 we present the model of quantum system and its solution. In Section 3 we define the quantum resources and discuss the numerical. The final section provides a summary of the key findings.

Six-level atom–field model and wave function

We introduce a system that consists of a six-level ladder-type atom, with transition energies $\omega_j\,(j = 1,\cdots,6)$ and $\omega_1 > \omega_1 > .... > \omega_6$, interacting with a cavity field. This atom interacts with a single mode cavity field. The total system Hamiltonian $\widehat{H}$, In the context of rotating wave approximation, can be formulated as:

$$\widehat{H} = \widehat{H}_{A-F} + \widehat{H}_{IN} \tag{1}$$

Here, $\widehat{H}_{A-F}$ designs the free Hamiltonian of the field-atom system and $\widehat{H}_{IN}$ represents the interaction Hamiltonian given by

$$\widehat{H}_{A-F} = \sum_j \omega_j \widehat{\sigma}_{j,j} + \Omega \widehat{a}^\dagger \widehat{a}. \tag{2}$$

where $\widehat{a}^+(\widehat{a})$ represents the creation (annihilation) operator and Ω denotes the field frequency.

We assume the interaction between atomic system and the field is affected via five photons that are needed to accomplish the transitions. In the non-resonant case, the interaction part in the presence of time-dependent coupling can be provided as

$$\widehat{H}_{IN}(\tau) = f(\tau)\sum_{j=1}^{5} b_j[\widehat{a}|j\rangle\langle j+1| + h.c.], \tag{3}$$

where b_j is the atom–field coupling constant for $f(\tau) = 1$ and $f(\tau) = \sin(\tau)$ in the case of time-dependent coupling. Here, we consider the case of identical coupling $b_j = b_{j+1} = b$ with $j = 1\cdots 4$.

The initial preperation of the system is assumed that the six-LA starts from its upper state $|1\rangle$ and the field from the coherent states' superposition. The state of the interacing system is

$$\begin{aligned}|\psi(0)\rangle &= \frac{1}{\sqrt{1+r+r^2\langle\alpha|-\alpha\rangle}}(|\alpha,1\rangle + r|-\alpha,1\rangle\,) \\ &= \sum nQ_n(1+r(-1)^n)|n,1\rangle,\end{aligned} \tag{4}$$

where

$$Q_n = \frac{\alpha^n\sqrt{\exp(-\alpha^2)}}{\sqrt{n!}\sqrt{(1+r^2+r\exp(-2\alpha^2))}}.$$

Now, we focus on solving the model of the considered system by writting the six-LA–field wave function at an arbitrary scaled time $T = b\tau$ in the form:

$$|\psi(T)\rangle = \sum_m Q_n\left[\sum_{J=1}^{6}\beta_j(T)|m+j-1\rangle\otimes|j\rangle\right]. \tag{5}$$

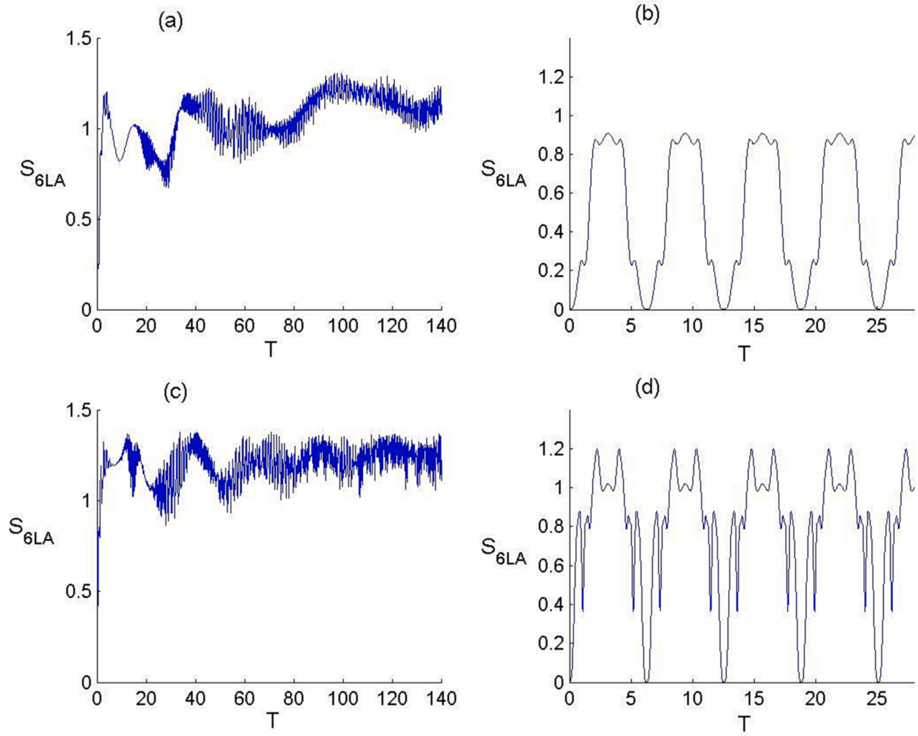

Fig. 2. The time variation of the von Neumann entropy S_{6LA} of a six-LA in the upper state, where the field initially in the coherent state (r = 0, $\alpha = 5$) for the panels (a,b) and in a superposition state (r = 1, $\alpha = 5$) for the panels (c,d). Panels (a,c) correspond to $f(\tau) = 1$ and (b,d) correspond to $f(\tau) = \sin(\tau)$.

The wave function can be determined through solving the system of differential equations obtained by using the Schrödinger equation in the interaction picture $\left(\frac{d}{dT}|\psi(T)\rangle = -i\widehat{H}_{IN}(T)|\psi(T)\rangle\right)$ and acting by the operators $\widehat{a}$ and $\widehat{a}^{\dagger}$ on the Fock state $|n\rangle$

$$i\frac{d}{dT}\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\\ \beta_6\end{pmatrix} = \begin{pmatrix} 0 & \mu_1(T) & 0 & 0 & 0 & 0\\ \mu_1(T) & 0 & \mu_2(T) & 0 & 0 & 0\\ 0 & \mu_2(T) & 0 & \mu_3(T) & 0 & 0\\ 0 & 0 & \mu_3(T) & 0 & \mu_4(T) & 0\\ 0 & 0 & 0 & \mu_4(T) & 0 & \mu_5(T)\\ 0 & 0 & 0 & 0 & \mu_5(T) & 0 \end{pmatrix}\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\\ \beta_6\end{pmatrix} \quad (6)$$

where

$$\mu_j(\mathrm{T}) = b\,sin(T)\sqrt{n+j}, \qquad j = 1\cdots 5. \quad (7)$$

Quantumness measures and results

In this section, we show and discuss the time variation of the population coherence and quantum entanglement considering a quantum system that consists a six-LA in the presence of a quantized field describes by an ordinary coherent state or a superposition state.

Atomic population

We assume that the atomic population of the six-LA system defined through the probability for which the atom is in an upper state

$$\rho_{11} = \langle 1|\rho_{6LA}|1\rangle \quad (8)$$

where $\rho_{6LA} = Tr_F[|\psi(T)\rangle\langle\psi(T)|]$ describes the density operator of the atomic state.

Fig. 1 displays the time variation of the population, ρ_{11}, of six-LA system with and without time-varying coupling for $r = 0$ and $r = 1$. In general, it seems that the dynamics of the function ρ_{11} is affected by the function f and parameter r. For $r = 0$ and $f(\tau) = 1$, we find that the function ρ_{11} displays fast oscillations with revival and collapse phenomena. When $r = 1$, we obtain the same behavior as in the case of $r = 0$ with population amplitude increases and collapse period occurring for small intervals of time. In the presence of time-varying coupling effect, the behavior of function ρ_{11} becomes regular, presenting a periodic function with time. Form these results, we can notice that the superposition of the field state can enhance the amplitude of population oscillations in the absence of time-varying coupling and that the existence of this effect results a periodic behavior of ρ_{11} and ignore the impact of the superposition effect of the field state on the population dynamics.

Six-Level atom–field entanglement

The von Neumann entropy defined in terms of the density operator, ρ, describing a given quantum state as

$$S(\rho) = -Tr(\rho\ln\rho). \quad (9)$$

The function S gets zero value for all pure states satisfying the condition $\widehat{\rho}^2 = \widehat{\rho}$. For a six-LA system, the von Neumann entropy can be written as

$$S_{6LA} = -Tr\{\rho_{6LA}\ln[\rho_{6LA}]\}, \quad (10)$$

where the reduced density matrix is

$$\rho_{6LA} = Tr_{field}\{|\psi(t)\rangle\langle\psi(t)|\}. \quad (11)$$

We can use the basis for which the density matrix of the atom is diagonal and we have

$$S_{6LA} = -\sum_{j=1}^{6}\xi_j\ln(\xi_j), \quad (12)$$

where ξ_j represents the eigenvalues of the six-level atomic density operator (11). The function S_{6LA} varies from 0 for a separable to ln6 for a

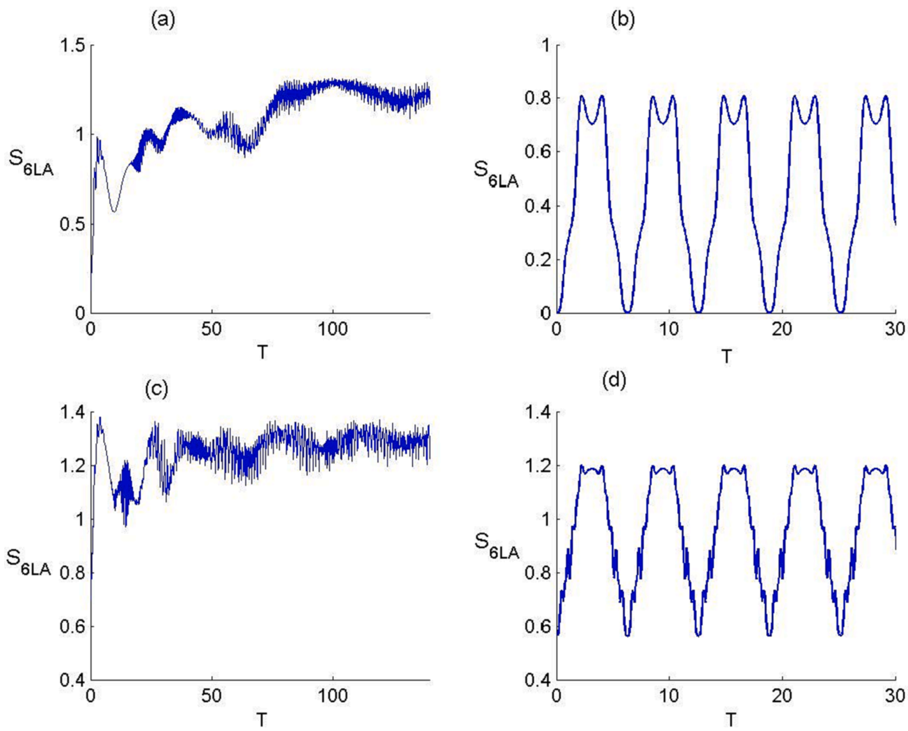

Fig. 3. The time variation of the von Neumann entropy S_{6LA} of a six-LA in the superposition state, $|\phi_{6LA}\rangle = \frac{1}{2|1\rangle} + \frac{\sqrt{3}}{2}|2\rangle$, where the field initially in the coherent state ($r = 0$, $\alpha = 5$) for the panels (a,b) and in a superposition state ($r = 1$, $\alpha = 5$) for the panels (c,d). Panels (a,c) correspond to $f(\tau) = 1$ and (b,d) correspond to. $f(\tau) = \sin(\tau)$.

maximally entangled state of the atom–field system.

In Fig. 2, we display the time evolution of the function S_{6LA} against the time for an initial field prepared in a Glauber coherent state ($r = 0$) or in a cat state ($r = 1$) considering constand- and time-varying coupling. In general, we find that the dynamical behavior of S_{6LA} is strongly dependent on the system's parameters. In the case of $r = 0$ and $f(\tau) = 1$, we find that the entanglement initially increases with time due to the six–LA-field interaction and thereafter the entanglement measure

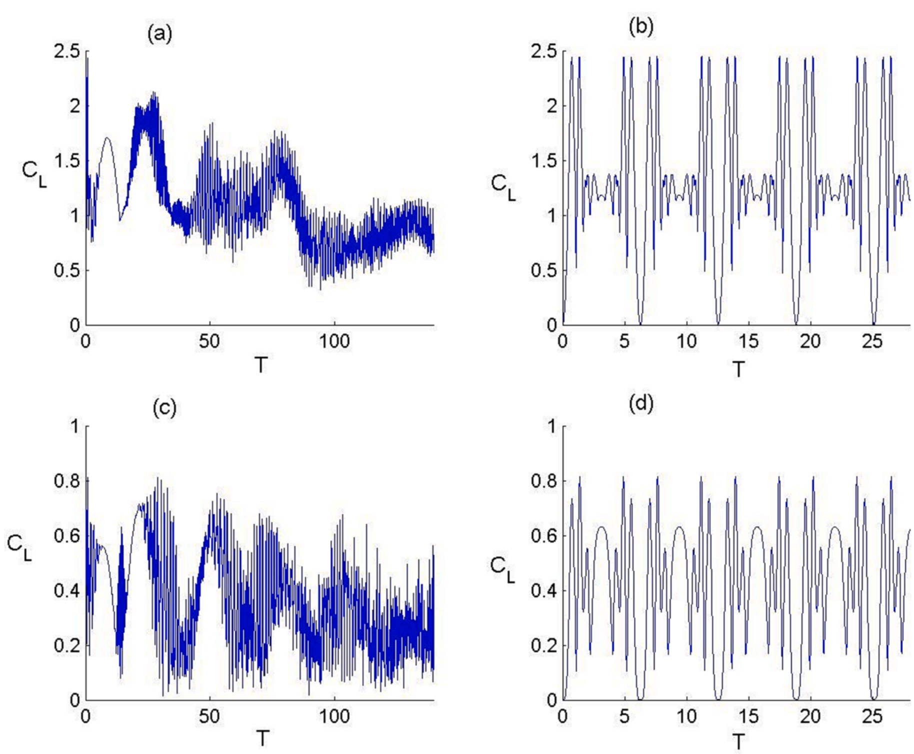

Fig. 4. The time variation of the quantum coherence C_L of a six-LA in the upper state, where field initially in the coherent state ($r = 0$, $\alpha = 5$) for the panels (a,b) and in a superposition state ($r = 1$, $\alpha = 5$) for the panels (c,d). Panels (a,c) correspond to $f(\tau) = 1$ and (b,d) correspond to. $f(\tau) = \sin(\tau)$.

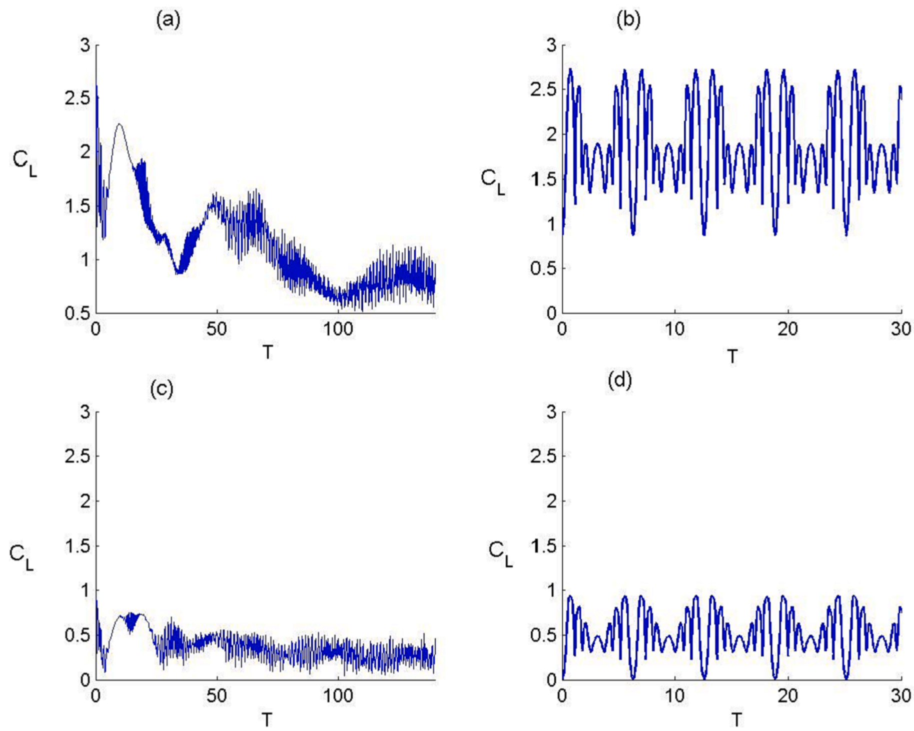

Fig. 5. The time variation of the quantum coherence C_L of a six-LA in the superposition state, $|\phi_{6LA}\rangle = 1/2|1\rangle + \sqrt{3}/2|2\rangle$, where the field initially in the coherent state (r = 0, $\alpha = 5$) for the panels (a,b) and in a superposition state (r = 1, $\alpha = 5$) for the panels (c,d). Panels (a,c) correspond to $f(\tau) = 1$ and (b,d) correspond to $f(\tau) = \sin(\tau)$.

exhibits an oscillatory behavior during the dynamics with rapid oscillations. When the time becomes large, the entanglement measure stabilizes in a steady behavior showing that the six-LA system is trapped by the quantized field. For $r = 0$ and $f(\tau) = \sin(\tau)$, the function S_{6LA} exhibits periodic behaviour with entanglement phenomena of sudden death and sudden birth during the evolution. In this context, six-LA–field entanglement attains the maximal value. When the field starts from a cat state, we can observe that the entanglement measure of six-LA–field state presents the same variation with the time as the case of field defined in an ordinary coherent state for $f(\tau) = 1$. The absence of time-varying coupling leads to organize the dynamical behavior of the function with the same periodicity time as the case of $r = 0$ but with greater values of the entanglement. This shows that the coupling function is responsible for the oscillations of the entangelement measure via the exchange of the energy between the field and six-LA system. Fig. 3 displays the time evolution of the function S_{6LA} against the time for an initial atom prepared in a superposition state, $|\phi_{6LA}\rangle = 1/2|1\rangle + \sqrt{3}/2|2\rangle$. We can observe that the change in the initial setting state of the atom don't strongly affect the behavior of S_{6LA} with respect to the model parameters. The obtained results illustrate a good understanding of the impact of the system parameters on the of entanglement of the Six-LA–field state considering the effect of the initial state setting of the field and its coupling to the atom.

Quantum coherence

The diagonal elements of the state ρ of a quantum system are responsible for the majority of the properties that define the coherence of quantum systems. The L_1 norm of the coherence determines whether or not there is coherence based on the elements that are not diagonal. The shortest distance between the system state of interest and its incoherent state is what is meant when referring to the definition of the coherence measure that is based on the relative entropy which formulated as [11]:

$$C_R = S(\rho||\rho_{\text{diag}}) = \text{Tr}(\rho_{\text{diag}} \ln \rho_{\text{diag}}) + \text{Tr}(\rho \ln \rho), \tag{13}$$

where the first and second terms of Eq. (12) repress von Neumann entropy in terms of the diagonal elements of the atomic density matrix (12) in terms of the incoherent state and density operator respectively.

The quantum coherence C_L in terms of the off-diagonal elements is given by [39]

$$C_L = \min_{\delta \in M} \left\| \rho - \varepsilon \right\|_{L_1} = \sum_{k \neq j} |\rho_{kj}|, \tag{14}$$

where L_1 and M describes a norm and set of incoherent states.

The functions C_L and C_R satisfy the property of monotonicity for different states. For pure states, it has been verified that the function C_L describes the upper bound of C_R.

Let us now explore the influence of the model parameters on the dynamics of the six-LA coherence. Fig. 4 depicts the variation of the coherence measure against time with respect to the values of r and f. In general, we can observe some significant dynamical features of the six-LA coherence with the presence of an oscillatory behavior. The time variation of coherence is very sensitive to the initial field state setting and six-LA–field coupling. Interestingly, we find the parameter r and function f act on the coherence in similar way as on the entanglement, where the function C_L exhibits an irregular behaviour with rapid oscillation in the absence of time-varying coupling for both cases of $r = 0$ and $r = 1$. The exitence of time-varying coupling leads to reduce the oscillations of C_L with periodic oscillation behaviour. Futhermore, we can observe that the measures of entanglement and coherence exihibit the same time period with more oscillations for the function C_L. Fig. 5 displays the time evolution of the function C_L against the time for an initial atom prepared in a superposition state, $|\phi_{6LA}\rangle = 1/2|1\rangle + \sqrt{3}/2|2\rangle$. We can observe that the change in the initial setting state of the atom don't strongly affect the behavior of C_L. The obtained results indicate that these features make quantum coherence a good candidate for describing the dynamics of quantum correlation in the proposed system, and then its application in diverse fields of quantum information and processing.

Conclusions

We have investigated the dynamics of the population, coherence and quantum entanglement considering a quantum system that consists a six-LA in the presence of a quantized field describes by an ordinary coherent state or a superposition state. We have analyzed the time variation of the quantifiers under the effect of the time-varying coupling of the six-LA–field system and make comparison among the quantifiers according to the system parameters. Futhermore, we have shown how the quantifiers can be controlled according these system parameters. We have observed that the measures of the coherence and entanglement increase from its minimum initial value and then exhibits rapid irregular oscillations that become periodic with time in the presence of time-varying coupling. Finally, we have proven that the six-LA coherence and entanglement exhibit the same behavior with respect to the system parameters during dynamics. The obtained results illustrated that the proposed system provides interesting values of the quantum coherence and entanglement which can be considered for understanding these phenomena in multilevel quantum systems. Here, we mention that we examine here only the coherence and entanglement dynamics in the presence of a mode field with one photon transition. Certainly, a study of six-LA systems in the presence of muti-mode field with more than one photon transtion will make a helpful contribution to understanding the dynamics of entanglement and coherence in the proposed quantum system.

Data availability Statement:

Not Applicable

CRediT authorship contribution statement

Mariam Algarni: Conceptualization, Methodology, Writing – original draft, Writing – review & editing. **Kamal Berrada:** Conceptualization, Methodology, Writing – original draft. **Sayed Abdel-Khalek:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing. **Hichem Eleuch:** Validation, Investigation, Writing – original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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