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Thermal Fisher and Wigner–Yanase information correlations in two-qubit Heisenberg XYZ chain

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ABSTRACT

Using Wigner–Yanase and Fisher information [including local quantum uncertainty (LQU) and local quantum Fisher information (LQFI)] as well as log-negativity, we investigate the thermal non-local correlations of two-qubit Heisenberg XYZ non-X states produced in the presence of the Dzyaloshinskii–Moriya (DM) interaction. The two-spin XYZ-Heisenberg non-local correlation decay can be weakened by increasing the DM interaction as well as the spin–spin XYZ-Heisenberg interactions. The LQFI correlation has greater resistance to thermal degradation. The spin–spin y-component antiferromagnetic coupling has a higher ability to protect the generated spin–spin nonlocal correlations against high temperatures. The emergence of the phenomena of the sudden death of log-negativity and the sudden change of the LQFI and LQU depend on the spin–spin XYZ-Heisenberg and DM interactions as well as on the bath temperature. The generated asymmetric correlations resulting due to the x, y-component spin interactions confirm that the antiferromagnetic coupling has a high ability to generate a maximally correlated two-qubit state. While the generated correlations, due to two-spin z-component interaction, present symmetric dynamics, and its amount depends on the x-component spin interaction.

Introduction

Exploring the existence of two-qubit quantum effects (as: quantum correlations, quantum coherence and nonlocality) in real quantum solid state systems (including two-qubit Heisenberg systems and two-qubit superconducting circuits) present important tools to the building blocks of quantum information [1,2] and computation [3]. For analyzing the quantum effects of spin chains, the Heisenberg model is the most practical. The Heisenberg systems have the ability to generate a high two-qubit correlation [4–6]. Heisenberg XXX-chain model has been used to explore two-qubit entanglement [7], where, the entanglement exists only for antiferromagnetic case. Furthermore, in the presence of homogeneous and inhomogeneous magnetic field, the generation of quantum correlations are investigated in different two-qubit Heisenberg systems as two-qubit XXX-, XXZ- and XY-Heisenberg systems [8–10]. By using an external magnetic field, the two-qubit Heisenberg XYZ chain's two-qubit quantum correlation is enhanced [11]. One significant type of the spin-1/2 Heisenberg chain is the two-qubit Heisenberg XYZ system. Two-qubit Heisenberg systems with spin–orbit interactions [12,13] induce other forms of anisotropy, such as Dzyaloshinskii–Moriya (DM) and Kaplan–Shekhtman–Entin–Wohlman–Aharony (KESA) interactions [14,15]. The DM interaction can be realized by an antisymmetric superexchange interaction in La_2CuO_4 [16].

In the research focusing on quantum information areas [17–19] such as: quantum computation [20,21], quantum estimation [22], teleportation [23], and quantum memory assisted entropic uncertainty [24], understanding the nature of quantum correlations in real two-qubit systems [25,26] is gaining high prominence. The investigations of thermal two-qubit-Heisenberg discord like quantum correlations (including Wigner–Yanase skew information [27–29] and quantum Fisher information [30,31] correlations) have been appeared,

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heavily, after introducing quantum discord [32] as a new type of quantum correlation beyond the entanglement [33]. In different two-qubit-Heisenberg models, the thermal two-qubit-Heisenberg correlations have been explored by using uncertainty-induced nonlocality and local quantum uncertainty (LQU) of the Wigner–Yanase skew information [34,35] (under DM and KSEA interactions), and local quantum Fisher information (LQFI) [35]. LQU is widely utilized to realize coherence and correlation in a wide range of two-qubit systems [36,37]. LQU and LQFI are directly related via an essential relation between quantum Fisher-information and Wigner–Yanase skew information theories [38,39]. The thermal pair-spin LQU and LQFI correlations have been realized with dipolar and Dzyaloshinskii–Moriya interactions [40].

The recent literatures have focused on the effects of spin–spin interactions in the absence of x, y, and z-component couplings and in the presence of DM and KSEA interactions in the z-direction for the X-state [5,41–46]. Yet, by using the thermal density matrix of the system at thermal equilibrium, we have investigated thermal non-local correlations of a two-qubit XYZ-Heisenberg non-X-state in the presence of the Dzyaloshinskii–Moriya interaction in our work. It is worth noting that the effects of DM and KSEA interactions in the x and y-directions for non-X-state/general state have not been explored previously. This study aims to investigate the impact of y-direction DM interactions on the thermal non-local correlations of two-qubit Heisenberg XYZ non-X states using Wigner–Yanase and Fisher information [including local quantum uncertainty and local quantum Fisher information] as well as log-negativity.

The paper is structured as follows: In Section "Two-spin Heisenberg XYZ model", we cover the physical two-spin model. In Sections "Non-local correlation measures" and "Spin–spin XYZ-Heisenberg nonlocal correlations", various measures of two-spin non-locality, namely local Fisher information, local quantum uncertainty, and logarithmic negativity are introduced and analyzed. Finally, in Section "Conclusion", we summarize our results.

Two-spin Heisenberg XYZ model

The Hamiltonian that describes a Heisenberg XYZ chain model in the presence of the Dzyaloshinskii–Moriya interaction is given by

$$\hat{H} = \sum_n (J_x \hat{\sigma}_n^x \hat{\sigma}_{n+1}^x + J_y \hat{\sigma}_n^y \hat{\sigma}_{n+1}^y + J_z \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z) \tag{1}$$

$$+\vec{D}.(\vec{\sigma}_n \times \vec{\sigma}_{n+1}), \tag{2}$$

where $J_k (k = x, y, z)$ represent the exchange coupling of the two-spin interactions, $\vec{D} = (D_x, D_y, D_z)$ designs the vector of the antisymmetric DM exchange interaction. $\hat{\sigma}_B^k (k = x, y, z)$ are Pauli k-spin operators. The two spins have antiferromagnetic behavior if $J_k > 0$ and they have ferromagnetic behavior for $J_k < 0$.

To study the behavior of thermal LQFI, LQU, and log-negativity two-spin XYZ-Heisenberg correlations with y-direction DM interaction, we use the thermal equilibrium density matrix M_{AB} of the two-spin Heisenberg XYZ system at the temperature T_E,

$$M_{AB} = \frac{\exp[-\frac{H}{T}]}{\mathrm{Tr}\{\exp[-\frac{H}{T}]\}}, \tag{3}$$

where the bath temperature $T = k_B T_E$ is normalized by considering Boltzmann's constant $k_B = 1$ for the sake of convenience. H represents the Hamiltonian of the two-spin Heisenberg XYZ (A and B) in the presence of the y-direction DM interaction with coupling strength D_y,

$$\hat{H} = \sum_{k=x,y,z} J_k \hat{\sigma}_A^k \hat{\sigma}_B^k + D_y(\sigma_A^z \sigma_B^x - \sigma_A^x \sigma_B^z), \tag{4}$$

By using the two-spin asymmetric and symmetric Bell states: $|S_{1,2}\rangle = \frac{1}{\sqrt{2}}(|0_A 1_B\rangle \pm |1_A 0_B\rangle)$ and $|S_{3,4}\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle \pm |1_A 1_B\rangle)$, respectively, the eigenstates $|V_i\rangle (i = 1, 2, 3, 4)$ and the eigenvalues V_i of the two-spin Heisenberg XYZ Hamiltonian of Eq. (4) are

$$\begin{aligned} |V_1\rangle &= |S_1\rangle, \ |V_2\rangle = |S_4\rangle, \\ |V_3\rangle &= \sin\beta_1 |S_3\rangle - \cos\beta_1 |S_2\rangle, \\ |V_4\rangle &= \sin\beta_2 |S_3\rangle - \cos\beta_2 |S_2\rangle, \end{aligned} \tag{5}$$

with

$$\beta_{1,2} = \tan^{-1}\{\frac{2D_y}{J_x + J_z \mp \sqrt{4D_y^2 + J_s^2}}\},$$

where $J_s = J_x + J_y$. The eigenvalues V_k are,

$$\begin{aligned} V_{1,2} &= \pm J_x + J_y \mp J_z, \\ V_{3,4} &= -J_y \pm \sqrt{4D_y^2 + J_s^2}. \end{aligned}$$

By using the eigenstates $|V_i\rangle (i = 1, 2, 3, 4)$ of Eq. (5) and the eigenvalues V_i of Eq. (6), the thermal two-spin Heisenberg DM-XYZ matrix M_{AB} can be expressed as

$$M_{AB} = \frac{1}{W}\sum_{i=1}^{4} \exp[\frac{-V_i}{T}]|V_i\rangle\langle V_i|, \tag{6}$$

$$W = \mathrm{Tr}\{\sum_{i=1}^{4} \exp[\frac{-V_i}{T}]|V_i\rangle\langle V_i|\}.$$

In the next section, we will determine the thermal density matrix M_{AB} of the system in the equilibrium to investigate the influence of the bath temperature, the spin–spin interaction, and the DM y-component interaction on the thermal two-spin Heisenberg XYZ non-local correlations.

Nonlocal correlation measures

The thermal two-spin Heisenberg DM-XYZ nonlocal correlations will be measured by following quantifiers:

- **Local quantum Fisher information (LQFI)**
  LQFI represents the minimal quantum Fisher information [30,31] determined by local unitary evolution of one part of the thermal two-spin Heisenberg XYZ. LQFI is a good quantifier of the two-spin Heisenberg XYZ nonlocal correlation beyond entanglement. It was recently suggested as another essential resource. For the two-spin Heisenberg XYZ state $M_{AB} = \sum_m \pi_m |V_m\rangle\langle V_m|$ with $\pi_m \geq 0$ and $\sum_m \pi_m = 1$, the closed form of the LQFI is

  $$FI = 1 - \pi_S^{\max}, \tag{7}$$

  $\pi_W^{\max}$ is the highest eigenvalue of the symmetric matrix $S = [s_{ij}]$ with the elements,

  $$s_{ij} = \sum_{\pi_m + \pi_n \neq 0} \frac{2\pi_m \pi_n}{\pi_m + \pi_n} V_{mn}^i (V_{nm}^j)^\dagger, \tag{8}$$

  which is based on the Pauli matrices σ^i $(i = 1, 2, 3)$, the operators $\hat{P}_i = I \otimes \sigma^i$ as well as the elements $V_{mn}^i = \langle V_m|\hat{P}_i|V_n\rangle$. For a maximally correlated two-spin state, the $FI = 1$. It is zero-value for separable state. Otherwise, in the range $0 < FI < 1$, it indicates the partial two-spin nonlocal correlation.
- **Local Wigner–Yanase uncertainty**
  Another two-spin correlation is quantified using the local quantum uncertainty (LQU) [28] of the Wigner–Yanase skew information [27]. For the two-spin state M_{AB} of Eq. (6), the LQU of the thermal two-spin Heisenberg DM-XYZ state is identified by M_{AB} can be written as [28]

  $$QU = 1 - \lambda_{max}(\Lambda_{AB}), \tag{9}$$

  λ_{max} designs the largest eigenvalue of the 3×3-matrix $\Lambda_{AB} = [\gamma_{ij}]$, where

  $$\gamma_{ij} = \mathrm{Tr}\{\sqrt{M_{AB}}(\sigma_i \otimes I)\sqrt{M_{AB}}(\sigma_j \otimes I)\}.$$

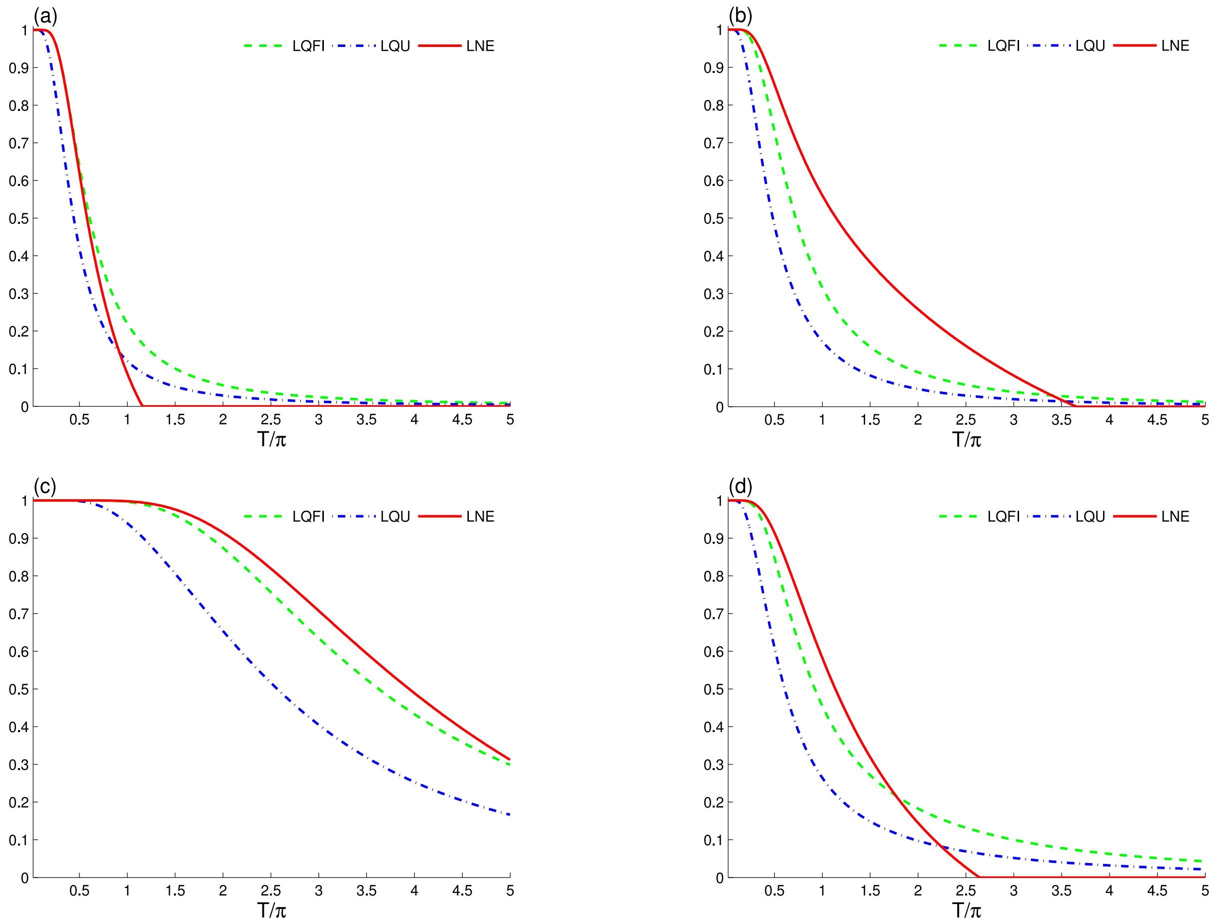

Fig. 1. Thermal two-spin XYZ-Heisenberg nonlocal correlations are shown against the bath temperatures for $D_y = 0$ and for different spin–spin coupling cases: $J_k = 1$ in (a), $(J_x, J_y, J_z) = (10, 1, 1)$ in (b), $(J_x, J_y, J_z) = (1, 10, 1)$ in (c), and $(J_x, J_y, J_z) = (1, 1, 10)$ in (d).

- **Log-negativity entanglement (LNE)**
  We employ the logarithmic negativity (LN) [47] as a measure of the two-spin entanglement to compare the behaviors of the thermal two-spin LQFI, and LQU nonlocal correlations with the generated two-spin entanglement. The negativity μ_T [47], which is defined as the absolute sum of the matrix's negative eigenvalues $(M_{AB})^T$ of the partial transposition of the two-spin density matrix M_{AB} of Eq. (6), is used to define the logarithmic negativity for the two-spin density matrix M_{AB}. The logarithmic negativity can be expressed as:

$$LN = \log_2[1 + 2\mu_T], \tag{10}$$

  The LN has zero value for uncorrelated states. We have also $0 \leq LN \leq 1$.

Spin–spin XYZ-Heisenberg nonlocal correlations

The behaviors of the two-spin XYZ-Heisenberg LQFI, LQU, and LNE will be explored for a particular temperature rang under the effects of the spin–spin interaction exchange couplings $J_k (k = x, y, z)$ as well as the antisymmetric DM y-direction exchange coupling D_y.

In Fig. 1, the thermal spin LQFI, LQU, and log-negativity evolution is displayed for the particular temperature rang $T \in [0, 5]\pi$ in the absence of the Dzyaloshinskii–Moriya interaction effect ($D_x = 0$) and for different spin–spin couplings in the case of the antiferromagnetic exchange $J_k > 0$. For small values of the spin–spin interaction antiferromagnetic-exchange couplings $J_k = 1$, Fig. 1a shows that the two-spin XYZ-Heisenberg LQFI, LQU, and log-negativity are exponentially decaying by increasing the temperatures, and they vanish at different specific high temperatures. The increase of the temperatures leads to the decay of LQFI, LQU, and log-negativity. Their dynamical evolutions are similar, and they are monotonous with exponential decay. The two-spin log-negativity entanglement disappear sharply at a certain temperature, $T_{SDE} \simeq 1.162$, this temperature is called "*the temperature of sudden-death-entanglement, T_{SDE}*". At and after, respectively, the temperature $T_{SDE} \simeq 1.162$, thermal sudden-death log-negativity entanglement phenomenon [48] occurs and then the log-negativity two-spin entanglement disappears completely with higher temperatures, $T > T_{SDE}$. The LQFI, LQU results of all figures agree with the hierarchy principle of quantum-Fisher and Wigner–Yanase skew information, which determines the relation between the quantum-Fisher information and Wigner–Yanase skew information nonlocal correlations, in which, the minimized values of the QFI (LQFI's values) are always larger than those of the Wigner–Yanase skew information (LQU's values) [22,38]. For $T > T_{SDE}$, the degenerated separable two-spin states can store partial LQFI and LQU nonlocal correlations. The exponential and asymptotical decay of the LQFI, LQU, and log-negativity amplitudes is due to a flow of the nonlocal correlations into the thermal environment [49].

Fig. 1(b–c) show the ability of the increasing spin–spin interaction antiferromagnetic-exchange couplings J_k to protect the spin–spin LQFI, LQU, and log-negativity nonlocal correlations against the higher temperatures. The spin–spin interaction components are individually increased as: $(J_x, J_y, J_z) = (10, 1, 1)$ in (b), $(J_x, J_y, J_z) = (1, 10, 1)$ in (c),

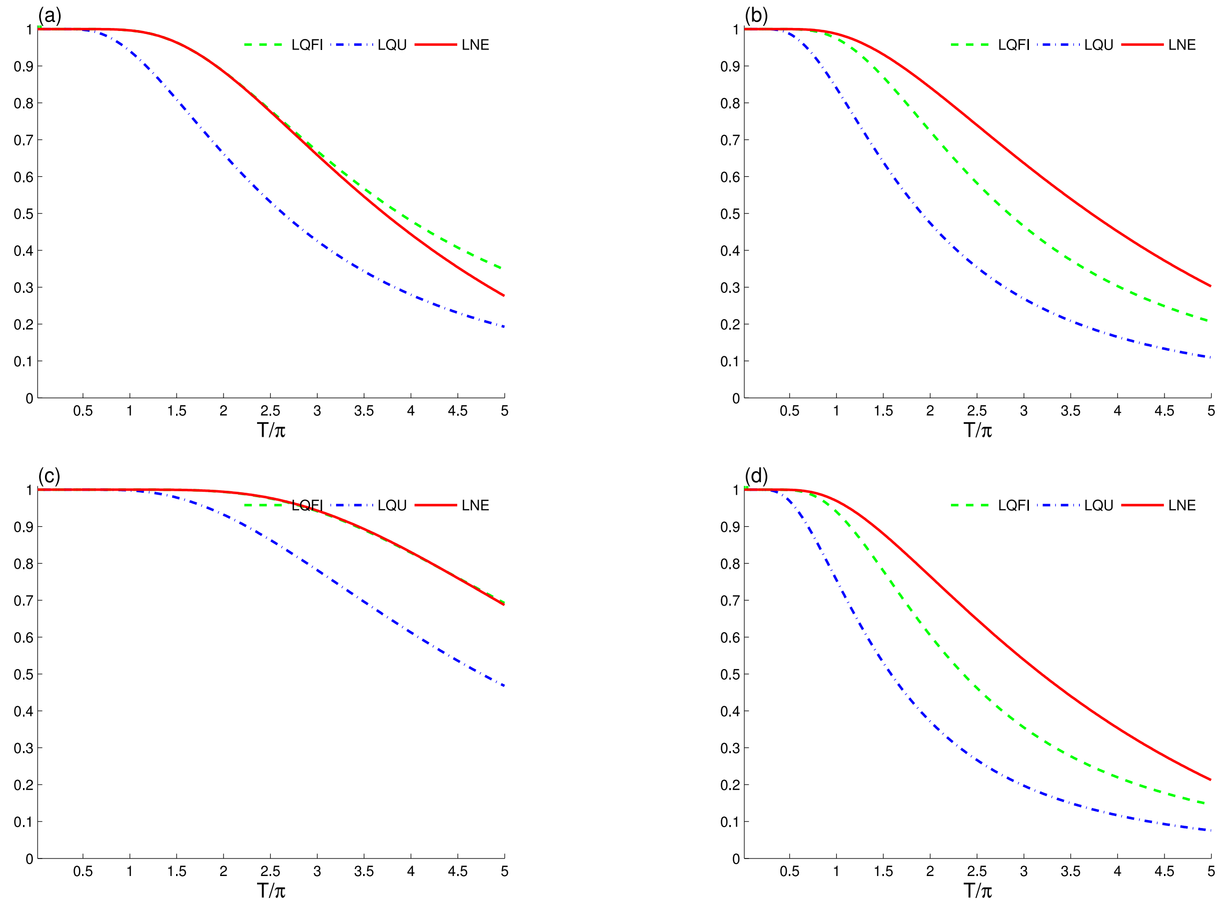

Fig. 2. Thermal spin–spin XYZ-Heisenberg nonlocal correlations of Fig. 1 are shown but for a strong DM interaction, $D_y = 10$.

and $(J_x, J_y, J_z) = (1, 1, 10)$ in (d). We find that the amplitudes of the generated spin–spin LQFI, LQU, and log-negativity nonlocal correlations can be enhanced by increasing the spin–spin interaction. The sudden-death-entanglement temperature T_{SDE} can be delayed by increasing spin–spin interaction antiferromagnetic-exchange couplings. We note that the log-negativity two-spin entanglement always prevails over the LQFI and LQU, almost, before the value's temperature of T_{SDE} for the cases $J_x = 10$ and $J_y = 10$, however, for $J_z = 10$ it is seen as less than the LQFI and LQU during a particular temperature range. Fig. 1(c) illustrates that the spin–spin antiferromagnetic interaction y-component has a high ability to protect the spin–spin LQFI, LQU, and log-negativity nonlocal correlations against the higher temperatures, compared to the other spin–spin interaction components. Thermal sudden-death log-negativity entanglement phenomenon disappears completely.

Fig. 2 illustrates the effect of a strong Dzyaloshinskii–Moriya interaction's coupling of $D_y = 10$ on the spin–spin LQFI, LQU, and log-negativity nonlocal correlations. From Fig. 2a, we find that: (1) the increase of the Dzyaloshinskii–Moriya interaction's coupling exhibits the degradation of the spin–spin LQFI, LQU, and LNE amplitudes (that is due to increasing the thermal environment effect), therefore, thermal sudden-death log-negativity entanglement phenomenon disappears completely. This means that the robustness of the initial spin–spin LQFI, LQU, and LNE's values (at $T = 0$), against the higher temperatures, is enhanced by increasing the Dzyaloshinskii–Moriya interaction. (2) For the temperature $T < 4\pi$, we observe that the spin–spin states encodes the same information of the LQFI and the log-negativity nonlocal correlation, this type of correlation can be called "*LQFI-LNE correlation*". Fig. 2(b–c) show the accompanied effects of the increase of the spin–spin antiferromagnetic interaction components (J_x, J_y, and J_z) with Dzyaloshinskii–Moriya interaction have a high ability to enhance the nonlocal correlation robustness. The increase of the J_y-component spin–spin antiferromagnetic interaction enhances the robustness against the decay resulted from the higher temperature effects.

Fig. 3 displays the evolution of the spin–spin LQFI, LQU, and LNE at low bath temperature $T = \pi$ to the spin–spin interaction components J_k: $|J_x| \leq 6\pi$ in (a) in (a), $|J_y| \leq 6\pi$ in (b), $|J_z| \leq 6\pi$ in (c) as well as with respect to the y-component Dzyaloshinskii–Moriya interaction $|D_y| \leq 6\pi$ in (d). Fig. 3a shows that the ability of the spin–spin x-component interaction antiferromagnetic $J_x > 0$ to generate spin–spin LQFI, LQU, and log-negativity nonlocal correlations is larger than that of the ferromagnetic $J_x < 0$. We also notice that LQFI, LQU, and LNE have asymmetric dynamics and the two-spin log-negativity always prevails over the LQFI and LQU. For the antiferromagnetic $J_x > 0$, LQFI, LQU, and LNE correlations are enhanced rapidly, and becoming independent on the spin–spin x-component interaction. The generated two-spin states have maximal LQFI and LNE (approximately) nonlocal correlations ($FI = LN \approx 1$) as well as stable partial LQU correlation. At a particular antiferromagnetic spin–spin Heisenberg XYZ interaction ($J_x = 0.3248\pi$), the local quantum uncertainty changes suddenly indicating the phenomenon of sudden-change local quantum uncertainty. The sudden-change phenomenon has been coined theoretically [50] and observed experimentally [51]. After the sudden-change LQU point, the generated two-spin states have stationary partial LQU correlation, and its nonlocal correlations are independent of the antiferromagnetic spin-Heisenberg XYZ J_x-coupling. For the ferromagnetic $J_x < 0$, the spin–spin state has different LQFI, LQU, and LNE correlations.

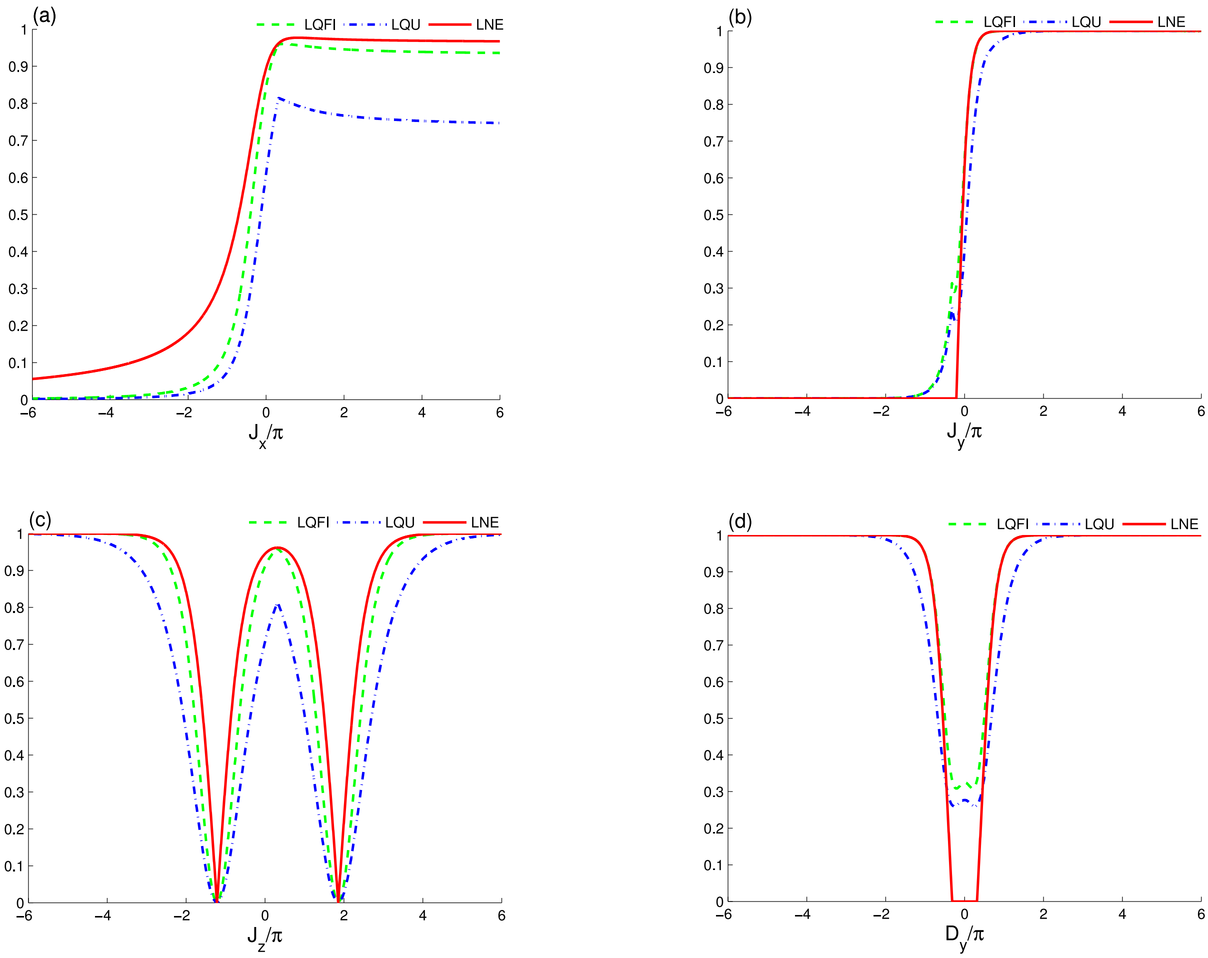

Fig. 3. Thermal spin–spin XYZ-Heisenberg nonlocal correlations are shown at $T = \pi$ for different cases: $|J_x| \le 6\pi$ and $J_y = J_z = D_y = 1$ in (a), $|J_y| \le 6\pi$ and $J_x = J_z = D_y = 1$ in (b), $|J_z| \le 6\pi$ and $J_x = J_y = D_y = 1$ in (c) as well as $|D_y| \le 6\pi$ and $J_{x,y} = -1.219$ with $J_z = 1$ in (d).

Fig. 3(b) illustrates the dependence of the thermal spin–spin LQFI, LQU, and LNE correlations on the spin–spin y-component interaction. The y-component interaction antiferromagnetic has a high ability to generate stable maximally correlated two spin-qubit states (the LQFI, LQU, and LN reach their maximal nonlocal correlations, quickly). The generated stable maximally correlated two spin-qubit states have numerous quantum technology applications [52]. While the ability of the y-component interaction ferromagnetic to generate stable partially correlated two spin-qubit state is very weak. The results of Fig. 3(c) show that the spin–spin z-component interaction can generate rapidly symmetric spin–spin LQFI, LQU, and log-negativity nonlocal correlations with respect to a particular z-component coupling value, P_{sym} (symmetry point). This symmetry point can be shifted right and left by increasing and decreasing the spin–spin x-component coupling J_x, respectively. After the symmetry point P_{sym}, the increase of the spin–spin z-component coupling leads to the thermal spin–spin nonlocal correlations being dropped sharply by increasing interaction's z-component J_z. They vanish instantaneously at a particular value of the spin–spin interaction's z-component. After that, by increasing the z-component two-spin coupling, the two-spin state tends rapidly to a stable maximally correlated two-qubit state. The LQFI, LQU, and LN reach their maximal values ($LQFI = LQU = LN = 1$), and the correlations are independent of J_z.

The aim of Fig. 3(d) is to explore the ability of the DM interaction to improve the LQFI, LQU, and log-negativity nonlocal correlations of the case where ferromagnetic for $J_{x,y} = -1.219$. For the y-component of the DM coupling range $|D_y| \le 6\pi$, with a low bath temperature ($T = \pi$), the spin–spin ferromagnetic x,y-component couplings $J_{x,y} = -1.219$ leads to that the generated spin-qubits Heisenberg XYZ correlations are enhanced quickly. They are symmetric to $D_y = 0$. The thermal sudden-birth log-negativity entanglement phenomenon occurs at the end of the DM coupling range $|D_y| \le 0.3121\pi$. In this range, the separable spin states can have partial LQFI and LQU, nonlocal correlations. Outsides the disentanglement range, directly, LQFI, LQU, and log-negativity nonlocal correlations grow suddenly and the two spin-qubits have stationary maximal nonlocal correlations.

Fig. 4 shows thermal spin–spin LQFI, LQU, and LNE dynamics with similar parameters as Fig. 3 but for a high temperature $T = 4\pi$. Fig. 4a displays the dependence of the produced spin–spin nonlocal correlations on the spin–spin x-component coupling for $|J_x| \le 6\pi$. By comparing the obtained results with the ones of Fig. 3(a), we note that the high bath temperature ($T = 4\pi$) leads to the decrease of LQFI, LQU, and LNE. The phenomenon of sudden change emerge in the LQFI and LQU dynamics, however, LNE is more robust against the temperature. While Fig. 4b confirms that the y-component spin coupling ($|J_y| \le 6\pi$) has a noticeable effect. The generation of LQFI, LQU, and log-negativity nonlocal correlations is better realized by increasing y-component spin coupling J_y. For $|J_z| \le 6\pi$, Fig. 4c shows that the amplitudes and the symmetric stationary maximal nonlocal correlation intervals decrease at a high bath temperature ($T = 4\pi$), see Figs. 3c and 4a. The phenomena of the sudden changes of the LQFI and LQU, and the thermal sudden death–birth log-negativity entanglement appear clearly. Fig. 4d

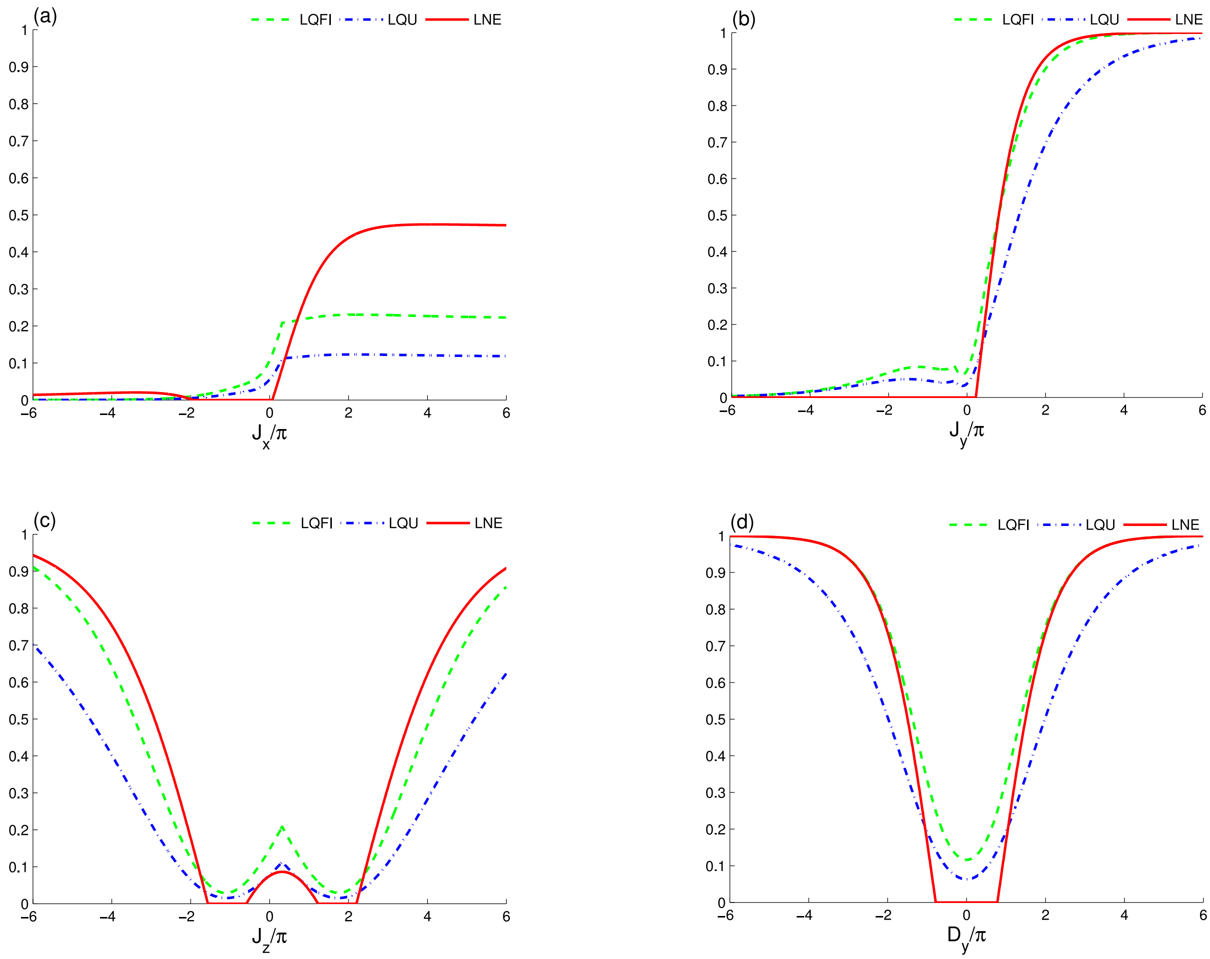

Fig. 4. Thermal spin–spin XYZ-Heisenberg nonlocal correlations of Fig. 3 are shown but for at high temperature $T = 4\pi$.

displays the dependence of the LQFI, LQU, and log-negativity on the DM y-component interaction. The high temperature leads to expanding the intervals of the sudden death–birth phenomenon of the two-spin-qubits entanglement and decreasing the correlation amplitudes. We can deduce that the combined effects of the strong spin–spin antiferromagnetic interaction with the DM y-component interaction improve and protect the LQFI, LQU, and log-negativity nonlocal correlations.

Conclusion

In this study, we have investigated the thermal non-local correlations of a two-qubit XYZ-Heisenberg non-X-states in the presence of the Dzyaloshinskii–Moriya interaction. We have used the thermal density matrix of the system at thermal equilibrium. Fisher, Wigner–Yanase, and log-negative correlations have all been utilized to measure the non-local correlations. They have been analyzed under the DM y-component interaction and for different spin–spin x, y, z-components couplings. The thermal two-qubit XYZ-Heisenberg non-X-state correlation dynamics is shown to be controlled by the spin–spin and Dzyaloshinskii–Moriya interactions, as well as by the thermal effect. The two-spin XYZ-Heisenberg LQFI, LQU, and log-negativity decrease exponentially and monotonically as the temperature rises. The non-local LQFI correlation is more robust against the thermal effect. The non-local LQFI correlation decay can be weakened by adjusting the intensity of the DM and the spin–spin XYZ-Heisenberg couplings. Compared to the other spin–spin interaction components, the spin–spin antiferromagnetic y-component interaction has a stronger ability to protect the spin–spin LQFI, LQU, and log-negativity correlations against the temperature. Strong DM and spin–spin XYZ-Heisenberg interactions can be used to prevent the thermal two-spin XYZ-Heisenberg LQFI, LQU, and log-negativity reductions that the bath temperature causes. The resistance of the DM y-component interaction to higher temperatures is improved by the J_y-component spin–spin antiferromagnetic interaction. In the two-spin XYZ-Heisenberg correlations, several significant quantum phenomena have been observed, including the robustness of the generated nonlocal correlations, the sudden death of the two-spin XYZ-Heisenberg entanglement, the sudden change of the LQFI and LQU, and the $LQFI = LNE$ correlation intervals. The appearance of these phenomena depends on the coupling strengths of the spin–spin XYZ-Heisenberg and the DM couplings as well as on the bath temperature. When compared to ferromagnetic interactions, the x- and y-components antiferromagnetic coupling present a higher capacity to produce a stable maximally correlated two-qubit state. Due to the y-component of the Dzyaloshinskii–Moriya coupling the resulting symmetric spin–spin nonlocal correlation is improved and protected.

CRediT authorship contribution statement

Abdel-Haleem Abdel-Aty: Conceptualization, Formal analysis, Investigation, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **A.-B.A. Mohamed:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **Nuha Al-Harbi:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **Hichem Eleuch:** Conceptualization, Formal analysis, Investigation, Methodology, Software, Validation, Writing – original draft, Writing – review & editing.

Declaration of competing interest

We declare that we have no significant competing financial, professional, or personal interests that might have influenced the work.

Data availability

No data was used for the research described in the article.

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