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Plithogenic Soft Set Models Based on Neutrosophic and Intuitionistic Fuzzy Sets for Multi-Attribute Time Series Forecasting

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Abstract

Forecasting multi-attribute time series (MATS) data remains a challenging task owing to complex data patterns and various sources of uncertainty. To address this, this study proposes two advanced forecasting models, plithogenic-intuitionistic fuzzy soft set (P-IFSS) and plithogenic-neutrosophic soft set (P-NSS), which extend conventional soft set frameworks by integrating the plithogenic concept. This integration enables a more effective representation of the degree of contradiction among the attributes, thereby improving the accuracy and robustness of the forecast. Using Indonesian bond-yield data as a case study, the performance of the proposed models was evaluated against traditional intuitionistic fuzzy soft set (IFSS) and neutrosophic soft set (NSS) models. The experimental results demonstrated that both P-IFSS and P-NSS consistently outperformed their non-plithogenic counterparts across multiple training periods and parameter settings. The findings highlight the potential of plithogenic soft set extensions as an effective framework for handling complex uncertainties in multi-attribute time-series forecasting, particularly in financial data analysis and decision-making applications.

Keywords: Plithogenic soft set, Intuitionistic fuzzy set, Neutrosophic soft set, Time-series forecasting, Bond yield, Uncertainty modeling.

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1. Introduction

Time-series forecasting is commonly used in many fields, particularly finance. In this area, historical data are typically analyzed to aid in decision-making, planning, and risk management. The goal of time-series forecasting is to identify hidden patterns within historical data and use this information to make informed predictions regarding future values. Financial instruments such as bonds (debt securities issued by corporations or governments) are key applications of time-series forecasting. Bond yields are of particular interest because they are closely related to investment returns and market conditions (https://www.rba.gov.au/education/resources/explainers/bonds-and-the-yield-curve.html). The bond yield curve reflects current interest rate conditions and expectations of future movements, which makes it useful for investors to formulate more informed investment strategies [1]. Furthermore, movements in the yield curve can provide signals regarding market confidence and may be influenced by economic and geopolitical factors that affect the overall economic stability [2].

Various forecasting methods have been developed, including statistical approaches such as smoothing, regression, stochastic models (e.g., ARIMA [3]), and machine-learning-based approaches [4]. Several methods have been studied extensively in the context of bond yield forecasting, including the Nelson-Siegel model [5], Bayesian VAR [6], and long short-term memory (LSTM) [7]. Recently, fuzzy time series-based forecasting has been more widely used because of its ability to deal with different data patterns while still providing accurate results [8].

Fuzzy logic, introduced by Zadeh [9] in 1965, and its extensions, play an important role in modeling systems involving uncertainty and ambiguity. This concept was extended to intuitionistic fuzzy sets (IFS) [10], neutrosophic sets (NS) [11–13], and plithogenic theory [14]. These extensions were designed to handle increasingly complex forms of uncertainty. Although research on fuzzy and neutrosophic logic has progressed rapidly, to the best of our knowledge, the application of plithogenic theory remains relatively limited, particularly in the context of multi-attribute time series forecasting.

Existing soft set-based forecasting models, such as the neutrosophic soft set (NSS) model proposed by Guan [15], have been widely used to model multi-attribute fluctuation patterns using fuzzy fluctuation logical relationships. Although these models demonstrate strong practical performance, several limitations remain.

First, in the NSS framework, historical fluctuation patterns that satisfy the same logical relationship are treated as having equal influence. This is the case, even though some past patterns may be more similar to the current pattern to be forecasted than others. Historical patterns are not equally informative in practical forecasting. Patterns closer to the current situation necessarily have a stronger impact on the prediction, whereas less relevant patterns are expected to contribute less. In Guan's original model, similarity information is primarily used at the matching stage and is not further incorporated as a weighting factor in the aggregation process. Consequently, the model may become less robust, particularly when different historical patterns share the same linguistic structure but differ in magnitude or direction.

Second, although NSS and intuitionistic fuzzy soft set (IFSS) are designed for multi-attribute data, they do not explicitly distinguish the degree of agreement or disagreement between historical and current multi-attribute patterns. In these frameworks, matched patterns are treated uniformly once a logical relationship is satisfied. Plithogenic soft sets introduce the concept of contradiction degree, which allows the level of agreement or opposition between patterns to be quantified in a graded manner. This information can be incorporated into the forecasting process to adjust the contribution of each historical pattern so that patterns that are more consistent with the current situation have a greater influence on the prediction.

Based on these observations, this study examined whether incorporating plithogenic-based similarity, expressed through contradiction degrees, into IFSS and NSS forecasting models can improve their predictive performance for multi-attribute time series.

To address this issue, we propose two enhanced forecasting frameworks, namely the plithogenic–intuitionistic fuzzy soft set (P-IFSS) and the plithogenic–neutrosophic soft set (P-NSS) models. The P-NSS model was developed as an extension of Guan's original neutrosophic soft set-based forecasting framework, whereas the P-IFSS model adapted the same forecasting mechanism to an IFSS. In both frameworks, plithogenic concepts are incorporated to introduce contradiction degree-based weighting into the forecasting process.

Through this mechanism, historical patterns that are more consistent with the current situation are allowed to have a stronger influence on the prediction, whereas less-relevant patterns contribute to a smaller extent. Thus, the proposed models extend the original IFSS and NSS formulations by introducing a graded weighting scheme instead of relying on the uniform aggregation of matched historical patterns.

The main contributions of this study are summarized as follows.

- We adapted Guan's neutrosophic soft set-based forecasting framework [15] to the IFSS context and further extended both frameworks by incorporating plithogenic weighting mechanisms.

- We constructed plithogenic soft set logical relationships, referred to as P-IFSSLR and P-NSSLR, as extensions of the classical fuzzy fluctuation logical relationship (FFLR) framework.

- We conducted experiments using Indonesian government bond yield data and analyzed their performance using mean absolute error (MAE), mean absolute percentage error (MAPE), mean squared error (MSE), and root mean squared error (RMSE) metrics.

Overall, the results indicate that incorporating plithogenic characteristics improves the ability of soft set-based forecasting models to distinguish relevant historical patterns, resulting in a more stable predictive behavior across different error criteria.

2. Literature Review

2.1 Related Works

The application of neutrosophic logic to time-series forecasting has been developed in recent studies, particularly for financial applications. An important study involved the neutrosophic soft set (NSS) forecasting model proposed by Guan et al. [15]. The model combined the concepts of neutrosophic logic and soft sets. It was applied for stock price prediction using the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) as the main variable, and amplitude and trading volume as additional factors. In this study, Guan et al. [15] compared the NSS model with several commonly used approaches, including classical fuzzy time-series models [16, 17], machine learning-based methods [18], and rough-set-based approaches [19]. The experimental results showed that the NSS model could capture multiday market fluctuations while maintaining relatively low computational complexity. In [20], the same framework was applied to Indonesian government bond yield forecasting, and good prediction accuracy was achieved.

Many studies have extended the neutrosophic logic to more complex and uncertain environments. Smarandache [21] introduced the development of plithogenic theory and plithogenic-neutrosophic soft sets. This theory has been explored mainly in decision-making and evaluation, where degrees of contradiction are used to provide more flexibility. For example, Ansari and Kant [22] integrated this concept with an analytic hierarchy process model and demonstrated its ability to improve the handling of multi-criteria uncertainty.

Further applications of plithogenic logic have been reported in multi-criteria decision-making (MCDM). Sudha et al. [23] studied its application in complex decision-making problems. The concept of a lithogenic sociogram has been applied to food processing in various industrial scenarios [24]. Abdel-Basset et al. [25] proposed a plithogenic-based approach for evaluating hospital services and demonstrated that it can provide more expressive representations than all fuzzy, intuitionistic fuzzy, or neutrosophic models. This direction was later extended by integrating plithogenic aggregation with objective weighting and ranking mechanisms such as the plithogenic TOPSIS-CRITIC framework [26].

Several recent studies have explored the use of fuzzy, intuitionistic, and neutrosophic soft sets in financial analysis and forecasting. Basheri and Ragab [27] proposed an intuitionistic-neutrosophic soft set model for financial market share prediction that emphasized the presence of multi-attribute uncertainty. Chai et al. [28] proposed an improved similarity measure for neutrosophic sets and demonstrated its effectiveness in capturing structural relationships under uncertainty. More recent contributions include a multimodal forecasting approach proposed by Mou et al. [29]. A fintech-oriented classification framework based on rough sets with neutrosophic intervals was covered by Kuznetsov et al. [30], illustrating a broader trend toward hybrid and uncertainty-aware modeling.

Other ideas have also appeared in fuzzy time-series forecasting for actuarial and financial planning. For example, Raouf and Elsaieed [31] applied several classical fuzzy time-series models to forecast social insurance benefits in Egypt and showed that suitable partitioning strategies can significantly improve the forecasting accuracy. Despite these advances, most studies focused on similarity measures or partitioning designs. They did not incorporate contradiction degree-based or similarity-driven adaptive weighting within a soft set-based forecasting framework. This gap has motivated this study.

Although the NSS forecasting model proposed by Guan et al. [15] demonstrated good predictive performance, several limitations remain. First, historical fluctuation patterns that satisfied the same logical relationship were treated as having equal influence. However, in practice, historical patterns may differ in their relevance to the forecasted current patterns. Therefore, patterns that share the same linguistic structure but differ substantially in magnitude may contribute equally to the prediction, which can reduce the robustness in volatile financial data.

Second, soft set-based forecasting typically involves multiple attributes, such as the opening yield, closing yield, and amplitude, which together form a multi-attribute fluctuation pattern. In the classical NSS and related frameworks, once a logical relationship is satisfied, differences in the degree of agreement between historical and current patterns are not further distinguished. Consequently, structurally different patterns with similar directional labels could be treated uniformly.

Finally, although plithogenic soft sets introduce the concept of a contradiction degree to quantify agreement and opposition in a graded manner, this concept has not yet been explored in time-series forecasting. Most existing applications of plithogenic theory focus on decision-making problems rather than forecasting. This observation encourages the development of P-IFSS and P-NSS multi-attribute forecasting models that use contradiction degree-based weighting to represent the relevance of past patterns.

2.2 Fuzzy Logic and Its Generalization

Uncertainty and vagueness often appear in many real-world situations; thus, modeling approaches that can handle imprecise information are required. The concept of fuzzy logic was first introduced by Zadeh [9] in 1965 to handle the imprecision associated with natural language and human reasoning. In contrast to classical set theory, in fuzzy logic, an element can have a partial membership value (ranging from 0 to 1) instead of only a binary (0 or 1) membership.

A fuzzy set A in a universe of discourse X is defined as follows:

$$A = \{(x, \mu_A(x)) \mid x \in X\},$$

where $\mu_A : X \to [0,1]$ denotes the membership function of the fuzzy set A. This value $\mu_A(x)$ represents the degree to which an element x belongs to the fuzzy set A.

Fuzzy logic has been widely applied in various domains such as control systems [32], decision-making and support systems [33, 34], and pattern recognition [35]. In general, the fuzzy-logic-based solution approach is performed by forming fuzzy rules, fuzzy aggregation, or hybridizing them with other models. This is because it facilitates in dealing with uncertainty mathematically. Unfortunately, classical fuzzy sets have limitations when dealing with more complex forms of uncertainty, which has led to the development of generalized fuzzy theories.

One such generalization is the IFS introduced by Atanassov [10] in 1986. An IFS A on a universe of discourse X is defined by a membership function μ_A, nonmembership function ν_A, and hesitation degree π_A that satisfies

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1, \quad \text{for all } x \in X,$$

where $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ and $\pi_A = 1 - \mu_A - \nu_A$.

Further generalizations include the neutrosophic set introduced by Smarandache [11] in 1998, which introduced three membership degrees: truth ($T_A(x)$), indeterminacy ($I_A(x)$), and falsity ($F_A(x)$) for each element x.

$$A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}.$$

These generalizations have proven effective in addressing a wider range of uncertainties by allowing for more flexible and independent representations of membership, nonmembership, and indeterminacy.

2.3 Plithogenic Soft Set

Traditional fuzzy and neutrosophic sets have limitations in capturing complex real-world situations, where attribute values may not only be uncertain, but may also partially agree or oppose each other. Existing models do not explicitly represent such graded disagreements, which has motivated the development of plithogenic soft sets. The plithogenic sets, introduced by Smarandache [21] in 2017, generalizes the fuzzy, intuitionistic fuzzy, and neutrosophic sets by introducing a degree of contradiction that quantifies the level of agreement or opposition of attribute values with respect to a dominant value.

2.3.1 Plithogenic set

A plithogenic set P is characterized by a universe of discourse X, a set of attributes $\{a_1, a_2, \ldots, a_n\}$, and a contradiction degree $c \in [0,1]$ associated with the values of each attribute a_i. It can be represented as

$$P = \{(x, \mu_{a_1}(x), \mu_{a_2}(x), \ldots, \mu_{a_n}(x), c) \mid x \in X\},$$

where $\mu_{a_i} : X \to [0,1]$ denotes the membership degree of element x with respect to attribute a_i, and c represents the degree of contradiction of an attribute value relative to a dominant value.

2.3.2 Plithogenic soft set

The plithogenic soft set (P-SS) extends the plithogenic set by incorporating the soft set theory introduced by Molodtsov in 1999. A plithogenic soft set is defined over a universe of discourse X, where each attribute a_i is associated with a set of possible values V_i, and a soft set F maps each attribute to a subset of X. A plithogenic soft set (F, P) can be represented as

$$(F, P) = \{(x, \mu_{a_1}(x), \mu_{a_2}(x), \ldots, \mu_{a_n}(x), c) \mid x \in X\},$$

where $\mu_{a_i} : X \to [0,1]$ denotes the membership degree of element x with respect to attribute a_i, $i = 1, ..., n$, and $c \in [0,1]$ represents the degree of contradiction associated with attribute values relative to a dominant value. This structure is suitable for modeling situations in which attribute evaluations are vague or uncertain and may exhibit graded agreement or opposition.

2.3.3 Plithogenic-intuitionistic soft set

The P-IFSS extends the plithogenic soft set by integrating the concept of IFS, which account for the degrees of membership (μ) and nonmembership (ν), as well as the hesitation degree (π).

$$(F, P) = \{(x, (\mu_{a_1}(x), \nu_{a_1}(x), \pi_{a_1}(x)), \ldots, (\mu_{a_n}(x), \nu_{a_n}(x), \pi_{a_n}(x)), c) \mid x \in X\},$$

where $\pi_{a_i}(x)$ denotes the hesitation degree, defined as

$$\pi_{a_i}(x) = 1 - \mu_{a_i}(x) - \nu_{a_i}(x).$$

The P-IFSS captures uncertainty and hesitation in decision-making while incorporating the graded disagreement of attribute values through the contradiction degree.

2.3.4 Plithogenic-neutrosophic soft set

By extending the P-SS framework, the P-NSS incorporates neutrosophic logic, which represents information using three independent membership degrees: truth (T), indeterminacy (I), and falsity (F). The P-NSS is defined as

$$(F, P) = \{(x, (T_{a_1}(x), I_{a_1}(x), F_{a_1}(x)), \ldots, (T_{a_n}(x), I_{a_n}(x), F_{a_n}(x)), c) \mid x \in X\}.$$

In this context, $T_{a_i}(x)$, $I_{a_i}(x)$, and $F_{a_i}(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively, for attribute a_i. The degree of contradiction c quantifies the level of agreement or opposition of the attribute values relative to a dominant value.

The P-NSS provides a flexible framework for modeling highly uncertain and partially conflicting information and is suitable for applications involving complex and ambiguous data.

3. Prediction Model for Time-Series Data Using P-IFSS and P-NSS

In this section, we discuss the forecasting procedure of Guan et al. [15] by incorporating a plithogenic framework through contradiction degree-based similarity weighting. The complete forecasting workflow is shown in Figure 1.

We first analyzed the plithogenic structure of Indonesian bond yield data and subsequently constructed two forecasting models: the P-IFSS and P-NSS models. Both models extend the neutrosophic soft set forecasting model introduced by Guan et al. [15] and Zhao et al. [36] but differ by embedding a similarity weighting mechanism derived from distance measures (Euclidean and Hamming).

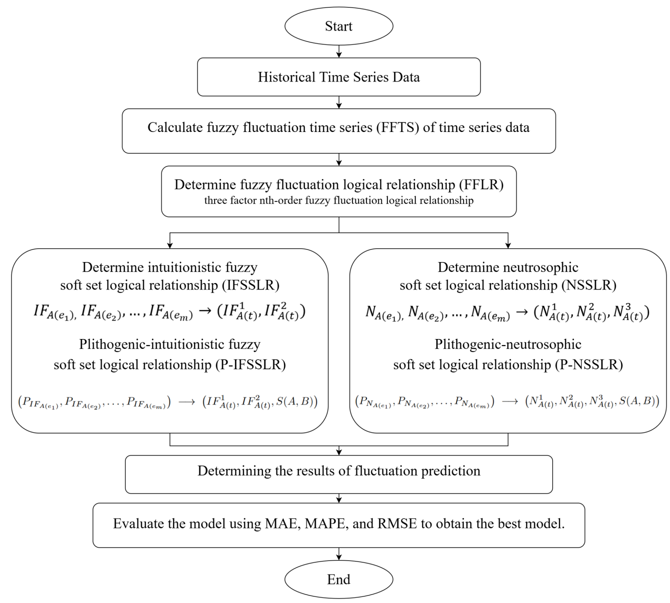

Figure 1. Time-series forecasting stages of the proposed methods.

The overall modeling steps are summarized as follows.

Step 1 (Calculate fuzzy fluctuation time series [FFTS])**.** We let $F_\varphi(t)$, $t = 1, 2, \ldots, T$ be a univariate time series for attribute φ (e.g., closing yield). The fluctuation values $G(t)$ were computed as the difference between the current $F(t)$ and the previous value $F(t-1)$.

$$G_\varphi(t) = F_\varphi(t) - F_\varphi(t-1), \quad t = 2, 3, \ldots, T. \quad (1)$$

Average absolute fluctuation

$$l_\varphi = \frac{1}{T-1} \sum_{t=2}^{T} |G_\varphi(t)|$$

was used to define the following three fuzzy linguistic states:

$$u_{\varphi 1} = (-\infty, -l_\varphi), \;\; u_{\varphi 2} = [-l_\varphi, l_\varphi], \;\; u_{\varphi 3} = (l_\varphi, +\infty).$$

Each $G_\varphi(t)$ was then mapped to a fuzzy state $S_\varphi(t) \in \{D, S, U\}$, corresponding to the *downward*, *steady*, and *upward* movements, respectively.

Step 2 (Constructing FFLR)**.** For a given order n, the FFLRs were constructed by grouping n consecutive fuzzy states as the left-hand side (LHS) and the next state as the right-hand side

(RHS). For a single attribute φ

$$\big(S_\varphi(t-n+1),\ldots,S_\varphi(t)\big)\to S_\varphi(t+1).$$

In a multi-attribute case (e.g., closing yield, opening yield, and amplitude), the LHS is a vector of fuzzy states, and each FFLR aggregates the joint fluctuation pattern over n days. These FFLRs form the basis for the subsequent IFSS and NSS representations.

Step 3 (Baseline IFSS and NSS probabilities)**.** For each distinct RHS pattern $A(t)$ in the training set, we observed all occurrences of the corresponding LHS patterns and recorded the empirical frequencies of the downward, steady, and upward movements on the RHS. Let

$$n_D(A),\ n_S(A),\ n_U(A)$$

denote the numbers of downward, steady, and upward outcomes, respectively, and

$$n(A)=n_D(A)+n_S(A)+n_U(A).$$

The empirical probabilities are

$$p_D(A)=\frac{n_D(A)}{n(A)},\ \ p_S(A)=\frac{n_S(A)}{n(A)},\ \ p_U(A)=\frac{n_U(A)}{n(A)}. \tag{2}$$

These probabilities were used to construct the intuitionistic and neutrosophic representations.

Intuitionistic fuzzy soft set (IFSS). For the IFSS model, we defined membership (upward tendency) and non-membership (downward tendency) degrees as

$$\mu_A=p_U(A)+\tfrac{1}{2}p_S(A),\ \ \nu_A=p_D(A)+\tfrac{1}{2}p_S(A), \tag{3}$$

where the hesitation degree is defined as

$$\pi_A=1-\mu_A-\nu_A. \tag{4}$$

Here, μ_A and ν_A describe tren of upward and downward, respectively, whereas π_A captures the remaining uncertainty.

Neutrosophic soft set (NSS). In the NSS model, each RHS pattern $A(t)$ is represented by a neutrosophic triple

$$N_A(t)=(T_A,I_A,F_A)=\big(p_U(A),\,p_S(A),\,p_D(A)\big), \tag{5}$$

where T_A, I_A, and F_A correspond to the degrees of truth (upward), indeterminacy (steady), and falsity (downward), respectively.

Step 4 (Search for a similar set in the training data)**.** The next step was to search for a similar set in the training data based on the similarity measurements. To incorporate the plithogenic aspect, we introduced a degree of similarity (or, equivalently, a contradiction) between the two fluctuation patterns A and B. In these experiments, we used the Euclidean distance to determine the similarity degree.

Euclidean distance. For intuitionistic value (μ,ν,π), the Euclidean distance is defined as

$$d_{\mathrm{IF}}(A,B)=\sqrt{\frac{(\mu_A-\mu_B)^2+(\nu_A-\nu_B)^2+(\pi_A-\pi_B)^2}{3}}. \tag{6}$$

For neutrosophic triples (T,I,F), we followed [15] and defined

$$d_{\mathrm{NS}}(A,B)=\sqrt{\frac{(T_A-T_B)^2+(I_A-I_B)^2+(F_A-F_B)^2}{3}}. \tag{7}$$

Similarity degree. In both cases, the distance was transformed into a similarity (plithogenic weighting) degree as follows:

$$S(A,B)=\frac{1}{1+d(A,B)}, \tag{8}$$

where $d(A,B)$ is given by either Eqs. (6), (7), or a Hamming-based variant. A higher similarity indicates a smaller contradiction, and therefore, a larger weight in the forecasting stage.

Step 5 (P-IFSSLR and P-NSSLR construction)**.** For each historical pattern $A(e_1),\ldots,A(e_m)$ in the training set, the plithogenic-intuitionistic fuzzy soft set logical relationship (P-IFSSLR) is expressed as

$$\begin{aligned}&\big(P_{IF_{A(e_1)}},P_{IF_{A(e_2)}},\ldots,P_{IF_{A(e_m)}}\big)\\ &\quad\longrightarrow\ \big(IF^1_{A(t)},IF^2_{A(t)},S(A,B)\big),\end{aligned} \tag{9}$$

where $IF^1_{A(t)}=\mu_A$ and $IF^2_{A(t)}=\nu_A$ were obtained from Eq. (3).

Similarly, the plithogenic-neutrosophic soft set logical relationship (P-NSSLR) is expressed as

$$\begin{aligned}&\big(P_{N_{A(e_1)}},P_{N_{A(e_2)}},\ldots,P_{N_{A(e_m)}}\big)\ \longrightarrow\\ &\qquad\big(N^1_{A(t)},N^2_{A(t)},N^3_{A(t)},S(A,B)\big),\end{aligned} \tag{10}$$

where $N^1_{A(t)}=T_A$, $N^2_{A(t)}=I_A$, and $N^3_{A(t)}=F_A$ are defined in Eq. (5).

Step 6 (Forecast computation)**.** For each test pattern B in the forecasting horizon, we searched the training set for the most similar historical pattern(s) A_k according to the similarity degree (Eq. (8)). In this study, we used the nearest pattern ($k = 1$), although the framework can be extended to K-nearest neighbors.

P-IFSS forecast. For the P-IFSS model, the expected fluctuation associated with pattern A was computed as

$$E_{\text{IFSS}}(A) = \mu_A - \nu_A, \tag{11}$$

to describe the balance between the upward and downward tendencies. The predicted fluctuation for test pattern B was

$$G'_{\text{IFSS}}(t+1) = S(A,B) \cdot E_{\text{IFSS}}(A) \cdot l_{\varphi}, \tag{12}$$

and the forecasted value was

$$F'_{\text{IFSS}}(t+1) = F_{\varphi}(t) + G'_{\text{IFSS}}(t+1). \tag{13}$$

P-NSS forecast. For the P-NSS model, the expected fluctuation was obtained from the neutrosophic triple as follows:

$$E_{\text{NSS}}(A) = T_A - F_A, \tag{14}$$

and the predicted fluctuation becomes

$$G'_{\text{NSS}}(t+1) = S(A,B) \cdot E_{\text{NSS}}(A) \cdot l_{\varphi}, \tag{15}$$

with

$$F'_{\text{NSS}}(t+1) = F_{\varphi}(t) + G'_{\text{NSS}}(t+1). \tag{16}$$

The original NSS models of Guan et al. [15] were recovered as special cases of (12) and (15) when $S(A,B) = 1$, that is, when no similarity-based weighting was applied.

Step 7 (Model evaluation)**.** The final stage was the evaluation of the model performance using three metrics: MAE, MAPE, and RMSE. Using actual values y_i and forecasts $\hat{y}_i$, $i = 1, \ldots, n$, we computed

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|, \quad \text{MAPE} = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|,$$
$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}.$$

These metrics were used to compare the performance of IFSS, NSS, P-IFSS, and P-NSS models. The following section discusses the computational time required for the proposed method.

Computational complexity analysis

T denotes the length of the time-series data, m is the number of attributes, and n is the order of the fuzzy fluctuation logical relationship. The fuzzy fluctuation states and fuzzy fluctuation logical relationships were constructed by scanning the data once for each attribute. Therefore, this step has time complexity $\mathcal{O}(T \cdot m)$.

In the plithogenic extension, additional computational cost arises from incorporating the similarity (or contradiction degree) values as weighting factors in the forecasting stage rather than only for pattern matching, as in the baseline IFSS and NSS models. Because each similarity computation involves a fixed number of arithmetic operations on intuitionistic or neutrosophic components, this step has a complexity $\mathcal{O}(m)$ per comparison. Consequently, the overall complexity of the similarity-based weighting step is $\mathcal{O}(T \cdot m)$ in the worst-case scenario. Because the m value is relatively small compared to T, we can conclude that the computational time is linear or $\mathcal{O}(T)$, which is efficient. This is consistent with the baseline IFSS and NSS frameworks. The introduction of plithogenic weighting only adds a constant computational overhead and does not change the scalability of the original soft set-based forecasting process.

4. Results and Discussion

In this section, we discuss the results of applying four soft set-based forecasting models, namely, IFSS, P-IFSS, NSS, and P-NSS, to multi-attribute time-series data. The main contribution of this study is the introduction of a contradiction degree-based weighting mechanism into intuitionistic and neutrosophic fuzzy soft set frameworks. Based on the experimental results, the plithogenic variants (P-IFSS and P-NSS) exhibited an improvement over their baseline models, particularly with respect to accuracy and stability, confirming the benefit of introducing similarity weighting into the construction of logical relationships.

Figure 2 shows the overall variation in MAPE across different order values and training window lengths. Instead of focusing on individual subplots, we observed a clear global pattern: across almost all experimental settings, the plithogenic models yielded lower MAPE values than their non-plithogenic counterparts. This improvement became more apparent as the order value increased when a larger number of historical patterns were involved in the forecasting process. In such cases, contradiction degree weighting played an important role in dis-

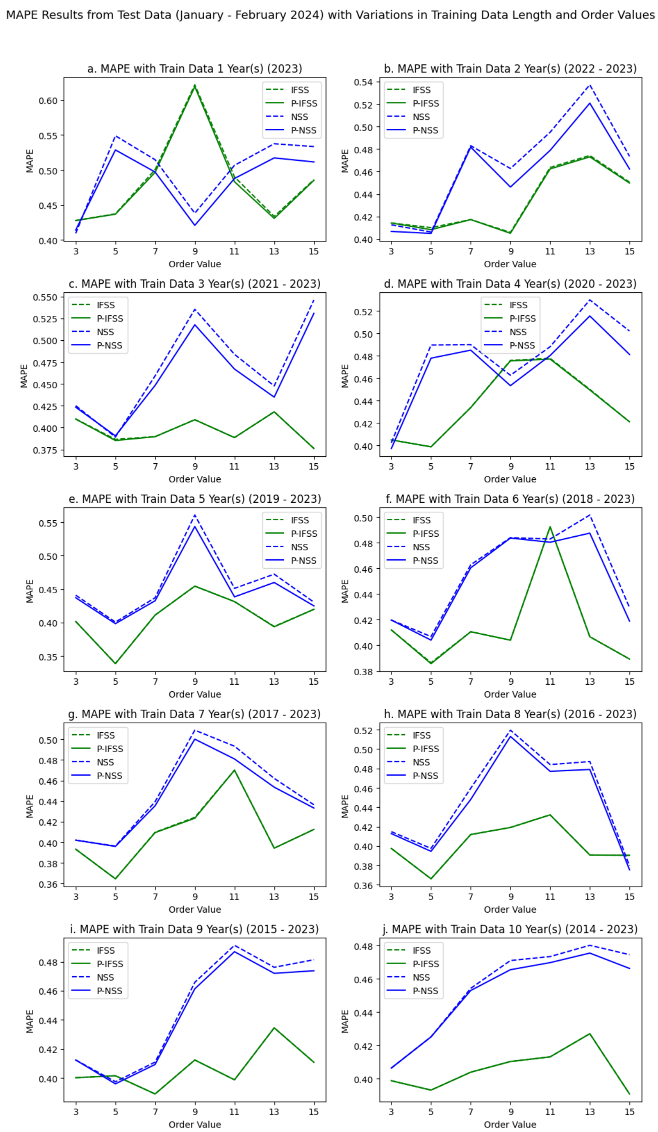

Figure 2. MAPE trends over different order values for each training-window length (1-10 years).

tinguishing genuinely relevant patterns from those that were linguistically similar.

From a broader perspective, the plithogenic mechanism enabled both P-IFSS and P-NSS to avoid the equal-weighting bias inherent in IFSS and NSS. By assigning a higher influence to patterns that were closer to the current situation, the proposed models produced smoother and more stable error trends across different parameter configurations. This reduced sensitivity to the choice of the order value is particularly desirable for practical forecasting applications.

The violin plot in Figure 3 shows a global view of the model behavior for all the experimental scenarios. Overall, both plithogenic models exhibited narrower error distributions and

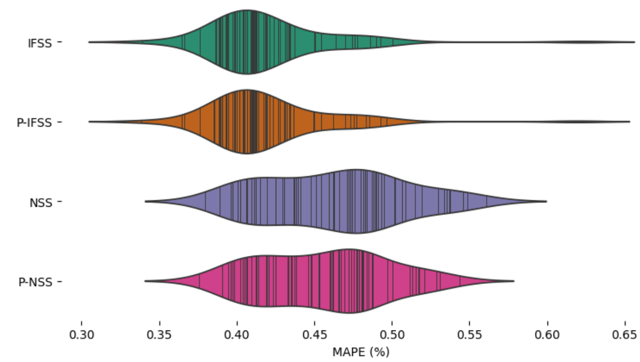

Figure 3. Distribution of MAPE values across all models.

lower median MAPE values than their base models. P-IFSS exhibited a dense concentration of low-error outcomes. This indicates the consistency of performance under parameter variation. Furthermore, the P-NSS was more effective in reducing the occurrence of extremely high-error cases.

This indicates an important distinction between the two plithogenic variants. Although P-IFSS improved the average performance, P-NSS exhibited stronger control on extreme deviations. This behavior aligns well with the neutrosophic representation, where truth, indeterminacy, and falsity are explicitly modeled and further refined by the contradiction-degree weighting mechanism.

The stabilities of the plithogenic models are further illustrated by the heatmaps shown in Figure 4. The plithogenic variants displayed wider and more continuous regions with low errors. This indicates that the P-IFSS and P-NSS are less sensitive to hyperparameter choices and exhibit a more reliable performance when the optimal parameters are not known in advance.

The proposed models are less sensitive to changes in parameters such as the order value and training window length because the plithogenic weighting assigns a smaller influence to less similar historical patterns. As a result, changes in the parameter settings do not cause large fluctuations in forecasting errors. This behavior was consistent with the smoother error trends and more compact error distributions observed in the experimental results.

From a risk-oriented perspective, these observations can be interpreted in terms of the error intervals. Each model induces a range of forecasting errors bounded by its minimum and maximum values. If the goal is to adopt a conservative (i.e., most cautious) forecasting strategy, the preferred model is the one with the smallest upper bound because it provides the tightest limit on the worst-case forecasting error.

Table 1 summarizes the minimum, maximum, and mean

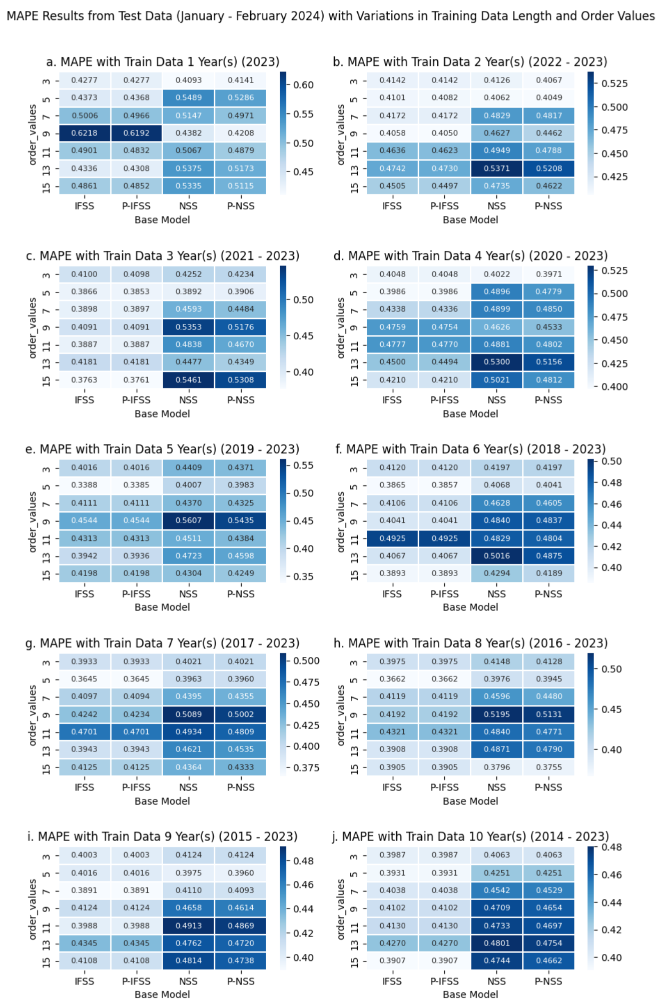

Figure 4. MAPE heatmaps for all experimental settings over different order values for each training-window length (1-10 years).

error values obtained across all experimental configurations. P-IFSS achieved the smallest minimum and average errors. This indicates a higher performance in terms of overall accuracy. In contrast, the P-NSS model consistently produced the smallest maximum errors across all metrics and the tightest upper bounds among the other models.

Furthermore, based on the minimum, average, and maximum error data presented in Table 1, we can conclude that the P-NSS is more suitable for conservative forecasting scenarios, particularly in cases where limiting the worst-case error is more important than optimizing the average performance. The combination of the power of neutrosophic representation with plithogenic contradiction degree weighting appears to enable P-NSS to suppress inconsistent historical patterns, resulting in improved robustness against extreme forecasting errors.

Overall, both the visual and numerical analyses support the conclusion that incorporating plithogenic characteristics enhances the forecasting performance of the IFSS and NSS models. In addition to increased accuracy, the proposed framework enables more informed model selection under different risk preferences, ranging from performance-oriented to risk-averse decision making.

Although the empirical evaluation performed in this study focused on government bond-yield data, the proposed framework is not restricted to financial time-series data. Because this methodology relies on multi-attribute fluctuation patterns and similarity weighting, it can be adapted to other domains. This proposed method can be applied to various other types of time-series data as long as the data can be represented numerically so that it can be transformed into FFLR form.

Table 1. Minimum, maximum, and average error values of all testing results grouped by model

Base model		MAE	MAPE (%)	MSE	RMSE
IFSS	<u>min</u>	<u>0.022411</u>	<u>0.338835</u>	0.001162	0.034094
	max	0.041157	0.621825	0.002641	0.051394
	<u>mean</u>	<u>0.027783</u>	<u>0.419812</u>	<u>0.001501</u>	<u>0.038659</u>
NSS	min	0.025095	0.379612	<u>0.001132</u>	<u>0.033652</u>
	<u>max</u>	<u>0.037078</u>	<u>0.560678</u>	<u>0.002394</u>	<u>0.048926</u>
	mean	0.030617	0.462728	0.001747	0.041650
P-IFSS	**min**	**0.022388**	**0.338479**	0.001162	0.034086
	max	0.040980	0.619173	0.002624	0.051223
	mean	**0.027754**	**0.419379**	**0.001499**	**0.038634**
P-NSS	min	0.024823	0.375507	**0.001111**	**0.033338**
	max	**0.035942**	**0.543505**	**0.002262**	**0.047563**
	mean	0.030100	0.454930	0.001696	0.041066

The best performance for each metric is highlighted in bold, while the second-best result is indicated by underlining.

5. Conclusion and Future Works

In this study, we proposed two forecasting models, namely the P-IFSS and P-NSS, as extensions of the existing IFSS and NSS approaches. The main contribution of the proposed framework is the integration of a contradiction degree-based weighting mechanism based on a plithogenic concept. This allows the model to distinguish between linguistically similar but structurally different historical patterns. This framework provides a more adaptive and realistic representation of similarity in multi-attribute time series.

The overall forecasting workflow of the proposed framework can be summarized as follows. The time series data are first transformed into fuzzy representations, followed by the construction of fluctuation patterns and logical relationships. Plithogenic weighting is then applied to account for similarities between historical and current patterns. The final forecast values are obtained based on the resulting weighted relationships.

Based on the experimental results of the Indonesian government bond-yield data, the plithogenic variants consistently outperformed their baseline model across different training window lengths and order values. These improvements were not only in terms of reducing errors such as MAE, MAPE, and RMSE, but also in the overall stability of the forecasting performance. In particular, P-IFSS exhibited the best average accuracy, particularly in shorter training windows, whereas P-NSS exhibited increasing robustness as the amount of historical data increased.

There are several avenues for future research. We will explore alternative forms of plithogenic weighting, including nonlinear similarity functions or adaptive contradiction measures that evolve with the data. Furthermore, the proposed forecasting framework can be extended by incorporating other soft-computing paradigms or fuzzy extensions. The proposed method can also be applied to other types of time-series data.

Overall, the proposed P-IFSS and P-NSS models demonstrated that plithogenic soft set extensions offer a flexible and effective framework for multi-attribute time-series forecasting. This model can accommodate the possibility of selecting a model based on different risk preferences. The proposed approach extends the application of soft set-based forecasting methods to real-world decision-making environments.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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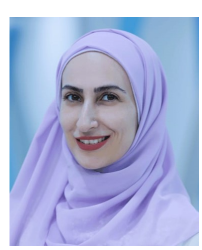

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