Results in Physics 31 (2021) 104975

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Local two-atom correlations induced by a two-mode cavity under nonlinear media: Quantum uncertainty and quantum Fisher information

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ARTICLE INFO

Keywords:
Local Fisher information
Kerr like medium
Stark shift
Two atoms

ABSTRACT

The nonlinearity effect on the various formed quantum correlations between two atoms interacting locally with a bimodal cavity is explored in this work. The local quantum Fisher information, local quantum uncertainty, and negativity are used to investigate the quantum correlation's sensitivity to the cavity modes characteristics. It is demonstrated that when the ratio of the Stark shift of the two modes increases, the atomic quantum correlations rise. During the time intervals of sudden death negativity entanglement, the three two-atom correlation quantifiers reveal that the two-atom system can contain local Fisher information and local uncertainty correlations. The Stark shift may be utilized to amplify the produced atomic correlation, while the Kerr-like medium can be used to suppress it. The regularity, the amount of generated atomic correlations, and their immunity to the Stark shift as well as the Kerr-like medium non-linearities are improved when the mean photon number is increased. As a result, the cavity modes' nonlinear parameters might be utilized as a controller to optimize atomic quantum correlations.

Introduction

Quantifying the quantum correlations (QC) of two-qubit states is substantial for quantum technologies [1]. The entanglement is the most know type of quantum correlations which used to be measured by the concurrence. Recently, other correlations quantifiers were introduced to measure the QCs [2–4]. Two-qubit measurement-induced disturbance [5], two-qubit discord [6] and geometrical two-qubit correlation measures which based on p-norm [7] and skew information [8,9]. The two-qubit QCs are deployed in various quantum information and quantum engineering applications [10–14].

The skew information quantity was utilized to establish the local quantum uncertainty (LQU), which may be regarded one of the discord-like measure families and disappears for all zero discord states [8,9]. The LQU is a quantum correlations quantifier that determines the local observable's lowest measurement uncertainty and meets all of the relevant conditions. LQU is a key concept in quantum metrology protocols because it is related to quantum Fisher information (QFI). Skew information (SI) has been demonstrated to be a specific case of quantum Fisher information [15].

QFI is useful quantifier to investigate multi-qubit entanglement [16], quantum speedup limit time [17], uncertainty relations [18], and quantum phase transition [19]. The multi-qubit entanglement of the QFI is convenient for quantum metrology [20], quantum topological states [21,22] and quantum phase-transition points [23]. In estimation protocols, QFI is deployed to get the best accuracy [24–26]. The QFI can implement quantum correlation and quantum metrology [27]. Local quantum Fisher information (LQFI) was introduced to quantify another type of two-qubit correlations [28,29]. The minimum quantum Fisher information is used to define it.

Several quantum phenomena, such as quantum correlations, are produced by interactions between an atomic system (qubit) and a cavity field, and are crucially important as quantum information resources [30–33]. The superconducting circuits, ion traps, and quantum dots are deployed to realize these interactions. Several generalizations for these interactions have been proposed, including two photons and

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https://doi.org/10.1016/j.rinp.2021.104975
Received 1 October 2021; Received in revised form 29 October 2021; Accepted 1 November 2021
Available online 12 November 2021

media nonlinearity terms of Kerr like medium with Stark shift [34]. The media-type nonlinearity (the Kerr-like and the Stark-shift terms) effects have considerable effect on qubit–cavity interactions [35]. When several photons are present, the Stark shift effect becomes more apparent and powerful, causing significant changes in the qubit–cavity dynamics [36]. The research works of [37,38] explored the generation of nonlinear Stark shift of k-photon in single-mode and non-degenerate Raman transitions. Recently, the influence of the nonlinearity of the Stark-shift and Kerr-Like Medium on quantum coherence and non-classical correlation dynamics were investigated for multi-level atomic systems interacting with a cavity field [39–41].

The entangled pair coherent states (PCS) are very useful for primary tasks in quantum information theory [42–44], such as teleportation, Bell nonlocality, and non-classical features of sub-Poissonian distribution and squeezing [45,46]. PCSs can be realized by different nonlinear processes in a two-photon medium [45].

As a result, we are interested in exploring the quantum correlations that can emerge between atomic subsystems. Quantifying the quantity of quantum correlations is proposed here using three distinct measures: local Fisher information, local uncertainty, and negativity. We study the sensitivity and robustness of these quantum correlations to the cavity's parameters as well as the cavity's initial state settings, and we analyze the influence of the cavity's parameters on the behavior of these correlations.

The manuscript is organized as following: Section "The proposed model and an analytical solution" describes the suggested model and the obtained analytical solution. The quantum correlation quantifiers; negativity, local Fisher information, and local uncertainty are discussed in Section "Quantum correlation quantifiers". Section "Atomic correlation dynamics' investigates the influence of the proposed model parameters on quantum correlation. In "Conclusions", the results are discussed.

The proposed model and an analytical solution

The proposed model consists of two atoms (each atom represents a qubit with upper and lower states) interacting locally with two cavity modes in the presence of the medium nonlinearity of the Kerr-like and the Stark-shift terms [47]. Therefore, the qubit–cavity system Hamiltonian is given by

$$\begin{aligned}\hat{H} &= \omega_1 \hat{a}_1^\dagger \hat{a}_1 + \omega_2 \hat{a}_2^\dagger \hat{a}_2 + \chi_1 \hat{a}_1^{\dagger 2} \hat{a}_1^2 + \chi_2 \hat{a}_2^{\dagger 2} \hat{a}_2^2 + \frac{1}{2}\Omega_A \hat{\sigma}_z^A \\ &+ \frac{1}{2}\Omega_B \hat{\sigma}_z^B + \sum_{k=1,2}\{\frac{1}{2}(\xi_1 \hat{a}_1^\dagger \hat{a}_1 \hat{\sigma}_+^k \hat{\sigma}_-^k + \xi_2 \hat{a}_2^\dagger \hat{a}_2 \hat{\sigma}_-^k \hat{\sigma}_+^k) \\ &+ \lambda(\hat{a}_1 \hat{a}_2 \hat{\sigma}_+^k + \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{\sigma}_-^k)\}, \end{aligned} \tag{1}$$

$\hat{\sigma}_+^k(\hat{\sigma}_-^k)$ and $\hat{\sigma}_z^{(k)}$ $(k = 1,2)$ represent the k-qubit Pauli operators, which follow the relations $[\hat{\sigma}_z^k, \hat{\sigma}_\pm^k] = \pm 2\hat{\sigma}_\pm^k$ and $[\hat{\sigma}_+^i, \hat{\sigma}_-^k] = \hat{\sigma}_z^k \delta_{ik}$. While $\hat{a}_i^\dagger$ and $\hat{a}_i (i = 1,2)$ represent the two-mode cavity field operators, ω_i and Ω_i design the two-mode cavity and the qubits frequencies, respectively. $\lambda = \sqrt{\xi_1 \xi_2}$ is the cavity–qubit coupling. ξ_i and χ_i are the nonlinearity coefficient of the Kerr-like and the Stark-shift types, respectively.

By using the Heisenberg relationships and the Hamiltonian (1), the motion equations of the two-atom system $\hat{\sigma}_z^k$ and the cavity system $\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k$ can be written as:

$$\begin{aligned}\frac{d\hat{n}_k}{dt} &= -i\lambda \sum_{k=1,2} (\hat{a}_1 \hat{a}_2 \hat{\sigma}_+^k - \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{\sigma}_-^k), j = 1,2 \\ \frac{d\hat{\sigma}_z^k}{dt} &= 2i\lambda(\hat{a}_1 \hat{a}_2 \hat{\sigma}_+^k - \hat{a}_1^\dagger \hat{a}_2^\dagger\ \hat{\sigma}_-^k),\ k = 1,2. \end{aligned} \tag{2}$$

The constants of motion are given by;

$$\hat{C}_k = \hat{n}_k + \frac{1}{2}(\hat{\sigma}_z^1 + \hat{\sigma}_z^2), k = 1,2 \tag{3}$$

Therefore, the cavity–qubit system Hamiltonian (1) becomes,

$$\hat{H} = \omega_1 \hat{C}_1 + \omega_2 \hat{C}_2 + \hat{D}, \tag{4}$$

with $\hat{D}$ is given by

$$\hat{D} = \sum_{k=1}^{2} \Delta_k(\hat{n}_1, \hat{n}_2)\hat{\sigma}_z^k + \lambda \sum_{k=1}^{2} \left(\hat{a}_1 \hat{a}_2 \hat{\sigma}_+^k + \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{\sigma}_-^k \right), \tag{5}$$

where the cavity–qubit detunings, Δ_k, takes the form

$$\Delta_k(\hat{n}_1, \hat{n}_2) = \lambda(\hat{n}_1 + \hat{I})\beta - \frac{\lambda(\hat{n}_2 - \hat{I})}{2\beta} + 2\chi_k(\hat{I} - 2\hat{n}_k) \tag{6}$$

where $\beta = \sqrt{\frac{\xi_1}{\xi_2}}$. Here, we assume the following initial cavity–qubit conditions:

$$|\psi(0)\rangle = |e_A, e_B\rangle \otimes |p\rangle, \tag{7}$$

where $|e_A, e_B\rangle$ denotes the upper two-qubit states. $|p\rangle$ is the initial two-mode cavity state that represents a pair of entangled coherent states [45], which is given by,

$$\begin{aligned}|p\rangle &= \sum_{l=0}^{\infty} \chi_n |l+q, l\rangle,\ \chi_l = \frac{p^l}{A\sqrt{l!(l+q)!}}, \\ A^2 &= \sum_{l=0}^{\infty} \frac{|p|^{2l}}{l!(l+q)!}. \end{aligned} \tag{8}$$

where p is the eigenvalue of the operator $\hat{a}_1 \hat{a}_2$ and q is the eigenvalue of the difference between the two operators $\hat{a}_1^\dagger \hat{a}_1$ and $\hat{a}_2^\dagger \hat{a}_2$.

For the case where $(\Delta_1 + \Delta_2 = 0)$ and for the above mentioned initial condition (Eq. (7)), the general solution for the cavity–qubit state $|\psi(t)\rangle$ is

$$\begin{aligned}|\psi(t)\rangle = &\sum_{l=0}^{\infty} a_l^1 |l+q, l, \varpi_1\rangle + a_l^2 |l+q+1, l+1, \varpi_2\rangle \\ &+ a_l^3 |l+q+1, l+1, \varpi_3\rangle + a_l^4 |l+q+2, l+2, \varpi_4\rangle, \end{aligned} \tag{9}$$

where $\{|\varpi_1\rangle = |e_A, e_B\rangle, |\varpi_2\rangle = |e_A, g_B\rangle, |\varpi_3\rangle = |g_A, e_B\rangle, |\varpi_4\rangle = |g_A, g_B\rangle\}$ is the basis of the two atoms A and B. The system of differential equations of the coefficients a_l^j are given by

$$\begin{bmatrix} a_l'^1 \\ a_l'^2 \\ a_l'^3 \\ a_l'^4 \end{bmatrix} = \begin{bmatrix} 0 & -i\alpha_1 & -i\ \alpha_1 & 0 \\ -i\alpha_1 & -i\delta & 0 & -i\alpha_2 \\ -i\alpha_1 & 0 & i\delta & -i\alpha_2 \\ 0 & -i\alpha_2 & -i\alpha_2 & 0 \end{bmatrix} \begin{bmatrix} a_l^1 \\ a_l^2 \\ a_l^3 \\ a_l^4 \end{bmatrix} \tag{10}$$

where $(' = \frac{d}{dt})$

$$\begin{aligned}\alpha_j &= \lambda\sqrt{(l+q+j)(l+j)},\ j = 1,2 \\ \delta &= \Delta_1(l+q+1, l+1) + \Delta_1(l+q, l). \end{aligned}$$

The coefficients $a_j(l)$ $j = 1-4$ can be given by,

$$\begin{aligned}a_l^1 &= \frac{\chi_l}{\mu_1^2}[\mu_1^2 - 2\alpha_1^2(1 - \cos\mu_1 t)], \\ a_l^2 &= -\chi_l \left[\frac{\delta\alpha_1(1 - \cos\mu_1 t)}{\mu_1^2} + i\frac{\alpha_1 \sin\mu_1 t}{\mu_1} \right], \\ a_l^3 &= -a_l^{2*}, \\ a_l^4 &= \frac{2\chi_l \alpha_1 \alpha_2}{\mu_1^2}(\cos\mu_1 t - 1), \end{aligned} \tag{11}$$

where $\mu_j = \sqrt{\delta^2 + (3-j)\mu_3^2}$ and $\mu_3 = \sqrt{\alpha_1^2 + \alpha_2^2}$. The analytical expression of the final qubit–cavity state $|\psi(t)\rangle$ is utilized to explore the local two-qubit correlations. If we consider $\hat{\Lambda}_{ij} = |\varpi_i\rangle\langle\varpi_j|$, then the dynamics of the two-qubit reduced density matrix $\rho^{AB}(t)$ is given by

$$\begin{aligned}\rho^{AB}(t) &= \mathrm{Trace}_{fields}\{|\psi(t)\rangle\langle\psi(t)|\} \\ &= \sum_{n=0}^{\infty} |a_{n+1}^1|^2 \hat{\Lambda}_{11} + [a_n^1 a_{n+1}^{2*} \hat{\Lambda}_{12} + a_n^1 a_{n+1}^{3*} \hat{\Lambda}_{13} \\ &+ a_n^1 a_{n+2}^{4*} \hat{\Lambda}_{14} + h.c] + |a_{n+1}^2|^2 [\hat{\Lambda}_{22} + \hat{\Lambda}_{33}] \end{aligned}$$

$$+ [a^2_{n+1} a^{3*}_{n+1} \hat{A}_{23} + a^2_{n+1} a^{4*}_{n+2} \hat{A}_{24} + h.c.] + [a^3_{n+2} a^{4*}_{n+1} \hat{A}_{34} + h.c.] + |a^4_{n+2}|^2 \hat{A}_{44}. \quad (12)$$

Quantum correlation quantifiers

The generated local two-qubit correlations are quantified via the local Fisher information and the local quantum uncertainty as well as the negativity. The two-qubit negativity based on the negative eigenvalues λ_k of the partial transpose matrix of the atomic matrix $\rho^{AB}(t)$ [48]. It is given by

$$N(t) = -2 \sum_k \lambda_k. \quad (13)$$

If $N(t) = 1$, the two-qubit state is in a maximally entangled state while $N(t) = 0$ means that the two-qubit state is in a pure state. Otherwise, $0 < N(t) < 1$, the two-qubit system has partial entanglement.

Local quantum Fisher information

For a two-qubit matrix $\rho^{AB} = \sum_m \pi_m |v_m\rangle\langle v_m|$ (which have the eigenvalues and the eigenvectors: $\{\pi_m\}$ and $\{|v_m\rangle\}$), the analytically form of the local quantum Fisher information (LQFI) is given by [28,29,49]

$$Q(t) = 1 - \pi_W^{\max}, \quad (14)$$

where $\pi_W^{\max}$ is the highest symmetric matrix eigenvalue $W = [w_{ij}]$, the elements w_{ij} are given by

$$w_{ij} = \sum_{m,n:\pi_m+\pi_n>0} \frac{2\pi_m \pi_n}{\pi_m + \pi_n} \langle v_m|\Lambda|v_n\rangle\langle v_n|\Lambda^\dagger|v_m\rangle,$$

where $\Lambda = I_A \otimes \sigma^i_B$.

Local quantum uncertainty

Based on the skew information quantity [50] and a local observable operator $\hat{O}$, the local quantum uncertainty (LQU) [8] for the two-atom state $\rho^{AB}(t)$ is defined as [9]

$$U(\rho^{AB}(t)) = \max_{\hat{O}} I(\rho^{AB}(t), \hat{O}), \quad (15)$$

where $I(\rho^{AB}(t), \hat{O})$ is the atomic skew information quantity,

$$I(\rho^{AB}(t), \hat{O}) = -\frac{1}{2} \mathrm{Tr}\Big\{ \Big(\sqrt{\rho^{AB}(t)}\hat{O} - \hat{O}\sqrt{\rho^{AB}(t)} \Big)^2 \Big\}. \quad (16)$$

The closed form of the LQU for the two-qubit matrix, $\rho^{AB}(t)$, is reduced to be [8],

$$U(t) = 1 - \lambda_{max}(R), \quad (17)$$

where λ_{max} is the largest eigenvalue of the 3x3-matrix $R = [r_{ij}]$ determined by the following matrix elements:

$$r_{ij} = \mathrm{Tr}\big\{ \sqrt{\rho^{AB}(t)}(\sigma_i \otimes I)\sqrt{\rho^{AB}(t)}(\sigma_j \otimes I) \big\}, \quad (18)$$

and $\sigma_{i(j)}$, $i = 1,2,3$ are the Pauli operators.

Atomic correlation dynamics

In this section, we examine the sensitivity of the LQFI, LQU and the negativity as measures of the generated quantum correlations between the atomic subsystems. We discuss the effect of the non-linearity of the field on the behavior of these measures during the interaction. In the absence of the Kerr-like medium (KLM), the behavior of the atomic correlations described in Fig. 1. It is clear that, the effect of the ratio $\beta = \sqrt{\xi_1/\xi_2}$ on the three measures: local quantum Fisher information $Q(t)$, the local uncertainty $U(t)$ and negativity, $N(t)$ are similar. The maximum and the minimum values of entanglement that determined by the negativity are much larger than the quantum correlation that determined by $Q(t)$ and $U(t)$. The upper bounds of the local Fisher information are much larger than those of the local uncertainty. The phenomena of the sudden death/birth of the entanglement is due to its sensitivity to the decoherence, meanwhile the quantum correlations of $Q(t)$ and $U(t)$ do not vanish during the disentanglement time.

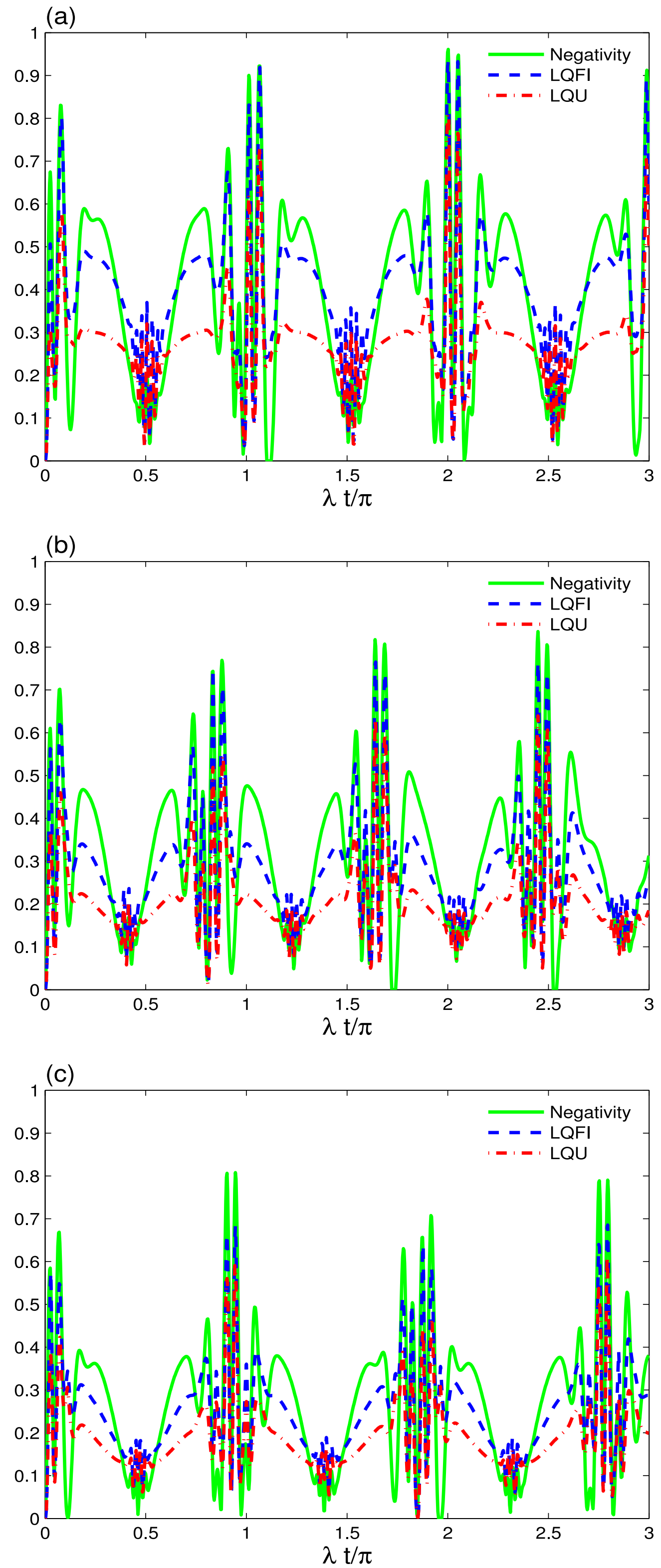

Fig. 1. The LQFI, LQU, and the negativity dynamics are presented for $p = 8$, $q = 1$, $\chi_1 = \chi_2 = 0$ and different cases for the SS parameter: $\beta = 1$ in (a), and $\beta = 0.5$ in (b) and $\beta = 1.5$ in (c).

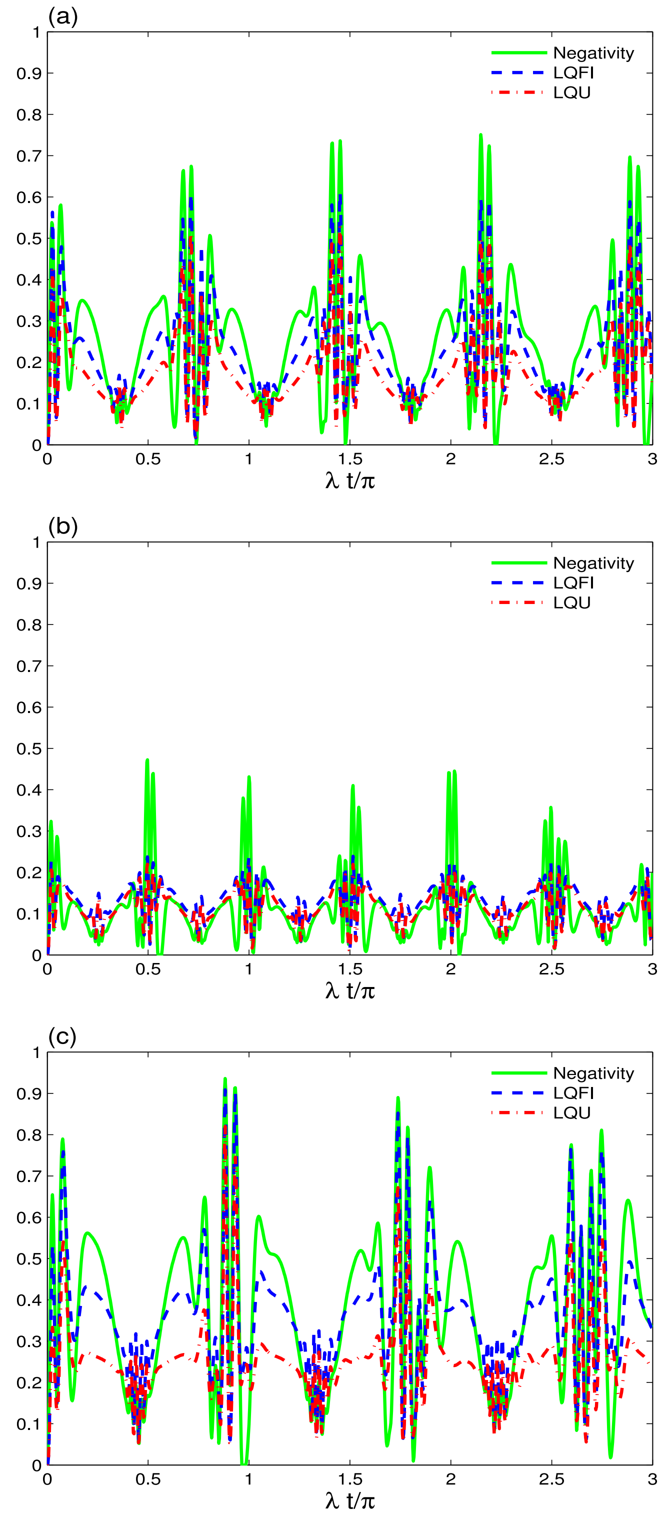

Fig. 2. The same as Fig. 1, but we set $\chi_1 = \chi_2 = 0.5$.

The effect of the ratio β on the behavior of the entanglement can be seen by comparing Figs. 1a, 1b, and 1c where different values of β are considered. However, the maximum/minimum values of the quantum correlation depends on this ratio, where the largest amount of quantum correlations are predicted if the two modes are prepared with equal strengths of the Stark shift, namely $\xi_1 = \xi_2$. While, the smallest values of entanglement are displayed when $\xi > \xi_2$. The ratio β has a slightly effect on the upper bounds of negativity as a measure of entanglement. On the other hand the effect of the ratio β appears clearly on the lower bounds of the negativity, where the possibility of sudden changes (death/birth) increases.

From Fig. 1, we conclude that the upper bounds of entanglement that generated between the atomic system are much larger than the generated quantum correlations. The phenomenon of the sudden death/birth presents only for the entanglement, while the quantum correlations are more robust. The amount of generated quantum correlations can be increased by preparing the two modes such that the Stark shift strengths are equal. Therefore, the Stark shift parameters can be used as control parameters, where by setting $\xi_1 \le \xi_2$, the amount of correlation that is generated between the atomic subsystems is increased.

From Fig. 1, we observe that the generated LQFI and LQU correlations agree with the fact that always the amount of quantum correlations quantified by LQFI is greater than the correlations captured by LQU according the inequality [51,52]: $U(t) \le Q(t) \le 2U(t)$.

In Fig. 2, we discuss the Kerr-like medium effect on the behavior of the correlation quantifiers, where different values of the ratio ξ_2 are considered. The behavior of $Q(t)$, $U(t)$ and $N(t)$ is similar to that displayed in Fig. 1. However, the oscillations amplitudes decrease and consequently, the maximum bounds of correlations that calculated by the three quantifies are reduced. Moreover, due to the presence of the KLM, the quantum correlation and the entanglement decrease. As it is illustrated from Fig. 2b, where we set small values of the Stark-shift ratio and the Kerr-like medium, the behavior of the local Fisher information and the local uncertainty is almost similar. The difference between the two measures is slight observable. However, the entanglement that displayed by the negativity is much larger than the amount of correlation that are quantified by $Q(t)$ and $U(t)$. In Fig. 2c, where $\xi_1 > \xi_2$, the results are changed dramatically, where the effect of the Stark shift SS and that for Kerr-like medium on the quantum correlations turns into amplifiers effect. Therefore, as it is displayed from Fig. 2, the smallest quantum correlations are displayed at smaller values of β, i.e. $\xi_1 < \xi_2$.

From Figs. 1 and 2, the non-linearity of the cavity modes presents different effect on the behavior of the generated atomic correlations. The Stark shift has an amplifying effect on quantum correlations, whereas the Kerr-like medium has a reducing effect on the generated entanglement. By controlling both parameters, it is possible to enhance the correlation of the atomic system by setting the strength of the non-linear stark shift larger than that for the second cavity mode.

The effect of the parameter q, which represents the two mean photon numbers (of the operators $\hat{a}_1^\dagger \hat{a}_1$ and $a_2^\dagger \hat{a}_2$), on the entanglement of the atomic system is displayed in Fig. 3 in the presence of the Stark shift and the Kerr-like medium nonlinearities. It is clear that, the collapse behavior of the entanglement increases as q increases. Moreover, the maximum values of the generated entanglement are much larger than those displayed in Fig. 2. Also, the amount of quantum correlation that has been determined by using the local Fisher information is larger than that quantified by using the local uncertainty measure. Therefore, the q parameter can be used to a robustness factor against of the Kerr-like medium effect that is leads to reducing the quantum correlations. Moreover, the negative impact of the large values of the parameter q is appeared on the entanglement behavior, where it increases the sudden death phenomena of entanglement. At larger interaction time, the upper bounds of entanglement decrease.

The effect of the mean photon numbers on the quantified correlation via $Q(t)$, $U(t)$ and $N(t)$ is illustrated in Fig. 4. It is clear that, the increase of the number of photons improves the quantum correlation of the atomic system on three aspects. The first, the maximum bounds of the correlations are risen as the mean photon number increases. However, by increasing the mean photons number inside the cavity, the maximum peaks appear regularly during the interaction time. In addition, we observe that the regularity behavior of the entanglement is improved due to the high squeezing degree. The third positive impact

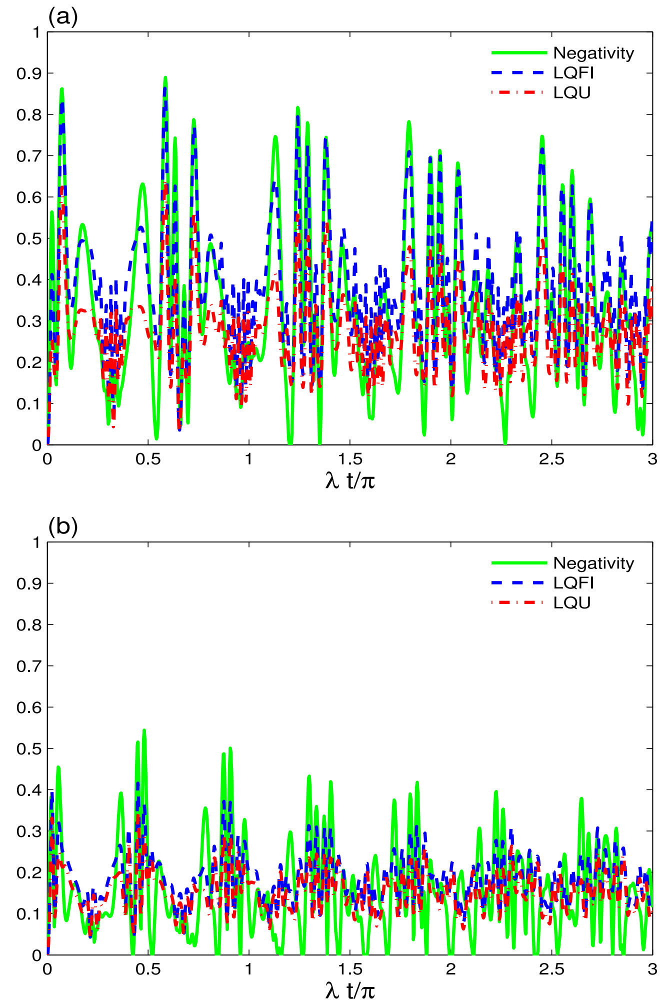

Fig. 3. Dynamics of the LQFI, LQU and the negativity are displayed for $p = 8$ with the cases $(\beta, \chi_i, q) = (1, 0.5, 20)$ and $(\beta, \chi_i, q) = (0.5, 0.5, 20)$ in (d).

of the large mean photon number, is the enhancement of the survival behavior of entanglement.

As can be seen in Figs. 3 and 4, increasing the degeneracy parameter causes the quantum correlations' oscillations and squeezing time to rise. The temporal squeezing intervals, on the other hand, diminish with the regular behavior of the generated correlations between the atomic subsystems for large values of q.

Conclusions

In the presence of nonlinear media, we have investigated two-atom correlations generated by two atoms interacting locally with two cavity modes. The local Fisher information, the local uncertainty, and the negativity are used to quantify the produced atomic quantum correlations. The amount of quantum correlation predicted by local Fisher information is shown to be significantly greater than the amount of local uncertainty. The negativity entanglement is significantly bigger than the other measures' quantifications. It is demonstrated that the Stark shift's nonlinearity could be utilized as a control parameter to magnify the generated correlation, whereas the Kerr-like medium is a decreasing factor that can be controlled as the Stark shift ratio grows. The sudden death/birth phenomenon is easily noticeable. The Stark shift ratio has a significant impact on its emergence. The amplitude of oscillations rises when the degeneracy parameter is enhanced, and consequently, the upper bound of entanglement increases. The mean photon number is crucial in achieving the regular quantum correlation predicted by both quantifiers. The immunity of entanglement to the nonlinearity of the Stark shift and the Kerr-like medium is improved further by a large number of photons. By raising the mean photon number inside the cavity, it is possible to improve the generated quantum correlations.

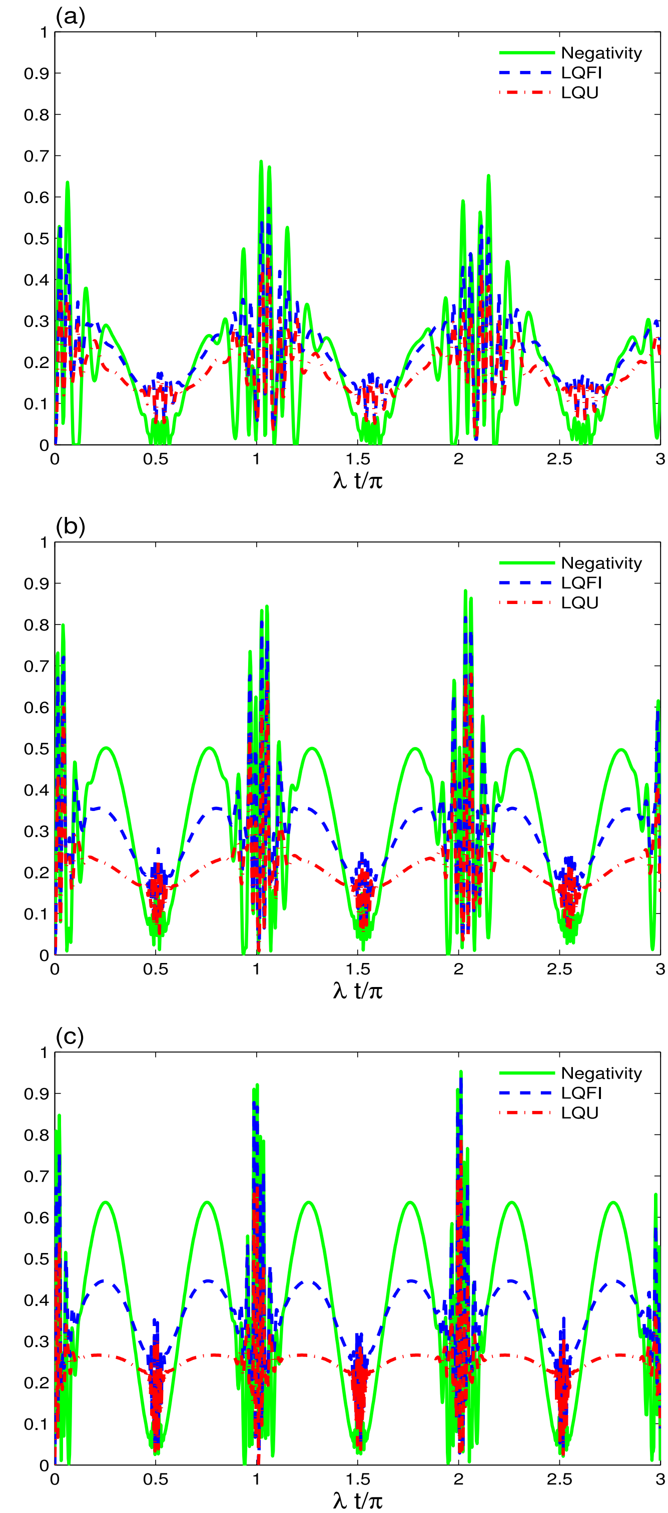

Fig. 4. The dynamics of the LQFI, LQU, and the negativity is illustrated under the effect of the mean photon number in the absence of the KLM with $\beta = 1, q = 10$, where (a) $p = 8$, (b) $p = 15$, and (c) $p = 30$.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This Research was supported by Taif University Researchers Supporting Project Number (TURSP-2020/17), Taif University, Taif, Saudi Arabia. So the authors are thankful of Taif university for this support.

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