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Article *in* Applied Mathematics and Computation · January 2015

DOI: 10.1016/j.amc.2014.10.113

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Applied Mathematics and Computation 250 (2015) 798–804

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

LMI-based stability for singularly perturbed nonlinear impulsive differential systems with delays of small parameter

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ARTICLE INFO

Keywords:
Nonlinear impulsive differential systems
Delays of small parameter
Global stability
Singular perturbation
Linear matrix inequalities (LMIs)

ABSTRACT

In this paper, a class of singularly perturbed nonlinear impulsive delay differential systems is considered, where the time delays include a small parameter. Based on Lyapunov–Krasovskii functional and free weighting matrix method, some novel delay-dependent global asymptotic stability results are derived in terms of linear matrix inequalities (LMIs) which can be easily solved using efficient convex optimization algorithms. The results show that the system stability is very sensitive to the singular perturbation and the small parameter in time delays. A numerical example is given to show the effectiveness of the proposed results.

1. Introduction

Nonlinear singular delayed systems often arise in the modeling of several physical and biological phenomena such as the optically bistable devices [1], lossless transmission lines [2], human pupil light reflex [3] and variational problems in control theory [4]. A large amount of results about the dynamics of nonlinear singular delayed systems have appeared in the literature, see e.g. [5–9].

On the other hand, impulsive phenomena exist ubiquitously in real world such as biological neural networks, dosage supply in pharmacokinetics, artificial intelligence, automatic control systems, and flying object motions, see [10–12]. To reflect a more realistic dynamics, it is necessary to investigate the dynamics of nonlinear singular delayed systems with impulsive perturbations. The first work on stability analysis of nonlinear singularly perturbed systems with impulse effects were given in [13,14]. And then some interesting results are reported in succession, see [15–18]. In [15], the exponential stability of singularly perturbed impulsive delay differential equations were studied via delay differential inequality, which is very useful in real applications. In [16], the stability properties of singularly perturbed switched systems with time delay and impulsive effects were studied via the multiple Lyapunov functions technique and the dwell time approach. The results show that impulses do contribute to the stability problem. Then the exponential stability of a class of singularly perturbed stochastic time delay systems with impulse effect were considered in [17] via comparison principle and time scale decomposition, which improved some existing works [7,13,18]. However, so far very few LMI-based stability results have been reported for singularly perturbed impulsive delay differential equations. As is well known, the result given in terms

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http://dx.doi.org/10.1016/j.amc.2014.10.113
0096-3003/

of LMI can be easily checked by the MATLAB LMI toolbox. In addition, there are some real examples where the corresponding singularly perturbed systems with delays of some small parameters, such as sunflower equations [5,19] and neurons models [20,21]. In book [21], Halanay also studied the linear case of singularly perturbed systems with delays of some small parameters and developed an estimate for the convergence of the fundamental matrix solution to zero, see [21, Section 4.17] for detailed information. In [5], Artstein considered a more general case and improved some results in [21]. We has to say, however, so far the relative research is little.

Motivated by the above discussion, in this paper we shall study a class of singularly perturbed nonlinear impulsive delay differential systems, where the time delays include a small parameter. Some novel global asymptotic stability results which are dependent on singular perturbation and time delays of small parameter are derived. To reduce the conservatism of the stability results, the free weighting matrices method is introduced in this paper. The obtained results are derived in terms of LMIs which can be efficiently solved by using various convex optimization algorithms. A numerical example and its simulations are given to illustrate the effectiveness of the results.

2. Preliminaries

Notations. Let $\mathbb{R}$ $(\mathbb{R}^+)$ denote the set of (positive) real numbers, $\mathbb{Z}_+$ denote the set of positive integers, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional and $n\times m$-dimensional real spaces equipped with the Euclidean norm $\|\bullet\|$, respectively. $\mathcal{A}>0$ or $\mathcal{A}<0$ denotes that the matrix $\mathcal{A}$ is a symmetric and positive definite or negative definite matrix. The notation $\mathcal{A}^T$ and $\mathcal{A}^{-1}$ mean the transpose of $\mathcal{A}$ and the inverse of a square matrix. I denotes the identity matrix with appropriate dimensions. For any interval $J\subseteq\mathbb{R}$, set $S\subseteq\mathbb{R}^k(1\leqslant k\leqslant n), C(J,S)=\{\phi: J\to S$ is continuous$\}$ and $PC^1(J,V)=\{\varphi: J\to V$ is continuously differentiable everywhere except at finite number of points t, at which $\varphi(t^+),\varphi(t^-),\dot\varphi(t^+)$ and $\dot\varphi(t^-)$ exist and $\varphi(t^+)=\varphi(t)$, $\dot\varphi(t^+)=\dot\varphi(t)$, where $\dot\varphi$ denotes the derivative of $\varphi\}$. In particular, let $\mathbb{PC}_r^1 \doteq PC^1([-r,0],\mathbb{R}^n)$ for some constant $r>0$. For $\varphi\in\mathbb{PC}_r^1$, the norm is defined as $\|\varphi\|_r=\max\{\sup_{-r\leqslant s\leq 0}\|\varphi(s)\|,\sup_{-r\leqslant s\leq 0}\|\dot\varphi(s)\|\}$. The impulse times t_k satisfy $0\leqslant t_0<t_1<\ldots<t_k<\ldots$, $\lim_{k\to+\infty}t_k=+\infty$. In addition, the notation $\star$ always denotes the symmetric block in one symmetric matrix.

Consider the following nonlinear impulsive delay systems:

$$\begin{cases}\varepsilon\dot x(t)=A(\varepsilon)x(t)+f(t,x(t-\tau(t,\varepsilon))), & t>0,\ t\neq t_k,\\ \Delta x(t_k)=x(t_k)-x(t_k^-)=-D_kx(t_k^-), & k\in\mathbb{Z}_+,\\ x(t)=\phi(t), & t\in[-\tau\varepsilon,0],\end{cases}\tag{1}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in\mathbb{R}^n$ is the state vector; $\dot x$ denotes the right-hand derivative of x; $\varepsilon>0$ is the perturbed parameter; $\tau(t,\varepsilon)$ is the time-varying delay and satisfies $0\leqslant\tau(t,\varepsilon)\leqslant\tau\varepsilon,\tau\geqslant0$ is a real constant; $\phi\in\mathbb{PC}^1_{\tau\varepsilon}; A(\varepsilon)\in\mathbb{R}^{n\times n}$ is a real matrix; $D_k\in\mathbb{R}^{n\times n},k\in\mathbb{Z}_+$, denote the impulsive matrices; f denotes the nonlinear term of system (1) which satisfies $f(t,0)=0$ and

$$\|f(t,x)\|\leqslant\beta\|x\|,\tag{2}$$

where β is a nonnegative real constant. The existence and uniqueness of the solution of (1) has been given by Liu and Ballinger [22].

Lemma 2.1 [23]. *For any real matrices $a\in\mathbb{R}^n,b\in\mathbb{R}^m,N\in\mathbb{R}^{n\times m},X\in\mathbb{R}^{n\times n},Y\in\mathbb{R}^{n\times m},Z\in\mathbb{R}^{m\times m}$, if*

$$\begin{bmatrix}X & Y\\ \star & Z\end{bmatrix}\geqslant0,$$

then

$$\pm2a^TNb\leqslant\begin{bmatrix}a\\ b\end{bmatrix}^T\begin{bmatrix}X & Y-N\\ \star & Z\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}.$$

3. Main results

Theorem 3.1. *Given scalars $\tau\geqslant0$ and $\varepsilon>0$, system (1) is globally asymptotically stable if there exist a constant $\delta>0$, two positive definite matrices $P>0,Q>0$, three real matrices F_1,F_2,F_3, three semi-positive-definite matrices*

$$X=\begin{bmatrix}X_{11} & X_{12}\\ \star & X_{22}\end{bmatrix}\geqslant0,\quad Y=\begin{bmatrix}Y_{11} & Y_{12}\\ \star & Y_{22}\end{bmatrix}\geqslant0,\quad Z=\begin{bmatrix}Z_{11} & Z_{12}\\ \star & Z_{22}\end{bmatrix}\geqslant0$$

and an appropriately dimensional matrix $H=[H_1^T\ \ H_2^T]^T$ such that the following LMIs hold:

$$\begin{bmatrix}Z & -H\\ \star & Q\end{bmatrix}\geqslant0,\quad\begin{bmatrix}P & (I-D_k)^TP\\ \star & P\end{bmatrix}\geqslant0,\quad k\in\mathbb{Z}_+\tag{3}$$

and

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ \star & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ \star & \star & \Pi_{33} & \Pi_{34} \\ \star & \star & \star & \Pi_{44} \end{bmatrix} < 0, \tag{4}$$

where

$$\Pi_{11} = PA + A^T P + X_{11} - F_1 A - A^T F_1 - H_1 - H_1^T + \tau\varepsilon Z_{11}, \quad \Pi_{12} = \tau A^T Q + F_1\varepsilon - A^T F_2^T,$$
$$\Pi_{13} = H_1 - H_2^T + \tau\varepsilon Z_{12}, \quad \Pi_{14} = X_{12} - P - F_1 - A^T F_3^T, \quad \Pi_{22} = Y_{11} + 2\varepsilon F_2, \Pi_{23} = \Pi_{34} = 0,$$
$$\Pi_{24} = Y_{12} - \tau Q - F_2 + \varepsilon F_3^T, \quad \Pi_{33} = H_2 + H_2^T + \delta\beta^2 + \tau\varepsilon Z_{22}, \quad \Pi_{44} = X_{22} + Y_{22} - F_3 - F_3^T - \delta I.$$

Proof. Let $x(t) = x(t, 0, \phi)$ be a solution of system (1) through $(0, \phi)$, where $\phi \in \mathbb{PC}_\rho^1$. Construct a Lyapunov–Krasovskii functional candidate as

$$V(x, t, \varepsilon) = \varepsilon x(t)^T P x(t) + \int_{-\tau\varepsilon}^{0} \int_{t+u}^{t} \dot{x}^T(s) Q \dot{x}(s) ds du. \tag{5}$$

Employing Lemma 2.1 and calculating the derivative of $V(x, t, \varepsilon)$ along the trajectory of system (1), one can deduce that

$$\begin{aligned} D^+V(x,t,\varepsilon) &= 2x(t)^T P[A(\varepsilon)x(t) + \mathcal{F}] + \tau\varepsilon\dot{x}^T(t)Q\dot{x}(t) - \int_{t-\tau\varepsilon}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \\ &\leqslant 2x(t)^T PA(\varepsilon)x(t) + 2x(t)^T P\mathcal{F} + \tau\dot{x}^T(t)Q[A(\varepsilon)x(t) + \mathcal{F}] - \int_{t-\tau(t,\varepsilon)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \\ &\leqslant 2x(t)^T PA(\varepsilon)x(t) + \tau\dot{x}^T(t)QA(\varepsilon)x(t) - \int_{t-\tau(t,\varepsilon)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \\ &\quad + \begin{bmatrix} x(t) \\ \mathcal{F} \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} - P \\ \star & X_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \mathcal{F} \end{bmatrix} \\ &\quad + \begin{bmatrix} \dot{x}(t) \\ \mathcal{F} \end{bmatrix}^T \begin{bmatrix} Y_{11} & Y_{12} - \tau Q \\ \star & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \mathcal{F} \end{bmatrix}, \quad t \in [t_{k-1}, t_k),\ k \in \mathbb{Z}_+, \end{aligned} \tag{6}$$

where $\mathcal{F} = f(t, x(t - \tau(t, \varepsilon)))$.

To derive our desirable result, we need some auxiliary equations. First, for any free-weighting matrix $F = [F_1^T \;\; F_2^T \;\; F_3^T]^T$, the following zero equation holds:

$$0 = 2\left[x^T(t) \quad \dot{x}^T(t) \quad \mathcal{F}^T\right] F[\varepsilon\dot{x}(t) - A(\varepsilon)x(t) - \mathcal{F}]. \tag{7}$$

Second, using the Leibniz–Newton formula, we know

$$0 = -x(t) + x(t - \tau(t,\varepsilon)) + \int_{t-\tau(t,\varepsilon)}^{t} \dot{x}(s)ds,$$

which implies that

$$0 = 2\eta^T(t)H\left[-x(t) + x(t - \tau(t,\varepsilon)) + \int_{t-\tau(t,\varepsilon)}^{t} \dot{x}(s)ds\right], \tag{8}$$

where $\eta(t) = [x^T(t) \quad x^T(t - \tau(t, \varepsilon))]^T$, $H = [H_1^T \quad H_2^T]^T$.

Moreover, for the semi-positive-definite matrix $Z = (Z_{ij})_{2\times 2} \geqslant 0$, the following equality holds:

$$0 \leqslant \tau\varepsilon\eta^T(t)Z\eta(t) - \int_{t-\tau(t,\varepsilon)}^{t} \eta^T(t)Z\eta(t)ds. \tag{9}$$

In addition, it follows from (2) that for any $\delta > 0$,

$$0 \leqslant \delta\beta^2 x^T(t - \tau(t,\varepsilon))x(t - \tau(t,\varepsilon)) - \delta\mathcal{F}^T\mathcal{F}. \tag{10}$$

Now adding the terms on the right hands of (7)–(10) to (6), it can be deduced that

$$D^+V(x,t,\varepsilon) \leqslant \xi^T(t)\Delta\xi(t) + \xi^T(t) \begin{bmatrix} \tau\varepsilon Z_{11} & 0 & \tau\varepsilon Z_{12} & 0 \\ \star & 0 & 0 & 0 \\ \star & \star & \tau\varepsilon Z_{22} & 0 \\ \star & \star & \star & 0 \end{bmatrix} \xi(t) - \int_{t-\tau(t,\varepsilon)}^{t} [\eta^T(t), \dot{x}^T(s)] \begin{bmatrix} Z & -H \\ \star & Q \end{bmatrix} \begin{bmatrix} \eta(t) \\ \dot{x}(s) \end{bmatrix} ds, \quad t \in [t_{k-1}, t_k), k \in \mathbb{Z}_+,$$

where

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} \\ \star & \Delta_{22} & \Delta_{23} & \Delta_{24} \\ \star & \star & \Delta_{33} & \Delta_{34} \\ \star & \star & \star & \Delta_{44} \end{bmatrix}, \quad \xi(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x(t-\tau(t,\varepsilon)) & \mathcal{F}^T \end{bmatrix}^T,$$

$$\begin{aligned} &\Delta_{11} = PA + A^TP + X_{11} - F_1A - A^TF_1 - H_1 - H_1^T \quad \Delta_{12} = \tau A^TQ + F_1\varepsilon - A^TF_2^T, \\ &\Delta_{13} = H_1 - H_2^T, \quad \Delta_{14} = X_{12} - P - F_1 - A^TF_3^T, \quad \Delta_{22} = Y_{11} + 2\varepsilon F_2, \quad \Delta_{23} = \Delta_{34} = 0, \\ &\Delta_{24} = Y_{12} - \tau Q - F_2 + \varepsilon F_3^T, \quad \Delta_{33} = H_2 + H_2^T + \delta\beta^2, \quad \Delta_{44} = X_{22} + Y_{22} - F_3 - F_3^T - \delta I. \end{aligned}$$

This, together with assumption (4), yields

$$D^+V(x,t,\varepsilon) \leqslant \xi^T(t)\Pi\xi(t) < 0, \quad t \in [t_{k-1}, t_k), \ k \in \mathbb{Z}_+. \tag{11}$$

On the other hand, it follows from (3) that

$$\begin{aligned} \begin{bmatrix} P & (I-D_k)^TP \\ \star & P \end{bmatrix} \geqslant 0 \iff &\begin{bmatrix} I & 0 \\ 0 & P^{-1} \end{bmatrix} \begin{bmatrix} P & (I-D_k)^TP \\ \star & P \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & P^{-1} \end{bmatrix} \geqslant 0 \\ \iff &P - (I-D_k)^TP(I-D_k) \geqslant 0, \end{aligned}$$

which implies that

$$V(x(t_k), t_k, \varepsilon) \leqslant V(x(t_k^-), t_k^-, \varepsilon), \quad k \in \mathbb{Z}_+. \tag{12}$$

By Lyapunov stability theorem [24], it follows from (11) and (12) that the trivial solution of system (1) is globally asymptotically stable. The proof of Theorem 3.1 is complete. □

Remark 3.1. In [13–18], the authors have studied the stability problem of singularly perturbed impulsive systems via different methods. However, all of those results are not given in terms of LMIs. As we know, the main advantages of the LMIs-based results include that firstly it only needs tuning of parameters and/or matrices, and secondly it can be solved numerically using efficient convex optimization. Moreover, the development results can be applied to the singularly perturbed impulsive systems with time delays of a small parameter. Hence, our developed results in this paper are more effective than those reported in [13–18]. In addition, from the proof of Theorem 3.1, one may find that the second LMI in (3) can be removed if $D_k \equiv d \cdot I, k \in \mathbb{Z}_+$, where $d \in [0, 2]$ is a constant.

Remark 3.2. In particular, let $\mathcal{F} = B(\varepsilon)x(t-\tau(t,\varepsilon))$, where $B(\varepsilon) \in \mathbb{R}^{n\times n}$ is a real matrix, then system (1) becomes a linear case of

$$\begin{cases} \varepsilon\dot{x}(t) = A(\varepsilon)x(t) + B(\varepsilon)x(t-\tau(t,\varepsilon)), & t > 0, \ t \neq t_k, \\ \Delta x(t_k) = x(t_k) - x(t_k^-) = -D_kx(t_k^-), & k \in \mathbb{Z}_+, \\ x(t) = \phi(t), & t \in [-\tau\varepsilon, 0]. \end{cases} \tag{13}$$

Corollary 3.1. *Under the assumptions in Theorem 3.1, system (13) with $0 \leqslant \tau(t,\varepsilon) \leqslant \tau\varepsilon$ is globally asymptotically stable if $B^T(\varepsilon)B(\varepsilon) \leqslant \beta^2I$.*

If there is no singular perturbed parameter ε on time delays, then system (1) becomes

$$\begin{cases} \varepsilon\dot{x}(t) = A(\varepsilon)x(t) + f(t, x(t-\tau(t))), & t > 0, \ t \neq t_k, \\ \Delta x(t_k) = x(t_k) - x(t_k^-) = -D_kx(t_k^-), & k \in \mathbb{Z}_+, \\ x(t) = \phi(t), & t \in [-\tau, 0], \end{cases} \tag{14}$$

where $0 \leqslant \tau(t) \leqslant \tau; \phi \in \mathbb{PC}_\tau^1$. For system (14), we have the following result.

Theorem 3.2. *Given scalars $\tau \geqslant 0$ and $\varepsilon > 0$, system (14) is globally asymptotically stable if there exist a constant $\delta > 0$, two positive definite matrices $P > 0, Q > 0$, three real matrices F_1, F_2, F_3, three semi-positive-definite matrices*

$$X = \begin{bmatrix} X_{11} & X_{12} \\ \star & X_{22} \end{bmatrix} \geqslant 0, \quad Y = \begin{bmatrix} Y_{11} & Y_{12} \\ \star & Y_{22} \end{bmatrix} \geqslant 0, \quad Z = \begin{bmatrix} Z_{11} & Z_{12} \\ \star & Z_{22} \end{bmatrix} \geqslant 0$$

and an appropriately dimensional matrix $H = [H_1^T \ \ H_2^T]^T$ such that the following LMIs hold:

$$\begin{bmatrix} Z & -H \\ \star & \varepsilon Q \end{bmatrix} \geqslant 0, \quad \begin{bmatrix} P & (I-D_k)P \\ \star & P \end{bmatrix} \geqslant 0, \quad k \in \mathbb{Z}_+$$

and

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ \star & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ \star & \star & \Pi_{33} & \Pi_{34} \\ \star & \star & \star & \Pi_{44} \end{bmatrix} < 0,$$

where

$$\Pi_{11} = PA + A^T P + X_{11} - F_1 A - A^T F_1 - H_1 - H_1^T + \tau Z_{11},$$
$$\Pi_{12} = \tau A^T Q + F_1 \varepsilon - A^T F_2^T, \quad \Pi_{13} = H_1 - H_2^T + \tau Z_{12},$$
$$\Pi_{14} = X_{12} - P - F_1 - A^T F_3^T, \quad \Pi_{22} = Y_{11} + 2\varepsilon F_2, \Pi_{23} = \Pi_{34} = 0,$$
$$\Pi_{24} = Y_{12} - \tau Q - F_2 + \varepsilon F_3^T, \quad \Pi_{33} = H_2 + H_2^T + \delta\beta^2 + \tau Z_{22},$$
$$\Pi_{44} = X_{22} + Y_{22} - F_3 - F_3^T - \delta I.$$

Proof. Let $x(t) = x(t, 0, \phi)$ be a solution of system (14) through $(0, \phi)$, where $\phi \in \mathbb{PC}_\tau^1$. Construct a Lyapunov–Krasovskii functional candidate as

$$V(x,t,\varepsilon) = \varepsilon x(t)^T P x(t) + \varepsilon \int_{-\tau}^{0} \int_{t+u}^{t} \dot{x}^T(s) Q \dot{x}(s) ds du.$$

Then similar to the proof of Theorem 3.1, it can be deduced that

$$D^+ V(x,t,\varepsilon) \leqslant \xi^T(t)\Delta\xi(t) + \xi^T(t) \begin{bmatrix} \tau Z_{11} & 0 & \tau Z_{12} & 0 \\ \star & 0 & 0 & 0 \\ \star & \star & \tau Z_{22} & 0 \\ \star & \star & \star & 0 \end{bmatrix} \xi(t) - \int_{t-\tau(t)}^{t} [\eta^T(t), \dot{x}^T(s)] \begin{bmatrix} Z & -H \\ \star & \varepsilon Q \end{bmatrix} \begin{bmatrix} \eta(t) \\ \dot{x}(s) \end{bmatrix} ds, \quad t \in [t_{k-1}, t_k), \ k \in \mathbb{Z}_+,$$

where the definitions of ξ, η and Δ is same as those in Theorem 3.1. By Lyapunov stability theorem [24], we can conclude that the trivial solution of system (14) is global asymptotic stability. $\square$

Corollary 3.2. *Under the assumptions in Theorem 3.2, system (14) with $0 \leqslant \tau(t,\varepsilon) \leqslant \tau$ is globally asymptotically stable if $B^T(\varepsilon)B(\varepsilon) \leqslant \beta^2 I$.*

4. Example

Example 4.1. Consider the following singularly perturbed systems

$$\begin{cases} \varepsilon \dot{x}(t) = A(\varepsilon)x(t) + f(t, x(t-\tau\varepsilon)), & t > 0, \ t \neq t_k, \\ \Delta x(t_k) = x(t_k) - x(t_k^-) = -D_k x(t_k^-), & k \in \mathbb{Z}_+, \\ x(t) = \phi(t), & t \in [-\tau\varepsilon, 0], \end{cases} \tag{15}$$

where $\tau \geqslant 0, \varepsilon > 0$ are two real constants, $\|f(t,x)\| \leqslant \|x\|$ for any $x \in \mathbb{R}$ and

$$A(\varepsilon) = \begin{bmatrix} -2+\varepsilon & 0.3 + 0.2\cos 2\varepsilon \\ 0.4 - 0.1\sin\varepsilon & -3 + 0.5\varepsilon \end{bmatrix}, \quad D_k = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \ k \in \mathbb{Z}_+.$$

In this case, it is obvious that $\beta = 1$. For some given ε, the upper bounds of τ to guarantee the global asymptotic stability are listed in Table 1 by solving the LMIs in Theorem 3.1. In Table 1, "–" means that the result is not applicable to the corresponding case.

Table 1
Allowable upper bounds of τ with different values of ε.

ε	10^{-4}	10^{-3}	10^{-2}	10^{-1}	$10^{-0.1}$	1
Max τ	0.1601	0.1600	0.1591	0.1502	0.0431	–

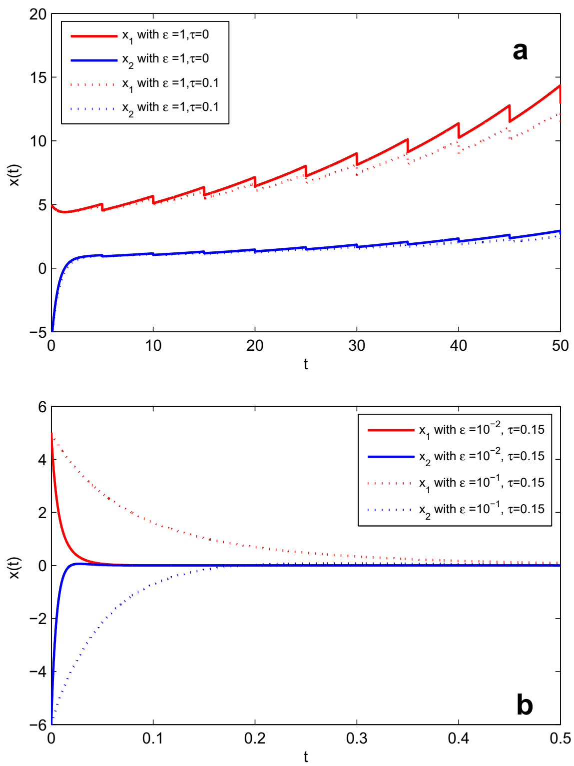

Fig. 1. (a) State trajectories of system (15) with different values of ε and τ .

Remark 4.1. From Table 1, one note that the LMIs in Theorem 3.1 is unfeasible for any $\tau \geqslant 0$ if there is no singular perturbation, i.e., $\varepsilon = 1$. In this case, it is interesting to see that system (15) is unstable (see Fig. 1(a)). However, it becomes stable under small singular perturbation (see Fig. 1(b)). From the simulations, one may observe that system stability is very sensitive to the singular perturbation and the small parameter in time delays and moreover proper singular perturbation can contribute to stability behavior of system.

Remark 4.2. It is easy to check that all the results in [13–18] are invalid for Example 4.1.

Acknowledgements

The authors are deeply grateful to the Associate Editor and two reviewers for their careful reading of this paper and helpful comments, which have been very useful for improving the quality of this paper.

This work was jointly supported by National Natural Science Foundation of China (No. 11301308, 11171085), the Natural Science Foundation of Zhejiang Province of China (No. LY14A010024), China Postdoctoral Science Foundation founded project (2014M561956) and Research Fund for International Cooperation Training Programme of Excellent Young Teachers of Shandong Normal University.

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