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Analysis of voltage and current flow of electrical transmission lines through mZK equation

M. Ali Akbar [a], Md. Abdul Kayum [a], M.S. Osman [b,c,*], Abdel-Haleem Abdel-Aty [d,e], Hichem Eleuch [f,g,h]

[a] Department of Applied Mathematics, University of Rajshahi, Bangladesh
[b] Department of Mathematics, Faculty of Science, Cairo University, Giza 12613, Egypt
[c] Department of Mathematics, Faculty of Applied Science, Umm Alqura University, Makkah 21955, Saudi Arabia
[d] Department of Physics, College of Sciences, University of Bisha, P.O. Box 344, Bisha 61922, Saudi Arabia
[e] Physics Department, Faculty of Science, Al-Azhar University, Assiut 71524, Egypt
[f] Department of Applied Physics and Astronomy, University of Sharjah, Sharjah, United Arab Emirates
[g] College of Arts and Sciences, Abu Dhabi University, Abu Dhabi 59911, United Arab Emirates
[h] Institute for Quantum Science and Engineering, Texas A&M University, College Station, TX 77843, USA

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A B S T R A C T

Discrete networks, nonlinear networks, high speed computer data buses, computer network connections, and cable signal distribution are analyzed using the modified Zakharov-Kuznetsov (mZK) equation as a model governing the propagation in the electrical transmission lines. In this study, we establish viable extensive traveling wave solutions with the familiar singular kink type, kink type, bell-type solution, singular bell-type solution, t-type solution, and some advanced soliton solutions through the modified simple equation (MSE) and the sine-Gordon expansion methods. The established solutions are highly stable, resilient and capable to travel long distance. The solutions obtained can determine the voltage and current flow that can be used to design the transmission lines. The density plot and three-dimensional diagrams of the solutions obtained illustrate the voltage and current distributions in the transmission line. The traveling wave solutions are found in closed-form which simplify their utilization. The implemented methods are compatible for the extraction of traveling wave solutions.

Introduction

The phenomena that ascend in applied and theoretical physics with applications in engineering and scientific disciplines can be described by nonlinear evolution equations (NLEEs) [1–11]. Soliton is a universal concept applicable to a large class of wave propagation phenomena. Thus the investigation of soliton solutions of NLEEs contributes significantly in scientific and technical studies [12], such as electric control theory, nonlinear transmission lines, system identification, mechanical engineering, signal processing, chemical engineering, optical fibers, biology, plasma physics, gas dynamics, ocean engineering, optical telecommunication, biomedical problems, electromagnetism, nanofiber technology, nuclear physics etc [13,14]. Different types of solutions can be identified through analytical approaches, namely rational wave solutions, rogue waves, breather waves, periodic waves, singular solutions, optical solutions and solitary wave solutions [15]. A number of analytic techniques used to construct the soliton solutions are: the method of Painleve analysis [16], the method of Riemann-Hilbert [17], the method of Darboux transformation [18], the method of Hirota bilinear [19], the inverse scattering method [20], the technique of $\exp(-\Phi(\xi))$-expansion [21], the technique of auxiliary equation [22], the technique of $\tan(\phi(\xi))$-expansion [23], the technique of improved (G'/G)-expansion [24], the method of generalized (G'/G)-expansion [25], the method of improved F-expansion [26], the technique of fractional sub-equation [27], the technique of generalized Kudryashov [28], the technique of $(G'/G, 1/G)$-expansion [29], the technique of extended Jacobi elliptic function expansion [30], the trial solution approach [31], the technique of Exp-function [32], the technique of Jacobi elliptic function expansion [33], the technique of reproducing kernel discretization [34], the Lie symmetry analysis [35], the technique of

* Corresponding author at: Department of Mathematics, Faculty of Science, Cairo University, Giza 12613, Egypt.
E-mail address: mofatzi@sci.cu.edu.eg (M.S. Osman).

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modified simple equation [36,37], the technique of sine-Gordon expansion [38] and Many other techniques [39–43]. The MSE technique [36,37] is one of the dynamic methods to draw out soliton solutions from algebraic and differential equations and the method of sine-Gordon expansion (SGE) [38,44], on the other hand, is an underlying approach to obtain soliton solutions unraveling algebraic equations derived from the NLEEs.

The MSE technique has been introduced to develop soliton type solutions to the Biswas-Arshed model [45], Kundu-Mukherjee-Naskar model [46], Biswas-Arshed equation [47], Kundu-Eckhaus equation [48], Lakshmanan-Porsezian-Daniel model [49], complex Ginzburg-Landau equation [50], derivative nonlinear Schrödinger equation [51], resonant nonlinear Schrödinger equation & nonlinear Schrödinger equation with the dual power law nonlinearity [52], the long-short wave resonance equations [53], nonlinear fractional Sharma-Tasso-Olver equation [54], Boussinesq and Fisher equations [55] etc. Also, the SGE technique has been introduced to obtain soliton type solutions to the conformable time fractional equations in RLW-class [56], the Fokas-Lenells equation [57], coupled Boussinesq-Burgers equations [58], the Biswas-Milovic model in nonlinear optics [59], the fractional perturbed Gerdjikov-Ivanov equation [60], new coupled Konno-Oono equation [61], Tzitzéica type equations [62], the Lonngren-wave equation [63] among others.

The mZK equation in electrical transmission line delivers a significant contribution in studying the dynamic behaviors of the computer network connections, microwave, the high speed computer data buses, cable signal distribution, radio receivers and transmitters with their antennas, radio-frequency engineering, voltage transformer, stub filters [64] etc. By means of the method of sine–cosine, the method of extended tanh-function and $(G^{'}/G)$-expansion method, Sardar *et al.* [65] established the multiple traveling wave solutions like as kink type, periodic type, singular type and shock type solutions. Wang and Zheng [66] presented the dynamical system approach for finding the periodic waves and solitary waves on coupled nonlinear electrical transmission lines. Some exact solutions to the mZK equation are obtained in [67]. Taking the advantage of the Jacobi elliptical function method, Tala *et al.* [68] ascertained the soliton solutions to the mZK equation. Some new solutions of the mZK equation are attained in [69] by means of the extended trial equation method and the improved Bernoulli sub-ODE method. Manafian and Lakestani [70] used the improved $\tan(\phi/2)$-expansion method, the $\exp(-\Omega(\xi))$-expansion method, the semi-inverse variational principle to find the soliton solutions to the mZK equation. Ali *et al.* [71] described the conservation laws, lie point symmetry and the exact solutions of electrical transmission line model. The three-dimensional nonlinear mZK equation of ion-acoustic waves in a magnetized plasma has been studied by Seadawy in [72]. Recently, Jhangeer *et al.* [73] investigated the conservation laws, chaotic, quasi-periodic, periodic, super nonlinear and solitonic waves to the mZK equation in transmission line using the extended algebraic method. The numerical and analytic solutions are studied in [74] by using Hamiltonian system.

To our best knowledge, the mZK equation has not been contrived by the modified simple equation technique and the sine-Gordon expansion approaches. The objective of this work is to obtain the soliton and inclusive broad-ranging wave solutions to the mZK equation to analyze the current and voltage in the NETLs. The solutions might be helpful to understand the wave nature in transmission line problems.

The rest of this study is organized as following: We introduce the modified simple equation technique and its application to mZK equation in section 2. In Section 3, we present the SGE method and its applications for the mZK equation. We present the three dimensional and density plots in section 4. The conclusion is pointed out in section 5.

The MSE method

The MSE technique [45–55] for the mZK equation of transmission lines is can be explained as the following:

Consider the nonlinear partial differential equation in the form:

$$F\left(v, v_t, v_x, v_y, v_{tt}, v_{xx}, v_{xt}, \cdots\right) = 0 \tag{1}$$

where $v = v(x, y, t)$ is an unknown function and F is a polynomial of v and its derivatives, in which the nonlinear terms and highest order derivatives are included. The MSE method has several steps to obtain traveling wave solutions which are described below:

Step 1: We present the wave transformation with compound variable

$$v(x, y, t) = w(\xi), \xi = \lambda_1 x + \lambda_2 y - \omega t \tag{2}$$

where λ_1, λ_2 and ω are respectively the wave number and wave speed, ξ is the compound wave variable which is the function of real variables x, y and t. By means of the transformation (2), the equation (1), yields the following ordinary differential equation

$$R(w, w^{'}, w'', w^{'''}, \ldots) = 0 \tag{3}$$

where the prime (') denotes differentiation with respect to ξ and $\mathscr{R}$ is a function of $w(\xi)$ and its derivatives.

Step 2: We assume the solution of Eq. (3) in the form

$$w(\xi) = \sum_{i=0}^{N} r_i \left[\frac{\varphi^{'}(\xi)}{\varphi(\xi)}\right]^i \tag{4}$$

where $r_i (i = 0, 1, 2, 3, \ldots)$ are unidentified constants to be evaluated, but the condition is that r_N is not equal to zero, $\varphi(\xi)$ is unknown function to be calculated in which the first derivative of this function is not equal to zero. The solutions are introduced by several auxiliary functions in the generalized Kudryashov method, first integral method, improved $\tan(\varphi/2)$-expansion method, the method of improved F-expansion, the method of Jacobi elliptic function, the method of sine–cosine, the method of *tanh*-function etc., but in the MSE method, $\varphi(\xi)$ is neither a solution of any pre-defined differential equation nor a predefined function. Therefore, this method is easy and unique.

Step 3: The positive integer N of Eq. (4) can be evaluated via the homogeneous balance principle after comparing the suitable nonlinear term and the highest linear derivative term of the ordinary differential equation.

Step 4: We determine all the necessary derivatives $w^{'}, w'', w^{'''}, \ldots$ of the function $w(\xi)$ and substituting equation (4) into (3). We find a polynomial in $(1/\varphi^i)$, $i = 0, 1, 2, 3, \cdots$. Equalizing the coefficient of $\varphi^0(\xi)$, $\varphi^{-1}(\xi)$, $\varphi^{-2}(\xi)$,… to zero, we accomplish some differential and algebraic equation. By solving these algebraic and differential equation we obtain the values of $r_0, r_1, r_2, \ldots$ and $\varphi(\xi)$. Substituting these values into Eq. (4), therefore we achieve the solution of Eq. (1).

Circuit equation model

In this subsection, we describe the circuit equation model for mZK in electrical transmission line. We will concentrate our study on the propagation in the NETL. In Fig. 1, there are several similar dispersive

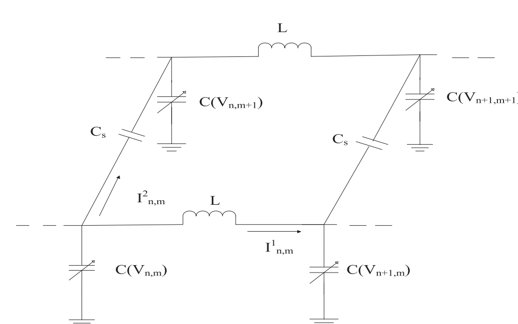

Fig. 1. Graphical illustration of the NETL.

lines bound by the capacitance C_s to each node. In the series branch, each section of a line consists of a constant inductor L. Also, in the shunt branch, each section of a line consists of a capacitance $C(V_{n,m})$, where $V_{n,m}$ is the voltage through the capacitor. There are two different co-ordinates for the nodes, n and m, where n marks the lines in the direction of wave propagation and m marks the lines in the direction of a transverse wave.

Using the Kirchhoff law on the model the following discrete partial differential equation is obtained:

$$\frac{\partial^2 Q_{n,m}}{\partial T^2} = \frac{1}{L}\left(V_{n-1,m} - 2V_{n,m} + V_{n+1,m}\right) + C_s\frac{\partial^2}{\partial T^2}\left(V_{n,m-1} - 2V_{n,m} + V_{n,m+1}\right) \quad (5)$$

where $V_{n,m} = V_{n,m}(T)$ is the voltage and $Q_{n,m}$ is the nonlinear charge. The nonlinear charge in terms of voltage is given by

$$Q_{n,m} = C_0\left(\frac{\beta_2}{3}V_{n,m}^3 - \frac{\beta_1}{2}V_{n,m}^2 + V_{n,m}\right) \quad (6)$$

where β_1, β_2 are constants. Using equation (6) into equation (5), we attain

$$C_0\frac{\partial^2}{\partial T^2}\left(\frac{\beta_2}{3}V_{n,m}^3 - \frac{\beta_1}{2}V_{n,m}^2 + V_{n,m}\right)$$

$$= \frac{1}{L}\left(V_{n-1,m} - 2V_{n,m} + V_{n+1,m}\right) + C_s\frac{\partial^2}{\partial T^2}\left(V_{n,m-1} - 2V_{n,m} + V_{n,m+1}\right) \quad (7)$$

Inserting $V_{n,m}(T) = V(n,m,T)$ into Eq. (7), we obtain

$$C_0\frac{\partial^2}{\partial T^2}\left(\frac{\beta_2}{3}V_{n,m}^3 - \frac{\beta_1}{2}V_{n,m}^2 + V_{n,m}\right) = \frac{1}{L}\frac{\partial^2}{\partial n^2}\left(V + \frac{1}{12}\frac{\partial^2 V}{\partial n^2}\right) + C_s\frac{\partial^4}{\partial T^2\partial m^2}\left(V + \frac{1}{12}\frac{\partial^2 V}{\partial m^2}\right) \quad (8)$$

Using the independent variable transformations

$$V(n,m,T) = \chi v(x,y,t), t = \chi^{\frac{1}{2}}T, y = \chi^{\frac{1}{2}}m, x = \chi^{\frac{1}{2}}(n - v_s T) \quad (9)$$

where χ is a formal parameter, $v_s^2 = \frac{1}{LC_0}$. Using the reductive perturbation method, Eq. (8) can be reduced to the subsequent modified mZK equation [57–67]:

$$v_t + Avv_x + Bv^2v_x + Mv_{xxx} + Nv_{xyy} = 0 \quad (10)$$

with

$$A = -\beta_1 v_s, B = -\beta_2 v_s, M = \frac{1}{24\beta_1 L v_s}, N = \frac{\beta_1}{288L^2 v_s C_0^2} \quad (11)$$

where x, y and t represent the spatial and temporal variables, $v(x,y,t)$ describes the first-order perturbation voltage in the transmission lines. Eq. (10) is useful to represent the voltage for the transmission line in the continuum limit.

Implementations of MSE method to the mZK equation

In this subsection, we use the MSE method to determine the different kinds of soliton solutions through MSE method.

We use the transformation $v(x,y,t) = w(\xi)$ and $\xi = \lambda_1 x + \lambda_2 y - \omega t$ to transform Eq. (10) to the subsequent nonlinear ordinary differential equation:

$$-\omega w + A\lambda_1\frac{w^2}{2} + B\lambda_1\frac{w^3}{3} + \left(M\lambda_1^3 + N\lambda_1\lambda_2^2\right)w'' = 0 \quad (12)$$

By balancing the terms w'' and w^3, yield $N = 1$.

Thus, the solution shape of Eq. (12) becomes

$$w(\xi) = r_0 + r_1\left(\frac{\varphi'(\xi)}{\varphi(\xi)}\right) \quad (13)$$

where $\varphi(\xi)$ is an unknown character to be determined, r_0 and r_1 are undetermined constants to be evaluated, such that $\varphi'(\xi) \neq 0$ and $r_1 \neq 0$.

It is easy to determine that

$$w' = r_1\left(\frac{\varphi''(\xi)}{\varphi(\xi)} - \left(\frac{\varphi'(\xi)}{\varphi(\xi)}\right)^2\right) \quad (14)$$

$$w'' = r_1\left(\frac{\varphi^{(3)}(\xi)}{\varphi(\xi)} + 2\left(\frac{\varphi'(\xi)}{\varphi(\xi)}\right)^3 - \frac{3\varphi'(\xi)\varphi''(\xi)}{\varphi(\xi)^2}\right) \quad (15)$$

Substituting (13) and (15) into equation (12) and equalizing the coefficients of $\varphi^0(\xi)$, $\varphi^{-1}(\xi)$, $\varphi^{-2}(\xi)$ and $\varphi^{-4}(\xi)$ to zero, we obtain the ensuing differential and algebraic equations:

$$\varphi^0(\xi) : 3Ar_0^2\lambda_1 + 2Br_0^3\lambda_1 - 6r_0\omega = 0 \quad (16)$$

$$\varphi^{-1}(\xi) : 6Ar_0r_1\lambda_1\varphi' + 6Br_0^2r_1\lambda_1\varphi' - 6r_1\omega\varphi' + 6r_1(M\lambda_1^3 + N\lambda_1\lambda_2^2)\varphi^{(3)} = 0 \quad (17)$$

$$\varphi^{-2}(\xi) : 3Ar_1^2\lambda_1\varphi'^2 + 6Br_0r_1^2\lambda_1\varphi'^2 - 18r_1(M\lambda_1^3 + N\lambda_1\lambda_2^2)\varphi'\varphi'' = 0 \quad (18)$$

$$\varphi^{-3}(\xi) : 2Br_1^3\lambda_1 + 12r_1(M\lambda_1^3 + N\lambda_1\lambda_2^2) = 0 \quad (19)$$

We use the computational software to unravel the algebraic Eqs. (16) and (19), we attain the subsequent values

$$r_0 = 0, r_0 = \frac{-3A\lambda_1 - \sqrt{3}\sqrt{3A^2\lambda_1^2 + 16B\lambda_1\omega}}{4B\lambda_1}$$

and

$r_1 = \pm\frac{\sqrt{6}\sqrt{-M\lambda_1^2 - N\lambda_2^2}}{\sqrt{B}}$, since $r_1 \neq 0$.

Solving Eq. (18) to extract the wave function $\varphi(\xi)$, we obtain

$$\varphi(\xi) = c_2 + \frac{6c_1 e^{\frac{(Ar_1 + 2Br_0r_1)}{6(M\lambda_1^2 + N\lambda_2^2)}\xi}(M\lambda_1^2 + N\lambda_2^2)}{Ar_1 + 2Br_0r_1} \quad (20)$$

Case 1:. *When* $r_0 = 0$ *and* $r_1 = \pm\frac{\sqrt{6}\sqrt{-M\lambda_1^2 - N\lambda_2^2}}{\sqrt{B}}$.

Therefore, we attain the value of wave velocity after putting the values of r_0, r_1 and $\varphi(\xi)$ into equation (17)

$$\omega = -\frac{A^2\lambda_1}{6B}$$

We acquire the ensuing solitary wave solution by using the values of r_0, r_1 and $\varphi(\xi)$ into equation (13)

$$w(\xi) = -\frac{Ac_1}{B(c_1 \mp Ae^{\pm S\xi}c_2)} \quad (21)$$

where c_1 and c_2 are arbitrary constants and $S = \frac{A}{\sqrt{6}\sqrt{B}\sqrt{-M\lambda_1^2 - N\lambda_2^2}}$.

Transforming the solution (21) into hyperbolic identities, we attain

$$w(\xi) = -\frac{Ac_1\left(\cosh\left(\frac{S}{2}\xi\right) \mp \sinh\left(\frac{S}{2}\xi\right)\right)}{B\left((c_1 \mp Gc_2)\cosh\left(\frac{S}{2}\xi\right) \mp (c_1 \pm Gc_2)\sinh\left(\frac{S}{2}\xi\right)\right)} \quad (22)$$

Since c_1 and c_2 are unknown constants, thus we can take the value of constants arbitrarily. If we choose $c_1 = \pm 1$ and $c_2 = \pm\frac{1}{S}$ (or $c_1 = \mp 1$ and $c_2 = \mp\frac{1}{S}$), therefore we achieve the subsequent solitary wave solutions:

$$w(\xi) = -\frac{A}{2B}\left(1 \mp \coth\left(\frac{S}{2}\xi\right)\right) \quad (23)$$

$$w(\xi) = -\frac{A}{2B}\left(1 \pm \tanh\left(\frac{S}{2}\xi\right)\right) \quad (24)$$

By using the transformation $\xi = \lambda_1 x + \lambda_2 y - \omega t$, the solutions (23) and (24) reduces to

$$w(x,y,t) = -\frac{A}{2B}\left(1 \mp \coth\left(\frac{S}{2}(\lambda_1 x + \lambda_2 y - \omega t)\right)\right) \tag{25}$$

$$w(x,y,t) = -\frac{A}{2B}\left(1 \pm \tanh\left(\frac{S}{2}(\lambda_1 x + \lambda_2 y - \omega t)\right)\right) \tag{26}$$

Transforming the solutions (25) and (26) from hyperbolic identities into trigonometric identities, we attain

$$w(x,y,t) = -\frac{A}{2B}\left(1 \mp i\cot\left(i\frac{S}{2}(\lambda_1 x + \lambda_2 y - \omega t)\right)\right) \tag{27}$$

$$w(x,y,t) = -\frac{A}{2B}\left(1 \mp i\tan\left(i\frac{S}{2}(\lambda_1 x + \lambda_2 y - \omega t)\right)\right) \tag{28}$$

The solutions (25), (26), (27) and (28) represent the singular kink, kink, anti-kink and singular bell shaped solutions.

Case 2:. *When* $r_0 = \frac{-3A\lambda_1 - \sqrt{3}\sqrt{3A^2\lambda_1^2 + 16B\lambda_1\omega}}{4B\lambda_1}$ *and* $r_1 = \pm\frac{\sqrt{6}\sqrt{-M\lambda_1^2 - N\lambda_2^2}}{\sqrt{B}}$.

Using the values of r_0, r_1 and $\varphi(\xi)$ into Eq. (13), we attain the next solitary wave solution

$$w(\xi) = \pm\frac{AQc_4}{B(c_3 e^{\pm Q\xi} \pm Qc_4)} \tag{29}$$

where c_3 and c_4 are arbitrary constants and $Q = \frac{A\lambda_1\sqrt{-M\lambda_1^2 - N\lambda_2^2}}{\sqrt{6}\sqrt{B(M\lambda_1^3 + N\lambda_1\lambda_2^2)}}$.

Transforming the Eq. (29) into hyperbolic identities, we attain

$$w(\xi) = \pm\frac{AQc_4\left(\cosh\left(\frac{Q}{2}\xi\right) \mp \sinh\left(\frac{Q}{2}\xi\right)\right)}{B\left((c_3 \pm Qc_4)\cosh\left(\frac{Q}{2}\xi\right) \pm (c_3 \mp Qc_4)\sinh\left(\frac{Q}{2}\xi\right)\right)} \tag{30}$$

Since c_3 and c_4 are unknown constants, thus we can take the value of constants arbitrarily. If we choose $c_3 = \pm Q$ and $c_4 = \pm 1$ (or $c_3 = \mp Q$ and $c_4 = \mp 1$), therefore we achieve the subsequent solitary wave solutions:

$$w(\xi) = \frac{A}{2B}\left(1 \mp \tanh\left(\frac{Q}{2}\xi\right)\right) \tag{31}$$

$$w(\xi) = \frac{A}{2B}\left(1 \pm \coth\left(\frac{Q}{2}\xi\right)\right) \tag{32}$$

By using the transformation $\xi = \lambda_1 x + \lambda_2 y - \lambda_3 t$, the solutions (31) and (32) reduces to

$$w(x,y,t) = \frac{A}{2B}\left(1 \mp \tanh\left(\frac{Q}{2}(\lambda_1 x + \lambda_2 y - \omega t)\right)\right) \tag{33}$$

$$w(x,y,t) = \frac{A}{2B}\left(1 \pm \coth\left(\frac{Q}{2}(\lambda_1 x + \lambda_2 y - \omega t)\right)\right) \tag{34}$$

Transforming the solutions (33) and (34) from hyperbolic identities into trigonometric identities, we obtain

$$w(x,y,t) = \frac{A}{2B}\left(1 \pm i\tan\left(i\frac{Q}{2}(\lambda_1 x + \lambda_2 y - \omega t)\right)\right) \tag{35}$$

$$w(x,y,t) = \frac{A}{2B}\left(1 \pm i\cot\left(i\frac{Q}{2}(\lambda_1 x + \lambda_2 y - \omega t)\right)\right) \tag{36}$$

These solutions represent the bell shape, singular bell shape, kink and singular kink type solitons, which narrates the current and voltage in the NETL problems.

The SGE method

We suppose the ensuing sine-Gordon equation [56–63]:

$$u_{xx} - u_{tt} = \tau^2 \sin(u) \tag{37}$$

where τ is a constant and $u = u(x,t)$ is the unknown wave function.

The partial differential equation (37) transforms into an ordinary differential equation by using the wave transformation $u(x,t) = V(\lambda(x - ct)) = V(\xi)$, where the independent variable ξ is the function of x, t. Therefore, Eq. (37) becomes

$$V'' = \frac{\tau^2}{\lambda^2(1 - c^2)}\sin(V) \tag{38}$$

where $V = V(\xi)$, c is the velocity of travelling wave, λ is the wave number and, ξ is the wave variable.

Integrating equation (38) with respect to ξ after multiplication by V', we obtain

$$\left(\frac{V'}{2}\right)^2 = \frac{\tau^2}{\lambda^2(1 - c^2)}\sin^2\left(\frac{V}{2}\right) + Q \tag{39}$$

where Q is constant of integration.

The relation (40) is obtained by substituting the assumption $w(\xi) = \frac{V}{2}$, $Q = 0$ and $d^2 = \frac{\tau^2}{\lambda^2(1-c^2)}$ into (39)

$$w' = d\sin(w) \tag{40}$$

If we choose $d = 1$, therefore the relation (40) becomes

$$w' = \sin(w) \tag{41}$$

The relation (41) is solvable because it is variable separable. Therefore, the relation (42) and (43) are obtained by solving Eq. (41)

$$\sin(w) = \sin(w(\xi)) = \left.\frac{2qe^{\xi}}{q^2 e^{2\xi} + 1}\right|_{q=1} = \operatorname{sech}(\xi) \tag{42}$$

$$\cos(w) = \cos(w(\xi)) = \left.\frac{2qe^{\xi}}{q^2 e^{2\xi} + 1}\right|_{q=1} = \tanh(\xi) \tag{43}$$

here q is an integral constant, such that $q \neq 0$.

Now, consider a general NLEE with dependent variable u and independent variable x, t in the form

$$Q(u, u_x, u_t, u_{xx}, u_{tt} \cdots) = 0 \tag{44}$$

By means of traveling wave variable $\xi = \lambda(x - ct)$, $u(x,t) = V(\xi)$, we can reduce Eq. (44) into the ensuing ODE

$$P(V, V', V'', V''', \cdots) = 0 \tag{45}$$

Now, we assume the following solution of the Eq. (45)

$$V(\xi) = A_0 + \sum_{j=1}^{N} \tanh^{j-1}(\xi)\left[A_j \tanh(\xi) + B_j \operatorname{sech}(\xi)\right] \tag{46}$$

Substituting the Eqs. (42) and (43) into Eq. (46), we acquire the following result

$$V(w) = A_0 + \sum_{j=1}^{N} \cos^{j-1}(w)\left[A_j \cos(w) + B_j \sin(w)\right] \tag{47}$$

The positive integer N of Eq. (47) can be evaluated via balancing suitable nonlinear terms and the highest linear derivative term appearing in equation (45). After using the solution (47) into equation (45), we obtain some algebraic equation by equalizing the coefficients of $\cos^i(w)\sin^j(w)$ to zero. We found the values of A_0, A_j, B_j, λ and c via solving these equations with the help of Maple 2019. We acquire various types of soliton solutions in a unique and simple way by substituting these values into (46).

Implementation of the SGE method

In this subsection, the soliton solutions to the modified Zakharov-Kuznetsov equation in the electrical transmission line are established via the SGE method.

Using the classical wave transformation $v(x,y,t) = u(\xi)$,

$\xi = r_1x + r_2y - r_3t$ where r_3 is the wave speed and r_1, r_2 are the wave number, Eq. (10) converts into the subsequent ODE:

$$-r_3u + Ar_1\frac{u^2}{2} + Br_1\frac{u^3}{3} + \left(Mr_1^3 + Nr_1r_2^2\right)u'' = 0 \tag{48}$$

where the second prime of $u(\xi)$ specifies the second derivative.

The balance between the terms u'' and u^3, yield $N = 1$.

Therefore, the solution shape of (48) is

$$u(\xi) = A_0 + B_1 sech(\xi) + A_1 tanh(\xi) \tag{49}$$

and therefore from (47), we derive

$$u(w) = A_0 + B_1 sin(w) + A_1 cos(w) \tag{50}$$

where A_0, B_1 and A_1 are unknown constants to be evaluated, such that both A_1 or B_1 can't be zero at the same time, but A_1 or B_1 can be zero.

Inserting this solution shape into Eq. (48) yields

$$\begin{aligned}&12A_1r_1\left(r_2^2N + \frac{1}{6}BA_1^2 - \frac{1}{2}BB_1^2 + Mr_1^2\right)\cos^2(w)\\&+3\left(4B_1\left(r_2^2N + \frac{1}{2}BA_1^2 - \frac{1}{6}BB_1^2 + Mr_1^2\right)sin(w) + (A_1 - B_1)(A_1 + B_1)(2BA_0 + A)\right)r_1\cos^2(w)\\&+6\left(B_1r_1(2BA_0 + A)sin(w) - 2Mr_1^3 + \left(BA_0^2 + BB_1^2 - 2Nr_2^2 + AA_0\right)r_1 - r_3\right)A_1cos(w)\\&+6B_1\left(-Mr_1^3 + \left(-r_2^2N + A_0^2B + \frac{1}{3}BB_1^2 + AA_0\right)r_1 - r_3\right)sin(w)\\&+\left((6BA_0 + 3A)B_1^2 + 2BA_0^3 + 3AA_0^2\right)r_1 - 6A_0r_3 = 0\end{aligned} \tag{51}$$

The subsequent system of algebraic equations is obtained after collecting the coefficients of $\cos^i(w)\sin^j(w)$ and setting them to zero

$$\text{Constant}: \left((6BA_0 + 3A)B_1^2 + 2BA_0^3 + 3AA_0^2\right)r_1 - 6A_0r_3 = 0 \tag{52}$$

$$\cos(w): 6A_1\left(-2Mr_1^3 + \left(BA_0^2 + BB_1^2 - 2Nr_2^2 + AA_0\right)r_1 - r_3\right) = 0 \tag{53}$$

$$\cos^2(w): 3(A_1 - B_1)(A_1 + B_1)(2BA_0 + A)r_1 = 0 \tag{54}$$

$$\cos^3(w): 12A_1r_1(r_2^2N + \frac{1}{6}BA_1^2 - \frac{1}{2}BB_1^2 + Mr_1^2) = 0 \tag{55}$$

$$\sin(w): 6B_1\left(-Mr_1^3 + \left(-r_2^2N + A_0^2B + \frac{1}{3}BB_1^2 + AA_0\right)r_1 - r_3\right) = 0 \tag{56}$$

$$\cos(w)\sin(w): 6A_1B_1r_1(2BA_0 + A) = 0 \tag{57}$$

$$\cos^2(w)\sin(w): 12B_1\left(r_2^2N + \frac{1}{2}BA_1^2 - \frac{1}{6}BB_1^2 + Mr_1^2\right)r_1 = 0 \tag{58}$$

Therefore, the solutions of the system of algebraic equations (52)-(58) through symbolic computation package can be described as

Case 1:. *When*

$$r_1 = \pm\frac{\sqrt{3}\sqrt{\left(BM\left(-12BNr_2^2 + A^2\right)\right)}}{6BM}, r_2 = r_2, r_3 = \mp\frac{A^2\sqrt{3}\sqrt{\left(BM\left(-12BNr_2^2 + A^2\right)\right)}}{36B^2M}, A_0 = -\frac{A}{2B}, A_1 = 0, B_1 = \pm\frac{\sqrt{2}A}{2B}$$

Therefore, we obtain the following soliton solutions to the mZK equation by using the earlier stated values into the solution (49)

$$u_1(\xi) = -\frac{A}{2B} \pm \frac{A}{\sqrt{2B}}\text{sech}(\xi) \tag{59}$$

Using the transformation $\xi = r_1x + r_2y - r_3t$, the solution (59) becomes

$$u_1(x,y,t) = -\frac{A}{2B} \pm \frac{A}{\sqrt{2B}}\text{sech}(r_1x + r_2y - r_3t) \tag{60}$$

which is the solitary wave solution of mZK equation in the electrical transmission line.

Transforming the hyperbolic identities $\text{sech}(\cdot)$ into trigonometric identities $\sec(i\cdot)$, the solution (60) can be written as

$$u_1(x,y,t) = -\frac{A}{2B} \pm \frac{A}{\sqrt{2B}}\sec(i(r_1x + r_2y - r_3t)) \tag{61}$$

which is the other solitary wave solution of mZK equation in the electrical transmission line.

Case 2:. *When*

$$r_1 = \pm\frac{\sqrt{\left(-6BM\left(24Nr_2^2B + A^2\right)\right)}}{12BM}, r_2 = r_2, r_3 = \mp\frac{A^2\sqrt{\left(-6BM\left(24Nr_2^2B + A^2\right)\right)}}{72B^2M}, A_0 = -\frac{A}{2B}, A_1 = \frac{A}{2B}, B_1 = 0$$

Thus, we determine the next soliton solutions to the mZK equation by replacing the earlier stated values into the solution (49)

$$u_2(\xi) = -\frac{A}{2B}(1 - \tanh(\xi)) \tag{62}$$

Substituting the transformation $\xi = r_1x + r_2y - r_3t$, the solution (62) becomes

$$u_2(x,y,t) = -\frac{A}{2B}(1 - \tanh(r_1x + r_2y - r_3t)) \tag{63}$$

which is the solitary wave solution of mZK equation in the electrical transmission line.

Transforming the hyperbolic identities $\tanh(\xi)$ into trigonometric identities $-i\tan(i\xi)$, the solution (63) can be written as

$$u_2(x,y,t) = -\frac{A}{2B}(1 + i\tan(i(r_1x + r_2y - r_3t))) \tag{64}$$

which is the other solitary wave solution of mZK equation in the electrical transmission line.

Case 3:. *When*

$$r_1 = \pm\frac{\sqrt{\left(-6BM\left(24Nr_2^2B + A^2\right)\right)}}{12BM}, r_2 = r_2, r_3 = \mp\frac{A^2\sqrt{\left(-6BM\left(24Nr_2^2B + A^2\right)\right)}}{72B^2M}, A_0 = -\frac{A}{2B}, A_1 = -\frac{A}{2B}, B_1 = 0$$

So, we achieve the subsequent soliton solutions to the mZK equation by substituting the earlier stated values into the solution (49)

$$u_3(\xi) = -\frac{A}{2B}(1 + \tanh(\xi)) \tag{65}$$

Putting the transformation $\xi = r_1x + r_2y - r_3t$, the solution (65) gives

$$u_3(x,y,t) = -\frac{A}{2B}(1 + \tanh(r_1x + r_2y - r_3t)) \tag{66}$$

which is the soliton solution of mZK equation in the electrical transmission line.

Transforming the hyperbolic identities $\tanh(\xi)$ into trigonometric identities $-i\tan(i\xi)$, the solution (66) can be written as

$$u_3(x,y,t) = -\frac{A}{2B}(1 - i\tan(i(r_1x + r_2y - r_3t))) \quad (67)$$

which is the other soliton solution of mZK equation in the electrical transmission line.

Case 4:. *When*

$$r_1 = \pm\frac{\sqrt{\left(-6BM\left(6Nr_2^2B + A^2\right)\right)}}{6BM}, r_2 = r_2, r_3 = \mp\frac{A^2\sqrt{\left(-6BM\left(6Nr_2^2B + A^2\right)\right)}}{36B^2M}, A_0 = -\frac{A}{2B}, A_1 = -\frac{A}{2B}, B_1 = \pm\frac{iA}{2B}$$

Consequently, we achieve the following travelling wave solutions to the mZK equation by replacing the earlier stated values into the solution (49)

$$u_4(\xi) = -\frac{A}{2B}(1 \mp i\mathrm{sech}(\xi) + \tanh(\xi)) \quad (68)$$

Substituting the transformation $\xi = r_1x + r_2y - r_3t$, the solution (68) gives

$$u_4(x,y,t) = -\frac{A}{2B}(1 \mp i\mathrm{sech}(r_1x + r_2y - r_3t) + \tanh(r_1x + r_2y - r_3t)) \quad (69)$$

which is the solitary wave solution of mZK equation in electrical transmission line.

Transforming the hyperbolic identities into trigonometric identities, the solution (69) can be written as

$$u_4(x,y,t) = -\frac{A}{2B}(1 \mp i\sec(i(r_1x + r_2y - r_3t)) - i\tan(i(r_1x + r_2y - r_3t))) \quad (70)$$

which is the other travelling wave solution of mZK equation in the electrical transmission line.

Case 5:. *When*

$$r_1 = \pm\frac{\sqrt{\left(-6BM\left(6Nr_2^2B + A^2\right)\right)}}{6BM}, r_2 = r_2, r_3 = \mp\frac{A^2\sqrt{\left(-6BM\left(6Nr_2^2B + A^2\right)\right)}}{36B^2M}, A_0 = -\frac{A}{2B}, A_1 = \frac{A}{2B}, B_1 = \pm\frac{iA}{2B}$$

Thus, we get the solitary wave solutions to the mZK equation by substituting the earlier stated values into the solution (49)

$$u_5(\xi) = -\frac{A}{2B}(1 \mp i\mathrm{sech}(\xi) - \tanh(\xi)) \quad (71)$$

which is the solitary wave solution of mZK equation in the electrical transmission line.

Substituting the transformation $\xi = r_1x + r_2y - r_3t$, the solution (71) turns out to be

$$u_5(x,y,t) = -\frac{A}{2B}(1 \mp isech(r_1x + r_2y - r_3t) - tanh(r_1x + r_2y - r_3t)) \quad (72)$$

Transforming the hyperbolic identities into trigonometric identities, the solution (72) can be written as

$$u_5(x,y,t) = -\frac{A}{2B}(1 \mp i\sec(i(r_1x + r_2y - r_3t)) + i\tan(i(r_1x + r_2y - r_3t))) \quad (73)$$

which is the other solitary wave solutions of mZK equation in the electrical transmission line.

In this article, we have founded some well-known and fresh soliton solutions which might help to analyze current and voltage in electrical transmission lines.

Graphical illustrations

In this section, we illustrate the three dimensional and density plots of the acquired results for different standards of the transmission line parameters. By applying the MSE method, we have achieved some renowned travelling wave solution as, bell type, singular bell type, kink type, singular kink type etc. Also, such soliton solutions of the mZK equation can be reproduced by the SGE method. For different values of the transmission line parameters of this equation we display have shown the density plot which highlight the dynamical behavior of the current and voltage (or energy) in electrical transmission line problems.

Graphical illustration of Eq. (10) solutions via MSE technique

From the mZK equation of the electrical transmission line, we obtain several types of solitary wave solutions which are illustratedd below in three-dimensional and density plots.

The graph of the solitons (23) and (25) are the singular kink shape solution which describes the formation of Rogue waves and illustrated only the soliton (25) for the values of the parameters $A = 0.77, B = 1.28$, $S = 1.34, \lambda_1 = 1.15$ and $\lambda_2 = 1.19 \mathrm{at} t = 0$. The center position of travelling wave is imaginary for singular type solution. Fig. 2 displays the three-dimensional and density graph within the interval $-10 \le x$, $y \le 10$.

The travelling wave solution (26) presents the t-shape soliton with the limit $-10 \le x,\ y \le 10$. Fig. 3 shows the density and three-dimensional sketch for the standards of the transmission line parameters $A = 1, B = 0.7, S = 1.26, \lambda_1 = 1.77$ and $\lambda_2 = 0.29$ at $t = 0$. The t-shape soliton descends from left to right.

The profile of the solution (26) in Fig. 4 is a kink type soliton which ascend from left to right. The density and three-dimensional graph are drawn for the values $A = 0.96, B = 0.66, S = 1.34, \lambda_1 = 1.59$ and $\lambda_2 = 1.89$ at $t = 0$ within the interval $-10 \le x,y \le 10$.

The density and three-dimensional graph of the soliton (27) in Fig. 5 represent the singular bell shape solution which describes the structure of Rogue waves. The middle space of the singular soliton is imaginary. The singular bell shaped solution is obtained for the values $A = 0.87$, $B = 0.28$, $S = 0.39$, $\lambda_1 = 1.28$ and $\lambda_2 = 1.4$ at $t = 0$ within the limit $-10 \le x,y \le 10$.

The soliton (28) presents stable soliton solutions, which describes the current and voltage in transmission line. Fig. 6 of the solution (28) displays the three-dimensional and density graph for the values $A = 0.26$, $B = 0.13$, $S = 0.15$, $\lambda_1 = -0.21$ and $\lambda_2 = 0.3$ at $t = 0$ within the interim $-10 \le x,y \le 10$.

The soliton (28) presents a single stable solution, which has unbounded wings on both sides. Fig. 7 of the solution (28) displays the three-dimensional and density graph for the values $A = -0.9$, $B = -1.05$, $S = 0.3$, $\lambda_1 = 1.59$ and $\lambda_2 = 0.9$ at $t = 0$ within the range $-10 \le x,y \le 10$.

Fig. 8 of the solution (33) shows the three-dimensional and density graph for the values $A = 1.36, B = 0.94, S = 0.09, \lambda_1 = 0.03$ and $\lambda_2 = 1.83$ at $t = 0$ within the interval $-5 \le x,y \le 5$. This is a flat stable soliton solution.

Fig. 9 of the solution (34) displays the three-dimensional and density graph for the values $A = 0.7, B = -1.6, S = 3, \lambda_1 = 11.1$ and $\lambda_2 = 7.9$ at $t = 0$ within the interval $-20 \le x,\ y \le 20$. This solution presents singular soliton solutions.

The absolute value of the solution (35) shows the three-dimensional and density graph for $A = -0.01, B = -0.9, S = 0.39, \lambda_1 = 1.72$ and $\lambda_2 = -1.71$ at $t = 0$ within the interval $-20 \le x,y \le 20$. Fig. 10 presents the kink soliton solutions.

The absolute value of the solution (36) gives the three-dimensional and density graph for $A = 3.08, B = 2.92, S = -0.1, \lambda_1 = 0.63$ and $\lambda_2 = 0.22$ at $t = 0$ within the interval $-10 \le x,y \le 10$. Fig. 11 presents the singular bell-shaped soliton solutions, which has infinite wings.

Thus, for different sets of the transmission line parameters, we

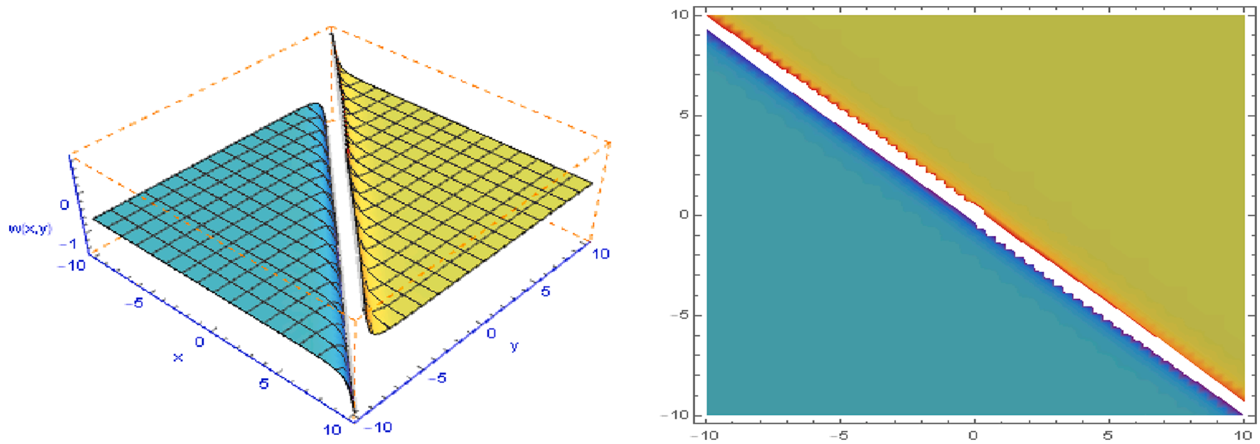

Fig. 2. The profile of the singular kink solution (25) for $A = 0.77$, $B = 1.28$, $S = 1.34$, $\lambda_1 = 1.15$ and $\lambda_2 = 1.19$ at $t = 0$.

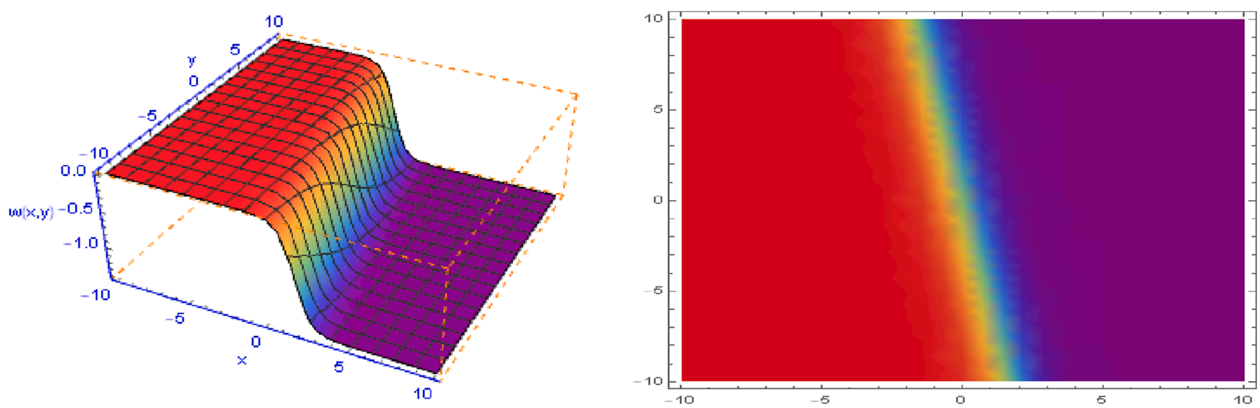

Fig. 3. The profile of the t-shape solution (26) for $A = 1$, $B = 0.7$, $S = 1.26$, $\lambda_1 = 1.77$ and $\lambda_2 = 0.29$ at $t = 0$.

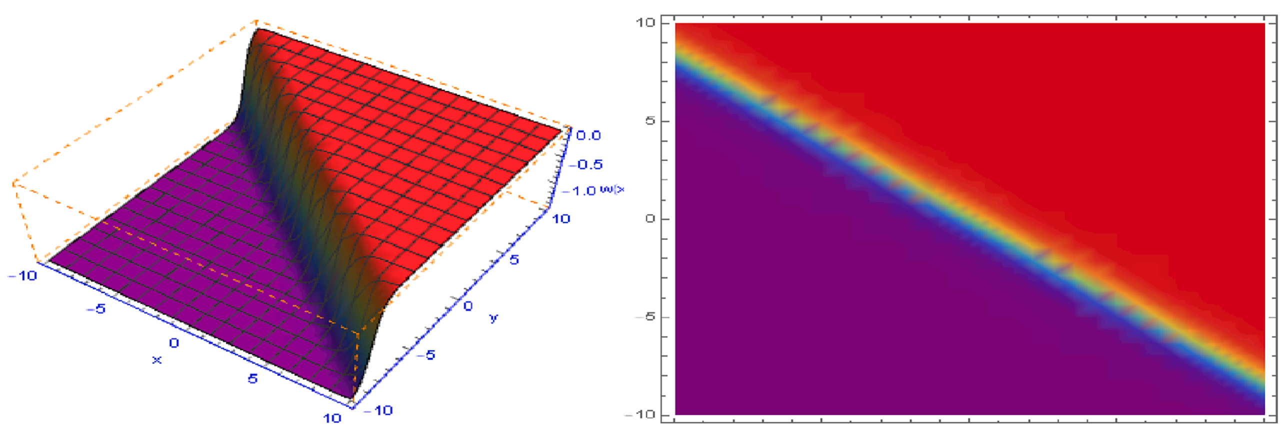

Fig. 4. The profile of the kink solution (26) for $A = 0.96$, $B = 0.66$, $S = 1.34$, $\lambda_1 = 1.59$ and $\lambda_2 = 1.89$ at $t = 0$.

achieve the different characteristics of the solution such as, singular bell shape, singular kink form, kink form, and some new forms of solitons.

Graphical illustration of Eq. (10) solutions via SGE method

The three-dimensional and density graph of the solution (60) presents the bell type solution for $B = 0.96$, $A = 1.09$, $r_1 = 0.9$ and $r_2 = 0.92$ at $t = 0$ with the interval $-10 \le x, y \le 10$. Fig. 12 shows the bell shape soliton with infinite wings on both sides.

The absolute value of the solution (61) gives the three-dimensional and density graph for the values $A = 0.62$, $B = 0.01$, $r_1 = 0.28$ and $r_2 = 0.26$ at $t = 0$ within the interval $-10 \le x, y \le 10$. Fig. 13 presents the three soliton solutions.

The soliton (63) presents stable kink shape solutions, which has infinite wings on both sides. Fig. 14 of the solution (63) shows the three-dimensional and density graph for the values $A = 1.34$, $B = 0.75$, $r_1 = 1.45$ and $r_2 = 1.4$ at $t = 0$ within the limit $-10 \le x, y \le 10$.

The soliton (66) presents stable kink shape solutions, which has infinite wings. Fig. 15 of the solution (66) shows the three-dimensional and density graph for the values $A = 1.34$, $B = 0.75$, $r_1 = 1.45$ and $r_2 =$

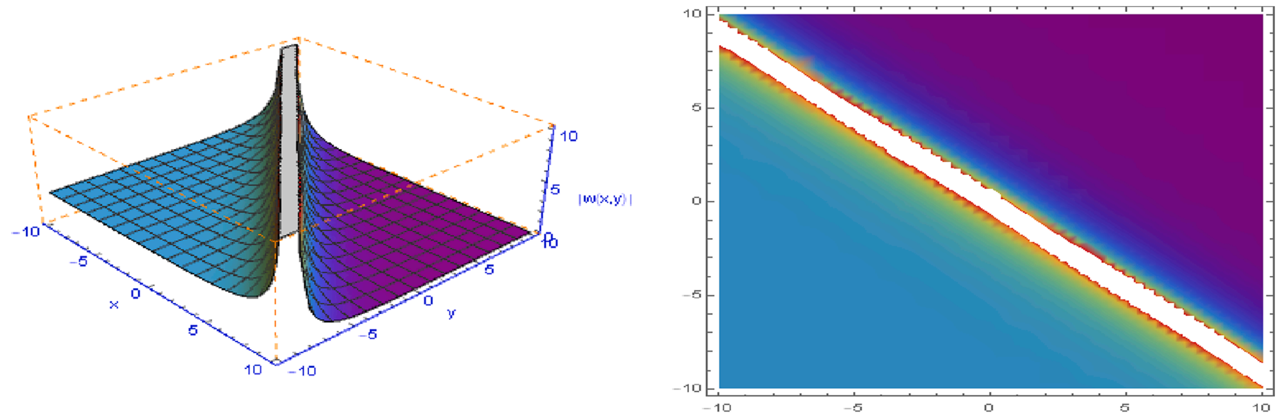

Fig. 5. The profile of the singular bell shape solution (27) for $A = 0.87$, $B = 0.28$, $S = 0.39$, $\lambda_1 = 1.28$ and $\lambda_2 = 1.4$ at $t = 0$.

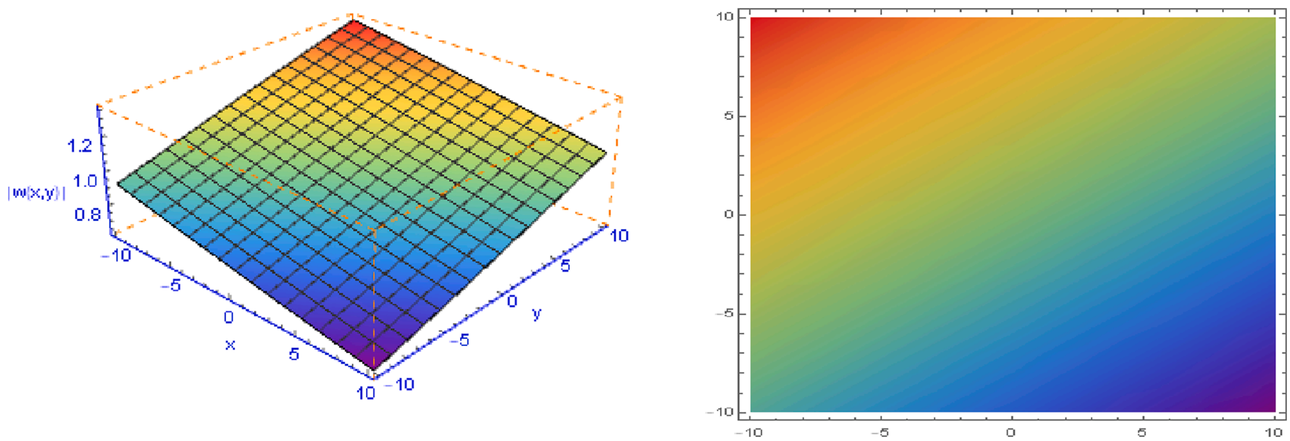

Fig. 6. The three dimensional and density plot of the general solution (28) for $B = 0.13$, $A = 0.26$, $S = 0.15$, $\lambda_1 = -0.21$ and $\lambda_2 = 0.3$ at $t = 0$.

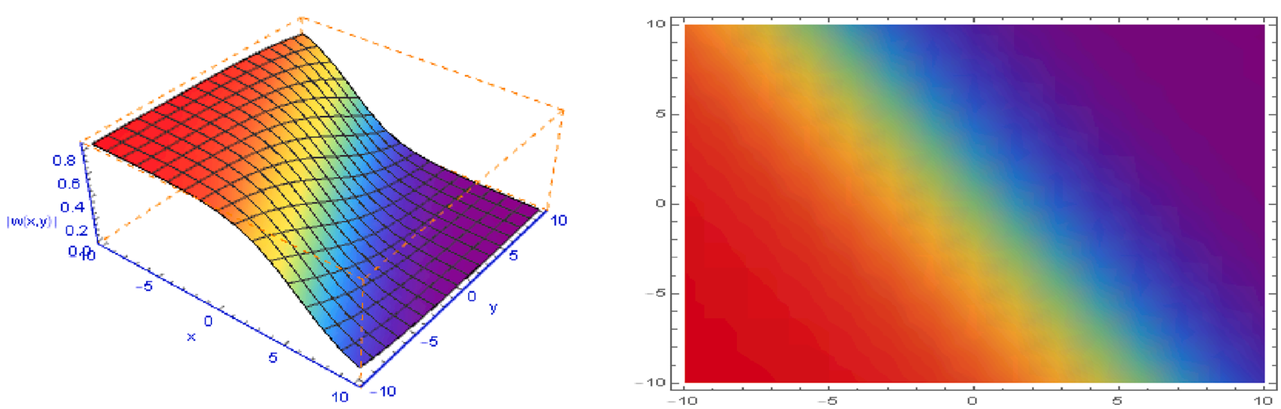

Fig. 7. The profile of the soliton solution (28) for $A = -0.9$, $B = -1.05$, $S = 0.3$, $\lambda_1 = 1.59$ and $\lambda_2 = 0.9$ at $t = 0$.

1.4 at $t = 0$ within the interim $-10 \leq x, y \leq 10$.

The absolute value of the solution (69) gives the three-dimensional and density graph for $A = 1.42$, $B = 1.42$, $r_1 = 0.13$ and $r_2 = -0.78$ at $t = 0$ within the interval $0 \leq x, y \leq 10$ and for density plot $-10 \leq x$, $y \leq 10$. Fig. 16 presents the one soliton solution.

The absolute value of the solution (72) gives the three-dimensional and density graph for $A = -0.84$, $B = -0.29$, $r_1 = -0.25$ and $r_2 = -0.06$ at $t = 0$ within the interval $-20 \leq x, y \leq 20$ and for density plot $-10 \leq x, y \leq 10$. Fig. 17 presents the one soliton solution.

Thus, for several sets of the transmission line parameters, we achieve different types solutions such as, bell type soliton, periodic type soliton, kink type soliton and some new type of solutions.

Conclusion

The singular kink shape, kink shape soliton, bell-shape soliton, singular bell-shape soliton, periodic soliton and some advanced traveling wave solutions have been introduced in this article to examine the voltage in transmission line problems by using the MSE and SGE techniques. Through the MSE technique, we have derived traveling wave solutions (25)-(28) and (33)-(36) to the modified mZK equation (10) and the solutions (60), (61), (63), (66), (69), (72) through the SGE method.

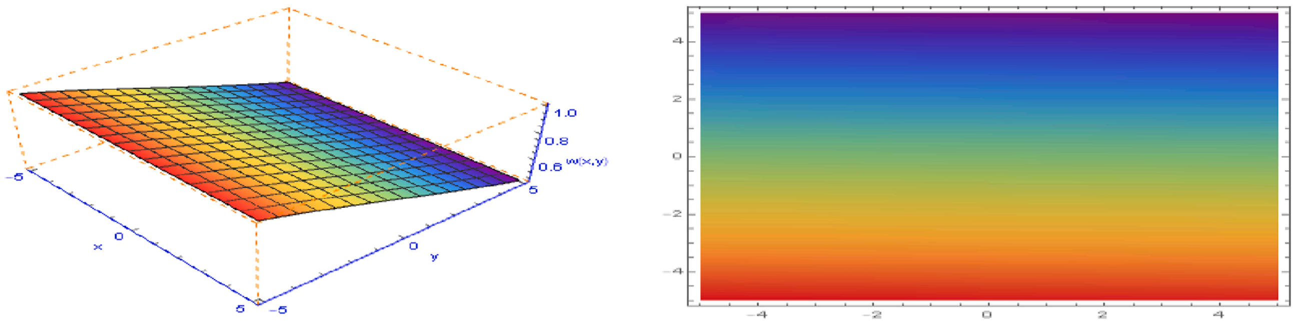

Fig. 8. The outline of the general solution (33) for $B = 0.94$, $A = 1.36$, $S = 0.09$, $\lambda_1 = 0.03$ and $\lambda_2 = 1.83$ at $t = 0$.

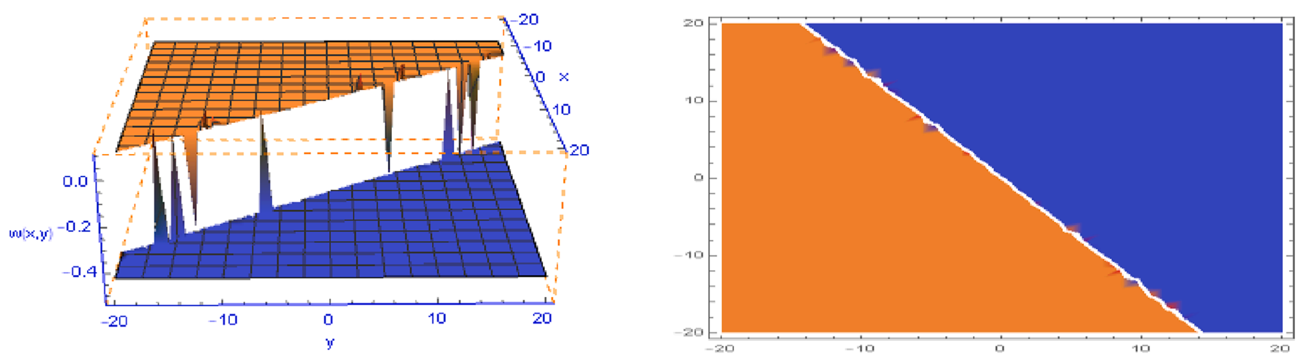

Fig. 9. The profile of the soliton solution (34) for $A = 0.7$, $B = -1.6$, $S = 3$, $\lambda_1 = 11.1$ and $\lambda_2 = 7.9$ at $t = 0$.

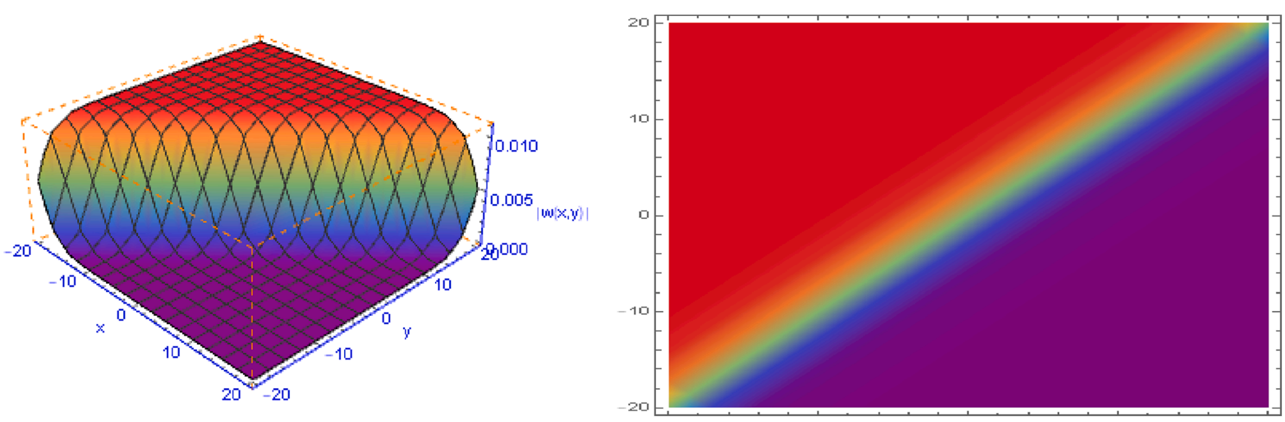

Fig. 10. The profile of the kink shape solution (35) for $A = -0.01$, $B = -0.9$, $S = 0.39$, $\lambda_1 = 1.72$ and $\lambda_2 = -1.71$ at $t = 0$.

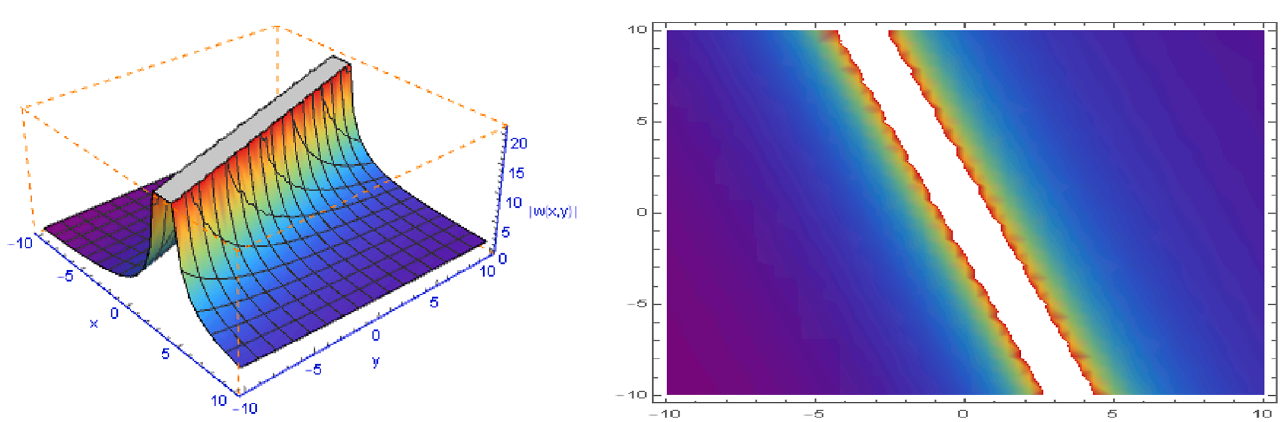

Fig. 11. The profile of the singular bell shape solution (36) for $A = 3.08$, $B = 2.92$, $S = -0.1$, $\lambda_1 = 0.63$ and $\lambda_2 = 0.22$ at $t = 0$.

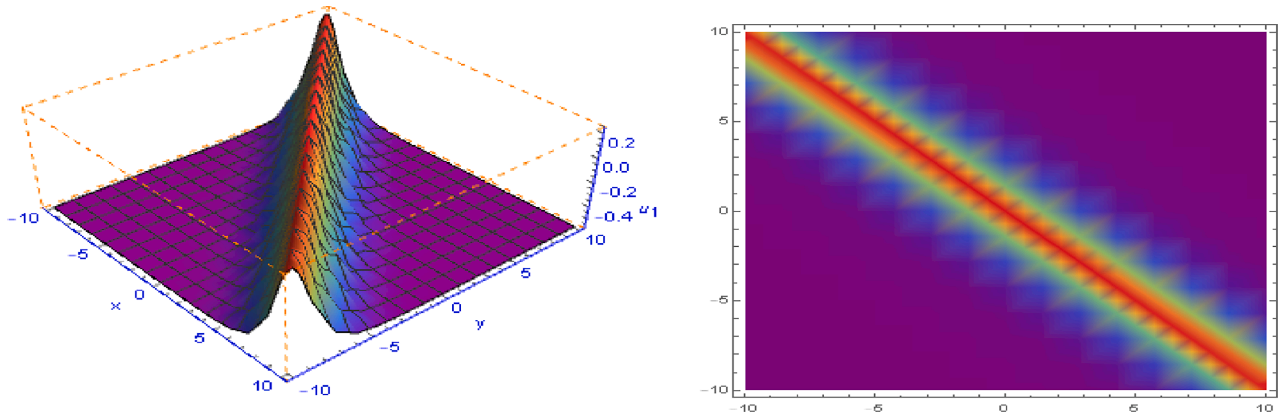

Fig. 12. The profile of the bell shape solution (60) for $A = 1.09$, $B = 0.96$, $r_1 = 0.9$ and $r_2 = 0.92$ at $t = 0$.

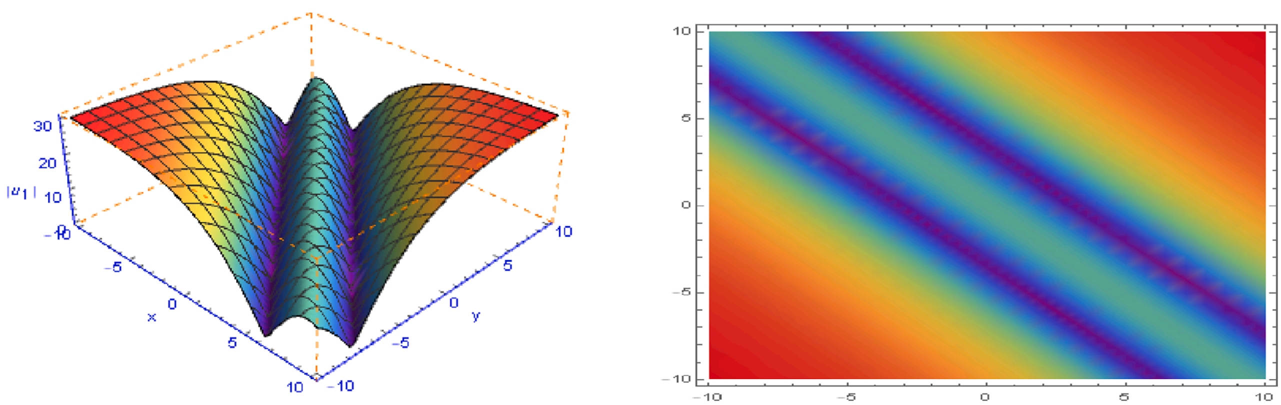

Fig. 13. The outline of the travelling wave solution (61) for $B = 0.01$, $A = 0.62$, r1 $= 0.28$ and $r_2 = 0.26$ at $t = 0$.

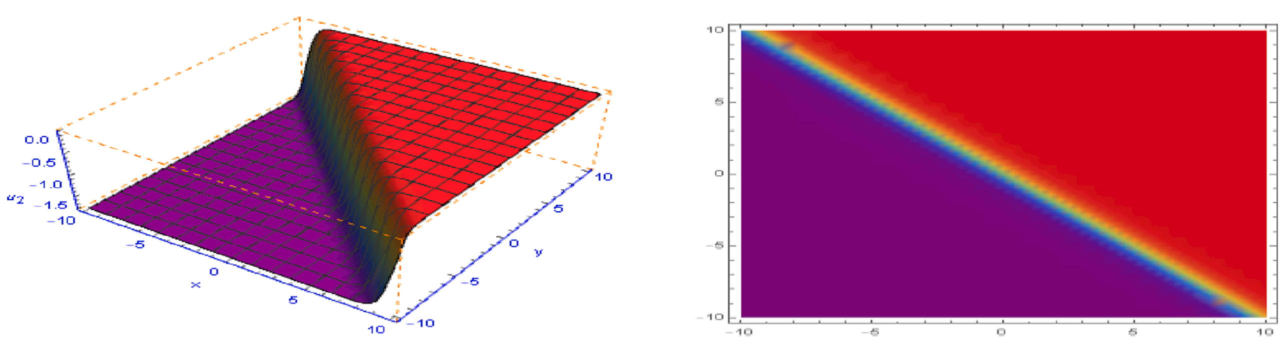

Fig. 14. The profile of the kink shape solution (63) for $A = 1.34$, $B = 0.75$, r1 $= 1.45$ and $r_2 = 1.4$ at $t = 0$.

The established solutions are stable, adaptable and capable to travel long distance. The obtained solutions illustrate the voltage and current flow that can be used to design the transmission lines. The density and 3D graph of the solutions have been presented. The developed solitons might play significant role in the modeling of long distance transmission lines in the engineering field including electric control theory, nonlinear transmission line, system identification, signal processing, optical fibers etc. This research confirms that the methods of SGE and MSE are efficient and useful in mathematical physics and engineering for extracting solitary wave solutions in other NLEEs. Our results show that the structures of the obtained wave solutions are multifarious in nonlinear dynamic system. In the next work, we will explore the non-autonomous wave solutions, that would be generated by different NLEEs when their coefficients are variables.

CRediT authorship contribution statement

M. Ali Akbar: Conceptualization, Methodology, Project administration, Supervision, Visualization, Writing - original draft. **Md. Abdul Kayum:** Data curation, Investigation, Methodology, Software, Writing - original draft. **M.S. Osman:** Conceptualization, Funding acquisition, Methodology, Resources, Software, Supervision, Validation, Writing - review & editing. **Abdel-Haleem Abdel-Aty:** Formal analysis, Investigation, Resources, Validation. **Hichem Eleuch:** Formal analysis, Funding acquisition, Resources, Software, Validation, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial

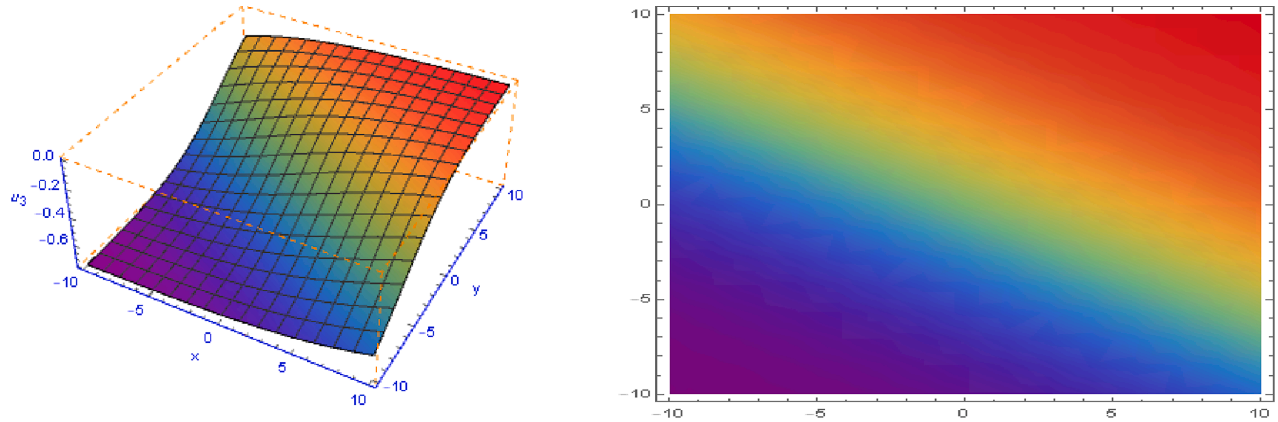

Fig. 15. The outline of the general solution (66) for $B = -0.97$, $A = -0.73$, r1 = -0.1 and $r_2 = -0.16$ at $t = 0$.

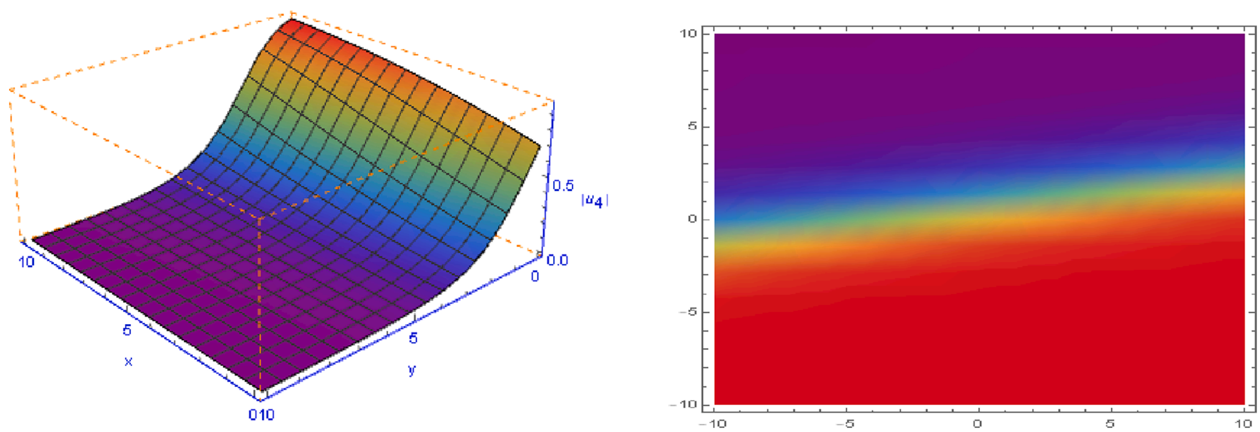

Fig. 16. The profile of the one-soliton solution (69) for $A = 1.42$, $B = 1.42$, r1 = 0.13 and $r_2 = -0.78$ at $t = 0$.

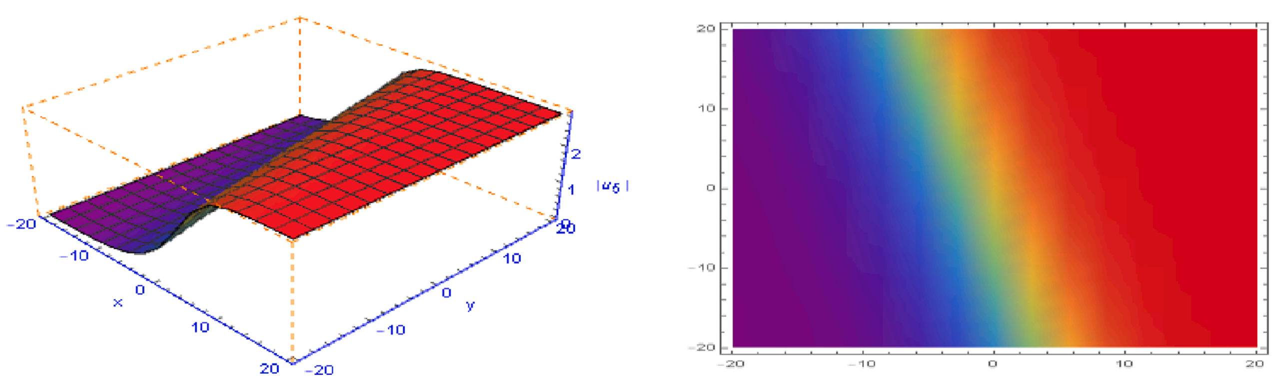

Fig. 17. The outline of the general solution (72) for $B = -0.29$, $A = -0.84$, r1 = -0.25 and $r_2 = -0.06$ at $t = 0$.

interests or personal relationships that could have appeared to influence the work reported in this paper.

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