

Nonclassical effects for a qubit coupled to a coherent two-mode cavity with intrinsic decoherence

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ABSTRACT

In this paper, we use the $su(1,1)$ -algebraic treatment to explore the effects of the intrinsic decoherence in a two-mode cavity containing a two-level atom. Each field is resonant with the qubit through a four-photon process. The role of the intrinsic decoherence and the superposition of the initial generalized Barut-Girardello coherent state on the quantum effects is investigated via different quantifiers as: atomic inversion, linear entropy and negativity. It is found that the non-linearity of the four-photon process leads to generate non-classical effects with a high oscillatory behavior. The superposition of the Barut-Girardello coherent state controls the dynamics of the purity loss and the entanglement. It is found that the nonclassical effects are very sensitive to the nonlinear interactions between the qubit and the two-mode cavity.

1. Introduction

Quantum coherence and quantum correlations, like purity, entanglement, discord, nonlocality and steering, are remarkable resources in quantum information [1–3]. Quantum entanglement (QE) is a specific type of quantum correlations [4–8], which has been extensively explored recently. Nowadays, quantum entanglement is considered as crucial to applications in quantum information, such as key distribution [9], quantum algorithm [10,11], cryptography [12] and quantum memory [13,14].

The qubit-field models were generalized to deal with the engineering nonlinear qubit-field couplings [15,16], which are applied in superconductor circuits [17]. The simplest theoretical description of these systems is the Jaynes-Cummings model (JC model) [18], that has impressive predictions for experimental effusive phenomena and applications [19], as: the atoms interacting with a multi-mode cavity, the multi-level atom inside a multi-mode cavity [20], multi-photon processes and Kerr-like medium [21].

Non-linear interaction between atoms and electromagnetic fields has important applications in the quantum information [22]. The nonlinear interactions allow the realization of the squeezed light [23].

The two-and three-mode cavity interaction can be converted to either parametric amplifier or parametric frequency converter [24,25].

Entangled states are important to the fields of quantum information. One of them is Barut-Girardello coherent state (B-GCS) that is the generalized entangled pair coherent state [26]. Many previous studies focused on exploring the non-classical properties of entangled quantum states, as they have many applications in quantum information theory [27]. In the same way of calculating standard coherent states, it is normal to generalize nonlinear single-mode coherent, to multi-mode, especially entangled pair coherent states [28,29]. Therefore, there are two types of two coherent states, the first is entangled pair coherent states and the second is separable coherent states. The entangled two-mode coherent states contain pair-coherent states [29]. B-GCS can be realized by using spherical harmonic functions [30]. Recently several methods to implement the B-GCS are proposed [31,32].

Quantum systems have decoherence problems due to their interactions with the external environment. Atomic inversion, the purity and the entanglement [33,34] are inevitably affected by the decoherences [35–38]. Even for close system the coherence could be lost. These type of decoherences is known as an intrinsic decoherence (ID). ID leads to that the purity and the entanglement are automatically deteriorated as

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the quantum system evolves [39]. The ID models are generated without the interaction of the qubit-cavity system with its surrounding environment, and the energy of the system is conserved, i.e., ID is realized without the dissipation. Where, the off-diagonal elements of the density matrix are intrinsically suppressed in the energy eigenstate basis. Most of the quantum effects originate from the decoherence [39]. Therefore, it is appropriate for investigating the decoherence effect on the quantum effects of the considered system. The intrinsic decoherence deteriorates the quantum effects when the system evolves. This ID has been extensively studied in [40–42].

The non-degenerate two-photon JC model [43] opens the door for the realization of the two-mode multi-photon laser. The quantum systems of the two-photon processes have revealed several nonclassical proprieties of multi-photon transitions. They have been experientially implemented for an atom-micromaser system [44]. With/without decoherence effect, the two-photon JC model is analytically described with and without the counter-rotating term [45]. The dynamics of the four-photon JC model were only studied without the damping effect [46].

Motivated by the intrinsic decoherence effects on the nonlinear interaction between a qubit and two-mode cavity, when the $su(1, 1)$ -system is initially in a superposition of generalized Barut-Girardello coherent state. We focus here on the application of the $su(1, 1)$ -algebraic concept to generalize the ID model describing the decoherence in a two-photon $su(1, 1)$ - $su(2)$ interaction. We then investigate the dynamics of some quantum effects namely, the atomic inversion, the purity loss and the entanglement for different initial coherent fields.

In this paper, the considered intrinsic damping model is presented in Sec. 2. While the dynamics of the entanglement and the mixture are discussed in Section 3. We present the conclusion in Section 4.

2. Hamiltonian and dynamics

Here, we consider two-mode cavity fields interacting with a qubit via the nondegenerate parametric amplifier terms [47]. The Hamiltonian of the qubit-cavity system is

$$\hat{H} = \frac{\omega_0}{4} \hat{\sigma}_z + \sum_{i=1}^2 \omega_i \left(\hat{\psi}_i^\dagger \hat{\psi}_i + \frac{1}{2} \right) + \tilde{\lambda} \left(\hat{\psi}_1^2 \hat{\psi}_2^2 \hat{\sigma}_+ + \hat{\psi}_1^{2\dagger} \hat{\psi}_2^{2\dagger} \hat{\sigma}_- \right), \quad (1)$$

where $\omega_i (i = 1, 2)$ represents the i th-mode frequency with the annihilation operators $\hat{\psi}_i$. $\tilde{\lambda}$ designs the coupling between the qubit and the two-mode cavity.

The dynamical operators can be calculated by the Heisenberg relationship. Therefore, the statistical results can be analyzed by employing these operators. The dynamical operator can be written as

$$\begin{aligned} i \frac{d}{dt} \hat{\psi}_j^\dagger \hat{\psi}_j &= -2\tilde{\lambda} \left(\hat{\psi}_1^2 \hat{\psi}_2^2 \hat{\sigma}_+ + \hat{\psi}_1^{2\dagger} \hat{\psi}_2^{2\dagger} \hat{\sigma}_- \right), \quad j = 1, 2 \\ -i \frac{d}{dt} \hat{\sigma}_z &= 2\tilde{\lambda} \left(\hat{\psi}_1^2 \hat{\psi}_2^2 \hat{\sigma}_+ + \hat{\psi}_1^{2\dagger} \hat{\psi}_2^{2\dagger} \hat{\sigma}_- \right). \end{aligned} \quad (2)$$

Therefore, the constants of motion are given by:

$$\begin{aligned} \hat{N}_j &= \hat{\psi}_j^\dagger \hat{\psi}_j + \hat{\sigma}_z, \quad j = 1, 2 \\ \hat{Q} &= \hat{\psi}_1^\dagger \hat{\psi}_1 - \hat{\psi}_2^\dagger \hat{\psi}_2. \end{aligned} \quad (3)$$

It worth mentioning that the Hamiltonian describes tripartite system (two fields coupled to an atom). Using Eq. (3) the two fields can be described by one mode and the system becomes an effective bipartite.

Here, the two modes are considered in the resonance case $\omega_1 = \omega_2 = \omega$, and the 4-photon resonance $\omega_0 = 4\omega$. We introduce the $su(1, 1)$ generators $(\hat{X}_\pm, \hat{X}_z)$ as follows: $\hat{X}_- = \frac{1}{2} \hat{\psi}_1 \hat{\psi}_2 = (\hat{X}_+)^{\dagger}$ and

$$\begin{aligned} \hat{X}_z &= \frac{1}{2} \left(\hat{\psi}_1^\dagger \hat{\psi}_1 + \hat{\psi}_2^\dagger \hat{\psi}_2 + 1 \right), \text{ which satisfy,} \\ [\hat{X}_+, \hat{X}_-] &= -2\hat{X}_z, \quad [\hat{X}_z, \hat{X}_\pm] = \pm \hat{X}_\pm, \end{aligned} \quad (4)$$

then $\hat{X}_\pm$ and $\hat{X}_z$ are the generators of the $su(1, 1)$ Lie algebra. The Casimir operator, $\hat{X}^2 = k(k-1)\hat{I}$ where k represents the Bargmann number, is given by

$$\hat{X}^2 = \hat{X}_z^2 - \frac{1}{2} (\hat{X}_+ \hat{X}_- + \hat{X}_- \hat{X}_+). \quad (5)$$

Therefore, the Hamiltonian of Eq. (1) becomes

$$\hat{H} = \omega \hat{\sigma}_z + \omega \hat{X}_z + \lambda [\hat{X}_-^2 \hat{\sigma}_+ + \hat{X}_+^2 \hat{\sigma}_-], \quad (6)$$

where $\lambda = 2\tilde{\lambda}$. The operators $\hat{X}_+$, $\hat{X}_-$ and $\hat{X}_z$ act on the state $|n, k\rangle$ as:

$$\begin{aligned} \hat{X}_- |n, k\rangle &= \sqrt{n(n+2k-1)} |n-1, k\rangle, \\ \hat{X}_+ |n, k\rangle &= \sqrt{(n+1)(n+2k)} |n+1, k\rangle, \end{aligned} \quad (7)$$

$$\hat{X}_z |n, k\rangle = (n+k) |n, k\rangle, \quad \hat{X}^2 |n, k\rangle = k(k-1) |n, k\rangle.$$

The eigenstates and the eigenvalues of $\hat{H}$ have the following expressions:

$$|\psi_n^\pm\rangle = \frac{1}{\sqrt{2}} \left(|e, n, k\rangle \pm |g, n+2, k\rangle \right), \quad (8)$$

and

$$\Lambda_n^\pm = \omega(n+k+1) \pm \lambda \sqrt{(n+1)(n+2)(n+2k)(n+2k+1)}, \quad (9)$$

It has been found that the influence of the surrounding environment [35–37] affects considerably the inversion, the purity, and the entanglement. Here, only the intrinsic decoherence is considered, that describes by Milburn equation [39]. This equation of the intrinsic decoherence reads

$$\frac{d}{dt} \hat{\rho}(t) = -i \left[\hat{H}, \hat{\rho}(t) \right] - \frac{\gamma}{2} \left[\hat{H}, \left[\hat{H}, \hat{\rho}(t) \right] \right], \quad (10)$$

where γ describes the decoherence rate.

The system is transformed into $su(1, 1)$ Lie group, which is more general. Therefore, Barut-Girardello coherent state is appropriate as initial state to measure the quantum phenomena of the considered system. The Barut-Girardello coherent state of $su(1, 1)$ Lie group which is the eigenstate of the operator $\hat{X}_-$ and given by [26],

$$|\alpha, k\rangle = \left(\frac{|\alpha|^{2k-1}}{I_{2k-1}(2|\alpha|)} \right)^{1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n! \Gamma(n+2k)}} |n; k\rangle, \quad (11)$$

where $I_n(x)$ is the n^{th} order modified Bessel function,

$$I_k(x) = \left(\frac{x}{2} \right)^k \sum_{n=0}^{\infty} \frac{(x/2)^{2n}}{n! \Gamma(n+k+1)}.$$

Here, we consider the case where the $su(1, 1)$ -system is initially in a superposition of generalized Barut-Girardello coherent state (B-GCS). The superposition of generalized B-GCS is defined as:

$$|\psi(0)\rangle = \frac{1}{A} \left[|\alpha, k\rangle + r |-\alpha, k\rangle \right] = \sum_{n=0}^{\infty} \frac{1}{A} \xi_n |n, k\rangle, \quad (12)$$

with

$$\xi_n = \alpha^n \left[1 + (-1)^n r \right] \sqrt{\frac{|\alpha|^{2k-1}}{n! I_{2k-1}(2|\alpha|) \Gamma(2k+n)}}. \quad (13)$$

Where, $A = 1 + r^2 + 2\langle \alpha, k | -\alpha, k \rangle$ with $r = 0$ for B-GCS and $r = 1$ for even B-GCS. Here, $k = \frac{1}{2}$, therefore, $|\alpha, \frac{1}{2}\rangle$ describes a nonlinear coherent state [48]. While, the qubit is prepared at the initial time in an upper state, i.e., $\rho^A(0) = |e\rangle\langle e|$.

Here, the used method to determine the solution of the Eq. (10) is

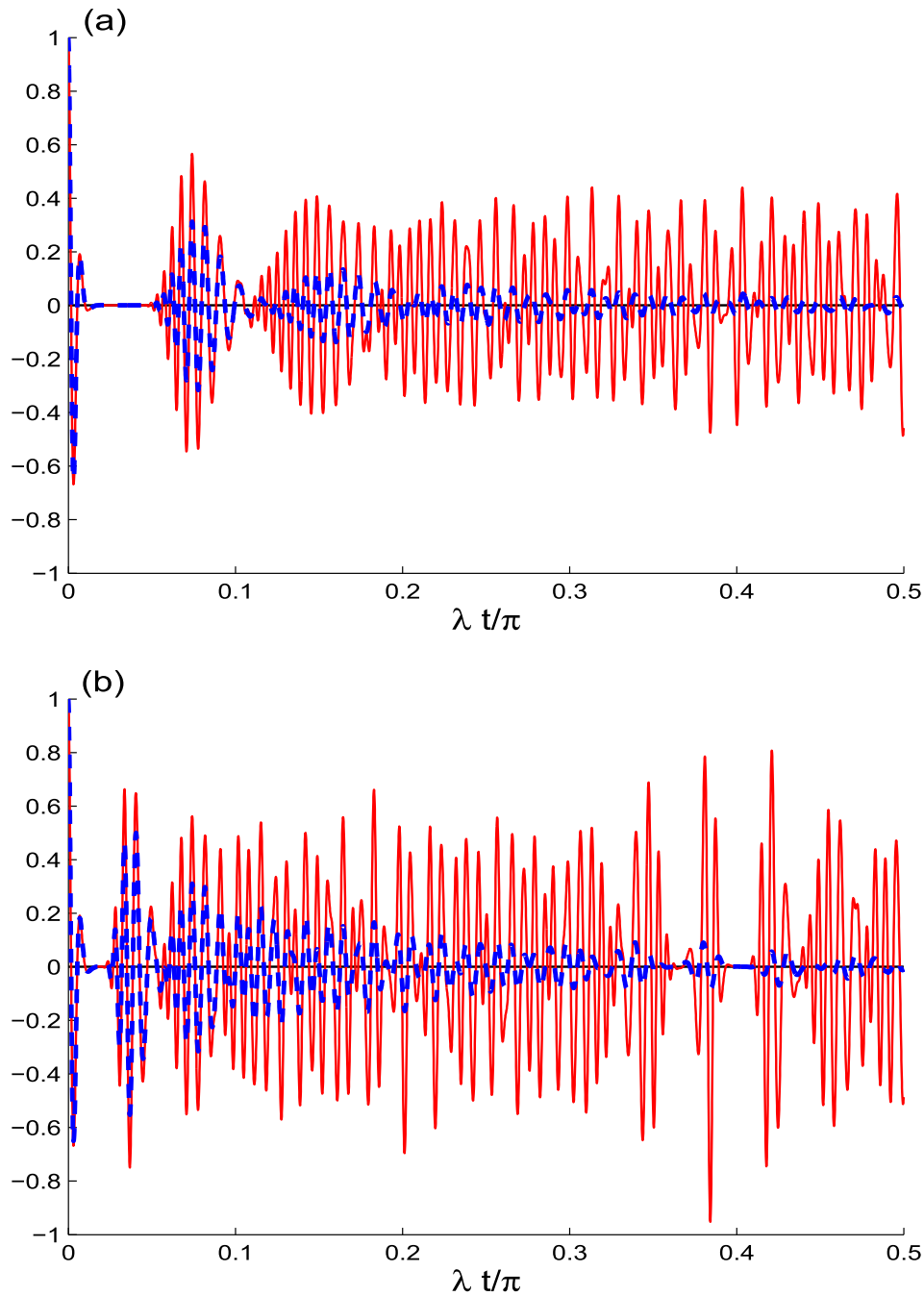


Fig. 1. Population inversion with $k = \frac{1}{2}$ and $\alpha = 20$ for $\gamma = 0$ (solid plots) and $\gamma = 0.00001\lambda$ (dashed plots) for different values of superposition parameter $r = 0$ in (a) and $r = 1$ in (b).

based on the dressed-state representation of the Hamiltonian Eq. (1). The initial state $\rho(0)$ of the entire system is rewritten in term of the dressed state $|\psi_i^\pm\rangle$ of Eq. (8) as:

$$\begin{aligned} \rho(0) &= \sum_{m,n=0} \frac{1}{A^2} \xi_m \xi_n \left| n, k \right\rangle \left\langle n, k \right| \otimes \left| e \right\rangle \left\langle e \right| \\ &= \sum_{m,n=0} \frac{1}{2A^2} \xi_m \xi_n \left[\left| \psi_m^+ \right\rangle \left\langle \psi_m^+ \right| + \left| \psi_m^- \right\rangle \left\langle \psi_m^- \right| \right] \times \left[\left| \psi_n^+ \right\rangle \left\langle \psi_n^+ \right| + \left| \psi_n^- \right\rangle \left\langle \psi_n^- \right| \right]. \end{aligned} \quad (14)$$

From Eq. (10), we obtain

$$\hat{\rho}(t) = e^{\hat{L}t} \hat{\rho}(0). \quad (15)$$

Consequently, the dynamics of the density matrices $|\psi_m^\pm\rangle\langle\psi_n^\pm|$ is given by

$$\begin{aligned} \left| \psi_m^\pm \right\rangle \left\langle \psi_n^\pm \right| &\rightarrow \rho_{ij}^{mn} \left| \psi_m^\pm \right\rangle \left\langle \psi_n^\pm \right| \\ &= e^{\left[-i \left(\Lambda_m^\pm - \Lambda_n^\pm \right) t - \frac{\gamma}{2} \left(\Lambda_m^\pm - \Lambda_n^\pm \right)^2 t \right]} \left| \psi_m^\pm \right\rangle \left\langle \psi_n^\pm \right|. \end{aligned} \quad (16)$$

In the space state of the total system, $\{|e, n, k\rangle, |g, n+1, k\rangle\}$, by using the Eqs. (14)–(16), the density matrix of the qubit-cavity system, $\hat{\rho}(t)$, is given by

$$\rho(t) = \sum_{i,j=0}^1 \frac{1}{A^2} \xi_i^* \xi_j^* \left\{ \begin{aligned} &U_{ij}^{++} + U_{ij}^{+-} + U_{ij}^{-+} + U_{ij}^{--} \\ &\otimes |e, i, k\rangle\langle e, j, k| + [U_{ij}^{++} - U_{ij}^{+-} + U_{ij}^{-+} - U_{ij}^{--}] \\ &\otimes |e, i, k\rangle\langle g, j+2, k| + [U_{ij}^{++} + U_{ij}^{+-} - U_{ij}^{-+} - U_{ij}^{--}] \\ &\otimes |g, i+2, k\rangle\langle e, j, k| + [U_{ij}^{++} - U_{ij}^{+-} - U_{ij}^{-+} + U_{ij}^{--}] \\ &\otimes |g, i+2, k\rangle\langle g, j+2, k|. \end{aligned} \right. \quad (17)$$

where .

Some of effects in the $\text{su}(2)\text{-su}(1, 1)$ system and its subsystems can be investigated by using the final state $\rho(t)$, the atom state $\rho^A(t)$ and $\text{su}(1, 1)$ field $\rho^F(t)$ state. $\rho^{A(F)}(t)$ are given by

$$\rho^{A(F)}(t) = \text{Tr}_{F(A)}\{\rho(t)\} \quad (18)$$

After that, we study the dynamics behaviors of some quantum effects (namely, the atomic inversion, the purity loss and the entanglement) by changing the parameters of the coherent states superposition and the decoherence. Physically, the combination of two or more coherent states generates more quantum coherence in the off-diagonal terms of the density matrix. So, we explore the superposition effect, we consider the two cases: coherent state and even coherent state. For the case, where the even coherent state, we will show next that the quantum effects are more robust against the decoherence than those for the coherent state. Finally, the parameter of $\gamma/\lambda > 0$ is used to see the amount of the diagonalization of the qubit-cavity density matrix, due to the decoherence, and its effect on the quantum phenomena.

3. Atomic inversion

The atomic population allows the investigation of the interaction mechanism for the considered system. It is defined by

$$W(t) = \langle e|\hat{\rho}_A|e\rangle - \langle g|\hat{\rho}_A|g\rangle. \quad (19)$$

The time revivals depends on the Rabi frequency,

$$R = \lambda\sqrt{(n+1)(n+2)(n+2k)(n+2k+1)}. \quad (20)$$

For large value of α the Rabi frequency is proportional to $\lambda n^2 \sim \lambda|\alpha|$, therefore the time revival $t_r \sim \frac{\pi}{\lambda(2|\alpha|+1)}$. Here, we consider that the qubit starts from the excited state and the two modes start from the B-GCS with $\alpha = 20$. For the B-GCS ($r = 0$), the collapse behavior of the population inversion is observed for long period of time. The revival occurred at $\lambda t = 0.7$ as expected from the above relation (20), followed by chaotic strong oscillations as observed in Fig.(1a).

When we take the intrinsic decoherence γ/λ into account the amplitude of the oscillations reduce gradually to reach the stationary state.

For even generalized Barut-Girardello coherent state (even B-GCS) ($r = 1$), the first collapse interval is reduced and the amplitude of the population oscillations increases, in addition another two collapse intervals around 0.4π appear. The period of the revivals decrease after taking the even B-GCS into account. The amplitude of the oscillations decrease more after considering the ID decay rate γ/λ . In the case $r = 0$, the atomic population reaches the stationary state quickly than the previous case of the B-GCS $r = 0$, i. e., the amplitudes of the atomic population enhances.

4. Linear entropies

In this section, the linear entropy and its partial functions are used to analyze the purity loss and the entanglement for the systems and its sub-systems.

(1)The linear entropy of the total system is calculated numerically by the total state $\hat{\rho}(t)$ of the Eq. (17) as:

$$L(t) = 1 - \text{Tr}\{\rho(t)^2\}. \quad (21)$$

(2)While the partial linear entropies are used to measure the purity and the entanglement [49] of some partitions. The reduced density matrices $\rho^{A(F)}(t)$ and their partial linear entropies are given by

$$L_i(t) = 1 - \text{Tr}\{\rho^i(t)^2\}, (i = A, F). \quad (22)$$

In Fig. (2a), we display the linear entropies $L(t)$, $L_A(t)$ and $L_F(t)$ in absence of the intrinsic decoherence terms $\gamma = 0$, and by considering the same conditions as the atomic inversion. For the B-GCS, $r = 0$, the purity of $L(t)$ for the total initial system state is not affected by the unitary interaction between the qubit and the two-mode cavity. The total entropy is time-dependent $L(t) = 0$. While the sub-entropies $L_A(t)$ and $L_F(t)$ grow, with the same irregular oscillatory behavior $L_A(t) = L_F(t)$, to the maximum value $L_A^{\max}(t) = L_F^{\max}(t) = 0.5$ of the maximally entangled state. So, one of them is enough for the proper measure of the entanglement between the qubit and the two-mode cavity states. The qubit-cavity system has maximal and partial generated entanglement regularly.

In Fig. (2b), the effect of the initial even B-GCS ($r = 1$) with $\gamma = 0$ is considered with the same conditions as before. The lower values of the entropies $L_A(t)$ and $L_F(t)$ tend to their maximum value 0.5 faster than the case $r = 0$. The frequencies of the oscillations increase compared to the previous case, as shown in Fig.(2a). It is worth to mention that, in the absence of the intrinsic damping, the linear entropies $L(t)$ and $L_A(t)$ have the same information about the entanglement and the mixture. While in the presence of the intrinsic damping term,

$D_{ij}(t) = e^{-\gamma\left(\Lambda_i^+ - \Lambda_j^+\right)^2 t}$, the linear entropies $L(t)$, $L_A(t)$ and $L_F(t)$ only quantify the mixture of the total system, the qubit and the two-mode cavity sub-systems, respectively.

In Fig. 3a, with $r = 0$ and intrinsic decoherence $\gamma = 0.00001\lambda$, the total entropy $L(t)$ grows from the initial pure state to a partial mixed state. As the γ/λ increases, $L_F(t)$ splits further from $L_A(t)$, and they have different damped oscillatory behaviors and different stationary values. The $\text{su}(2)$ -system reach its stationary mixedness state faster than the $\text{su}(1, 1)$ -system.

From Fig. (3) b, with $r = 1$, we note that the effect of the intrinsic decoherence on the purity loss of the total and $\text{su}(2)$ -system is more pronounced than on the purity loss of the $\text{su}(1, 1)$ -system. The linear entropy functions $L(t)$ and $L_A(t)$ increase quickly to their stationary values for $r = 1$. We note that the purity loss grows up gradually as the time evolves. The purity loss and its stationary state can be enhanced with the even B-GCS.

The high nonlinearity, that results from the coupling between the qubit and the two-mode cavity fields, leads to: (1) Speeding up the entropy functions to its extreme values. (2) High sensitivity to a small value of the intrinsic decoherence.

5. Entanglement negativity

The negativity can be regarded as an important measure of entanglement [50–52], for a quantum system has a state described by the density matrix $\hat{\rho}$, and define a bipartition into two subsystems with arbitrary dimensions d_1 and d_2 ($d_1 < d_2$) as the state of Eq. (17), the negativity is defined as

$$N(\rho) = \frac{1}{d_1 - 1} \left[\left\| \hat{\rho}^{\tau_1} \right\| - 1 \right], \quad (23)$$

where $\hat{\rho}^{\tau_1}$ is the matrix obtained by producing the partial transpose for $\hat{\rho}$ with respect to the d_1 -subsystem, and $\|\hat{\rho}^{\tau_1}\|$ is its trace norm. Here, the final state of Eq. (17) describes $2 \times n$ quantum system, and the partial transposition is with respect to the atomic system, where $d_1 = 2$. Therefore, the entanglement negativity can be given by

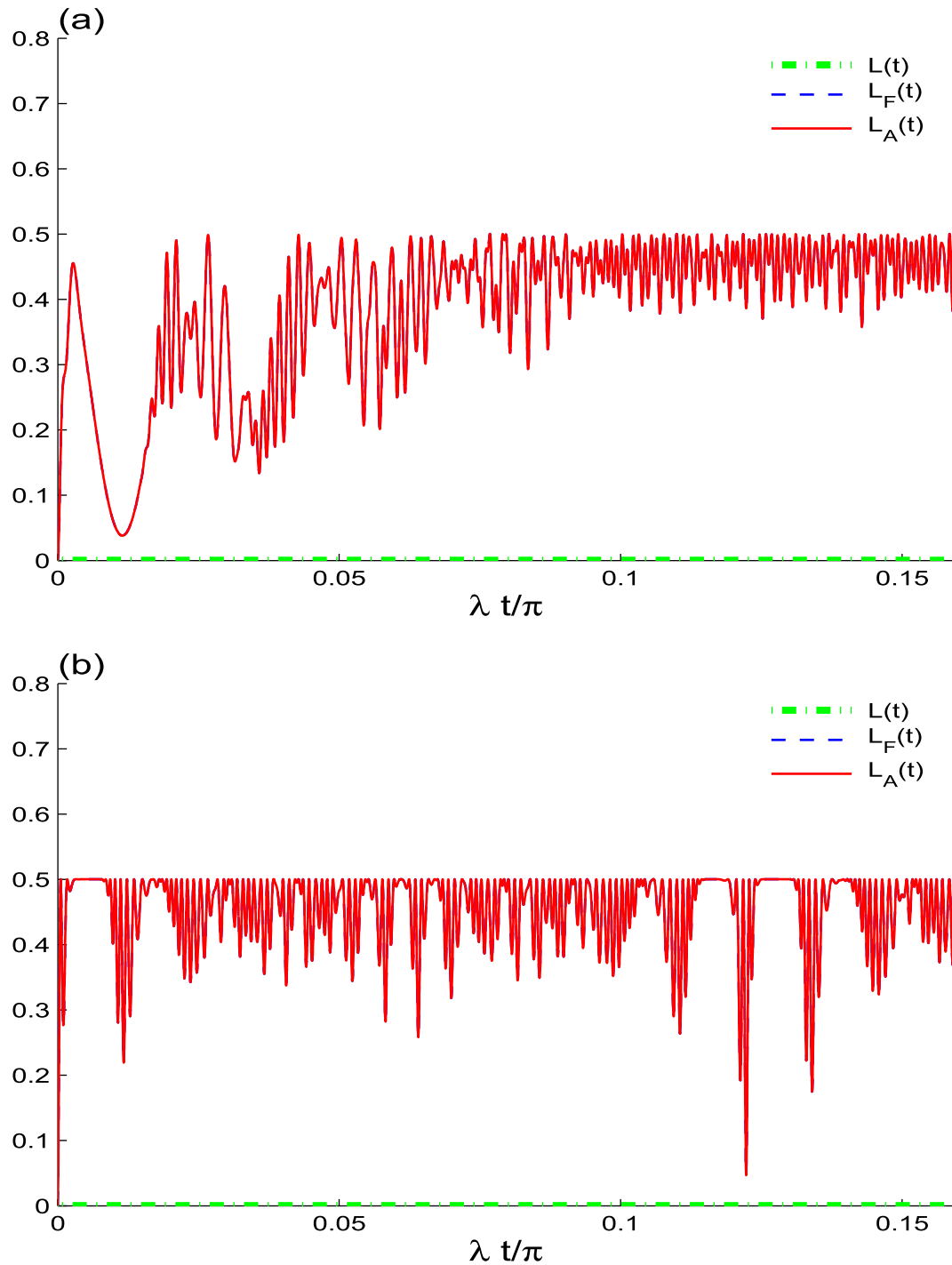


Fig. 2. Linear entropies in the case $\gamma = 0$, $L_A(t) = L_F(t)$ (solid curves) and $L(t)$ (dashed-dotted curves) for and $\alpha = 20$ for different values of r : $r = 0$ in (a) and $r = 1$ in (b).

$$N(t) = - \sum_k \pi_k, \quad (24)$$

where π_k are the negative eigenvalues of the partial transpose of the matrix $\rho(t)$. If $N(t) = \frac{1}{2}$, then the qubit-cavity state is a maximally entangled state. $N(t) = 0$ means that there is no qubit-cavity entanglement, otherwise, the $\text{su}(2)\text{-su}(1,1)$ states are in partial entanglement.

Fig. 4 shows the dynamic of the entanglement negativity for different values of the ID decay, $\gamma = 0, 0.00005\lambda, 0.0005\lambda$. In Fig. 4a, without the intrinsic decoherence and for B-GCS ($r = 0$), the solid curve shows that the entanglement negativity has irregular oscillations.

Partially and maximally entangled states are generated due to the unitary interaction between the qubit and the two-mode cavity as expected from entropy results in Fig. 2a. In addition, the negativity approaches the one of the pure states at the middle of the collapse intervals Fig. (4a).

Dashed and dashed dotted curves in Figs. 4b, show that as the intrinsic damping increases, the QE present more pronounced damped oscillatory behavior.

Now we are able to investigate the influence of the initial states on the negativity for the second case even B-GCS ($r = 1$). We observe that the fluctuations and the amplitude of the negativity are reduced.

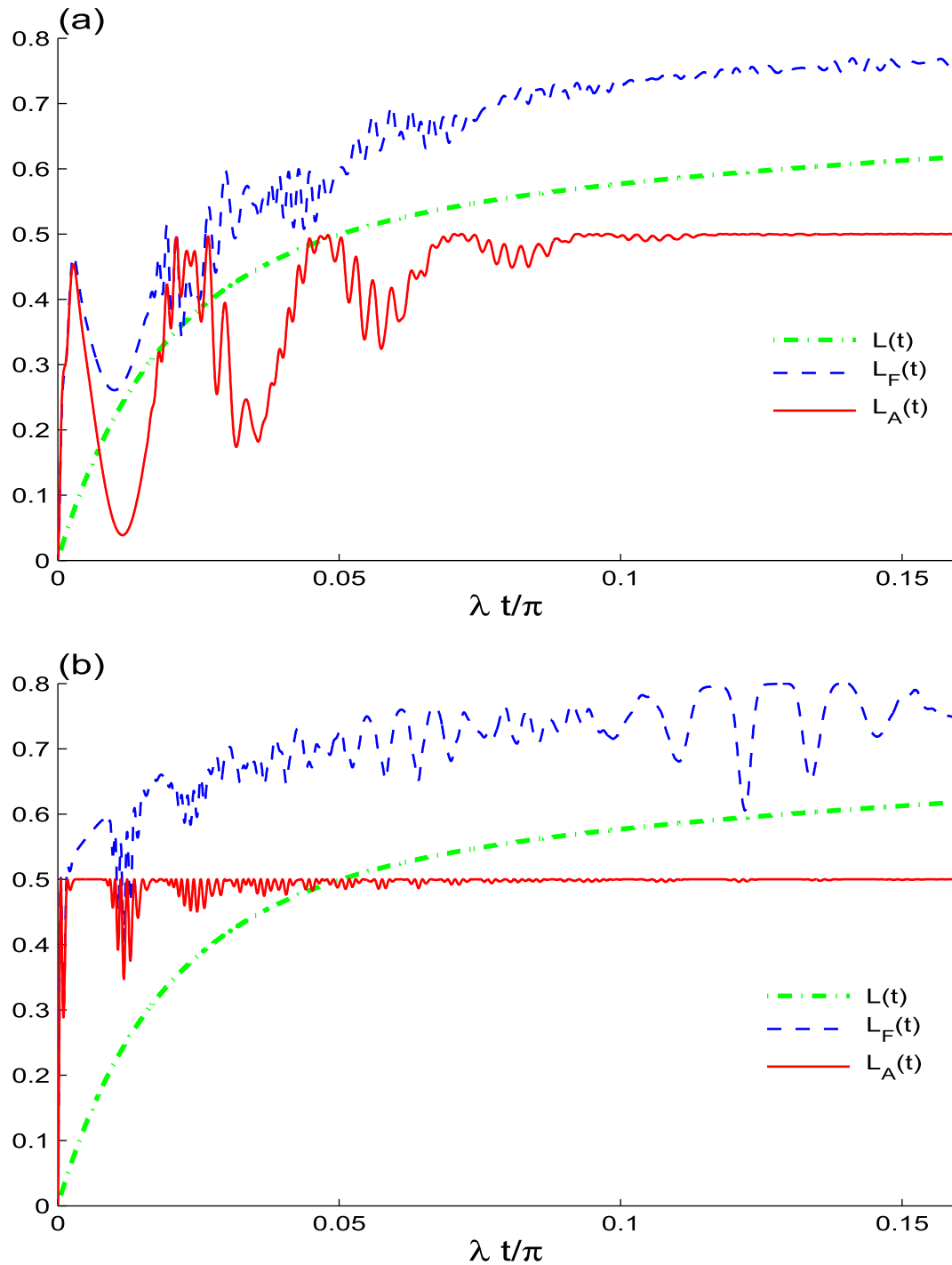


Fig. 3. Linear entropies $L_A(t)$ (solid curves), $L_F(t)$ (dashed curves) and $L(t)$ (dashed-dotted curves) for $k = \frac{1}{2}$, $\gamma = 0.00001\lambda$, and $\alpha = 20$ for $r = 0$ in (a) and $r = 1$ in (b).

Generally, the entanglement reaches its maxima and minima at the same time that determining the revivals of the atomic inversion. With the case of the even B-GCS $r = 1$, the ID terms, $D_{ij}(t) = e^{-\gamma(\Lambda_i^+ - \Lambda_j^+)^2 t}$, impose irregularity and more oscillations for the negativity. Also, the increase of the initial coherent field intensity of $r = 1$ leads to the qubit-cavity system is often stable in a maximally entangled state. The negativity decreases to present partial entanglement in some small intervals, in which it is affected by the intrinsic damping more than those the stable intervals as observed in Fig. (4b). In this case $r = 1$ the effect of the intrinsic decoherence on the rapid oscillations of the generated

entanglement is weak compared to the previous case $r = 0$. The stationary entanglement negativity is disappeared completely. Finally, we observe that the high nonlinearity leads to: (1) Speeding up the $\text{su}(2)$ - $\text{su}(1, 1)$ state to reach the maximal entanglement. (2) The generated entanglement has high sensitivity to a small intrinsic decoherence value.

6. Conclusion

In this article, we have explored analytically a two-mode cavity containing a qubit by using the $\text{su}(1,1)$ -algebraic concept. In this

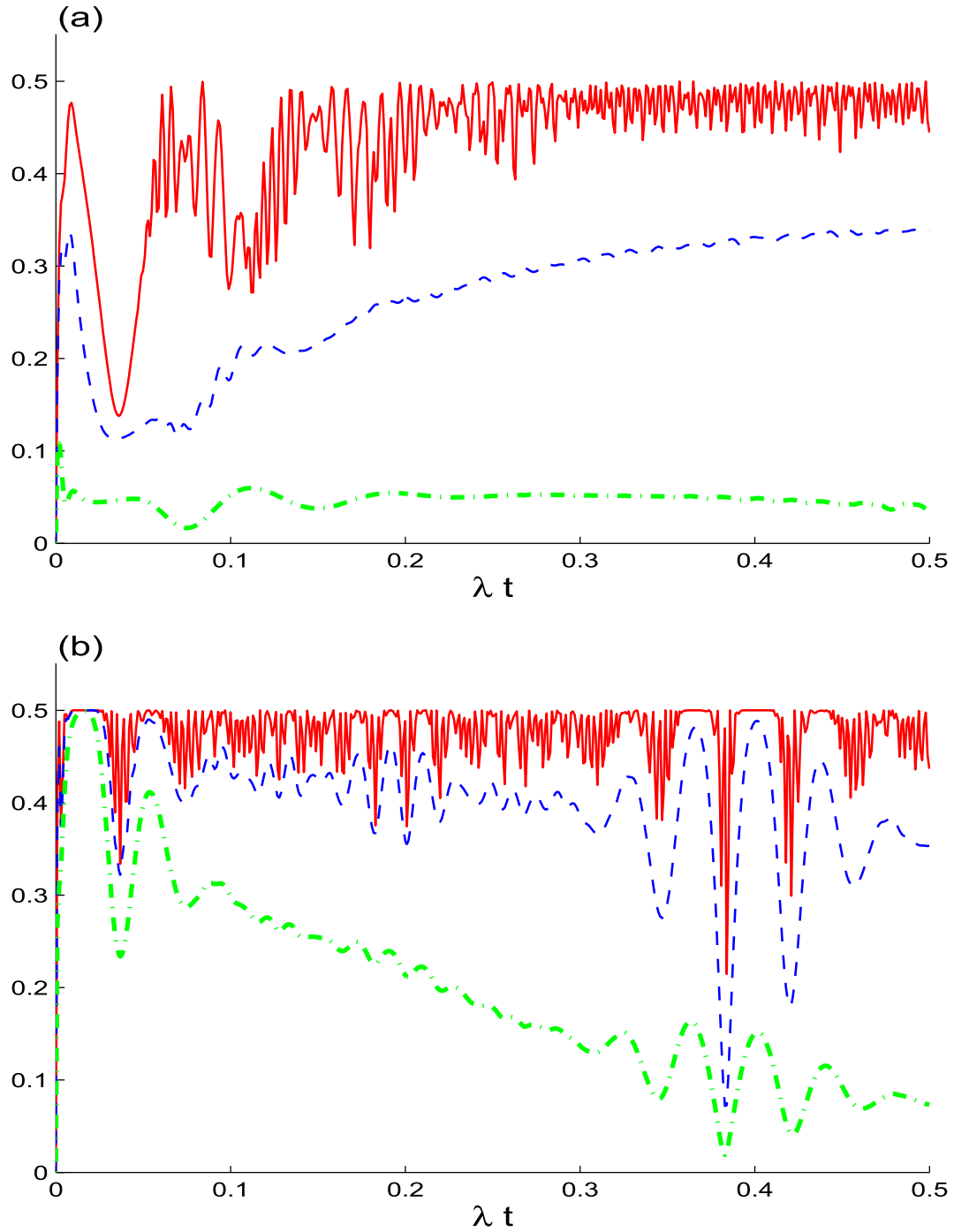


Fig. 4. Negativity with $k = \frac{1}{2}$ and $\alpha = 20$ for $\gamma = 0$ (solid curves), $\gamma = 0.00005\lambda$ (dashed curves) and $\gamma = 0.0005\lambda$ (dashed dotted curves) for different values of r : $r = 0$ in (a) and $r = 1$ in (b).

system, the two mode cavity interacting resonantly with the qubit through a four-photon processes. The population inversion, the purity loss and the entanglement are investigated under the intrinsic damping and the superposition of the initial generalized Barut-Girardello coherent states. The collapses and the oscillation amplitudes of the atomic inversion depend on the coherent density of the generalized Barut-Girardello coherent state and the intrinsic damping. The atomic inversion, the purity loss and the entanglement are enhanced and more robust for the even B-GCS. When the intrinsic damping effect is considered, the purity loss of the quantum states grows, while the entanglement degrades. The superposition of the B-GCS controls the completion acceleration of the stationary entanglement. It is found that

the quantifiers of the atomic inversion and the purity loss as well as the entanglement are very sensitivity to the high nonlinear couplings.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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