

Received July 15, 2020, accepted August 19, 2020, date of publication August 25, 2020, date of current version September 11, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3019454

Robustness of Generated Geometric Phase of Quantum Wells in Two Open Waveguide-Coupled Optical Cavities

ABDEL-BASET A. MOHAMED^{1,2}, AHMED FAROUK³, (Member, IEEE),
ABDEL-HALEEM ABDEL ATY^{4,5}, AND HICHEM ELEUCH^{6,7,8}

¹Department of Mathematics, College of Sciences and Humanities in Al-Aflaj, Prince Sattam Bin Abdulaziz University, Al-Aflaj 710-11912, Saudi Arabia

²Faculty of Science, Assiut University, Assiut 71516, Egypt

³Department of Physics and Computer Science, Faculty of Science, Wilfrid Laurier University, Waterloo, ON N2L 3C5, Canada

⁴Department of Physics, College of Sciences, University of Bisha, Bisha 61922, Saudi Arabia

⁵Physics Department, Faculty of Science, Al-Azhar University, Assiut 71524, Egypt

⁶Department of Applied Physics and Astronomy, University of Sharjah, Sharjah, United Arab Emirates

⁷Department of Applied Sciences and Mathematics, College of Arts and Sciences, Abu Dhabi University, Abu Dhabi, United Arab Emirates

⁸Institute for Quantum Science and Engineering, Texas A&M University at College Station, College Station, TX 77843, USA

Corresponding author: Abdel-Baset A. Mohamed (abdelbastm@yahoo.com)

This work was supported by the Deanship of Scientific Research at Prince Sattam Bin Abdulaziz University under Project 2020/01/11801.

ABSTRACT In this article, we study the geometric phase in a system formed by two spatially separated cavities interacting with the environment. Each cavity is filled by a linear optical medium and contains a quantum well. For different initial states, the robustness of the generated geometric phase is analyzed under the effects of the optical susceptibility, the dissipation of the cavities, the exciton-cavity and fiber-cavity couplings. Our results show that the geometric phase is extremely sensitive to the effects of the cavity-exciton and the fiber-cavity couplings as well as to the optical susceptibility. This opens new routes to understand the storage and manipulation of quantum data in a quantum network.

INDEX TERMS Geometric phase, microcavity damping, optical susceptibility.

I. INTRODUCTION

The explorations of the system-environment interactions open the door to study more critical quantum phenomena, which are primary resources in quantum information and quantum computation [1]–[5]. The geometric phase (GP) is the basis of quantum computation [6]. For example, the geometric phase was used to improve the quantum gate protocol in a superconducting qubits, and to realize a multi-target-qubit phase gate [7]–[10]. Recently, Wang *et al.* [9] propose a scheme of circuit QED that opens the possibility of implementing ultrafast quantum gates holding noise-resistant merits based on the advantages of geometric phases. GP is defined as the phase obtained for a quantum system during an adiabatic evolution. If the final time-dependent state of a quantum system returns to its initial state, the evolution of the quantum system is called “cyclic”. The GP was extended to the quantum states of nonadiabatic cyclic [11] and noncyclic [12], [13] evolutions. It has also contributed to the significant applications in quantum information and

computation for developing robust quantum systems such as: superconducting circuits [14], trapped ions [15] and atoms in cavity field [16]. The dynamics of the geometric phase were investigated in cavity-QED systems [17], phase-qubit system coupled to a LC circuit [18], trapped ion system with Stark shift [19], open qubit-cavity system [20] and a two-level system undergoing pure dephasing [21].

An ensemble of cavity-QED systems, linked by an optical waveguide were recently investigated to realize quantum gates and quantum networks. They contribute to the architecture of the distributed quantum computing [22] where the quantum network can be envisaged as spatially separated local qubits [23]–[26]. Unfortunately, these systems suffer from the loss of coherence due to the interaction with the environment [27]. One way to remedy this problem is to use geometric phase shifts [28]. The motivation of our work can be summarized into two main points: (1) Studying the contribution of the multi-qubit systems linked by an optical waveguide. This could be a guideline for the realization of quantum networks based on distributed quantum nodes (qubits), for the storage and manipulation of quantum data [29], [30]. (2) Analyzing the geometric phase under the influence of

The associate editor coordinating the review of this manuscript and approving it for publication was Chi-Yuan Chen¹.

the optical susceptibility and the dissipation. The paper is organized as follows: The physical model, along with the system Hamiltonian, is presented in Sec.2. The dynamics of the geometric phase for different initial correlated/uncorrelated are illustrated and analyzed in Sec.3. A conclusion of the proposed work is presented in Sec.4.

II. PHYSICAL MODEL

We study here, two spatially distant open cavity systems which are linked by a mode waveguide (see Fig.(1)). The system Hamiltonian can be written as

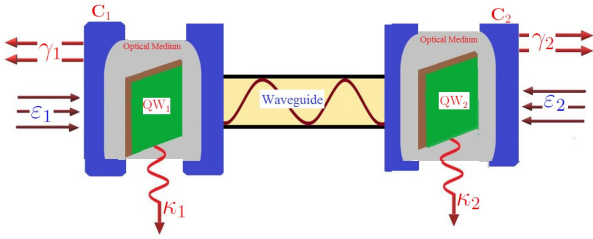


FIGURE 1. Two spatially distant cavity systems (C_1 and C_2) are linked by an optical waveguide. Each of them contains one semiconductor quantum well (QW) and filled with a linear optical medium.

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_f. \quad (1)$$

The two first terms of Eq.1 describe the cavity 1 and 2 respectively. Each cavity is filled with a linear optical medium and contains a quantum well. The quantum well is placed in a position that corresponds to the maximum of the electromagnetic cavity field [31]–[33]. The Hamiltonian $\hat{H}_k$, can be written as

$$\hat{H}_k = \omega_{cv,k} \hat{a}_k^\dagger \hat{a}_k + \omega_{ex,k} \hat{b}_k^\dagger \hat{b}_k + \chi_k \hat{a}_k^\dagger \hat{a}_k + \lambda_k (\hat{a}_k^\dagger \hat{b}_k + \hat{b}_k^\dagger \hat{a}_k) + \varepsilon_k (\hat{a}_k^\dagger e^{i\omega_{L,k}t} + \hat{a}_k e^{-i\omega_{L,k}t}). \quad (2)$$

where $\hat{a}_k$ ($\hat{a}_k^\dagger$), $\omega_{cv,k}$ and χ_k ($k = 1, 2$) are the annihilation (creation) operators, the frequency and the linear optical susceptibility (of the optical medium) linked to the cavity k . $\omega_{ex,k}$ and $\hat{b}_k$ design the frequency and the annihilation operator of the exciton, respectively. The forth term of the Hamiltonian describes the interaction between the k -exciton and the k -cavity with the coupling constants λ_k . The last term is denoting the optical pumping of the cavity. ε_k and $\omega_{L,k}$ are the amplitude and the frequency of the pump laser.

The $\hat{H}_f$ describes the coupling between the two-cavity systems and the optical fiber mode, it is given by

$$\hat{H}_f = \sum_{k=1,2} \omega_{f,k} \hat{f}_k^\dagger \hat{f}_k + \nu_k (\hat{f}_k \hat{a}_k^\dagger + \hat{f}_k^\dagger \hat{a}_k). \quad (3)$$

The $\omega_{f,k}$ are the frequencies of the fiber modes. Where $\hat{f}_k$ designs the annihilation operator for the fiber k -mode, the ν_k represent the coupling constants between the optical fiber mode and the cavity modes.

In this work we focus on the case where: (1) The photonic and excitonic modes are quantified along the normal direction

to the cavity. The pumping is perpendicular to the cavity in the direction of the symmetric axes of the cavity and the fiber that allows the excitation of the cavity photonic mode with the same direction of the photonic and excitonic modes. (2) Our analysis is developed in the rotating frame with the frequency $\omega_{L,k}$ and for the resonant ($\omega_{cv,k} = \omega_{ex,k} = \omega_{L,k}$).

We also limit our study to the case of the short fiber interaction which is realized in usual experimental situations [30], [34]. The condition of the short fiber limit is defined by $\frac{2l\bar{\nu}}{2\pi c} \ll 1$, where l represents the fiber length, c designs the light speed in the fiber, $\bar{\nu}$ is the cavity dissipation. We consider here that only one resonant mode $\hat{f}$ of the fiber is able to interact with the cavity mode. $\nu_k = \lambda_3$ denote the fiber-microcavity coupling constants. As mentioned above the frequencies of the microcavities, the excitons and the fiber modes are equal. In this case the interaction Hamiltonian can be written as

$$\begin{aligned} \hat{H}_{int} = & \sum_{k=1,2} \chi_k \hat{a}_k^\dagger \hat{a}_k + \lambda_k (\hat{a}_k^\dagger \hat{b}_k + \hat{a}_k \hat{b}_k^\dagger) \\ & + \varepsilon_k (\hat{a}_k^\dagger + \hat{a}_k) + \lambda_3 (\hat{f} \hat{a}_k^\dagger + \hat{f}^\dagger \hat{a}_k). \end{aligned} \quad (4)$$

The master equation of the system can be written by considering the dissipation as,

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & -i\hbar[H_{int}, \rho] + \sum_{k=1,2} \kappa_k (2\hat{a}_k \rho \hat{a}_k^\dagger - \hat{a}_k^\dagger \hat{a}_k \rho - \rho \hat{a}_k^\dagger \hat{a}_k) \\ & + \frac{\gamma_k}{2} (2\hat{b}_k \rho \hat{b}_k^\dagger - \hat{b}_k^\dagger \hat{b}_k \rho - \rho \hat{b}_k^\dagger \hat{b}_k). \end{aligned} \quad (5)$$

where $\frac{\gamma_k}{2}$ and κ_k identify the normalized excitonic spontaneous emission and the microcavity dissipation rates respectively.

The system is in the strong coupling regime between the exciton and the photon of the cavity. The coupling constant of the exciton-photon is higher than the cavity dissipation and the exciton emission rates. We also consider the weak pumping regime: $\varepsilon_k \ll \kappa_k$, the off-diagonal terms $2\hat{a}_k \rho \hat{a}_k^\dagger$ and $2\hat{b}_k \rho \hat{b}_k^\dagger$ can be ignored in Eq.(5) [35]–[37] and the purity of the system state is almost preserved. Therefore, we can obtain:

$$i \frac{d}{dt} \rho = \hat{H}_{eff} \rho - (\rho \hat{H}_{eff})^\dagger. \quad (6)$$

where the non-Hermitian operator H_{eff} is,

$$H_{eff} = H_{int} - i\hbar \sum_{k=1,2} \kappa_k \hat{a}_k^\dagger \hat{a}_k + \frac{\gamma_k}{2} \hat{b}_k^\dagger \hat{b}_k. \quad (7)$$

From Eq.(6), the dynamics of the system is governed by:

$$\rho(t) = |\psi(t)\rangle \langle \psi(t)|, \quad (8)$$

where $|\psi(t)\rangle$ satisfies

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}_{eff} |\psi(t)\rangle, \quad (9)$$

We consider the description of the exciton basis space as: $\{|E_1\rangle = |\downarrow\downarrow\rangle, |E_2\rangle = |\downarrow\uparrow\rangle, |E_3\rangle = |\uparrow\downarrow\rangle, |E_4\rangle = |\uparrow\uparrow\rangle\}$ and the two-cavity basis space as: $\{|C_1\rangle = |00\rangle, |C_2\rangle =$

$|01\rangle, |C_3\rangle = |10\rangle, |C_4\rangle = |11\rangle\}$, while the basis space of the optical-fiber states is defined by $\{|0_f\rangle, |1_f\rangle\}$. The model of Eq.(1) describes the cavity-exciton and the fiber-cavity interactions, therefore, the total number of the excitations is determined by the operator $\hat{f}^\dagger \hat{f} + \sum_k (\hat{b}_k^\dagger \hat{b}_k + \hat{a}_k^\dagger \hat{a}_k)$. Based on the fact that the system is in the strong coupling regime between the exciton and the photon of the cavity, the coupling constant of the exciton-photon is higher than the cavity dissipation. In this case for a short time there is approximately a constant excitation number N_e , we restrict N_e to $N_e = 3$ and consider only single photon processes. Therefore, the wave function $|\psi(t)\rangle$ can be represented by

$$|\psi(t)\rangle = \sum_{k=1}^4 |\Psi_k\rangle \otimes |E_k\rangle, \quad (10)$$

with

$$\begin{aligned} |\Psi_1\rangle &= [q_1|C_1\rangle + q_9|C_2\rangle + q_{13}|C_3\rangle + q_{17}|C_4\rangle]|0_f\rangle \\ &\quad + [q_5|C_1\rangle + q_{11}|C_2\rangle + q_{15}|C_3\rangle + q_{18}|C_4\rangle]|1_f\rangle, \\ |\Psi_2\rangle &= [q_2|C_1\rangle + q_{14}|C_3\rangle]|0_f\rangle + [q_6|C_1\rangle + q_{16}|C_3\rangle]|1_f\rangle, \\ |\Psi_3\rangle &= [q_3|C_1\rangle + q_{10}|C_2\rangle]|0_f\rangle + [q_7|C_1\rangle + q_{12}|C_2\rangle]|1_f\rangle, \\ |\Psi_4\rangle &= [q_4|0_f\rangle + q_8|1_f\rangle]|C_1\rangle. \end{aligned}$$

The time-dependent coefficients q_n are driven from Eq.(9) as,

$$\begin{aligned} \dot{q}_1 &= \varepsilon_1 q_{13} + \varepsilon_2 q_9, \\ \dot{q}_2 &= -i\lambda_2 q_9 - \tilde{\gamma}_2 q_2, \\ \dot{q}_3 &= -i\lambda_1 q_{13} - \tilde{\gamma}_1 q_3 + \varepsilon_1 q_{14} + \varepsilon_2 q_{10}, \\ \dot{q}_4 &= -i\lambda_1 q_{14} - i\lambda_2 q_{10} - (\tilde{\gamma}_1 + \tilde{\gamma}_2) q_4, \\ \dot{q}_5 &= -i\lambda_3 q_9 - i\lambda_3 q_{13} + \varepsilon_1 q_{15} + \varepsilon_2 q_{11}, \\ \dot{q}_6 &= -i\lambda_2 q_{11} - i\lambda_3 q_{14} - \tilde{\gamma}_2 q_6, \\ \dot{q}_7 &= -i\lambda_1 q_{15} - i\lambda_3 q_{10} - \tilde{\gamma}_1 q_7 + \varepsilon_1 q_{16} + \varepsilon_2 q_{12}, \\ \dot{q}_8 &= -i\lambda_1 q_{16} - i\lambda_2 q_{12} - (\tilde{\gamma}_1 + \tilde{\gamma}_2) q_8, \\ \dot{q}_9 &= -i\lambda_2 q_2 - i\lambda_3 q_5 - \kappa_2 q_9 - i\chi_2 q_9 + \varepsilon_1 q_{17} + \varepsilon_2 q_1, \\ \dot{q}_{10} &= -i\lambda_1 q_{17} - i\lambda_2 q_4 - i\lambda_3 q_7 - (\kappa_2 + \tilde{\gamma}_1) q_{10}, \\ &\quad - i\chi_2 q_{10} + \varepsilon_2 q_3, \\ \dot{q}_{11} &= -i\lambda_2 q_6 - i\lambda_3 q_{17} - \kappa_2 q_{11} - i\chi_2 q_{11} + \varepsilon_1 q_{18} + \varepsilon_2 q_5, \\ \dot{q}_{12} &= -i\lambda_1 q_{18} - i\lambda_2 q_8 - (\kappa_2 + \tilde{\gamma}_1) q_{12} - i\chi_2 q_{12} + \varepsilon_2 q_7, \\ \dot{q}_{13} &= -i\lambda_1 q_3 - i\lambda_3 q_5 - \kappa_1 q_{13} - i\chi_1 q_{13} + \varepsilon_1 q_1 + \varepsilon_2 q_{17}, \\ \dot{q}_{14} &= -i\lambda_1 q_4 - i\lambda_2 q_{17} - i\lambda_3 q_6 - (\kappa_1 + \tilde{\gamma}_2) q_{14}, \\ &\quad - i\chi_1 q_{14} + \varepsilon_1 q_2, \\ \dot{q}_{15} &= -i\lambda_1 q_7 - i\lambda_3 q_{17} - \kappa_1 q_{15} - i\chi_1 q_{15} + \varepsilon_1 q_5 + \varepsilon_2 q_{18}, \\ \dot{q}_{16} &= -i\lambda_1 q_8 - i\lambda_2 q_{18} - (\kappa_1 + \tilde{\gamma}_2) q_{16} - i\chi_1 q_{16} + \varepsilon_1 q_6, \\ \dot{q}_{17} &= -i\lambda_1 q_{10} - i\lambda_2 q_{14} - i\lambda_3 q_{15} - (\kappa_1 + \kappa_2) q_{17}, \\ &\quad - i(\chi_1 + \chi_2) q_{17} + \varepsilon_1 q_9 + \varepsilon_2 q_{13}, \\ \dot{q}_{18} &= -i\lambda_1 q_{12} - i\lambda_2 q_{16} - i\lambda_3 q_{11} - (\kappa_1 + \kappa_2) q_{18}, \\ &\quad - i(\chi_1 + \chi_2) q_{18} + \varepsilon_1 q_{11} + \varepsilon_2 q_{15}. \end{aligned} \quad (11)$$

where $\tilde{\gamma}_k = \gamma_k$ and λ_k are the coupling rates between excitons and photons, and ε_k are the laser pump amplitudes. Eqs.11 are

numerically solved to determine the wave function $|\psi(t)\rangle$ and the dynamics of the geometric phase.

III. GEOMETRIC PHASE

Here, we explore the the geometric phase. If the final state $|\psi(t)\rangle$ cannot be obtained from an unitary transformation of the initial state $|\psi(0)\rangle$, the evolution is then considered as a noncyclic. In this case the phase is not trivial, we use the Pancharatnam phase [38]–[43] approach to prescribe the total phase. In the open dynamics of the hybrid system, a pure state convert to a mixed state that often is described via its non-unitary evolution, say $\rho(t)$. For this case, Uhlmann [44] and Tong *et al.* [45] extended the geometric phase to the mixed states. But, with the considered approximations, the effective description of the open system is governed by the master equation that is derived by neglecting the non-diagonal terms, then the dynamics of the system is described by the non-Hermitian Hamiltonian $\hat{H}_{eff}$. Therefore, the evolution of the initial state $|\psi(0)\rangle$ is governed by the Schrödinger equation as

$$|\psi(0)\rangle \rightarrow |\psi(t)\rangle = (U(t) = e^{-i\hat{H}_{eff}t})|\psi(0)\rangle.$$

Our system in the strong regime that means the dissipation is small compared to the interaction coupling, i.e., we neglect the dissipation for a short time, so that the wave function conserves approximately the norm for a short time other than that the wave function can be renormalized. Pancharatnam's relative phase between the states $|\psi(0)\rangle$ and $|\psi(t)\rangle$ of Eq.10 is expressed by [38], [39]

$$G_P(t) = \arg\langle\psi(0)|\psi(t)\rangle. \quad (12)$$

Here, we will analyze the robustness of the generated geometric phase for the correlated and uncorrelated initial states by considering three different cases of the initial system state. First one, when the system is initially in the maximally entangled state and the other two cases are considered for the system in unentangled states. The wave function for the the first case can be written as

$$\begin{aligned} |\psi(0)\rangle_1 &= \frac{1}{\sqrt{5}} [(|C_1\rangle + |C_2\rangle + |C_3\rangle + |C_4\rangle) \otimes |1_f\rangle] \otimes |E_1\rangle \\ &\quad + e^{i\varphi} |C_1\rangle \otimes |1_f\rangle \otimes |E_4\rangle, \end{aligned} \quad (13)$$

and for the two other unentangled states, we consider:

$$\begin{aligned} |\psi(0)\rangle_2 &= |C_1\rangle \otimes |1_f\rangle \otimes |E_1\rangle, \\ |\psi(0)\rangle_3 &= |C_4\rangle \otimes |0_f\rangle \otimes |E_1\rangle. \end{aligned} \quad (14)$$

We focus here on the case where the phase angle $\varphi = \frac{\pi}{4}$. The type of entanglement of the initial states of Eq.14 is very useful for distributed quantum information [30].

In the numerical calculations, the time t represents a unitless time normalized to τ_c , where τ_c is the round trip time for a photon inside the cavity. We normalize also the parameters of the system to $1/\tau_c$ as: $\lambda_i = \lambda_i \tau_c$, $\kappa_i = \kappa_i \tau_c$, $\gamma_i = \gamma_i \tau_c$ and $\varepsilon = \varepsilon \tau_c$.

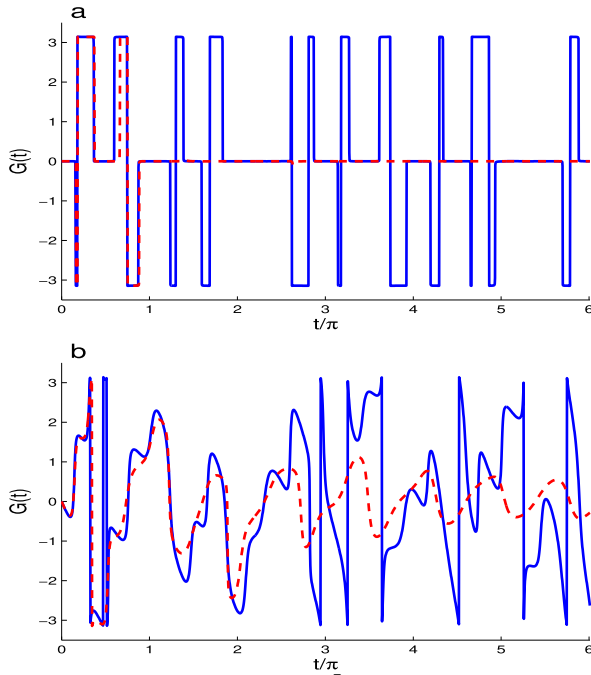


FIGURE 2. $G_P(t)$ for $|\psi(0)\rangle_1$ and $\epsilon = 10^{-3}$ with $\lambda_k = 2(k = 1 - 3)$ with $\gamma = 0$ (solid curves) and $\gamma = 0.4$ (dashed curves). For $\chi_k = 0.0$ in (a) and $\chi_k = 2$ in (b).

A. GP WITH THE INITIAL CORRELATED STATE $|\psi(0)\rangle_1$

The dynamics of the Pancharatnam geometric phase considering Eq.14 $|\psi(0)\rangle_1$, is given by

$$G_P(t) = \frac{1}{\sqrt{5}} \arg[q_5 + q_{11} + q_{15} + e^{i\varphi} q_{17} + q_{18}] \quad (15)$$

In Fig.2a, the PGP function is for the initial correlated state: $|\psi(0)\rangle_1$, $\epsilon = 10^{-3}$ and $\chi = 0.0$ when the interactions of the two cavity-excitons and the fiber-cavity are strong $\lambda_k = 2.0$ ($k = 1 - 3$) and for different values of the cavity dissipation rates $\kappa_k = \gamma_k = \gamma = 0.0, 0.4$ ($k = 1, 2$), respectively in (a) and (b).

In the case of the absence of the optical linear medium inside the cavity $\chi = 0.0$, we noted that: (1) The PGP function has rectangular oscillations verifying the inequality $-\pi < G(t) < \pi$. (2) The number of rectangular oscillations is enhanced with the increase of the coupling constants λ_k ($k = 1, 2, 3$) and uncertain intervals [17] appear, in which the GP values can not be unequivocally determined. (3) Dashed curve of the Fig.3 shows the dynamics of the PGP for the the dissipation rates $\kappa_k = \gamma_k = 0.4$. These dissipations lead to the reduction of the oscillation frequency.

Fig.2b. shown that the generated GP oscillations are affected by increasing the optical susceptibility, $\chi_k = 2$. We observed that: (1) The generated GP has more smoother oscillations (GP) unlike the case of $\chi_k = 0$ that achieved rectangular GP oscillations. (2) The phenomena of the collapses and revivals disappear entirely. (3) The optical susceptibility enhances the robustness of the generated GP against the dissipation rates.

In Fig.3, we demonstrated the same when the cavities are filled with an optical medium, $\chi = 0, 2$, and the coupling

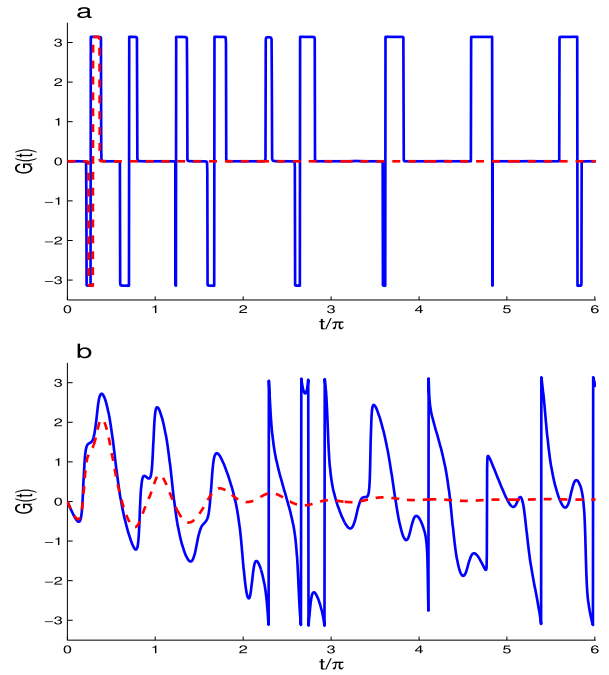


FIGURE 3. As Fig.2, but with $\lambda_3 = 0.5$.

strength of the fiber-cavity coupling is reduced to $\lambda_3 = 0.5$. We find that the amplitude of the GP is reduced compared to the previous case where $\lambda_3 = 2$. We also observe that the GP is fragile against the dissipation rates. The ability of the optical susceptibility to maintain the robustness of the GP is vanished for $\lambda_3 = 0.5$.

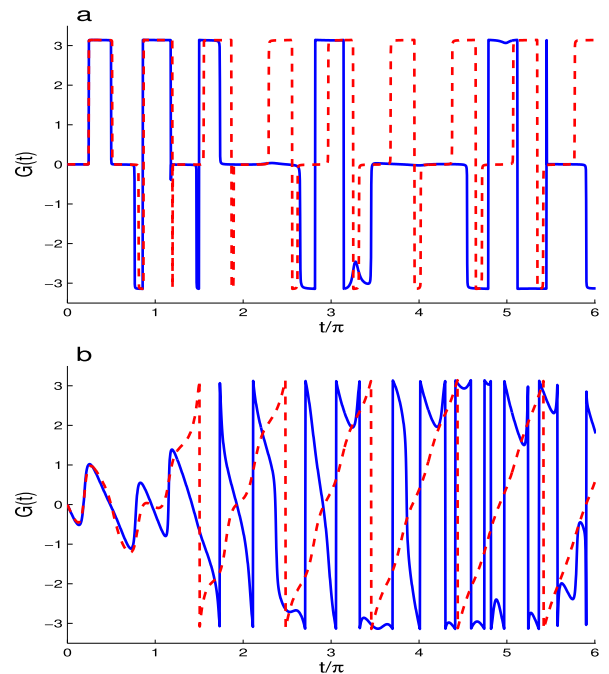


FIGURE 4. As Fig.2, but with $\lambda_1 = \lambda_2 = 0.5$.

Fig.4 illustrates the sensitivity of the generated GP oscillations to the exciton-microcavity coupling constants $\lambda_k = 0.5$. By employing the same values of the coupling constant to the environment $\kappa_k = \gamma_k = 0.4$, we note that the oscillation frequency of the GP is increased. If the cavities are filled with an optical medium, the effect of the optical susceptibility, $\chi_k = 2$ for $\lambda_k = 0.5$, leads to more GP oscillations. The effect of the excitonic and the microcavity dissipation is smoothing the shape of the GP which shows a regular dynamics. By increasing the Coupling constants λ_k , the generated GP oscillations become more more robust.

Finally, we deduce that the generated geometric phase is controlled by the external environment, the coupling constants of the exciton-microcavity and the fiber-microcavity as well as by the susceptibility of the optical medium.

B. DYNAMICS OF GP WITH UNCORRELATED STATES

For initial uncorrelated states $|\psi(0)\rangle_2$ and $|\psi(0)\rangle_3$, we investigate the effect of the couplings $\lambda_k (i = 1, 2, 3)$ to generate GP oscillations under the presence of the optical susceptibility χ_i and the dissipation rates $\kappa_k = \gamma_k$. The PGP of the initial uncorrelated states $|\psi(0)\rangle_i (i = 2, 3)$ is given by

$$G(t) = \begin{cases} \arg\{q_5(t)\}, & |\psi(0)\rangle_2; \\ \arg\{q_{17}(t)\}, & |\psi(0)\rangle_3. \end{cases} \quad (16)$$

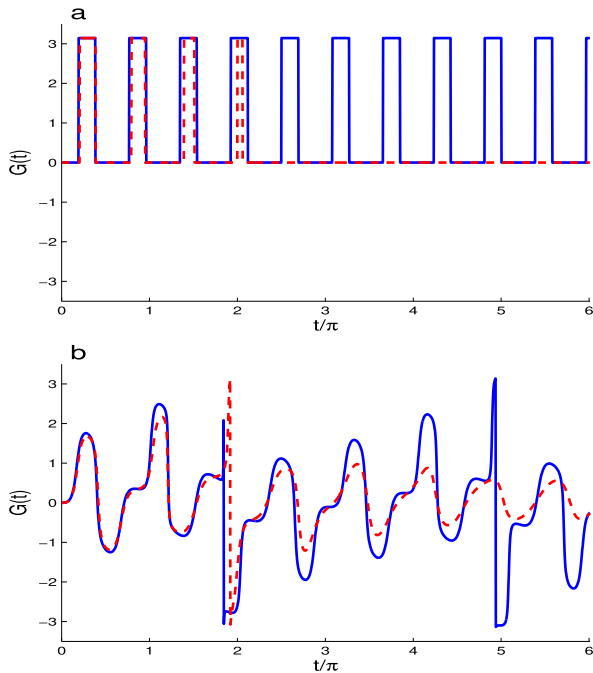


FIGURE 5. $G_P(t)$ for $|\psi(0)\rangle_2$ and $\epsilon = 10^{-3}$ with $\lambda_k = 2(k = 1 - 3)$ and $\kappa_k = \gamma_k = 0$ (solid curves) and $\kappa_k = \gamma_k = 0.4$ (dashed curves). For $\chi_k = 0.0$ in (a) and $\chi_k = 2$ in (b).

Fig.5a, shows the dynamics of the geometric phase $G(t)$ for the initial state $|\psi(0)\rangle_2$ when the interactions of the two cavity-excitons and the fiber-cavity are strong $\lambda_k = 2(k = 1 - 3)$ (as Fig.2) for different values of the cavity dissipation

rates $\kappa_k = \gamma_k$ and in the absence of the optical linear medium $\chi_k = 0$. We observed that the geometric phase has rectangular oscillations which represent the collapses/revivals phenomena. The geometric phase identified by $G(t)$, verifies the inequality $0 < G(t) < \pi$. Dashed curve of Fig.5a illustrates the robustness of the generated PGP oscillations against the cavity dissipations.

By comparing the dynamics of GP for the case $\chi_k = 0$ of Fig.5a, and for the case $\chi_k = 2$ of Fig.5b, we observe that: (1) The effect of the optical susceptibility leads to smoothing the shape of the oscillations. (2) The GP verifies the inequality $-\pi < G(t) < \pi$, and it is robust against the cavity dissipation rates.

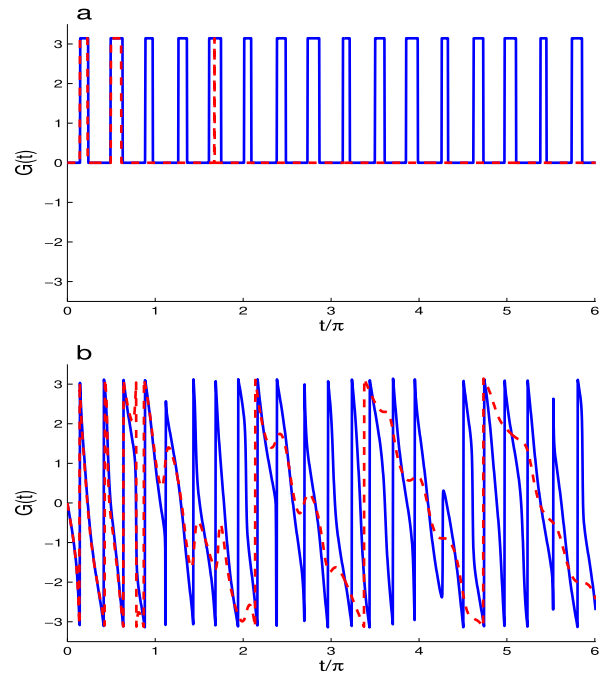


FIGURE 6. As Fig.5, but for $|\psi(0)\rangle_3$.

In Fig.6, the geometric phase for another initial uncorrelated state $|\psi(0)\rangle_3$ is depicted with $\lambda_k = 2(k = 1 - 3)$ and $\kappa_k = \gamma_k = 0, 0.4$. From Figs.6 and Figs.6, we observe that the initial state $|\psi(0)\rangle_3$ realizes PGP with more oscillations than the one with $|\psi(0)\rangle_2$. The amplitudes and the number of the GP oscillations increase for the initial state $|\psi(0)\rangle_3$. Dashed curves of the Fig.6 show the robustness of the generate GP for the initial state $|\psi(0)\rangle_3$ against the cavity dissipations rates. We deduce that the Geometric phase is more robust than the case with the initial uncorrelated state $|\psi(0)\rangle_2$. Finally, we can deduce that the robustness of the generated PGP against the cavity dissipations depends on susceptibility of the optical medium and the coupling constants of the exciton-microcavity and the fiber-microcavity interactions as well as on the initial states.

IV. CONCLUSION

In this work, we have investigated a system formed by two spatially separated open microcavities linked by a waveguide.

Each microcavity contains one-exciton and is filled by a linear optical medium. The robustness of the generated geometric phase of the entire system is investigated. It is shown that the generation and robustness of the GP oscillations are very sensitive to three parameters; first, the effects of the cavity-exciton; second, the fiber-cavity coupling rates and finally to the susceptibility of the linear medium. These parameters control the regularity, the amplitudes, and the frequencies of the GP oscillations. The dissipations are responsible of the GP vanishing. These effects can be delayed by the optical susceptibility and the coupling constants of the cavity-exciton and the fiber-cavity. These results could strengthen the understanding of the geometric phase and its impact on implementing a distributed quantum network.

The qubit-cavity systems linked by a waveguide present several potential applications for the quantum-gate realizations [7], the quantum correlations [27], distributed quantum computation [30] and quantum network [46].

ACKNOWLEDGMENT

The authors are very grateful to the referees and the editor for their constructive remarks, which have helped to improve the manuscript. This project was supported by the Deanship of Scientific Research at Prince Sattam Bin Abdulaziz University under the research project No. 2020/01/11801.

REFERENCES

- [1] W. H. Louisell, *Quantum Statistical Properties of Radiation*. New York, NY, USA: Wiley, 1973.
- [2] G. J. Milburn, "Intrinsic decoherence in quantum mechanics," *Phys. Rev. A, Gen. Phys.*, vol. 44, p. 5401, Nov. 1991.
- [3] A.-S. F. Obada, H. A. Hessian, and A.-B. A. Mohamed, "The effects of thermal photons on entanglement dynamics for a dispersive Jaynes—Cummings model," *Phys. Lett. A*, vol. 372, no. 20, pp. 3699–3706, May 2008.
- [4] A.-B. A. Mohamed, H. Eleuch, and C. H. Raymond Ooi, "Non-locality correlation in two driven Qubits inside an open coherent cavity: Trace norm distance and maximum bell function," *Sci. Rep.*, vol. 9, p. 19632, Dec. 2019.
- [5] A.-S. F. Obada, A.-B. A. Mohamed, M. Hashem, and M. M. Elkhateeb, "Generating non-classical correlations between two superconductor qubits confined in a transmission cavity in dispersive limit under intrinsic noise," *Phys. E: Low-Dimensional Syst. Nanostruct.*, vol. 117, Mar. 2020, Art. no. 113854.
- [6] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, "Geometric quantum computation using nuclear magnetic resonance," *Nature*, vol. 403, pp. 869–871, Feb. 2000.
- [7] T. Liu, X.-Z. Cao, Q.-P. Su, S.-J. Xiong, and C.-P. Yang, "Multi-target-qubit unconventional geometric phase gate in a multi-cavity system," *Sci. Rep.*, vol. 6, p. 21562, Feb. 2016.
- [8] C. Song, S.-B. Zheng, P. Zhang, K. Xu, L. Zhang, Q. Guo, W. Liu, D. Xu, H. Deng, K. Huang, D. Zheng, X. Zhu, and H. Wang, "Continuous-variable geometric phase and its manipulation for quantum computation in a superconducting circuit," *Nature Commun.*, vol. 8, p. 1061, Oct. 2017.
- [9] Y. Wang, C. Guo, G.-Q. G. Zhang, and G. Wang, and C. Wu, "Dynamic generation of multi-qubit entanglement in the ultrastrong-coupling regime," *Sci. Rep.*, vol. 7, p. 44251, 2017.
- [10] B. Douçot, R. Danneau, K. Yang, J.-G. Caputo, and R. Mélin, "Berry phase in superconducting multiterminal quantum dots," *Phys. Rev. B, Condens. Matter*, vol. 101, Jan. 2020, Art. no. 035411.
- [11] Y. Aharonov and J. Anandan, "Phase change during a cyclic quantum evolution," *Phys. Rev. Lett.*, vol. 58, p. 1593, Apr. 1987.
- [12] J. Samuel and R. Bhandari, "General setting for Berry's phase," *Phys. Rev. Lett.*, vol. 60, p. 2339, Jun. 1988.
- [13] K.-P. Marzlin, S. Ghose, and B. C. Sanders, "Geometric phase distributions for open quantum systems," *Phys. Rev. Lett.*, vol. 93, Dec. 2004, Art. no. 260402.
- [14] Y.-X. Liu, C. P. Sun, and F. Nori, "Scalable superconducting qubit circuits using dressed states," *Phys. Rev. A, Gen. Phys.*, vol. 74, 2006, Art. no. 052321.
- [15] L.-M. Duan, J. I. Cirac, and P. Zoller, "Geometric manipulation of trapped ions for quantum computation," *Science*, vol. 292, pp. 1695–1697, Jun. 2001.
- [16] A. Recati, T. Calarco, P. Zanardi, J. I. Cirac, and P. Zoller, "Holonomic quantum computation with neutral atoms," *Phys. Rev. A, Gen. Phys.*, vol. 66, 2002 Art. no. 032309.
- [17] A.-B. A. Mohamed and H. Eleuch, "Geometric phase in cavity QED containing a nonlinear optical medium and a quantum well," *J. Mod. Opt.*, vol. 62, no. 19, pp. 1630–1637, 2015.
- [18] A.-B. A. Mohamed and A.-S. F. Obada, "Asymptotic geometric phase and purity for phase qubit dispersively coupled to lossy LC circuit," *Ann. Phys.*, vol. 326, pp. 2369–2376, Sep. 2011.
- [19] M. A. Bouchene, M. Abdel-Aty, and S. Mandal, "Sensitivity of the population and the Pancharatnam phase for a trapped ion with Stark shift," *Phys. Rev. A, Gen. Phys.*, vol. 82, Aug. 2010, Art. no. 023409.
- [20] A.-B. A. Mohamed and I. Masmali, "Control of the geometric phase in two open qubit–cavity systems linked by a waveguide," *Entropy*, vol. 22, no. 1, p. 85, 2020.
- [21] S. Austin, S. Zahid, and A. Z. Chaudhry, "Geometric phase corrected by initial system–environment correlations," *Phys. Rev. A, Gen. Phys.*, vol. 101 Feb. 2020, Art. no. 022114.
- [22] J. I. Cirac, A. K. Ekert, S. F. Huelga, and C. Macchiavello, "Distributed quantum computation over noisy channels," *Phys. Rev. A, Gen. Phys.*, vol. 59, p. 4249, Jun. 1999.
- [23] M. Paternostro, M. S. Kim, and G. M. Palma, "Non-local quantum gates: A cavity-quantum-electrodynamics implementation," *J. Mod. Opt.*, vol. 50, no. 13, pp. 2075–2094, 2003.
- [24] T. Pellizzari, "Quantum networking with optical fibres," *Phys. Rev. Lett.*, vol. 79, p. 5242, Dec. 1997.
- [25] S. Mancini and S. Bose, "Ponderomotive entangling of atomic motions," *Phys. Rev. A, Gen. Phys.*, vol. 64, Aug. 2001, Art. no. 032308.
- [26] J. Cho and H.-W. Lee, "Generation of atomic cluster states through the cavity input-output process," *Phys. Rev. Lett.*, vol. 95, Oct. 2005, Art. no. 160501.
- [27] A.-B. A. Mohamed and H. Eleuch, "Generation and robustness of bipartite non-classical correlations in two nonlinear microcavities coupled by an optical fiber," *J. Opt. Soc. Amer. B, Opt. Phys.*, vol. 35, no. 1, pp. 47–53, 2018.
- [28] P. C. Haljan, K.-A. Brickman, L. Deslauriers, P. J. Lee, and C. Monroe, "Spin-dependent forces on trapped ions for phase-stable quantum gates and entangled states of spin and motion," *Phys. Rev. Lett.*, vol. 94, Apr. 2005, Art. no. 153602.
- [29] H. J. Kimble, "The quantum Internet," *Nature*, vol. 453, pp. 1023–1030, Jun. 2008.
- [30] A. Serafini, S. Mancini, and S. Bose, "Distributed quantum computation via optical fibers," *Phys. Rev. Lett.*, vol. 96, Jan. 2006, Art. no. 010503.
- [31] R. Houdre, C. Weisbuch, R. P. Stanley, U. Oesterle, and M. Ilegems, "Non-linear emission of semiconductor microcavities in the strong coupling regime," *Phys. Rev. Lett.*, vol. 85, p. 2793, Sep. 2000.
- [32] E. Giacobino, J.-P. Karr, G. Messin, H. Eleuch, and A. Baas, "Quantum optical effects in semiconductor microcavities," *Comp. Rendus Physique*, vol. 3, no. 1, pp. 41–52, 2002.
- [33] H. Eleuch, J. M. Courty, G. Messin, C. Fabre, and E. Giacobino, "Cavity QED effects in semiconductor microcavities," *J. Opt. B: Quantum Semi-class. Opt.*, vol. 1, no. 1, p. 1, 1999.
- [34] S. J. Van Enk, H. J. Kimble, J. I. Cirac, and P. Zoller, "Quantum communication with dark photons," *Phys. Rev. A, Gen. Phys.*, vol. 59, p. 2659, Apr. 1999.
- [35] H. Eleuch, "Photon statistics of light in semiconductor microcavities," *J. Phys. B, At., Mol. Opt. Phys.*, vol. 41, no. 5, 2008, Art. no. 055502.
- [36] A.-B. Mohamed and H. Eleuch, "Non-classical effects in cavity QED containing a nonlinear optical medium and a quantum well: Entanglement and non-Gaussianity," *Eur. Phys. J. D*, vol. 69, p. 191, Aug. 2015.
- [37] A.-B. A. Mohamed and H. Eleuch, "Quantum correlation control for two semiconductor microcavities connected by an optical fiber," *Phys. Scripta*, vol. 92, no. 6, 2017, Art. no. 065101.
- [38] S. Pancharatnam, "Generalized theory of interference and its applications," *Proc. Indian Acad. Sci. A*, vol. 44, no. 6, p. 398, 1956.

- [39] D. Chruscinski and A. Jamiolkowski, *Geometric Phases in Classical and Quantum Mechanics* (LLC). New York, NY, USA: Springer-Verlag, 2004.
- [40] A. G. Wagh, V. C. Rakhecha, J. Summhammer, G. Badurek, H. Weinfurter, B. E. Allman, H. K. Kaiser Hamacher, D. L. Jacobson, and S. A. Werner, "Experimental separation of geometric and dynamical phases using neutron interferometry," *Phys. Rev. Lett.*, vol. 78, p. 755, Feb. 1997.
- [41] A. G. Wagh, V. C. Rakhecha, P. Fischer, and A. Ioffe, "Neutron interferometric observation of noncyclic phase," *Phys. Rev. Lett.*, vol. 81, p. 1992, Sep. 1998.
- [42] H. Kobayashi, S. Tamate, T. Nakanishi, K. Sugiyama, and M. Kitano, "Observation of geometric phases in quantum erasers," *J. Phys. Soc. Jpn.*, vol. 80, no. 3, 2011, Art. no. 034401.
- [43] B. Lin, J. Guo, L. Lv, Z. Liu, X. Ji, and J. Wu, "An ultra-wideband reflective phase gradient metasurface using pancharatnam-berry phase," *IEEE Access*, vol. 7, pp. 13317–13325, 2019.
- [44] A. Uhlmann, "Geometric phases and related structures," *Rep. Math. Phys.*, vol. 36, nos. 2–3, pp. 461–481, 1995.
- [45] D. M. Tong, E. Sjöqvist, L. C. Kwek, and C. H. Oh, "Kinematic approach to the mixed state geometric phase in nonunitary evolution," *Phys. Rev. Lett.*, vol. 93, Aug. 2004, Art. no. 080405.
- [46] T. Pellizzari, "Quantum networking with optical fibres," *Phys. Rev. Lett.*, vol. 79, p. 5242, Dec. 1997.



ABDEL-BASET A. MOHAMED was born in Assiut, Egypt, in April 1974. He received the M.S. and Ph.D. degrees in applied mathematics from Assiut University, Egypt. He is currently a Professor with Assiut University and Prince Sattam Bin Abdulaziz University, Saudi Arabia. His research interests include applied mathematics and mathematical physics, including different directions in quantum information and computation.



AHMED FAROUK (Member, IEEE) received the M.Sc. and Ph.D. degrees from Mansoura University, Egypt. He is currently a Postdoctoral Research Fellow with Wilfrid Laurier University and Ryerson University, Canada. He is one of the top 20 technical co-founders of the Quantum Machine Learning Program by Creative Destruction Lab, University of Toronto. He is selected as Top 25 of InnovateTO150 Canada to showcase the best of Toronto's next generation of change-

makers, innovators, and entrepreneurs. He is exceptionally well known for his seminal contributions to theories of quantum mechanics, communications, and cryptography. The exceptional quality of his research is recognized nationally and internationally. He was awarded many full travel grants to attend prestigious scientific meetings, such as one sponsored by the IEEE Computer Society for attending First Global Chapter Summit: Engaging with Industry in Italy and other sponsored by the Okinawa Institute of Science and Technology for attending Information Security in the Quantum Age (ISQA) Workshop in Japan. He was nominated by the African Academy of Science to attend 69th Lindau Nobel Laureate Meeting dedicated to physics and 70th Lindau Nobel Laureate Meeting (Interdisciplinary) and selected by the Scientific Review Panel of the Council for the Lindau Nobel Laureate Meetings to participate in the 70th Lindau Nobel Laureate Meeting. He was appointed as the Chair of the IEEE Computer Chapter for the Waterloo-Kitchener Area and Editorial Board of *Scientific Reports* (Nature), *IET Quantum Communication*, and *IEEE ACCESS*.



ABDEL-HALEEM ABDEL ATY received the B.Sc. and M.Sc. degrees in physics from the Physics Department, Al-Azhar University, Egypt, in 2004 and 2009, respectively, and the Ph.D. degree in 2015. After finishing the master's degree in 2011, Universiti Teknologi PETRONAS, Malaysia, awarded him the Full Ph.D. Scholarship in theoretical physics (quantum information). In 2017, he received the Scholarship as a Visiting Researcher from the University of Oxford, U.K.

He is especially working in theories of quantum measurement, nanomechanical modeling, highly non-classical light, practical information security, and optical implementations of quantum information tasks. His current research interests include quantum resources, optical and atomic implementations of quantum information tasks and protocols, and mathematical modeling. He has published more than 40 articles in ISI journals, four book chapters, and ten papers in conference proceedings and indexed by Scopus and ISI. In 2015, he was awarded the Sultana Nahar's Prize for the Best Ph.D. Thesis in Egypt. In 2017, has been elected as a Junior Associate Membership from the African Academy of Sciences, Kenya. He is acting as the Managing Editor of some journals, such as *Quantum Physics Letters* and the *International Journal of New Horizons in Physics*. He is also acting as the Vice-President of Natural Sciences Publishing, USA.



HICHEM ELEUCH received the Ph.D. degree in quantum physics from the Kastler-Brossel Laboratory, École Normale Supérieure de Paris, Université Pierre et Marie Curie, France. He worked at and visited several prestigious universities and research institutes (Texas A&M University, the Max Planck Institute of Quantum Optics, Princeton University, McGill University, Auckland University, and so on). He joined Abu Dhabi University as a Professor of physics. He has authored or coauthored more than 170 articles. He has participated in over 60 international conferences and gave more than 30 invited talks. He has refereed articles for more than 40 journals of physics and mathematics. His research interests include quantum optics, quantum computing, mathematical physics, and complex systems.

...