

Solutions to the Konopelchenko-Dubrovsky equation and the Landau-Ginzburg-Higgs equation via the generalized Kudryashov technique

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ABSTRACT

The $(2 + 1)$ -dimensional Konopelchenko-Dubrovsky (KD) equation and the Landau-Ginzburg-Higgs (LGH) equation describe the nonlinear waves with weak scattering and long-range interactions between the tropical, mid-latitude troposphere, the interaction of equatorial and mid-latitude Rossby waves etc. This article studies the KD and LGH models stated earlier using the generalized Kudryashov technique. We obtained a variety of analytical solutions including unknown parameters. The figures of some of the obtained solutions are sketched with certain parameters. The derived results demonstrate the efficiency and reliability of the generalized Kudryashov technique for establishing systematic solutions to nonlinear evolution equations (NLEEs).

Introduction

The concept of solitary waves contributes a significant role for recognizing the intricate incidents associated with various technological fields, especially in plasma physics, solid state physics, chemical kinematics, elasticity theory, nonlinear optics, condensed matter physics, quantum mechanics, plasma waves, propagation of shallow water waves, nano-chemistry, quantum field theory, mathematical biology, fluid mechanics etc. [1–11]. The NLEEs emerge when interpreting these tangible incidents mathematically. These physical incidents can be better understood through investigating the exact solutions to these NLEEs. It is noticeable that there have some remarkable improvements for investigating analytic solutions to NLEEs in the recent years. Furthermore, nonlinear wave equations provide several examples of new solutions that are greatly different from linear wave problems. Water waves, shock waves and solitary waves are the best-known

example of the nonlinear wave equations. In the last twentieth century, the main attainment of science is the discovery of the inverse scattering transformation method and the soliton interactions to find the analytic solution for asymptotic perturbation analysis and canonical PDEs for the NLEEs [12]. Therefore, the solitary wave solutions to NLEEs are of extreme importance because of their potential applications in many physical areas such as neural physics, chaos, diffusion process, reaction process etc. There are several schemes, for instance, the modified Kudryashov method [13], the exp-function method [14,15], the auxiliary equation method [16,17], the dual mode fourth-order Burgers equation [18], the (G'/G) -expansion method [19–21], the first integral method [22], the new generalized (G'/G) expansion method [23], the improved Kudryashov method [24], the tanh-method [25,26], the extended trial equation method [27], the $(G'/G, 1/G)$ -expansion method [28], the sine-Gordon equation expansion method [29], the improved F-expansion method [30], the modified simple equation

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method [31–34], the improved Bernoulli sub-equation function method [35], the Hirota bilinear transformation method [36–41], the Darboux transformation method [42–44] etc. which have been used for searching stable closed-form wave solutions to NLEEs.

In this article, we search atypical stable soliton solutions of a couple of NLEEs, namely the $(2 + 1)$ -dimensional Konopelchenko-Dubrovsky equation and the Landau-Ginzburg-Higgs equation. In 1984, by using the inverse scattering transformation [45], Konopelchenko-Dubrovsky investigated some $(2 + 1)$ -dimensional NLEEs. The subsequent Konopelchenko-Dubrovsky [46] equation is one of them:

$$\frac{\partial v}{\partial t} - \frac{\partial^3 v}{\partial x^3} - 6nv \frac{\partial v}{\partial x} + \frac{3}{2}m^2 v^2 \frac{\partial v}{\partial x} - 3 \frac{\partial w}{\partial y} + 3mw \frac{\partial v}{\partial x} = 0, \quad (1a)$$

$$\frac{\partial v}{\partial y} = \frac{\partial w}{\partial x}, \quad (1b)$$

where m and n are real parameters. The KD model is fully integrable NLEE associated with two spatial variables and one temporal variable. If $\frac{\partial v}{\partial y} = 0$, the simplified model (1) is named as the Gardner equation (combined KdV and mKdV equation). On the other hand, if $m = 0$, the model (1) is converted to the well-known Kadmatsev-Petviashvili (KP) equation and for $n = 0$, it is converted to the modified KP equation.

The NLEE (1) has been examined by diverse groups of researchers; videlicet Wang and Zhang [47] implemented the further improved F-expansion method and found only a couple of wave solutions, namely periodic solution and general soliton solutions. They did not sketch the geometrical structures of these results. By means of the improved Bernoulli sub-equation function method, Dusunceli [48] obtained some explicit soliton solutions, such as singular soliton, kink soliton and periodic. These results have been expressed in terms of $\sinh$ and $\cosh$ functions. Wazwaz [49] executed three efficient techniques, namely the tanh-sech method, the cosh-sinh method and the exponential function method in order to ascertain the periodic, kink and other types of soliton shapes. He [50] put in use an effective scheme, namely the bifurcation theory which is not only provides the analytic solutions, but also formulates the dynamical features of the results. Periodic wave solutions involving Jacobi elliptic functions for NLEE (1) have been reported via the extended F-expansion method [51]. Zhang and Xia [52] utilized the generalized F-expansion method and derived some general soliton solutions including single and combined Jacobi elliptic functions.

The Landau-Ginzburg-Higgs equation [53] is stated as

$$\frac{\partial^2 q}{\partial t^2} - \frac{\partial^2 q}{\partial x^2} - g^2 q + h^2 q^3 = 0 \quad (2)$$

wherein $q(x, t)$ symbolizes the electrostatic potential of the ion-cyclotron wave x and t stand for the nonlinearized spatial and temporal coordinates and g and h are real parameters. The NLEE (2) was formulated by Lev Devidovich Landau and Vitaly Lazarevich Ginzburg with broad applications for the explanation of superconductivity and drift cyclotron waves in radially inhomogeneous plasma for coherent ion-cyclotron waves. Several methods have been adopted to evaluate the unique soliton solutions to the integrable NLEE (2). Bekir and Unsal [54] used the first integral method to search the NLEE (1) and accomplished some exponential function solutions. Iftikhar et al. [55] investigated different type of analytic solutions to NLEE (1) through putting in use of the $(G'/G, 1/G)$ -expansion method. They have ascertained general soliton solutions and kink shape soliton for different parametric choices. Islam and Akbar [56] derived a few types of stable solutions by using the IBSEF method. The obtained results provide many types of soliton shape like as kink wave shape, singular soliton, periodic and other soliton shapes which comprehend the unidirectional wave propagation in nonlinear science with dispersion systems.

Using the generalized Kudryashov approach, to our utmost understanding the Konopelchenko-Dubrovsky equation and the Landau-Ginzburg-Higgs equation were not analyzed. Therefore, the aim of this

article is to evaluate wide-ranging, advanced and contextual solutions to the considered wave equations by the use of the generalized Kudryashov technique. The application of this method is observed in the works [57–62]. The solutions could describe the dispersive waves, long range interactions between the tropical, mid-latitude troposphere, Rossby waves etc. Some recent works about the solitary wave solution have been discussed in [63–67].

The generalized Kudryashov method

In this paragraph, we interpret shortly the generalized Kudryashov method for extracting the consistent solutions to NLEEs. We firstly presume a NLEE of the general form

$$H\left(v, \frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \frac{\partial^2 v}{\partial x^2}, \frac{\partial^2 v}{\partial x \partial t}, \dots\right) = 0 \quad (3)$$

wherein $v = v(x, t)$ is an underdetermined function, H represents a polynomial in v and $\frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \dots$ imply the partial derivatives with respect to the space and temporal variables. The wave variable

$$v(x, y, t) = v(\psi), \psi = \mu x + \lambda y - ct, \quad (4)$$

whereinto c is the wave speed and μ, λ denote the wave numbers, alters the NLEE (3) into the following ordinary differential equation (ODE):

$$S\left(v, \frac{dv}{d\psi}, \frac{d^2 v}{d\psi^2}, \dots\right) = 0 \quad (5)$$

wherein S defines the polynomial in v . We deduce a general solution of ODE (5) in the form

$$v(\psi) = \frac{a_0 + \sum_{j=1}^Q a_j Y^j(\psi)}{b_0 + \sum_{l=1}^P b_l Y^l(\psi)} \quad (6)$$

wherein a_j and b_l are unidentified parameters to be calculated such that $a_Q \neq 0$ and $b_P \neq 0$ and $Y(\psi)$ comply with the Bernoulli's differential equation

$$\frac{dY(\psi)}{d\psi} = -Y(\psi) + Y^2(\psi) \quad (7)$$

which admits a solution of the form

$$Y(\psi) = \frac{1}{1 + \eta \exp(\psi)} \quad (8)$$

where η defines the integration constant whose values can be selected arbitrarily. The positive quantities Q and P involved in Eq. (6) can be determined by means of the balancing theory between the nonlinear term and the highest order derivative occurring in Eq. (5).

Inserting the solution (6) into Eq. (5) along with (7) gives a polynomial in $Y(\psi)$. A set of algebraic equations can be found by equalizing the coefficient of $Y(\psi)$ and setting them to zero. By using mathematical software such as Maple, the value of the unknowns $a_j (j = 1, 2, \dots, Q)$, $b_l (l = 1, 2, \dots, P)$ and the wave velocity c can be evaluated and the desired solutions to the NLEE (3) are identified.

Determination of solutions

A viable method, namely the generalized Kudryashov technique [68–70] has been used in this section to evaluate the explicit solutions to the Konopelchenko-Dubrovsky equation and the Landau-Ginzburg-Higgs equation respectively.

The Konopelchenko-Dubrovsky equation

Since our aim is to evaluate a number of stable and explicit solitary

wave solutions, a new wave variable for the NLEE (1) is specified initially:

$$v(x, y, t) = v(\psi), w(x, y, t) = w(\psi), \psi = x + y - ct, \quad (9)$$

where c refers to the velocity of the soliton which reduces the NLEE (1) into a one-dimensional ODE given below:

$$-c \frac{dv}{d\psi} - \frac{d^3 v}{d\psi^3} - 6nv \frac{dv}{d\psi} + \frac{3}{2} m^2 v^2 \frac{dv}{d\psi} - 3 \frac{dw}{d\psi} + 3mw \frac{dv}{d\psi} = 0, \quad (10)$$

$$\frac{dv}{d\psi} = \frac{dw}{d\psi}. \quad (11)$$

The solution of Eq. (11) is presented in the following

$$w(\psi) = v(\psi) + k, \quad (12)$$

where k is the constant of integration. Eliminating w and $\frac{dw}{d\psi}$ from the Eq. (10) and simplifying, we receive a new structure

$$2 \frac{d^3 v}{d\psi^3} - m^2 \frac{d}{d\psi} (v^3) + (6n - 3m) \frac{d}{d\psi} (v^2) + (2c + 6 - 6mk) \frac{dv}{d\psi} = 0. \quad (13)$$

Integrating equation (13) with respect to ψ and ignoring the integral constant provides

$$2 \frac{d^2 v}{d\psi^2} - m^2 v^3 + (6n - 3m)v^2 + (2c + 6 - 6mk)v = 0. \quad (14)$$

The homogeneous balance principle generates a relation as follows

$$Q = P + 1, \quad (15)$$

where P is a free parameter. Therefore, for any integer $P = 1$, the relation (15) reveals that $Q = 2$. The definite values of P and Q generate the solution stated as

$$v(\psi) = \frac{a_0 + a_1 Y(\psi) + a_2 Y^2(\psi)}{b_0 + b_1 Y(\psi)} \quad (16)$$

wherein a_0, a_1, a_2, b_0 and b_1 symbolizes the unspecified constants to be evaluated. Inserting the solution (16) along with Bernoulli equation (7) into the equation (14) yields a polynomial in $Y(\psi)$. Collecting the co-

$$v(x, y, t) = \frac{-2\eta}{(m - n - n\eta^2)\cosh(x + y - ct) - (m - n + n\eta^2)\sinh(x + y - ct) + (m - 2n)\eta} \quad (26)$$

efficients of similar power of $Y(\psi)$ and setting zero deliver the subsequent algebraic equations:

$$3ma_0^2 b_0 - 6na_0^2 b_0 - 2ca_0 b_0^2 + m^2 a_0^3 - 6a_0 b_0^2 + 6mka_0 b_0^2 = 0 \quad (17)$$

$$6mka_1 b_0^2 + 12mka_0 b_1 b_0 + 6ma_0 a_1 b_0 - 12na_0 a_1 b_0 - 4ca_0 b_1 b_0 - 8b_0^2 a_1 + 3ma_0^2 b_1 - 6na_0^2 b_1 - 10b_1 a_0 b_0 + 3m^2 a_1 a_0^2 - 2ca_1 b_0^2 = 0 \quad (18)$$

$$3m^2 a_2 a_0^2 - 2ca_0 b_1^2 - 12na_0 a_1 b_1 - 12na_0 a_2 b_0 + 6ma_0 a_1 b_1 + 6ma_0 a_2 b_0 + 6mka_1 b_0^2 - 4ca_1 b_1 b_0 + 6mka_2 b_0^2 + 12mka_0 b_1 b_0 - 8a_0 b_1^2 - 14a_2 b_0^2 + 6a_1 b_0^2 - 2ca_2 b_0^2 - 10a_1 b_1 b_0 - 6a_0 b_1 b_0 - 6na_1^2 b_0 + 3ma_1^2 b_0 = 0. \quad (19)$$

$$3ma_1^2 b_1 - 6na_1^2 b_1 - 2a_1 b_1 b_0 + 4a_0 b_1 b_0 - 2ca_1 b_1^2 - 18a_2 b_1 b_0 - 4a_1 b_0^2 + 6ma_0 a_2 b_1 - 6a_1 b_1^2 + 20a_2 b_0^2 + 12mka_2 b_1 b_0 + 6mka_1 b_1^2 - 4ca_2 b_1 b_0$$

$$+ m^2 a_1^3 + 6m^2 a_0 a_1 a_2 + 6ma_1 a_2 b_0 - 12na_1 a_2 b_0 + 12na_0 a_2 b_1 = 0. \quad (20)$$

$$6ma_1 a_2 b_1 - 12na_1 a_2 b_1 + 6mka_2 b_1^2 - 126a_2 b_0^2 - 8a_2 b_1^2 + 18b_1 a_2 b_0 + 6ma_2^2 b_0 - 6na_2^2 b_0 + 3m^2 a_0 a_2^2 + 3m^2 a_2 a_1^2 + 2ca_2 b_1^2 = 0. \quad (21)$$

$$6a_2 b_1^2 - 6na_2^2 b_1 + 3ma_2^2 b_1 + 3m^2 a_1 a_2^2 - 12a_2 b_1 b_0 = 0. \quad (22)$$

$$m^2 a_2^2 - 4a_2 b_1^2 = 0. \quad (23)$$

The above-mentioned algebraic equations provide solutions concerned with some unspecified parameters as below:

Set 1:

$$c = 3mk - 4, \quad a_0 = 0, \quad a_1 = -a_2, \quad a_2 = a_2, \quad b_0 = -\frac{1}{2}na_2, \quad b_1 = \frac{1}{2}ma_2$$

Set 2:

$$c = 3mk - 4, \quad a_0 = 0, \quad a_1 = -a_2, \quad a_2 = a_2, \quad b_0 = \frac{1}{2}(m+n)a_2, \quad b_1 = -\frac{1}{2}ma_2$$

Set 3:

$$c = \frac{3ma_1 b_1 - 6na_1 b_1 - 6b_1^2 + m^2 a_1^2 + 6mkb_1^2}{2b_1^2}, \quad a_0 = 0, \quad a_1 = a_1, \quad a_2 = a_1, \quad b_0 = 0, \quad b_1 = b_1.$$

In order to construct the exact solutions, we firstly substitute the estimation of the parameters accumulated in set 1 along with (8) into the solution (16) and developed the exponential function solution:

$$v(\psi) = \frac{-2\eta}{(m - n)\exp(-\psi) + (m\eta - 2n\eta) - n\eta^2 \exp(\psi)} \quad (24)$$

The hyperbolic identities modify the solution (24) into the successive form

$$v(\psi) = \frac{-2\eta}{(m - n - n\eta^2)\cosh(\psi) - (m - n + n\eta^2)\sinh(\psi) + (m - 2n)\eta} \quad (25)$$

The estimation (25) is expressed as follows in terms of space and time variables:

In solution (26), there involve an integral constant η , thus we might accomplish a number of closed form solutions by means of assigning some values of η . Consequently, if we pick $\eta = 1$, result (26) gives the bell shape soliton

$$v(x, y, t) = \frac{-2\text{sech}(x + y - ct)}{(m - 2n)\{1 + \text{sech}(x + y - ct)\} - m\text{tanh}(x + y - ct)}. \quad (27)$$

On the other hand, for the values of $\eta = -1$, solution (26) develops

$$v(x, y, t) = \frac{2\text{sech}(x + y - ct)}{(m - 2n)\{1 - \text{sech}(x + y - ct)\} - m\text{tanh}(x + y - ct)}. \quad (28)$$

The results (27) and (28) are connected with result (12), so we ascertain the following soliton structures

$$w(x, y, t) = \frac{-2\text{sech}(x + y - ct)}{(m - 2n)\{1 + \text{sech}(x + y - ct)\} - m\text{tanh}(x + y - ct)} + k \quad (29)$$

and

$$w(x, y, t) = \frac{2\operatorname{sech}(x + y - ct)}{(m - 2n)\{1 - \operatorname{sech}(x + y - ct)\} - m \tanh(x + y - ct)} + k. \quad (30)$$

Furthermore, choosing $\eta = \frac{1}{2}$, the solution (26) yields the kink shape soliton

$$v(x, y, t) = \frac{-4}{(4m - 5n)\cosh(x + y - ct) - (4m - 3n)\sinh(x + y - ct) + (2m - 4n)}. \quad (31)$$

Again choosing $\eta = -\frac{1}{2}$, solution (26) turns to

$$v(x, y, t) = \frac{4}{(4m - 5n)\cosh(x + y - ct) - (4m - 3n)\sinh(x + y - ct) - (2m - 4n)}. \quad (32)$$

The result (12) takes the following forms in accordance with the solutions (31) and (32)

$$w(x, y, t) = \frac{-4}{(4m - 5n)\cosh(x + y - ct) - (4m - 3n)\sinh(x + y - ct) + (2m - 4n)} + k \quad (33)$$

and

$$w(x, y, t) = \frac{4}{(4m - 5n)\cosh(x + y - ct) - (4m - 3n)\sinh(x + y - ct) - (2m - 4n)} + k. \quad (34)$$

Similarly, set 2 provides the subsequent estimation combining with hyperbolic function:

$$v(x, y, t) = \frac{-2\eta}{\{(m + n)\eta^2 + n\}\cosh(x + y - ct) + \{(m + n)\eta^2 - n\}\sinh(x + y - ct) + (m + n + n\eta)} \quad (35)$$

The result (35) is obviously a general soliton solution including some parameters and an integral constant η . We can determine atypical categorical solutions by assigning the special values of η . Therefore, if we select $\eta = 2$, the solution (35) turns out to be

$$v(x, y, t) = \frac{-4}{(4m + 5n)\cosh(x + y - ct) + (4m + 3n)\sinh(x + y - ct) + (m + 3n)}. \quad (36)$$

For $\eta = -2$, the solution (35) gives

$$v(x, y, t) = \frac{4}{(4m + 5n)\cosh(x + y - ct) + (4m + 3n)\sinh(x + y - ct) + (m - n)}. \quad (37)$$

Embedding the results (36) and (37) into (12) reveals the ensuing solutions as follows:

$$w(x, y, t) = \frac{-4}{(4m + 5n)\cosh(x + y - ct) + (4m + 3n)\sinh(x + y - ct) + (m + 3n)} + k \quad (38)$$

and

$$w(x, y, t) = \frac{4}{(4m + 5n)\cosh(x + y - ct) + (4m + 3n)\sinh(x + y - ct) + (m - n)} + k. \quad (39)$$

Another choice of $\eta = 1$, the solution (35) yields

$$v(x, y, t) = \frac{-2}{\{(m + 2n)(1 + \cosh(x + y - ct))\} + m \sinh(x + y - ct)}. \quad (40)$$

For $\eta = -1$, the solution (35) is shrink to the bell-shaped soliton

$$v(x, y, t) = \frac{2}{\{(m + 2n)\cosh(x + y - ct)\} + m\{1 + \sinh(x + y - ct)\}}. \quad (41)$$

The results (40) and (41) produce the subsequent soliton structures

$$w(x, y, t) = \frac{-2}{\{(m + 2n)(1 + \cosh(x + y - ct))\} + m \sinh(x + y - ct)} + k. \quad (42)$$

and

$$w(x, y, t) = \frac{2}{\{(m + 2n)\cosh(x + y - ct)\} + m\{1 + \sinh(x + y - ct)\}} + k. \quad (43)$$

Alternatively, by using the unspecified parameters concerned with solution set 3, we secure a new form of hyperbolic functions soliton solutions:

$$v(x, y, t) = \frac{a_1}{b_1} \left[\frac{2\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + \eta}{\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + \eta} \right] \quad (44)$$

The result provides different type of soliton structures by means of particular choice of integral constant η . Independently, if we opt $\eta = \frac{1}{3}$, the solution (44) is stated as the following soliton solutions

$$v(x, y, t) = \frac{a_1}{b_1} \left[\frac{6\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + 1}{3\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + 1} \right] \quad (45)$$

Moreover, if we opt $\eta = -\frac{1}{3}$, the solution (44) is stated as

$$v(x, y, t) = \frac{a_1}{b_1} \left[\frac{6\{\cosh(x + y - ct) - \sinh(x + y - ct)\} - 1}{3\{\cosh(x + y - ct) - \sinh(x + y - ct)\} - 1} \right] \quad (46)$$

Result (12) reduces to the exact solutions with the aid of the results (45) and (46).

$$w(x, y, t) = \frac{a_1}{b_1} \left[\frac{6\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + 1}{3\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + 1} \right] + k \quad (47)$$

and

$$w(x, y, t) = \frac{a_1}{b_1} \left[\frac{6\{\cosh(x + y - ct) - \sinh(x + y - ct)\} - 1}{3\{\cosh(x + y - ct) - \sinh(x + y - ct)\} - 1} \right] + k. \quad (48)$$

Another choice of $\eta = 1$, the solution (44) is minimized as

$$v(x, y, t) = \frac{a_1}{b_1} \left[\frac{2\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + 1}{\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + 1} \right] \quad (49)$$

For $\eta = -1$, the solution (44) becomes

$$v(x, y, t) = \frac{a_1}{b_1} \left[\frac{2\{\cosh(x + y - ct) - \sinh(x + y - ct)\} - 1}{\{\cosh(x + y - ct) - \sinh(x + y - ct)\} - 1} \right] \quad (50)$$

The result (12) along with the results (49) and (50) specifies the soliton structures concerned with $\sinh$ and $\cosh$ functions

$$w(x, y, t) = \frac{a_1}{b_1} \left[\frac{2\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + 1}{\{\cosh(x + y - ct) - \sinh(x + y - ct)\} + 1} \right] + k. \quad (51)$$

and

$$w(x, y, t) = \frac{a_1}{b_1} \left[\frac{2\{\cosh(x + y - ct) - \sinh(x + y - ct)\} - 1}{\{\cosh(x + y - ct) - \sinh(x + y - ct)\} - 1} \right] + k. \quad (52)$$

Moreover, we might formulate other ample explicit and stable wave solutions by means of different choices of η , but for straightforwardness, we have disregarded these solutions.

The Landau-Ginzburg-Higgs equation

A significant NLEE is the Landau-Ginzburg-Higgs equation, which deals with the internal processes of complex physical phenomena which occur to explain superconductivity and drift cyclotron waves in radially inhomogeneous plasma for coherent ion-cyclotron waves. To search compatible solitary solutions, we will now use the generalized Kudryashov method. Introducing the wave variable

$$q(x, t) = q(\psi), \psi = \mu x - ct, \quad (53)$$

where c refers to the velocity of soliton and μ implies the wave number, to the Eq. (2), we accomplish an ODE of the structure

$$(c^2 - \mu^2) \frac{d^2 q}{d\psi^2} - g^2 q + h^2 q^3 = 0. \quad (54)$$

Homogeneous balance principle gives the following relation

$$Q = P + 1 \quad (55)$$

If we indicate $P = 1$, the relation (55) reveals that $Q = 2$. Then the solution (6) becomes

$$q(\psi) = \frac{a_0 + a_1 Y(\psi) + a_2 Y^2(\psi)}{b_0 + b_1 Y(\psi)} \quad (56)$$

wherein a_0, a_1, a_2, b_0 and b_1 are random parameters to be calculated. Embedding the solution (56) with the help of equation (7) into the equation (54) yields a polynomial in $Y(\psi)$. We separate the similar power of $Y(\psi)$ and setting them to zero leads to a system of algebraic equations as follows:

$$h^2 a_0^3 - g^2 a_0 b_0^2 = 0 \quad (57)$$

$$c^2 a_1 b_0^2 - \mu^2 a_1 b_0^2 - g^2 a_1 b_0^2 + 3h^2 a_1 a_0^2 - c^2 a_0 b_1 b_0 - 2g^2 a_0 b_1 b_0 = 0. \quad (58)$$

$$4c^2 a_2 b_0^2 - 3c^2 a_1 b_0^2 + c^2 a_0 b_1^2 + 3\mu^2 a_1 b_0^2 - 4\mu^2 a_2 b_0^2 - \mu^2 a_0 b_1^2 - g^2 a_0 b_1^2 - g^2 a_2 b_0^2 + 3h^2 a_2 a_0^2 + 3h^2 a_0 a_1^2 + 3c^2 a_0 b_1 b_0 - c^2 a_1 b_1 b_0 - 3\mu^2 a_0 b_1 b_0 + \mu^2 a_1 b_1 b_0 - 2g^2 a_1 b_1 b_0 = 0. \quad (59)$$

$$10\mu^2 a_2 b_0^2 - 10c^2 a_2 b_0^2 + 2c^2 a_1 b_0^2 - c^2 a_0 b_1^2 - 2\mu^2 a_1 b_0^2 + \mu^2 a_0 b_1^2 - g^2 a_1 b_1^2 + h^2 a_1^3 + 3c^2 a_2 b_1 b_0 - 2c^2 a_0 b_1 b_0 + c^2 a_1 b_1 b_0 + \mu^2 a_0 b_0 b_1 - \mu^2 a_1 b_1 b_0 - 2g^2 a_2 b_1 b_0 + 6h^2 a_1 b_1 b_0 = 0. \quad (60)$$

$$3h^2 a_2 a_1^2 + c^2 a_2 b_1^2 - \mu^2 a_2 b_1^2 + 6c^2 a_2 b_0^2 - 6\mu^2 a_2 b_0^2 - g^2 a_2 b_1^2 + 3h^2 a_0 a_2^2 - 9c^2 a_2 b_1 b_0 + 9\mu^2 a_2 b_1 b_0 = 0. \quad (61)$$

$$6c^2 a_2 b_1 b_0 - 6\mu^2 a_2 b_1 b_0 + 3h^2 a_1 a_2^2 + 3\mu^2 a_2 b_1^2 - 3c^2 a_2 b_1^2 = 0. \quad (62)$$

$$2c^2 a_2 b_1^2 - 2\mu^2 a_2 b_1^2 + h^2 a_2^3 = 0. \quad (63)$$

Since the availability of computer system like Maple or Mathematica facilitates the complicated algebraic calculations, so we have put in use of Maple software and constructed the subsequent solution sets:

Set 1:

$$c = \pm \frac{1}{\sqrt{2}} \sqrt{2\mu^2 - g^2}, \quad a_0 = \frac{-gb_1}{2h}, \quad a_1 = \frac{gb_1}{h}, \quad a_2 = \frac{-gb_1}{2h}, \quad b_0 = \frac{-b_1}{2}, \quad b_1 = b_1$$

Set 2:

$$c = \pm \sqrt{\mu^2 + g^2}, \quad a_0 = 0, \quad a_1 = \pm \frac{2\sqrt{2}gib_0}{h}, \quad a_2 = \mp \frac{2\sqrt{2}gib_0}{h}, \quad b_0 = b_0, \quad b_1 = -2b_0$$

Set 3:

$$c = \pm \sqrt{\mu^2 - 2g^2}, \quad a_0 = \frac{-gb_1}{h}, \quad a_1 = \frac{g(2b_0 - b_1)}{h}, \quad a_2 = \frac{2gb_1}{h}, \quad b_0 = b_0, \quad b_1 = b_1$$

Set 4:

$$c = \pm \sqrt{\mu^2 - 2g^2}, \quad a_0 = \frac{gb_1}{2h}, \quad a_1 = \frac{g(b_1 - 2b_0)}{h}, \quad a_2 = \frac{-2gb_1}{h}, \quad b_0 = b_0, \quad b_1 = b_1$$

where g and h are real free parameters. Since the solution set 1 contains some free parameters, we might substitute the values of the parameters along with the values of $Y(\psi)$ into the solution (56) and obtain the following exponential function solution:

$$q(\psi) = \frac{2g}{h} \left\{ \frac{\eta^2 \exp(\psi) + \exp(-\psi) + \eta}{(\eta^2 \exp(\psi) + \eta)} \right\} \quad (64)$$

With the help of exponential and hyperbolic function formula, we attain following result

$$q(\psi) = \frac{2g}{h} \frac{(\eta^2 + 1) \cosh(\psi) + (\eta^2 - 1) \sinh(\psi) + \eta}{\eta^2 \cosh(\psi) + \eta^2 \sinh(\psi) + \eta} \quad (65)$$

Result (65) can be rewritten in terms of space and time variables as

$$q(x, t) = \frac{2g}{h} \frac{(\eta^2 + 1) \cosh(\mu x - ct) + (\eta^2 - 1) \sinh(\mu x - ct) + \eta}{\eta^2 \cosh(\mu x - ct) + \eta^2 \sinh(\mu x - ct) + \eta} \quad (66)$$

We observed that the estimation (66) allied with integration constant η . Consequently, if we choose the values of η randomly, we obtain some new types of explicit solutions. As for instance, the result (66) along with $\eta = 1$ gives the compacton soliton structure as

$$q(x, t) = \frac{2g}{h} \left\{ \frac{2 + \text{sech}(\mu x - ct)}{1 + \tanh(\mu x - ct) + \text{sech}(\mu x - ct)} \right\} \quad (67)$$

Again, for $\eta = -1$, the solution (66) reduces to

$$q(x, t) = \frac{2g}{h} \left\{ \frac{2 - \text{sech}(\mu x - ct)}{1 + \tanh(\mu x - ct) - \text{sech}(\mu x - ct)} \right\} \quad (68)$$

Likewise, for $\eta = 1/2$, result (66) reduces to the new form of soliton solution as follows

$$q(x, t) = \frac{2g}{h} \left\{ \frac{5 \cosh(\mu x - ct) - 3 \sinh(\mu x - ct) + 2}{\cosh(\mu x - ct) + \sinh(\mu x - ct) + 2} \right\} \quad (69)$$

Solution (66) provides the singular soliton by means of $\eta = -1/2$

$$q(x, t) = \frac{2g}{h} \left\{ \frac{5 \cosh(\mu x - ct) - 3 \sinh(\mu x - ct) - 2}{\cosh(\mu x - ct) + \sinh(\mu x - ct) - 2} \right\} \quad (70)$$

In the similar procedure, the set 2 comprehends the subsequent results in terms of sech and tanh functions

$$q(x, t) = \frac{2\sqrt{2}gi}{h} \left\{ \frac{\eta \text{sech}(\mu x - ct)}{(1 + \eta^2) \tanh(\mu x - ct) - (1 - \eta^2)} \right\} \quad (71)$$

The solution (71) also contains an integration constant η , accordingly we can independently designate $\eta = \sqrt{2}$. Thus, result (71) yields the flat kink shape soliton

$$q(x, t) = \frac{4gi}{h} \left\{ \frac{\text{sech}(\mu x - ct)}{1 + 3 \tanh(\mu x - ct)} \right\} \quad (72)$$

Again, by means of the definite values of $\eta = -\sqrt{2}$, result (71) reduces to

$$q(x, t) = -\frac{4gi}{h} \left\{ \frac{\text{sech}(\mu x - ct)}{1 + 3 \tanh(\mu x - ct)} \right\} \quad (73)$$

On the other hand, for the value $\eta = 3$, generates the following solitary wave solution

$$q(x, t) = \frac{3\sqrt{2}gi}{h} \left\{ \frac{\text{sech}(\mu x - ct)}{5 \tanh(\mu x - ct) - 4} \right\} \quad (74)$$

Moreover, if we choose $\eta = -3$, the solution (71) gives

$$q(x, t) = -\frac{3\sqrt{2}gi}{h} \left\{ \frac{\text{sech}(\mu x - ct)}{5 \tanh(\mu x - ct) - 4} \right\} \quad (75)$$

Furthermore, the values of the unknown constants accumulated in set 3, we derive a general soliton solution of the form

$$q(x, t) = \frac{g}{hb_0} \left\{ \frac{(4b_0 + b_1 + \eta^2 b_1) \cosh(\mu x - ct) + (\eta^2 b_1 - 4b_0 - b_1) \sinh(\mu x - ct) + \eta(2b_0 - b_1)}{(\eta^2 + 2) \cosh(\mu x - ct) + (\eta^2 - 2) \sinh(\mu x - ct) + 3\eta} \right\} \quad (76)$$

For the value $\eta = 0$, from the result (76), we found the hyperbolic structure which represents the smooth kink shaped soliton

$$q(x, t) = \frac{g}{hb_0} \frac{(4b_0 + b_1)(1 - \tanh(\mu x - ct))}{(2 - \tanh(\mu x - ct))} \quad (77)$$

But for the choice of $\eta = 2$, yields a new soliton solution with the combination of sinh and cosh functions

$$q(x, t) = \frac{g}{hb_0} \frac{(2b_1 + 4b_0) \cosh(\mu x - ct) + (b_1 - 4b_0) \sinh(\mu x - ct) + (4b_0 + b_1)}{6 \cosh(\mu x - ct) + 2 \sinh(\mu x - ct) + 6} \quad (78)$$

Again, for $\eta = -2$, the result (76) transformed to the following structure

$$q(x, t) = \frac{g}{hb_0} \frac{(2b_1 + 4b_0) \cosh(\mu x - ct) + (b_1 - 4b_0) \sinh(\mu x - ct) - (4b_0 + b_1)}{6 \cosh(\mu x - ct) + 2 \sinh(\mu x - ct) - 6} \quad (79)$$

In the similar procedure, for the other selection of the values of η , we can achieve a number of explicit solitary wave solutions. But the additional results are not reported here for minimalism. On the other hand, for values of parameters arranged in set 4, we might accomplish further solutions, but for succinctness the rest of the solutions are not displayed in this portion.

Graphical depictions and physical explanations of the results

The graphical representations describe the physical significance of the derived results to the considered NLEEs. The 3D and 2D diagrams of some of the particular findings obtained will be outlined in this section. A 3D plot is a three-dimensional graph that is useful for investigating the

desired response and conditions of operation and a 2D plot is a feature that allows a user to construct two-dimensional plots as a function of independent variable for a dependent variable. Here we observed that all the derived results contain some random parameters. The particular wave shape depends on the effect of the associated parameters. That is, for different particular quantities of the parameters generate different type of wave shapes. In the following, we depict the 3D and 2D profiles of some attained results to the subsequent NLEEs, videlicet the Konopelchenko-Dubrovsky equation and the Landau-Ginzburg-Higgs equation.

The Konopelchenko-Dubrovsky equation

For the Konopelchenko-Dubrovsky equation, we establish various

types of explicit and stable solitary wave solutions, including several free parameters. Different choices of the parameter yield a number of wave forms, such as the bell-shape, anti-bell shape, kink, flat kink shape and other forms of the soliton. We have shown the graphical structures of some derived results in Figs. 1–5 and then interpreted the nature of the depicted profiles.

The profile of result (27) is the bell-shaped soliton for the values of random parameters $m = 1.93, n = 1.95$. The wave shape propagates smoothly by generating the wings on both sides. The bell shape soliton describes the propagation femtosecond pulse through optical silica fiber in single mode. The 3D profile is shown in Fig. 1 within the interval $-5 \leq x \leq 5, -6 \leq y \leq 6, t \leq 4$ and 2D profile is depicted for $t = 0$.

Again, for the other choice $m = 0.32, n = -0.10$ of the parameters derive the anti-bell shaped soliton whose 3D profile is shown in Fig. 2 within the same interval.

On the other hand, it is observed that, the profile of result (31) is kink shape soliton corresponding to the values $m = n = -2$ of the parameters. The wave shape ascends from right to left and tends to a constant level at $t \rightarrow \infty$. The 3D profile is depicted in Fig. 3 within the interval $-10 \leq x \leq 10, -10 \leq y \leq 10, t \leq 8$ and 2D profile is depicted for $t = 0$.

For the values of the parameters $k = -1.92, m = n = -2$, from the result (33) we secure the smooth kink shape soliton. The wave shape propagates from left to right and whose 3D surface is portrayed in Fig. 4 within the same interval.

Choosing the negative values $k = -0.97, m = -0.76, n = -0.25$ of the parameters stands for the contracted bell-shaped soliton structure of the result (38). The upper part of this shape is too narrow. The 3D profile is sketched in Fig. 5 within the interval $-13 \leq x \leq 13, -13 \leq y \leq 13, t \leq 7$ and 2D profile is sketched for $t = 0$.

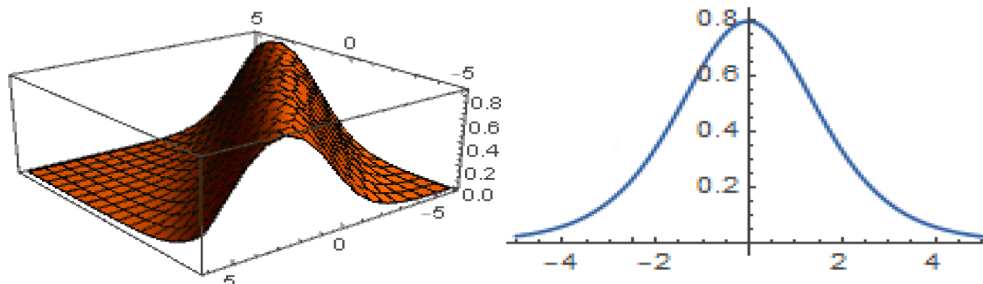


Fig. 1. Bell-shaped soliton structure of the result (27) for the values of $m = 1.93, n = 1.95$.

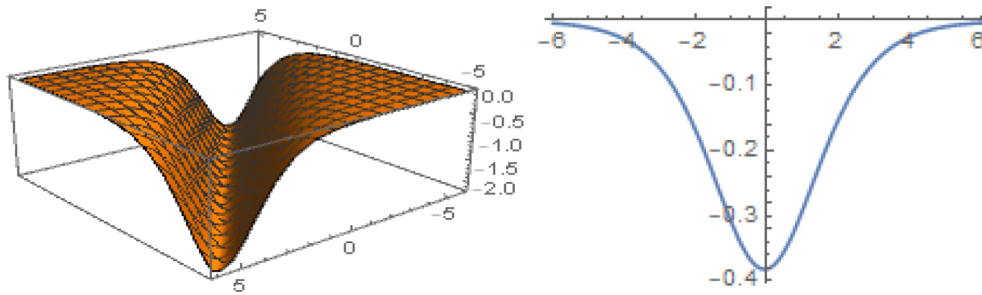


Fig. 2. Anti-bell shaped soliton structure of the result (27) for the values of $m = 0.32$, $n = -0.10$.

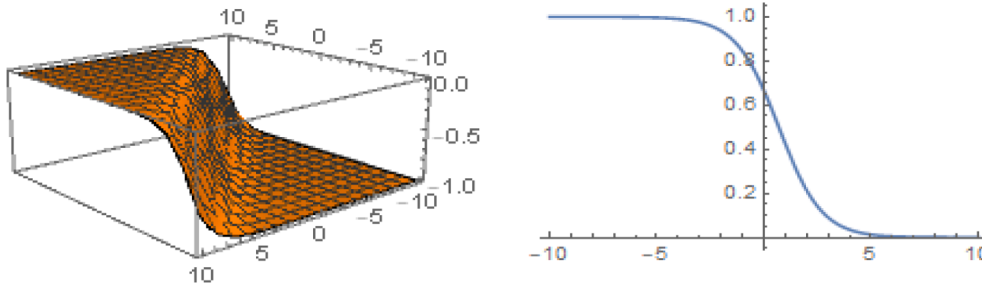


Fig. 3. Kink shape soliton of the result (31) for the values of $m = n = -2$.

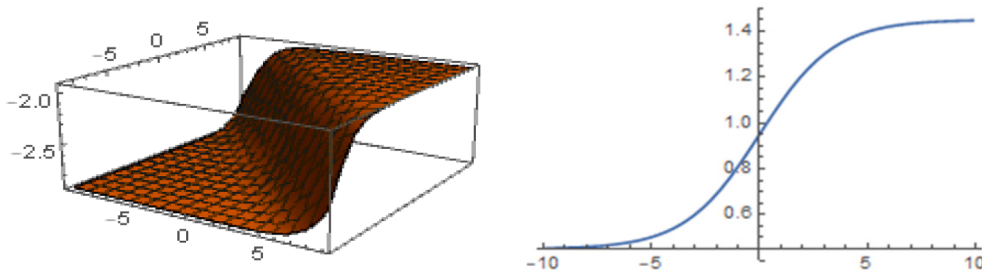


Fig. 4. Kink soliton structure of the result (33) for the values of $k = -1.92$, $m = n = -2$.

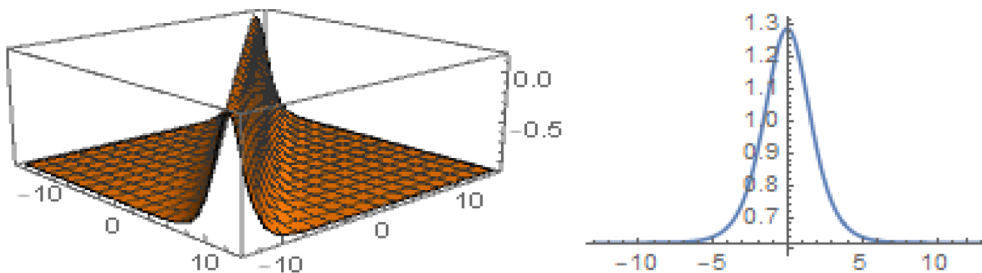


Fig. 5. Contracted bell-shaped soliton of the result (38) for the values of $k = -0.97$, $m = -0.76$, $n = -0.25$.

The Landau-Ginzburg-Higgs equation

For the Landau-Ginzburg-Higgs equation, we have acquired scores of stable and consistent results which are assembled in (65)–(79) concerned with hyperbolic functions. Since each result involves some open parameters, so we might achieve different type of general soliton shapes like compacton, kink shape, flat kink shape, singular soliton and other type solitons. In the subsequent, we have sketched the 3D and 2D plots of some derived results which are displayed in Figs. 6–11.

The result (67) consists of the parameter c, g, h, μ . For the wave speed $c = -0.12$ and parameters $g = -0.42$, $h = 0.01$, $\mu = 0.13$, we achieve the remarkable wave shape compacton. When two compactons interact,

they conserve their identities that is shape, velocity and amplitude. The 3D profile has been sketched in Fig. 6 within the interval $-8 \leq x \leq 8, t \leq 4$ and 2D profile has been depicted for $t = 0$.

Now for the result (69), we achieve a general soliton shape by means of the wave velocity $c = 0.03$ and parameters $g = -0.48, \mu = -2$. The 3D surface is depicted Fig. 7 within the interval $-5 \leq x \leq 5, t \leq 5$ and 2D surface is depicted for $t = 0$.

Fig. 8 signifies the soliton structure of the result (70) for the wave velocity $c = 0.03$ and parameters $h = 0.54, \mu = 1.10$. The lower portion of this structure decreases at infinity as $t \rightarrow \infty$. The 3D surface is sketched within the interval $-4 \leq x \leq 4, t \leq 4$ and 2D surface is sketched for $t = 0$.

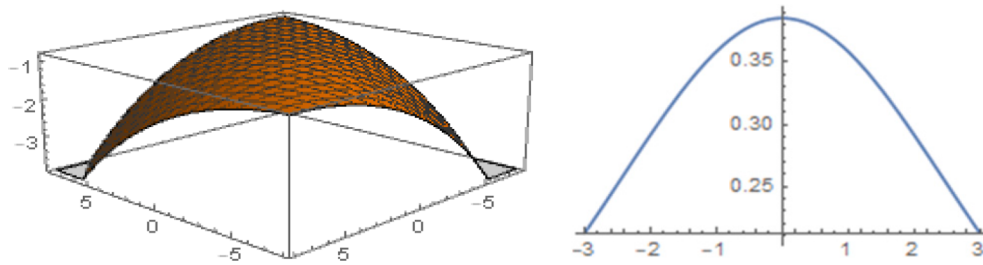


Fig. 6. Compacton structure of the result (67) for the values of $c = -0.12$, $g = -0.42$, $h = 0.01$, $\mu = 0.13$.

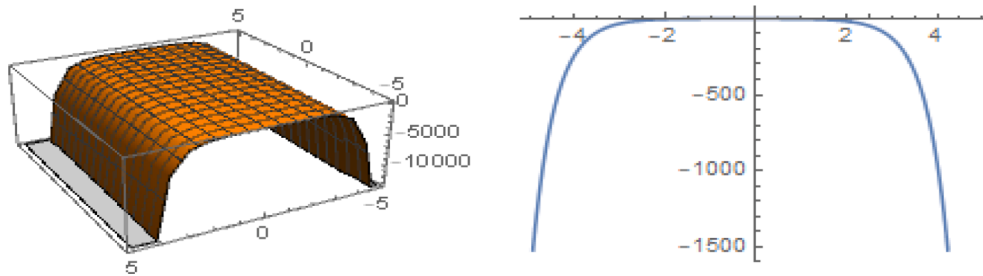


Fig. 7. Soliton structure of the result (69) for the values of $c = 0.03$, $g = -0.48$, $\mu = -2$.

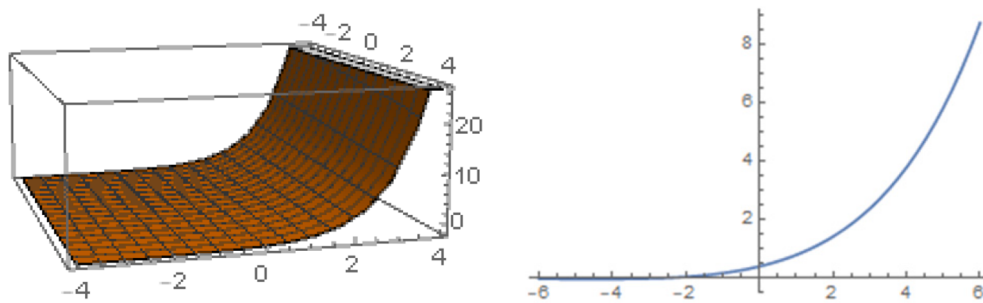


Fig. 8. Soliton structure of the result (70) for the values of $c = 0.03$, $h = 0.54$, $\mu = 1.11$.

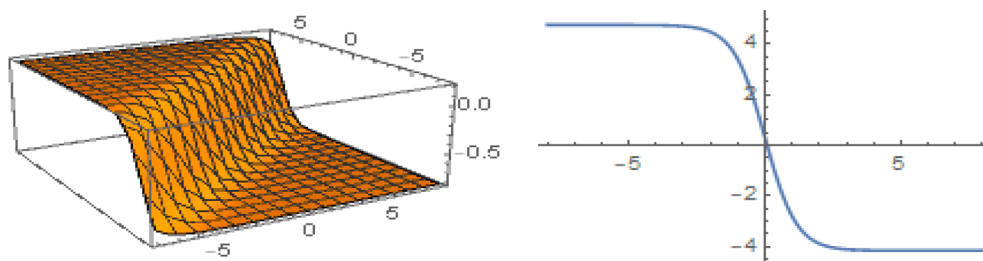


Fig. 9. Kink soliton structure of the result (71) for the values of $c = 0.64$, $h = 0.41$, $\eta = -0.50$, $\mu = 0.77$.

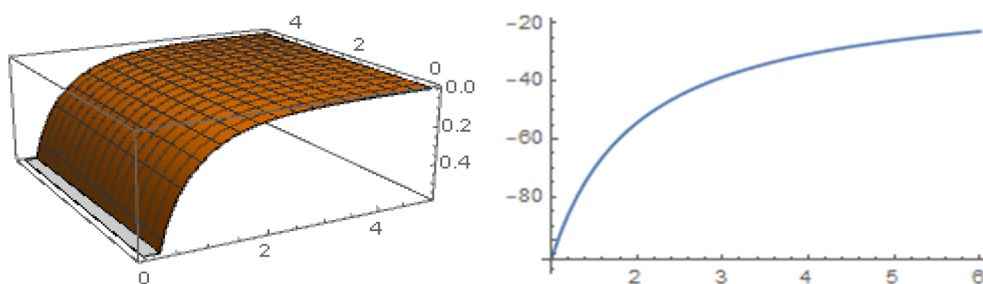


Fig. 10. Flat kink soliton structure of the result (72) for the values of $c = 0.01$, $g = 0.11$, $h = 0.35$, $\mu = 1.38$.

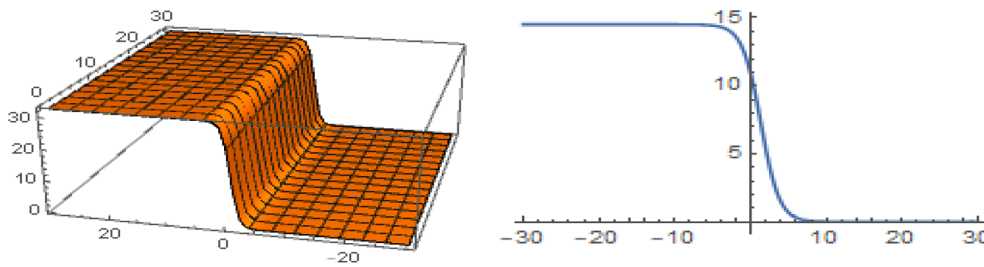


Fig. 11. Kink soliton structure of the result (77).

The solution (71) comprises the parameters c , h , η , μ and g . The wave velocity $c = 0.64$ and parameters $h = 0.41$, $\eta = -0.50$, $\mu = 0.77$, $g = 1$ produce the smooth kink wave shape displayed in Fig. 9. Initially, the wave shape flows parallel to the ground state and then it increases smoothly and finally it flows parallel to the initial level. The kink soliton plays a fundamental role in speculating the optical fiber communication and the propagation of ultra-short pulse in optical fibers. The 3D modulus plot has been presented within the interval $8 \leq x \leq -8$, $t \leq 6$ and 2D profile has shown for $t = 0$.

For the value of wave speed $c = -0.01$ and parameters $g = 0.11$, $h = 0.35$, $\mu = 1.38$, the profile of the result (72) is the flat kink shape soliton. This type of shape moves from left to right and finally propagates with constant velocity for $t \rightarrow \infty$. The 3D modulus plot has been depicted in Fig. 10 within the interval $5 \leq x$, $y \leq 0$, $t \leq 2$ and 2D profile has been depicted for $t = 0$.

The result (77) in Fig. 11 represents the kink shape soliton corresponding to the wave velocity $c = 0.01$ and parameters $g = 1.59$, $h = 0.15$, $\mu = -0.82$, $b_0 = 0.41$, $b_1 = 0.24$. The 3D profile is delineated within the interval $-30 \leq x$, $y \leq 30$, $0 < t \leq 20$ and 2D profile is delineated for $t = 0$.

Conclusion

In the present article, we have successfully figured out scores of advanced soliton solutions in terms of exponential and hyperbolic function associated with free parameters to the Konopelchenko-Dubrovsky equation and the Landau-Ginzburg-Higgs via the generalized Kudryashov method. The obtained solutions are further general, and the shape are in different forms, like bell-shape, anti-bell shape, kink, flat kink, compacton etc. Some of the obtained solutions are depicted in 3D and 2D profiles. The introduced method is effective, concise, and straightforward for solving NLEEs. The performance of the method is reliable, impressive, and computerized mathematical approach to conduct other NLEEs in the field of mathematical physics and applied sciences. Furthermore, this advantageous and influential technique can be used to investigate other NLEEs which frequently arise in various scientific real-world applications.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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