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Relative fatigue life estimation of cylindrical hollow rollers in general pure rolling contact

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Solid and hollow cylindrical rollers in pure rolling contact have been modelled. The two rollers are subjected to a combined normal and tangential loading. The tangential loading is one-third of the normal loading value. The finite element package, ABAQUS, is used to study the stress distribution and the resulting deformations in the bodies of the rollers. Then the Ioannides–Harris fatigue life model for rolling bearings is applied on the ABAQUS numerical results to investigate the fatigue life of the solid and hollow rollers. Using the fatigue life of the solid rollers as the reference fatigue life, the relative fatigue lives of hollow rollers are determined. Four main different hollowness percentages are been studied: 20, 40, 60 and 80%. The hollowness percentage is the ratio of the diameter of the hole to the outer diameter of the cylinder. For each of those hollowness percentages, two cases are studied – when the two rollers in contact are hollow and when one hollow roller is in contact with a solid roller. This study includes two main models: Model 1, where the two cylindrical rollers in contact are of the same size, and Model 2, where the two rollers in contact are not of the same size. The estimated relative fatigue lives of hollow rollers showed a great improvement of the fatigue life compared with solid rollers under the same loading conditions. This was a result of the redistribution of stresses in the contact zone in the case of hollow rollers. Redistribution of stresses over a larger volume of the roller body decreased the peak stress and reduced the volume under risk. Increasing the hollowness percentage from 20 to 60% increased the flexibility of the roller, and better stress distribution was achieved, which resulted in improving the fatigue life. Although 80% of hollowness rollers have more flexibility than 60% of hollowness rollers, the bending stresses (σ_b) on the inner surface of the rollers tend to decrease the fatigue life. Copyright © 2007 John Wiley & Sons, Ltd.

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INTRODUCTION

To date, there is no perfect investigation of the damage in rolling contacts. It is just known that repeated stressing of the machine elements causes irreversible changes in them, which results in the formation of cracks. These cracks occur after a number of load cycles based on the stress distribution inside the machine element. A sign of crack propagation is the formation of damage in the contact surface known as pitting or spalling.[1] As the stress distribution within the machine element is a main factor in determining fatigue life, design changes should be made to redistribute the contact stresses of the rolling elements.

Using hollow rollers in the roller bearings has the interest of many designers because of their advantages over solid rollers. Hollow roller bearings are single or double row radial bearings with an inner ring, outer ring and hollow or thin wall rollers. The thin wall in the rollers allows these bearings to be preloaded, as opposed to cylindrical roller bearings with solid rollers. This increases radial stiffness and reduces radial vibration and radial run out. For these reasons, hollow rollers are used in these roller bearings, but not yet in friction drives. In traction drives, two or more sets of rollers are used in contact between the inner race and the outer ring. A good example of the traction drives using cylindrical roller bearings is the self-actuating traction drive designed by Flugrad and Qamhiyah.[2] This traction drive is used as a basis for this work. Even so, the results can be generalised for cylindrical rollers used in roller bearings and other traction drives.

In spite of the many advantages of using traction drives over gears, such as less noise, ease of manufacture, and easy and cheap maintenance, there are some disadvantages. One of the main disadvantages is weight. For the same load application, the required traction drive is heavier than the required gear system. Thus, the main objective of this research is to resolve the limitations of using the cylindrical rollers, their fatigue life and their heavy weight. A solution to both these problems can be found by using hollow rollers instead of solid rollers. Using the hollow rollers means less weight, so the problems are partially solved. On the other hand, the fatigue life should be investigated. This paper presents results from a numerical simulation of two rollers in contact under pure rolling to study the fatigue life of hollow rollers and compare it with that of solid rollers.

LITERATURE REVIEW

For more than a century, the contact problem between two elastic bodies has had the attention of many researchers. Hertz[3] first investigated the problem of two elastic bodies subjected to pure normal loading. He used the Newtonian potential function to study the distribution of the stresses in the contact body and to study the distribution of the load over the contact area. He verified his solution by experimental results. Good results for the stresses in the contact region, called Hertzian stresses, were obtained. Smith and Liu[4] assumed the Hertzian distribution of the normal and tangential loads over the area of contact. The resulting stresses of applying the normal and tangential loads were represented in a closed form. The tangential loading effect on contact stresses was not taken into consideration until 1939. A general theory of two semi-infinite elastic bodies in contact was developed by Lundberg.[5] In his theory, each component of the load in each direction of the Cartesian coordinates is represented by a potential function. The tangential loading components were considered as frictional forces between the two bodies in contact, but he did not consider the stresses caused by those tangential components.

The interest in using hollow rollers in bearings started in the 20th century when researchers started to discuss the advantages and disadvantages of hollow rollers. Hanau[6] described the advantages of

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roller bearings with over 50% hollowness in high-speed applications. Given[7] pointed out the advantages of hollow roller bearings in driving turbine shafts at the required speed. Harris[8] utilised this advantage in his patent with 60–80% hollowness. In his design, he increased the hollowness to be able to increase the preloading and so reduce the roller skidding. Some experimental investigations were carried out on hollow roller bearings such as the one carried out in 1976 by Bamberger *et al.*[9] The experimental results were not sufficient to prove the advantage of using hollow rollers over solid ones. Bowen utilised the advantages of preloading the hollow rollers and their superior characteristics in his patents in 1976[10] and 1977.[11]

Many unique characteristics of the hollow roller were discussed by Bowen and Bhateja.[12] These authors experimentally investigated the 50–80% hollowness. They found that rollers with 50% hollowness are stiffer and have less deflection than rollers with 70% hollowness. Therefore, the 50% hollow roller behaves like a solid roller, and most of the deformations occur at the contact surface. Bowen and Bhateja believed the 50% hollow roller is not practically usable for most applications. They also found that the 80% hollow roller is so flexible that the load carrying capacity is severely limited. Their conclusion was that a hollowness range from 60 to 70% is the best to use for most applications.

The standard methods of calculating the fatigue life of bearings are based on the work of Lundberg and Palmgren,[13,14] who modified the original Lundberg–Palmgren (LP) theory for reliability, material and lubrication factors. One of the main shortcomings of the LP theory was the lack of a precise knowledge of the mechanics of lubrication for concentrated contacts. Life adjustment factors were used to improve the original standards of the Lundberg and Palmgren work. The LP theory risk volume extends from the surface to the point of maximum orthogonal shear stress. In 1985, a new model for the fatigue life prediction of bearings was described by Ioannides and Harris,[15] who tried to fix all the shortcomings found in the LP theory. A main difference between the Ioannides–Harris (IH) theory and the LP theory is that the IH theory defines a fatigue limit. If the stress is less than that limit, the bearing can have infinite life. Therefore, the definition of stressed volume, known as volume under risk, is the volume of the material that has a stress value greater than the fatigue limit. The IH theory was applied to beams in torsion, rotating beams and rolling bearings using the exponent parameter values of LP theory with some modifications. They had good agreement with the experimental results.

Harris and McCool[16] compared the fatigue life predictions by LP theory and IH theory for 62 applications. They found that the IH theory is more accurate in predicting the bearing fatigue life. Harris and Yu[17] demonstrated the risk volume used by the LP theory and found that their fatigue life prediction was not accurate.

The fatigue life of the hollow roller bearings compared with solid roller bearings has been studied by many researchers. The general trend has been to agree that the fatigue life is improved in the case of hollow roller bearings. However, researchers used different methods to prove that. Murthy and Rao[18] studied the contact of hollow cylindrical elements. The analytical method they used was not rigorous, but their experimental findings were still valuable. Using hollow rollers improves the wear characteristics of the contact surfaces of the rollers as presented by Somasundar and Krishnamurthy in 1984,[19] who investigated the surface durability of tufftrided hollow rolling elements. In 1996, Elsharkawy[20] developed a numerical model for two viscoelastic cylinders in contact. His analysis was to solve for the contact pressure and contact area. However, his analysis was restricted to normal contact only. The contact problem of hollow cylinders was analysed by Hong and Jianjun,[21] who used three hollowness percentages in their analysis: 50, 60 and 70%. In their experimental results, they found that when hollowness is less than 70%, fatigue life of contacting bearings can be improved. Another analysis aimed at comparing the stress distribution in solid rollers and hollow rollers was

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carried out by Zhao.[22] He simplified the bearing contact problem to a planar contact problem and used the virtual contact loading method to study the stress distribution within the roller bearing. His numerical solution agreed with the analytical solution in having a better load distribution for hollow roller bearings than the solid roller bearings.

According to the literature, the IH theory is one of the best recent theories for predicting the fatigue life of roller bearings. Therefore, it will be applied to the models developed in this work. Moreover, the approach and assumptions that Nikas[23] used in applying the IH theory on his numerical results will also be used.

PROBLEM STATEMENT AND SOLUTION TECHNIQUE

Problem statement

Although researchers have indicated that hollow rollers have longer fatigue life than solid rollers, no reliable mathematical model to estimate the hollow rollers fatigue life when subjected to a combined normal and tangential loading has been developed. On the other hand, the contact problem between two solid cylinders has been discussed by researchers. Smith and Liu[4] developed a mathematical model for solid cylinders. Trying to improve this model to work for hollow rollers resulted in unreliable assumptions. To study the fatigue life of hollow rollers, the stress distribution in the rollers' bodies needs to be determined, and then a fatigue life theory can be used to determine the fatigue life of those rollers. In pure rolling contact problems, the loading has two components, normal and tangential, which should be taken into consideration in the stress analysis. Sufficient coefficient of friction should be applied to make sure no slipping occurs.

In the self-actuating traction drive developed by Flugrad and Qamhiyah,[2] the cylindrical rollers are subjected to both normal and tangential loading. To replace those solid rollers by hollow rollers, the fatigue life of hollow rollers should be investigated. To determine the fatigue life of the hollow rollers, the stress and strain distribution should be studied and analysed. However, no reliable mathematical model has been found to estimate those stresses and the resulting deformations in the hollow roller body.

The solution to this problem consists of two stages. In the first stage, the finite element package ABAQUS is used to determine the values of stresses and the volumes under risk in the rollers in contact. ABAQUS has the ability to determine all stress components at certain grid nodes of the finite element mesh. Based on the endurance limit of the material used (680 MPa for CVD 52100), the volumes under risk can be determined by the regions where the stress value exceeds or is equal to the endurance limit. Then a fatigue life prediction theory can be applied on those numerical results to determine the fatigue life of those rollers in contact as the second and final stage of the solution. The fatigue life theory used in the paper is the IH theory. Two main models are studied. When the two rollers have identical size, called Model 1, and when the two rollers have different sizes, called Model 2. The two rollers have been assumed to run dry, with no lubricant. In each model of the two, two cases are studied: when the hollow roller is in contact with another hollow one with the same percentage of hollowness, and when that hollow roller is in contact with a solid roller.

Numerical simulation

One of the best advanced finite element software packages of those extensively used for structure analysis of contact problems and dynamic analysis is ABAQUS.

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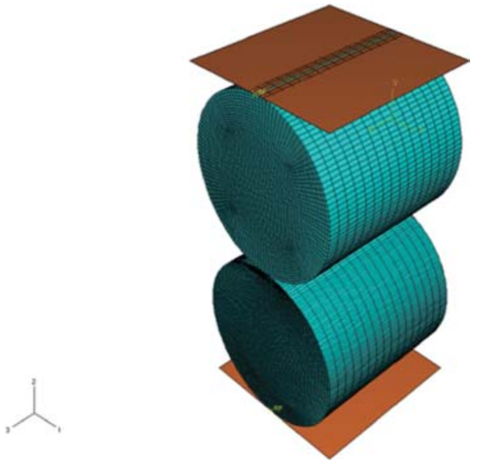

Figure 1. Two identically sized rollers in contact (M 1-00)

Using ABAQUS, two cylindrical rollers in contact were modelled. Those rollers were subjected to a combined loading of normal and tangential components. They were subjected to normal loading through two analytically rigid zero thickness sheet plates, as shown in Figure 1, in which two solid identically sized rollers are in contact (Model 1). The load was applied through those rigid bodies to ensure uniform distribution of loading on the roller surface and chosen to be analytically rigid so that they do not affect the results in the contact region between the two rollers. A tangential loading component was also applied to the two cylindrical rollers. The value of that tangential loading was one-third of the normal loading.

A sufficient coefficient of friction of 0.35 was assumed between the two rollers' surfaces to be more than one-third so that no slipping may occur. Analytically rigid bodies have been used to be able to run the analysis on ABAQUS. The stresses that have been used in the fatigue life analysis are the ones in the contact cylinders' halves, away from the rigid body effect. Two main models have been built. The first model, Model 1, was with rollers of the same size. The other model was with two rollers of different size such that one of the cylinders is twice as large as the other. This model is called Model 2. The analysis was started with both rollers solid. The results of both models have been verified by the solution of Smith and Liu[4] in the case of parallel rollers in combined rolling and sliding. Their results are shown in Norton.[24] This is a good check of the validity of the models and the boundary conditions applied on the model. After the validity of the ABAQUS results was checked, the two solid rollers were replaced by hollow rollers. The case of having one hollow roller with one solid roller has also been studied. Different hollowness percentages have been tested: 20, 40, 60 and 80%. The same procedure was applied to Models 1 and 2.

ABAQUS enabled us to describe all the stress components' values and distributions through the rollers' bodies. Highly stressed regions with Von Mises stresses have been determined. Those regions with stresses higher than the endurance limit of the material used for the two rollers are referred to as risk volumes. The endurance limit or the fatigue life of the material was defined by Shigley and Mischke[25] as the strength beyond which failure will not occur no matter how great the number of cycles. Both rollers are assumed to be made of CVD 52100 steel, with an endurance limit of 680 MPa.[26] For the identically sized model (Model 1), the outer diameter and the length of the roller were both 20 mm. In the non-identically sized rollers (Model 2), the diameter of the secondary roller is doubled with the same length of 20 mm. The secondary roller is the roller that was kept solid in

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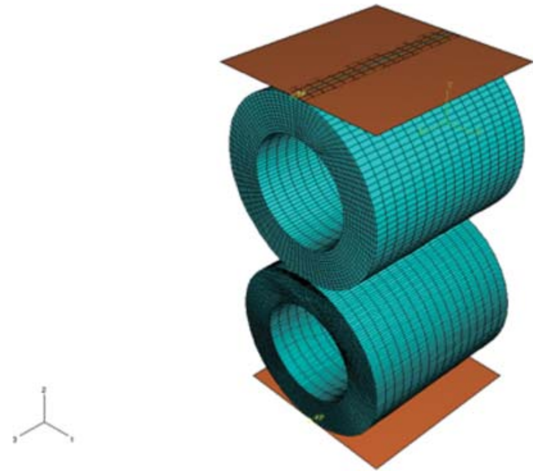

Figure 2. Two identically sized hollow rollers in contact (M 1-60 Two)

the second stage while the main roller is the one that was hollow in that stage. The behaviour of the main roller has been investigated when it is in contact with another identically sized roller, as is the case in Model 1, and when it is in contact with the inner race or the outer race, as is the case in the non-identically sized rollers model (Model 2).

A consistent convention is used for naming the models based on whether the rollers are of the same size or not, the percentage of hollowness, and whether one roller is hollow or both rollers are hollow. The name of each model starts with M, which refers to Model. As mentioned earlier, identically sized models are called Model 1, while non-identically sized roller models are called Model 2. Figure 2 shows a model of two identically sized rollers, with 60% of hollowness. Thus, it is called M 1–60. Then the name is followed by the word (One) or (Two) to refer to the number of hollow rollers in the model. In the case of a solid model, there is no need to use any of these two words. The letter T was added to the name of the models to indicate that those models are subjected to a combined normal and tangential loading.

IH theory

A new model for fatigue life prediction of roller bearings was suggested by Ioannides and Harris[15] in 1985. It was a modification of the LP life theory. It is based on the statistical relationship between the probability of survival, the fatigue life and the stress level above the endurance limit in a certain region of the roller, called the risk volume. In this theory, the fatigue life of a rolling bearing is calculated using the Weibull weakest link theory similar to the LP theory.[13] The basic equation of the IH theory is

$$\ln(1/\Delta S_i) = F(N, \sigma_i - \sigma_{ui})\Delta V_i \tag{1}$$

Therefore, integrating Equation (1) over the risk volume expresses the fatigue life using the following equation:

$$\ln(1/S) \approx \bar{A}N^e \int_{VR} ((\sigma - \sigma_u)^c / z'^h)dV \tag{2}$$

The IH theory assumes the two exponents e and c to be properties of the material that remain constant throughout the material's load history. Initially, the same values of c, e, and h were used as

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those used by LP. Then they were modified based on experimental results and on the material used. The IH theory assumes that the major part of the fatigue life is consumed during the initiation phase, which begins with the cyclic stressing that results in a macroscopic, self-propagating crack. Therefore, the fatigue life can be approximated by this initiation period, ignoring the time required for the crack growth phase. The IH theory modified the LP theory and eliminated some of its shortcomings. Although it was built on the same principles as the LP theory, the definition of the risk volume in the IH theory is different. Whereas the LP theory uses the risk volume that includes a predefined stress volume no matter how small the applied load is, the IH theory risk volume includes all the volume of the material with a stress value greater than the endurance limit of the material. This difference can be clearly seen in Equation (2), where σ_u was introduced as the lower limit of the fatigue limit. No failure can occur in the volume if the stresses are less than this fatigue limit. The IH theory predicts an infinite fatigue life for the bearing if the stresses do not exceed the endurance limit of the material used. Thus, it predicts a longer fatigue life than the LP theory. Recent research shows that bearings manufactured from clean steel, lubricated and kept free of contaminants may have infinite fatigue life.[16] This agrees with the IH theory of having infinite life of bearings with stresses less than the fatigue limit.

Using the IH theory to calculate the relative fatigue life of hollow rollers The first stage of the solution of investigating the fatigue life of hollow rollers was carried out using the ABAQUS simulation of two cylindrical rollers in contact. Then a fatigue life model was applied to those numerical values of stress distribution of the loaded rollers. Based on the previous discussion, the IH theory was to be the best predictor for the roller bearing fatigue life. Thus, for the stress results obtained from the simulation, the IH fatigue life model was applied with the depth weighting removed as discussed by Lubrecht *et al.*[27] and Tripp and Ioannides[28] and applied by Nikas.[23] Therefore, the z', which is the stress weighted average depth, is removed from Equation (2). However, this equation requires obtaining an experimental value for the constant $\bar{A}$. This constant is assumed to be of the same value for the same material under the same loading conditions for both solid and hollow rollers, which generally have the same geometry. The accuracy of the results obtained in this paper is mainly dependent on this assumption. This assumption was considered before by Nikas.[23] As we are only interested in seeing whether the fatigue life of the hollow rollers is improved or not with respect to the solid roller fatigue life, and not in the absolute fatigue life of the hollow rollers, the relative fatigue life of any hollow roller can be calculated from the following equation:

$$L_{rel}(ref) = \left[\int_{V_{ref}} \left(|\sigma_{ref}| - \sigma_u\right)^c \cdot dV \Big/ \int_{V_e} \left(|\sigma| - \sigma_u\right)^c \cdot dV \right]^{1/e} \tag{3}$$

The solid roller fatigue life is the reference life for the hollow rollers. Thus, it gives what is called the relative fatigue life (L_{rel}). As long as the value of the relative fatigue life is greater than 1.0, that means there is an improvement in the fatigue life by using hollow rollers. Therefore, the relative fatigue life estimated by Equation (3) is simply the summation of the stresses above the endurance limit in the solid roller (throughout the risk volume of the solid roller) divided by the summation of the stresses above the endurance limit in the hollow roller (risk volume of the hollow roller), raised to the power 1/(e). Based on the IH theory and the findings of Harris and McCool[16] for cylindrical roller bearings, the Weibull slope (e) was taken as 1.59 and the exponent c as 8.

As every point in the stress computational grid of the finite element simulation of the rollers in contact represents an elemental volume of the material with a constant fatigue limit, Equation (3) can be applied to the FE numerical results by replacing the integral with a summation of stresses at

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all grid points having a certain value of stress there greater than the fatigue limit of the material. The same density of grid points was used for all solid and hollow roller models. The relative fatigue lives of hollow rollers were calculated from the following equation:

$$L_{rel}(ref) = \left(\sum \left(\left(|\sigma_{ref}| - \sigma_u \right)^c \cdot \Delta V_{ref} \right) \Big/ \sum \left(\left(|\sigma| - \sigma_u \right)^c \cdot \Delta V \right) \right)^{1/e} \tag{4}$$

To apply this equation to the numerical simulation results, a fatigue-initiating stress criterion was needed. The ABAQUS results give all stress components on each point in the grid. In this research, the Deformation Energy (Von Mises) criterion was used for the stress. It is more accurate than other criteria because it uses the six components of the stress and it is more appropriate for rough contacts with high local stress concentrations.[23] According to this Von Mises criterion, the stress σ (or σ_{ref}) can be calculated from the following equation:

$$\sigma = \sqrt{((\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6 * (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2))/2} \tag{5}$$

However, the ABAQUS results of the stresses included the Von Mises stress value given by Equation (5) and all the six stress components at each point in the finite element mesh.

DISCUSSION OF THE RESULTS

Investigations have been carried out for both Model 1 and Model 2. Thus, results for each model are discussed separately. Then a comparison between results of both models is made. Graphical representation of the results is used to show the general trends and compare the results of hollow and solid rollers and hollow rollers with different hollowness percentages. Tabulated results are used to show the numerical representation of the results and the calculated values of the relative fatigue lives. To prove the validity of the results obtained by ABAQUS, the results from the solid models are compared with the analytical solution of Smith and Liu.[4]

Results of models with identically sized rollers (Model 1)

The results obtained by ABAQUS were verified by the solution of Smith and Liu for two cylinders in pure rolling and sliding contact. When the two cylinders were subjected to a combination of normal and tangential loading, the comparison is shown in Figure 3. The *x-axis* represents the depth normalised by the half patch width a. There is good agreement between the ABAQUS solution and the Smith and Liu[4] solution. The ABAQUS solution gave higher values of stress near the contact region than the Smith and Liu solution. This is likely related to the nature of the Smith and Liu solutions that are not valid for the surface contact region. The Hertz approach is used to obtain stresses there.

As the results from ABAQUS are verified for the solid model, we can be confident of the results from ABAQUS for the hollow models. Figure 4(a) represents all the identically sized roller models when subjected to a combination of both normal and tangential loading. Figure 4(a) is just for full scale, while Figure 4(b) is only for the region of the figure that represents the risk volume, where the stresses are higher than 680 MPa, the endurance limit for the material used. It can be clearly seen from the figure that the area under the curves of the hollow roller models is less that that of the solid roller model. That means the volume under risk is smaller in the case of the hollow roller models. Also, the figure gives an indication about the location of that volume that starts at the contact point and extends under the surface to the point where the curve intersects the *x*-axis. Figure 4(b) shows

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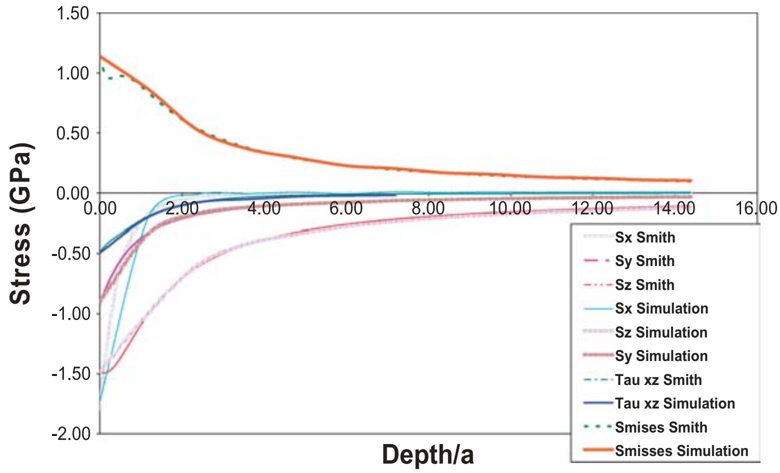

Figure 3. Smith proof for identical solid roller model under combined loading

the size of the volume under risk and that the values of stress in that volume are less for the hollow rollers than the solid one. Furthermore, the risk volume decreases as the hollowness percentage increases from 20 to 60%, where the smallest risk volume is found with lower stress values. In the case of 80% of hollowness, it is apparent that the risk volume is smaller than the 20% hollowness case, but the values of the stress there are much greater than other hollowness percentage cases. Even so, the volume under risk and the values of stress there in the 80% hollowness are less than for the case of the solid roller.

Small differences were found between models when only one roller is hollow and models with both rollers hollow. In general, rollers with both rollers hollow have a smaller risk volume with lower stress values there and so longer fatigue lives. Unlike the curve for the solid roller model, other curves start to go up again as shown in Figure 4(a). That is related to σ_b occurring on the inner surface of the hollow roller. σ_b increase with increasing hollowness percentage, but their values are less than the contact stresses on the surface in all cases.

The value of the tangential load was one-third of the normal load value. Sufficient coefficient of friction is used (0.35) to make sure no sliding occurs between the two rollers. The 60% hollowness percentage roller obtained the lowest stress values and the smallest risk volume, followed by the 40% hollowness roller. It can be seen from Figure 4(b) that the 20% hollowness roller has a larger risk volume than the solid roller, but the stresses there are much smaller than the case solid roller case. That is an indication of how stresses are distributed over a larger volume when the roller is hollow. The stresses are distributed over a larger volume and the peak value of the stress is less compared with the solid rollers. It should be noted that the risk volume of all rollers extends from the surface of contact when the rollers are subjected to normal and tangential loading. The maximum stress now is also on the contact surface for normal plus tangential loading. σ_b on the inner surfaces are less than the endurance limit of the roller. Therefore, they have no effect on the fatigue life as indicated by the IH theory.

The IH theory was applied on the stress values obtained by ABAQUS. The results of the fatigue life estimation are summarised in Table 1, which shows the values of the relative life of the hollow rollers compared with solid rollers as calculated from Equation (4). These numbers show how many times the solid roller bearing fatigue life can be increased using hollow rollers instead of solid rollers.

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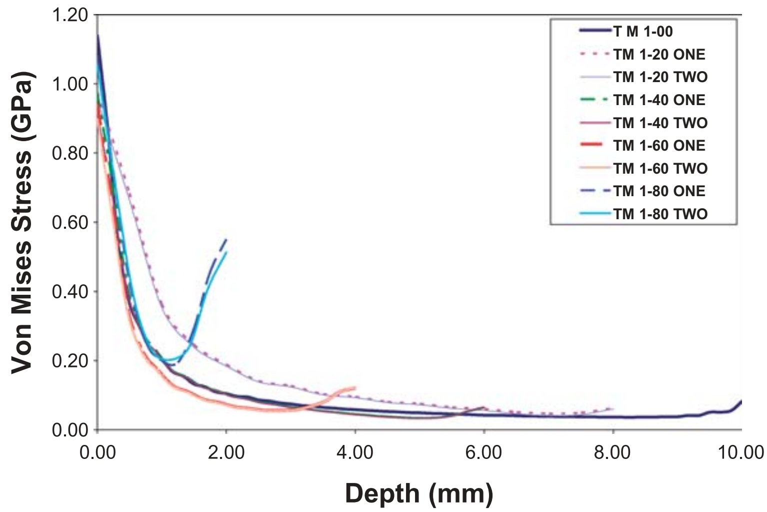
(a)



(b)

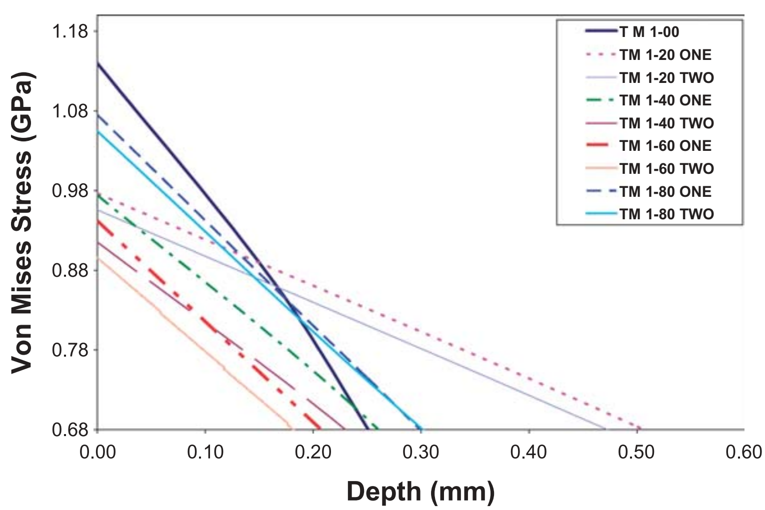


Figure 4. (a) Identically sized roller models under combined loading, (b) identically sized roller models under combined loading

Table 1. Fatigue life estimation of Model 1 using IH theory

Model 1	Relative fatigue life
M 1-20 One	4.1
M 1-20 Two	6.4
M 1-40 One	9.9
M 1-40 Two	35.6
M 1-60 One	23.9
M 1-60 Two	70.6
M 1-80 One	1.9
M 1-80 Two	2.5

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Under combined normal and tangential loading, the 60% of hollowness of both rollers will survive more than 70 times two solid rollers under the same loading conditions. The least improvement of the fatigue life is the case of using two rollers; only one of them hollow with 80% hollowness. The fatigue life in this case is almost twice as much as the case of using solid rollers. So, numerical results in the table show that using hollow rollers instead of solid rollers will improve the fatigue life. Even so, 60% hollowness with both rollers hollow gives the longest fatigue life. When the hollow roller is subjected to tangential and normal loading, the fatigue life improvement increases as the hollowness percentages increases, up to 60% hollowness. In the case of 80% hollowness, the improvement significantly decreases. Those results agree with experimental results reported by Bowen and Bhateja,[12] who found that the hollowness range of 60–70% is the best to use for most roller bearing applications.

Results of models with non-identically sized rollers (Model 2)

When Model 2 was subjected to a combination of tangential and normal loading, the results were as shown in Figure 5. Figure 5(a) explains what is happening when the roller is hollow. It is just a matter of redistribution of stresses. Instead of being concentrated near the contact area, it is distributed throughout the whole body of the roller. The 20% hollowness model is a good example of that. It can be seen in Figure 5(b) how it has lower stresses than the solid roller model under the contact area but higher stresses than the solid roller model when going down away from the contact region. That is the main issue with hollow rollers. Although the risk volume is larger in the case of 20% hollowness, the stress values there are much smaller than the solid model. That gives the hollow roller longer fatigue life. The advantage of having both rollers hollow is emphasised when the rollers are subjected to a combination of tangential and normal loading. For the 40% hollowness model the two hollow roller model responds much better than the model with one hollow roller as can be seen in Figure 5(b).

However, no significant difference can be seen between the 60% hollowness models with one hollow roller and with two hollow rollers. That indicates that the advantages that were there for one hollow roller are balanced by another factor when the rollers are subjected to tangential loading. The results of applying the IH theory on Model 2 are summarised in Table 2. The numerical numbers in this table show the relative fatigue life of hollow rollers compared with the solid rollers under the same loading condition. These values are calculated using Equation (5). The table shows that in the case of two hollow rollers in contact, with both rollers having 80% hollowness, subjected to a combination of tangential and normal loading, the fatigue life of the roller bearing will decrease to less than 50% of the life of the solid rollers. That is the only case found where hollow rollers have a shorter fatigue life than solid rollers. On the other hand, rollers with 60% hollowness will improve the fatigue life to more than 87 times in the case of combined normal and tangential loading.

Comparison between Model 1 and Model 2 results

When comparing the response of the hollow rollers in contact, with an identically sized roller, with a hollow roller in contact, with a roller of twice size, we can see many differences in their response to a combination of normal and tangential loading. Under the same loading, rollers in Model 2 have longer fatigue lives than rollers in Model 1. That can clearly be seen when comparing corresponding values in Table 1 to those in Table 2.

This result applies for all hollowness percentages except when both rollers have 80% hollowness. That might be related to increasing the flexibility and increasing the contact area and so decreasing

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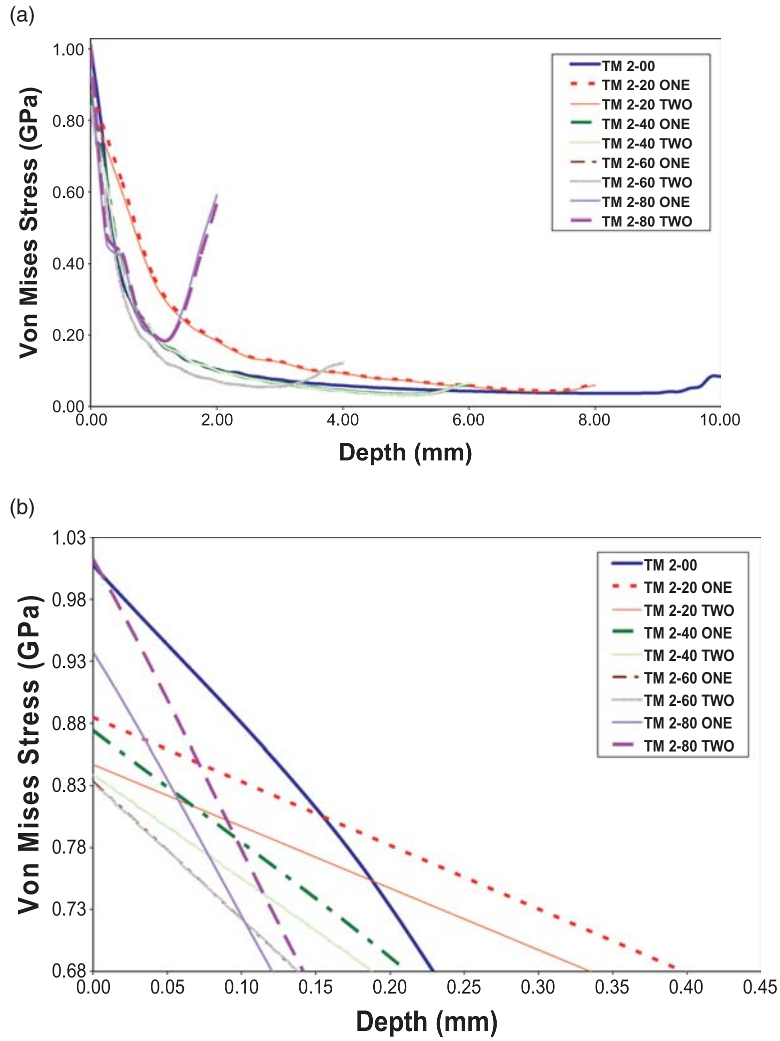

Figure 5. (a) Non-identically sized roller models under combined loading, (b) non-identically sized roller models under combined loading

Table 2. Fatigue life estimation of Model 2 using IH theory

Model 2	Relative fatigue life
M 2-20 One	5.2
M 2-20 Two	18.0
M 2-40 One	15.0
M 2-40 Two	47.6
M 2-60 One	85.5
M 2-60 Two	87.5
M 2-80 One	7.4
M 2-80 Two	0.5

the contact stresses of a roller in contact with a bigger sized roller. Increasing the contact width decreases the contact stress but has nothing to do with the shearing stress. The Von Mises stress contains all the stress components. Thus, decreasing the contact stress will decrease one of the components and decreases the Von Mises stress value. On the other hand, when the hollowness exceeds 60%, σ_b from the sides and the inner surfaces tend to increase the shearing stresses in the contact region and the risk volume, resulting in greater Von Mises stress values and decreased fatigue life.

Using ABAQUS figures to show the redistribution of contact stresses in hollow rollers

To show how stresses are redistributed in the larger volume of the roller body, two of the ABAQUS figures are included (Figures 6 and 7). Those figures show models of identically sized rollers. Two main cases were chosen to show how contact stresses are redistributed when making rollers with a

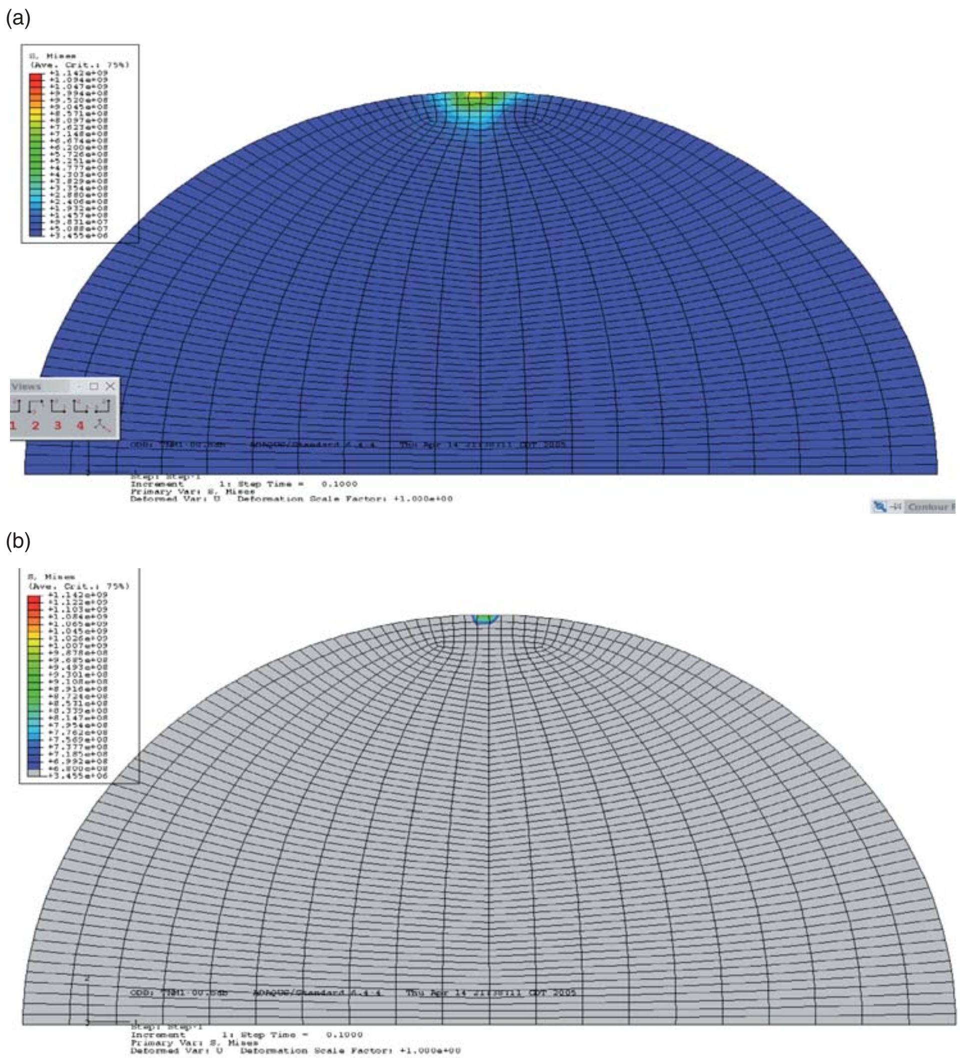

Figure 6. (a) ABAQUS figure of one of two identically sized solid rollers in pure rolling contact, (b) ABAQUS figure of one of two identically sized solid rollers in pure rolling contact

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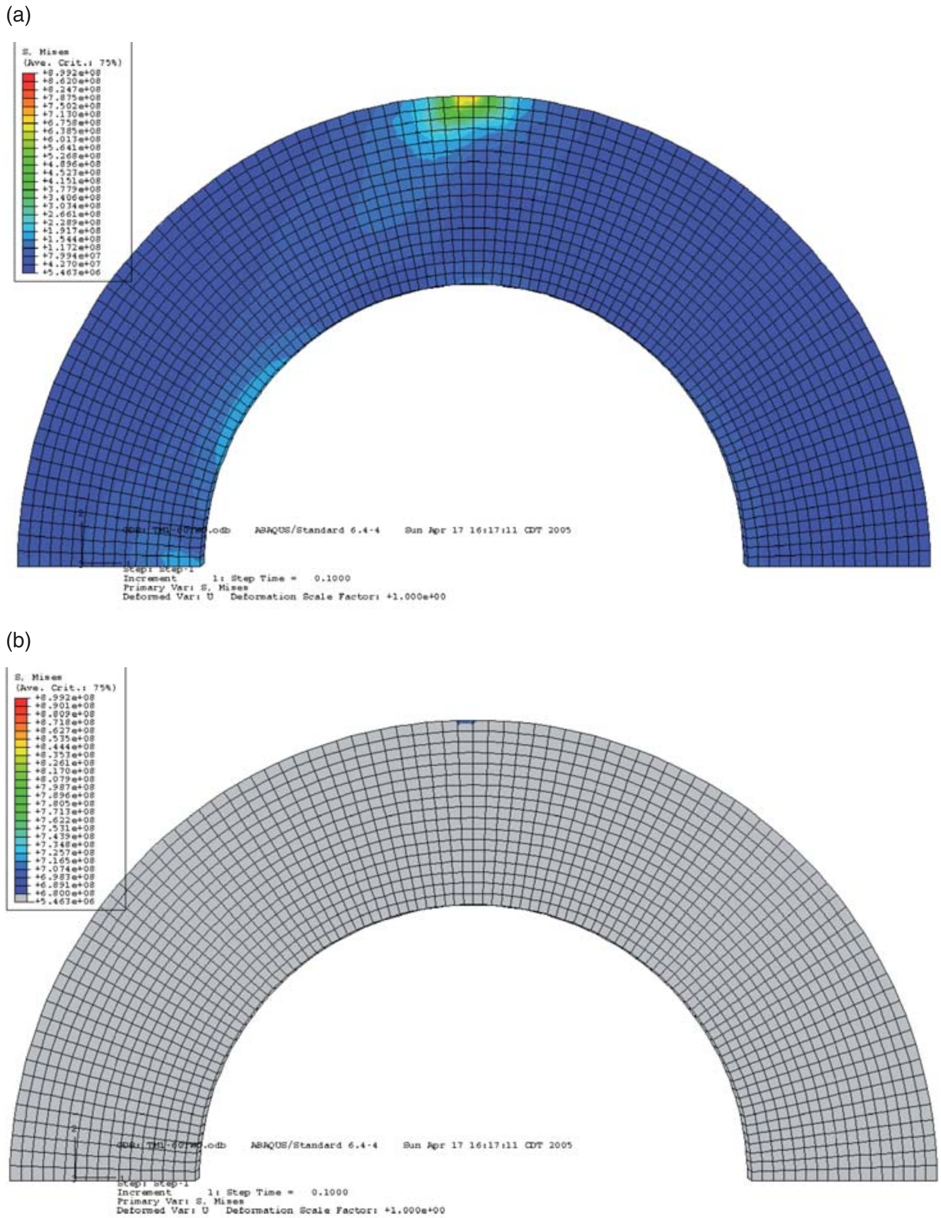

Figure 7. (a) ABAQUS figure of one of two identically sized 60% hollowness rollers in pure rolling contact, (b) ABAQUS figure of one of two identically sized 60% hollowness rollers in pure rolling

certain hollowness. The first case is of two solid rollers in pure rolling contact, and the second case is when those rollers are hollow with 60% hollowness. The 60% hollowness was chosen because it showed the best fatigue life results. Each case is represented by two figures. The first figure of each pair shows how stresses in the roller body are redistributed throughout a larger volume as the percentage of hollowness increases. The second figure shows the location and the size of the risk volume. Under combined loading, the maximum stress is shifted in the direction of the tangential loading for all solid and hollow rollers. The shift distance increases as the percentage of hollowness increases, and it is very apparent for the 80% hollowness case.

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Comparing Figure 6 with Figure 7, one can explain how the fatigue life is improved for 60% hollowness as the risk volume is much smaller than the case of a solid roller. In the case of the 60% hollowness roller, a typical distribution of the stresses made the risk volume very small and so improved the fatigue life of the roller. It has been found that although the stresses are redistributed in most of the roller body in the case of the 80% hollowness roller, it started to behave like a tube or ring, with a significant effect of σ_b on the inner surfaces.

CONCLUSIONS AND RECOMMENDATIONS

In general, hollow rollers have longer fatigue life than solid rollers when subjected to a combination of normal and tangential loading. The fatigue life is improved as the hollowness percentage increases up to 60%, where the longest fatigue life was obtained. That is related to a decrease in the contact stresses because of the flexibility of the hollow roller and increase of the contact width. Those two reasons result in redistribution of the stresses. The fatigue life improvements were decreased when the percentage of hollowness was 80%. The bending stress started to affect the stresses in the contact zone significantly, by increasing the shearing stress values there.

It was found that the risk volume is under the loading region, not in the bending stress regions. Therefore, the failure of the hollow rollers within the applied loading range of this work, and for the material used, is a result of contact stresses, not the (σ_b). The effect of (σ_b) is not significant for hollowness percentages less than 60%.

In the case of a roller in contact with two other rollers, one of identical size and the other larger, as is the case for the traction drive developed by Flugrad and Qamhiyah,[2] the fatigue life resulting from contact with the same size roller is shorter than the fatigue life resulting from contact with a bigger size roller. By using Miner's[29] rule, the fatigue life of that roller is determined by contact with both rollers, and the resulting fatigue life is shorter than that fatigue life determined by contact with the identically sized roller.

The maximum stress value and so the crack initiation point may be under the surface for solid and hollow rollers under pure normal loading. However, for rollers subjected to a combination of tangential and normal loading, the highest risk point shifts towards the contact surface.

Nomenclature

A	constant
a	half patch width and equals to 0.139 mm for Model 1 and 0.161 mm for Model 2
c	stress criterion exponent.
e	Weibull slope
h	depth exponent
i	volume element number
N	fatigue life
V_{ref}	volume of material where σ_{ref} is greater than σ_u
z	stress weighted average depth
ΔS_i	probability of survival
ΔV_i	volume element in which stresses are higher than the fatigue limit
σ_i	value of stress-related fatigue criterion in the volume element i
σ_{ui}	fatigue limit
σ_{ref}	denotes the stress used for the reference life computation, which is the stress in the solid roller
σ_b	bending stresses

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References

1. Gohar R. *Elastohydrodynamics*. Imperial College Press, 2001.
2. Flugrad D, Qamhiyah A. A self-actuating traction-drive speed reducer. *Proc. ASME PTG Conference*, Baltimore, MD, September 2000.
3. Hertz H. *Gesammelte Werke*, Vol. 1: Leipzig, Germany, 1895; English translation in Miscellaneous Papers, 1896.
4. Smith JO, Liu CK. Stresses due to tangential and normal loads on an elastic solid with application to some contact stress problems. *J. Appl. Mech.* 1953; **20**:57–166.
5. Lundberg G. Elastische Beruehrung Zweier Halbraeume. *Forschung Auf Dem Gebute Des Ingwieurwesens, Ausgabe A* 1939; **10**:201–11.
6. Hanau H. *New Concepts in Bearing Designs and Applications* Industrial Tectonics Inc., 1965, pp. 20–2.
7. Given PS. Main-shaft preloaded roller bearings. ASME Paper, 1965.
8. Harris TA, *et al.* Anti-skid bearing. U.S. Patent # 3, 410, 618, 1968.
9. Bamberger EN, Purker RJ, Dietrich MW. Flexural fatigue of hollow rolling elements. NASA Paper TH-D-8313, 1976.
10. Bowen WL. Full complement bearing having preloaded hollow rollers. U.S. Patent # 3, 930, 693, 1976.
11. Bowen WL. Roller bearing of superior run-out characteristics. U.S. Patent # 4, 002, 380, 1977.
12. Bowen WL, Bhateja CP. The hollow roller bearing. *ASME Trans.* 1980; **102**:222–8.
13. Lundberg G, Palmgren A. Dynamic capacity of rolling bearings. *Acta Polytech. Mech. Eng. Ser. R. Swed. Acad. Eng. Sci.* 1947; **1**(3):7.
14. Lundberg G, Palmgren A. Dynamic capacity of rolling bearings. *Acta Polytech. Mech. Eng. Ser. R. Swed. Acad. Eng. Sci.* 1952; **2**:4, 96.
15. Ioannides E, Harris TA. A new fatigue life model for rolling bearings. *J. Tribol.-Trans. ASME* 1985; **107**:367–78.
16. Harris TA, McCool JI. On the accuracy of rolling bearing fatigue life prediction. *J. Tribol.-Trans. ASME* 1996; **118**:297–310.
17. Harris TA, Yu WK. Lundberg–Palmgren fatigue theory: considerations of failure stress and stresses volume. *J. Tribol.-Trans. ASME* 1999; **121**:85–9.
18. Murthy CSC, Rao AR. Mechanics and behaviour of hollow cylindrical members in rolling contact. *Wear* 1983; **87**:287–96.
19. Somasundar HV, Krishnamurthy R. Surface durability of tufftrided rolling elements. *Wear* 1984; **97**:117–27.
20. Elsharkawy AA. A numerical solution for dry contact between two viscoelastic rollers. *Tribol. T.* 1996; **39**(3):627–35.
21. Hong L, Jianjun L. Analysis of contact problems on hollow cylindrical rollers. *Tribology* 1997; **17**(3):253–9.
22. Zhao H. Analysis of load distributions within solid and hollow roller bearings. *ASME Trans.* 1998; **120**:134–9.
23. Nikas GK. Fatigue life and traction modeling of continuously variable transmissions. *J. Tribol.-Trans. ASME* 2002; **124**:689–98.
24. Norton RL. *Machine Design*. Prentice-Hall, 2000.
25. Shigley JE, Mischke CR. *Machine Engineering Design*. McGraw-Hill Book Company, 1989.
26. Harris TA. *Rolling Bearing Analysis*. John Wiley & Sons, Inc., 2001.
27. Lubrecht AA, Jacobson BO, Ioannides E. Lundberg Palmgren revisited. Rolling element bearings – towards the 21st century. Seminar organized by the Institution of Mechanical Engineers (UK), November 16, 1990, pp. 17–20.
28. Tripp JH, Ioannides E. Effects of surface roughness on rolling bearing life. *Proc. Japan International Tribology Conference*, Nagoya, Japan, 1990, pp. 797–802.
29. Miner MA. Cumulative damage in fatigue. *J. Appl. Mech.* 1945; **12**:A159–64.

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