

Two-qubit Fisher information and Jensen–Shannon nonlocality dynamics induced by a coherent cavity under dipole, intensity-dependent, and decoherence couplings

A.-B.A. Mohamed^{a,b,*}, H. Eleuch^{c,d,e,1}

^a Department of Mathematics, College of Science and Humanities in Al-Aflaj, Prince Sattam bin Abdulaziz University, Saudi Arabia

^b Faculty of Science, Assiut University, Assiut, Egypt

^c Department of Applied Physics and Astronomy, University of Sharjah, Sharjah, United Arab Emirates

^d College of Arts and Sciences, Abu Dhabi University, Abu Dhabi 59911, United Arab Emirates

^e Institute for Quantum Science and Engineering, Texas A&M University, College Station, TX 77843, USA

ARTICLE INFO

Keywords:

Local Fisher information
Jensen–Shannon coherence
Entanglement of formation

ABSTRACT

This study investigates two-qubit local quantum Fisher information correlation (LQFI), entanglement of formation (EoF), and two-qubit Jensen–Shannon coherence as a result of cavity–qubit coupling via intensity dependent and qubit–qubit interactions. Initially, the cavity is filled with coherent and even coherent fields, and the two-qubit state is pure. The cavity–qubit intensity dependent interaction, in the presence of intrinsic decoherence, can produce two-qubit local quantum Fisher information, entanglement of formation, and Jensen–Shannon coherence. This ability depends on dipole–qubit interactions, intrinsic decoherence, and initial cavity states. The phenomena of sudden changes emerge in the dynamics behavior of the local quantum Fisher information, whereas the phenomenon of sudden birth–death occurs during the entanglement of formation dynamics. These phenomena can be enhanced by modifying the initial coherent field parameters, the mean photon number, and the non-classicality. The dipole qubit–qubit interaction improves the generated two-qubit correlation and coherence, while intrinsic decoherence decreases and stabilizes the generated two-qubit correlation and coherence. For a small initial coherent field intensity and dipole qubit–qubit interaction, the LQFI and EoF non-local correlations grow while the two-qubit Jensen–Shannon coherence diminishes. The qubit–qubit interaction coupling and the initial coherent field intensity can improve the produced stationary LQFI and EoF correlations and Jensen–Shannon coherence in the presence of the intrinsic decoherence.

Introduction

Non-local correlation and coherence in two-qubit states are recognized as a fundamental quantum information resources for various applications in the quantum technology [1–3]. Two-qubit entanglement is a type of non-local correlation that is required for certain types of quantum information processing, such as: quantum teleportation [4], dense coding [5] and quantum cryptography [6]. There are different quantifiers have been used to estimate entanglement dynamics, such as von Neumann entropy, entanglement of formation (EoF), negativity and concurrence [7–12]. Following the introduction of quantum discord [13] as a new type of non-local correlation beyond entanglement, other quantifiers proposed measuring other non-local correlations such as measurement-induced disturbance [14],

measurement-induced nonlocality [15], and local quantum Fisher information [16,17]. Beyond entanglement, local quantum Fisher is a suitable quantifier for estimating non-local correlation.

Quantum coherence, which is derived from the quantum pure state superposition principle, is often viewed as a more broad information resource than quantum entanglement [18]. It is crucial in emerging domains like nanoscale thermodynamics [19] and quantum biology [20]. Quantum coherence can exist localized individually in a qubit system [21,22] and as non-local correlation between qubits [23]. Quantum coherence and non-local correlation are all closely related concepts [24, 25]. Knowing their relationship can give a comprehensive insight in describing the quantum properties of nonclassical systems [26]. Various reliable metrics, such as von Neumann entropy, relative entropy, trace norm of coherence, and the square root of the Jensen–Shannon

* Corresponding author at: Department of Mathematics, College of Science and Humanities in Al-Aflaj, Prince Sattam bin Abdulaziz University, Saudi Arabia.
E-mail address: abdelbastm@aun.edu.eg (A.-B.A. Mohamed).

¹ All authors contributed equally to the manuscript preparation.

divergence (JSD) [27], have been suggested to quantify two-qubit quantum coherence. The $\sqrt{JSD}$ coherence measure has entropic nature and metric properties, which allows for the coherence to be decomposed into various contributions arisen from local and intrinsic coherences [28,29]. The superposition of quantum qubit/cavity states with strong nonclassicality can produce quantum coherence and non-local correlation, both of which have various potential uses in quantum information [30,31]. This superposition is required for the formation of many non-local correlations. Therefore, we will investigate the dynamics of two-qubit local quantum Fisher information, entanglement of formation, and Jensen–Shannon coherence caused by interacting the qubits with a cavity initially filled with a superposition of coherent fields while two-qubit state is initially in a pure state. This system is under the effect of the dipole two-qubit interaction and the intrinsic decoherence. These dipole two-qubit interactions have a great ability to enhance quantum coherence and non-local correlation generation [32–35]. There have been limited investigations on two-qubit quantum coherence and non-local correlation in the presence of intrinsic decoherence [36]. This intrinsic decoherence differs from what occurs in open cavity–qubit systems but constitutes a major obstacle when a closed quantum system evolves [36].

Currently, one of the main challenging goals in solid quantum materials is to realize qubits, which represent the most basic building blocks of quantum computation. Therefore, several entangled qubit systems have been realized experimentally as: superconducting qubits [37,37], trapped-ion qubits [38], quantum-dot qubits [39], and optomechanical qubit systems [40]. This paper investigates the two-qubit local quantum Fisher information, entanglement of formation, and two-qubit Jensen–Shannon coherence owing to cavity–qubit interactions through intensity dependent and qubit–qubit interactions under intrinsic decoherence. We investigate the possibilities to produce two-qubit non-local correlations by varying the key parameters of the physical system.

This paper is organized as follows: Section “The physical model” presents the proposed cavity–qubit intrinsic-decoherence model. In Section “Two-qubit correlation and coherence quantifiers”, we provide a quick overview of the two-qubit correlation and coherence quantifiers. Section “Dynamics of two-qubit correlation and coherence” shows the numerical results and discussions of the dynamics of two-qubit correlation and coherence. Finally, in Section “Conclusions”, our conclusions will be listed.

The physical model

In this paper, we consider the effect of the intrinsic decoherence [36] on the dynamics of a pure qubit–qubit state under nonlinear two-qubit interaction with a coherent cavity field through intensity dependent [41] and dipole qubit–qubit interactions. The intensity-dependent [42,43] and dipole qubit–qubit [44] interactions have been realized in qubit superconducting circuits. The dynamics of the system in the presence of the intrinsic decoherence is governed by the following Milburn equation [36],

$$\frac{d\rho(t)}{dt} = -i[\hat{H}, \rho] - \frac{\gamma}{2}[\hat{H}, [\hat{H}, \rho]], \quad (1)$$

where γ is the intrinsic decoherence rate. The system loses its quantum coherence and entanglement as it develops, which is described by the Milburn intrinsic decoherence equation. Intrinsic decoherence causes a quantum coherence loss without the typical energy loss associated with decay. The system-reservoir interactions described by other approaches’ Master equations in the Lindblad form [45,46] inevitably result in decoherence since the information is lost as soon as the system evolves. For the resonance case where the qubits and the cavity frequencies have the same value f_ω , the Hamiltonian $\hat{H}$ is given by

$$\begin{aligned} \hat{H} = & (\hat{c}^\dagger \hat{c} + \hat{\sigma}_z^A + \hat{\sigma}_z^B) f_\omega + J(\sigma_+^A \sigma_-^B + \sigma_+^B \sigma_-^A) \\ & + \sum_{i=A,B} \lambda (\hat{c}^\dagger \hat{c} \hat{\sigma}_+^i + I(\hat{c}^\dagger \hat{c}) \hat{\sigma}_-^i), \end{aligned} \quad (2)$$

where $\hat{\sigma}_\pm^i$ and $\hat{\sigma}_z^i$ are the i -qubit ($i = A, B$) creation, annihilation, and atomic inversion operators. $\hat{c}^\dagger(\hat{c})$ denotes the creation (annihilation) cavity-field operator. The second term describes the dipole two-qubit interaction with the coupling J . Furthermore, the last terms represent the cavity–qubit interactions with the coupling J and the intensity-dependent coupling $I(\hat{c}^\dagger \hat{c})$.

To find a particular solution for the intrinsic decoherence Milburn equation (1), we assume that the cavity is initially filled with coherent and even coherent fields while the two-qubit state is initially in a pure excited state, $|1_A 1_B\rangle\langle 1_A 1_B| = |11\rangle\langle 11|$. Therefore, the initial cavity–qubit density matrix can be written as:

$$\hat{\rho}(0) = |11\rangle\langle 11| \otimes \hat{\rho}^F(0) = \sum_{m,n=0}^{\infty} H_{mn} |11m\rangle\langle 11n|. \quad (3)$$

The two cavities are initially in two different cases. Case-1: the two cavities are in a coherent state $\hat{S}^F(0) = |\alpha\rangle\langle\alpha|$. Case-2: the two cavities are in an even coherent state, $\hat{\rho}^F(0) = (|\alpha\rangle + |-\alpha\rangle)(\langle\alpha| + \langle-\alpha|)/A$, with the normalization factor A . Where $|\alpha\rangle = \sum_n T_n |n\rangle T_n = e^{-\frac{1}{2}N^2} \frac{a^n}{\sqrt{n!}}$, $N = |\alpha|^2$ is the initial coherent field intensity (mean photon number).

The Milburn equation (1) can be solved by using the eigenstates $|\Psi_m^l\rangle (m = 1, 2, 3, 4)$, the eigenstates operator $\hat{\Psi} = |\Psi_m^l\rangle\langle\Psi_m^l|$ and the energies E_m^l of the matrix (2). In the field-qubit basis: $\{|B_1\rangle = |11, n\rangle, |B_2\rangle = |10, n+1\rangle, |B_3\rangle = |01, n+1\rangle, |B_4\rangle = |00, n+2\rangle\}$, the eigenstates $|\Psi_m^l\rangle$ are given by:

$$|\Psi_m^l\rangle = \sum_{l=1}^4 \Lambda_{mn} |B_n\rangle, (m = 1, 2, 3, 4), \quad (4)$$

the coefficients Λ_{ij} satisfy the condition of eigenvalue-problem: $\hat{H}|\Psi_m^l\rangle = E_m^l |\Psi_m^l\rangle$, where E_m^l are:

$$\begin{aligned} E_1^l &= f_\omega(l+1), & E_2^l &= \omega(l+1) - J, \\ E_3^l &= f_\omega(l+1) + \frac{1}{2}J - \frac{1}{2}\sqrt{J^2 + 8\mu}, \\ E_4^l &= f_\omega(l+1) + \frac{1}{2}J + \frac{1}{2}\sqrt{J^2 + 8\mu}. \end{aligned}$$

Here, we use consider the intensity-dependent coupling $I(\hat{c}^\dagger \hat{c}) = \sqrt{\hat{c}^\dagger \hat{c}}$, therefore, $\mu = (n^2 + 6n + 5)I(n)$. From the initial cavity–qubit density matrix of Eq. (3), the cavity–qubit density matrix can be expressed as

$$\hat{\rho}(t) = \sum_{m,n=0}^4 \sum_{l,k=1}^4 H_{mn} \Lambda_{m1} \Lambda_{n1} D(t) e^{-i\lambda(E_m^l - E_n^k)t} \hat{\Psi}, \quad (5)$$

where $D(t) = e^{-\frac{\gamma}{2}(E_m^l - E_n^k)^2 t}$ is intrinsic decoherence term.

To investigate the generated two-qubit correlation and the coherence of the two qubits, we need to define the reduced two-qubit density matrix, $\hat{\rho}^{AB}(t)$, by tracing the cavity field states as:

$$\hat{\rho}^{AB}(t) = \sum_{k=0}^{\infty} \langle k | \hat{\rho}(t) | k \rangle. \quad (6)$$

Now, based on the reduced two-qubit density matrix of Eq. (6), we can quantify the amount of the produced two-qubit correlation and coherence by different measures, local quantum Fisher information and entanglement of formation as well as two-qubit Jensen–Shannon coherence.

Two-qubit correlation and coherence quantifiers

The aim of this manuscript is to study the generated two-qubit correlations of the local quantum Fisher information and entanglement of formation as well as two-qubit Jensen–Shannon coherence due to cavity–qubit, intensity dependent, and qubit–qubit interactions.

Local quantum Fisher information (LQFI):

Beyond entanglement, the LQFI (local quantum Fisher information) quantifier is shown to be a good measure of novel two-qubit quantum information resources such as two-qubit coherence and two-qubit quantum correlation [16,17]. For the two-qubit state, the density matrix $S^{AB}(t)$ can be written as: $\hat{\rho}^{AB}(t) = \sum_k \xi_k |A_k\rangle\langle A_k|$ with $\xi_k \geq 0$ and $\sum_k \xi_k = 1$, the two-qubit LQFI function is given by [16,17]

$$L(t) = 1 - \max\{\lambda_1, \lambda_2, \lambda_3\}, \quad (7)$$

where λ_i is the eigenvalues of the generated symmetric matrix $G = [g_{ij}](i, j = 1, 2, 3)$, with the elements

$$g_{ij} = \sum_{\xi_m + \xi_n \neq 0} \frac{2\xi_m \xi_n}{\xi_m + \xi_n} Q_{mn}^i Q_{nm}^{j\dagger}. \quad (8)$$

The expressions of the coefficients $Q_{mn}^j = \langle A_m | \hat{H}_j | A_n \rangle$ where the matrices $\hat{H}_i = I \otimes \sigma^i$ and σ^i ($i = 1, 2, 3$) represent the Pauli matrices. For a maximally mixed/correlated two-qubit state, the LQFI reaches the unit, while for maximal two-qubit coherence (separable state), vanishes.

Two-qubit Jensen–Shannon coherence

To measure the generated two-qubit coherence, the square root of the Jensen–Shannon divergence ($\sqrt{JSD}$) [28,29,47] is used as the distance measure of two states and two-qubit coherence. The Jensen–Shannon information is defined as [28],

$$J(t) = \sqrt{S[S_T^{AB}] - \frac{1}{2}S[S^{AB}] - \frac{1}{2}S[S_d^{AB}]}, \quad (9)$$

where $S(\cdot)$ is the von Neumann entropy, $S_T^{AB} = [S^{AB} + S_d^{AB}]/2$, and S_d^{AB} is the diagonal two-qubit matrix.

Entanglement of formation (EOF)

To explore the amount of the generated two-qubit entanglement, we can consider the entanglement of formation [10] that is based on the concurrence [12]. The EOF is the only measure of entanglement for which an analytical expression is available for arbitrary systems of particular dimensions. It was found that the case of two-qubit systems. The EOF is a monotonically increasing function of the concurrence. The EOF function can be expressed as:

$$F(t) = \sum_{n=0,1} (n-K) \log(n - (-1)^{n+1} K), \quad (10)$$

$$K = \frac{1}{2}[1 + \sqrt{1 - C^2(t)}], \quad (11)$$

concurrence function is defined as,

$$C(t) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (12)$$

λ_i ($i = 1 - 4$) design the eigenvalues of the matrix: $R = \hat{\rho}^{AB}(\sigma_y \otimes \sigma_y) \hat{\rho}^{*AB}(\sigma_y \otimes \sigma_y)$, they listed in decreasing order.

Dynamics of two-qubit correlation and coherence

To investigate the amount, the speed, and the regularity of the generated two-qubit correlations of the local Fisher information and entanglement of formation as well as two-qubit Jensen–Shannon coherence due to cavity–qubit, intensity dependent, and qubit–qubit interactions, we have assumed that the two-qubit state is initially in the pure state $\rho^{AB}(0) = |1_A 1_B\rangle\langle 1_A 1_B|$. Therefore, the initial values of the correlations and coherence functions $L(t)$, $J(t)$, and $F(t)$ are equal to zero-value, which are in concordance with the numerical results. The behavior of the local Fisher information, Jensen–Shannon coherence, and entanglement of formation are shown when the cavity is initially filled with a coherent field with $|\alpha|^2 = 25$ in the absence

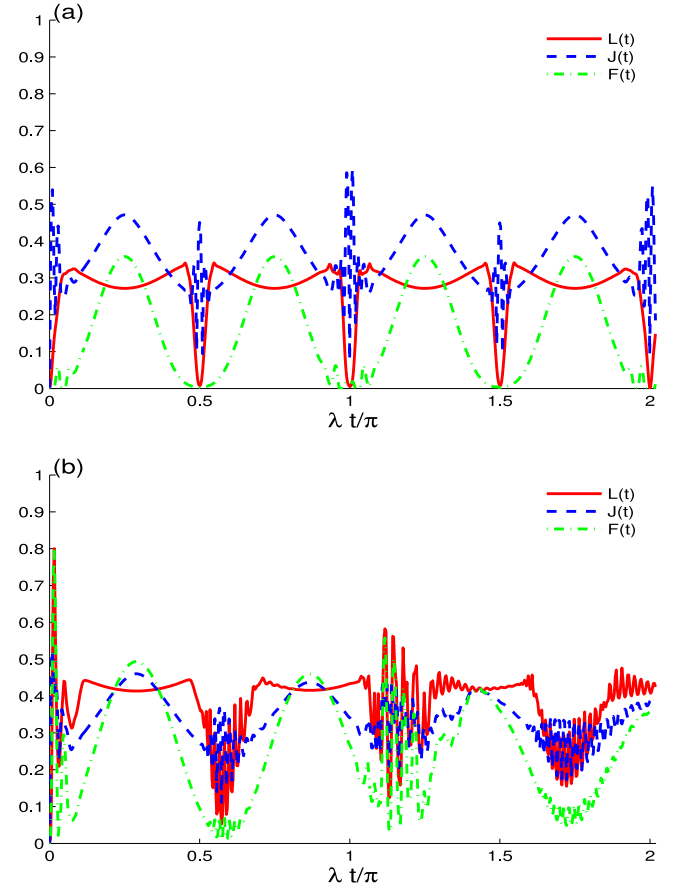


Fig. 1. Time evolutions of the local Fisher information $L(t)$, Jensen–Shannon information $J(t)$, and entanglement of formation $F(t)$ are shown when the cavity is initially filled with a coherent field with $|\alpha|^2 = 25$ in the absence of the decoherence for different values of the qubit–qubit coupling: $J = 0$ in (a) and $J = 60\lambda$ in (b).

of the decoherence for different values of the qubit–qubit interaction coupling: $J = 0$ in (a) and $J = 60\lambda$ in (b).

Fig. 1a illustrates the ability of the cavity–qubit interactions, through intensity-dependent coupling, to generate two-qubit correlation and coherence, in the absence of the qubit–qubit coupling, as well as the intrinsic decoherence. From Fig. 1a, we find that the non-local correlations and coherence functions $J(t)$, and $F(t)$ grow and oscillate periodically, $\sqrt{JSD}$ and EoF with period π whereas LQFI with period $\frac{1}{2}\pi$. Their partial values correspond to the generated partially correlated and coherent two-qubit states. In each period we note that:

- The local Fisher information measure demonstrates that, beyond the produced EoF, the uncorrelated two-qubit state evolves to exhibit regular partial LQFI non-local correlation. The generated LQFI non-local correlation, within interval period, is quasi-stable. The phenomenon of sudden changes appears in the dynamical behavior of the LQFI. The phenomenon of the sudden changes was coined in [48] and experimentally observed in [49]. During each period of the LQFI, two sudden changes (SCs) are observed. In the time intervals between the SCs, the dynamics of the $\sqrt{JSD}$ and EoF show maximal coherence and entanglement, respectively.
- The Jensen–Shannon information $J(t)$ reveals quicker and stronger two-qubit coherence compared to the other correlations quantifiers. The $\sqrt{JSD}$ amplitudes and oscillations are larger than these of the LQFI and EoF correlations. The two qubits reach their maxima and minima's Jensen–Shannon coherence and entanglement of formation at the same times $\lambda t = \frac{(2n-1)\pi}{4}$ (at the center of the LQFI period) and $\lambda t = \frac{n\pi}{2}$ ($n = 1, 2, \dots$), respectively.

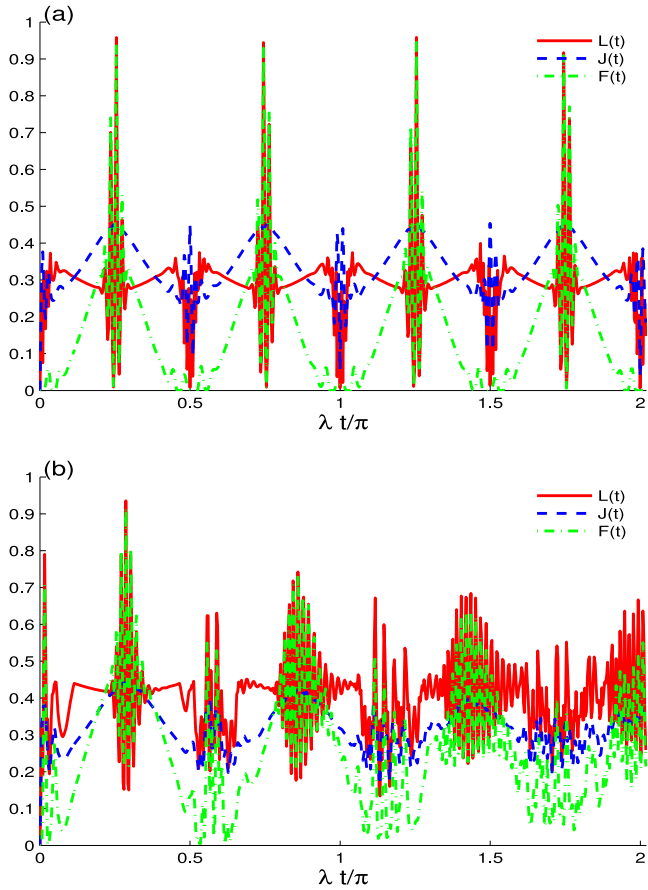


Fig. 2. As Fig. 1 but when the cavity is initially filled with an even coherent field with $|\alpha|^2 = 25$.

- The EoF shows that, the generation of two-qubit entanglement delays for a short time. After that, the EoF is generated and oscillates periodically. At the end of the period, the sudden birth and death phenomena [50,51] appear.

Fig. 1b examines the impact of dipole–dipole interaction coupling the amount, speed, and regularity of the created two-qubit correlations of local Fisher information and entanglement of formation, as well as two-qubit Jensen–Shannon coherence. In general, the behaviors of the two-qubit coherence and the two-qubit correlation quantifiers have notable changes due to the increase of the dipole–dipole interaction’s coupling. The coupling of the dipole–dipole interaction results in: (1) increasing the local Fisher information, Jensen–Shannon coherence, and entanglement created between the two qubits owing to the cavity–qubit intensity dependent interactions. The amplitudes and oscillations of non-local correlations and coherence get stronger. (2) When compared to Fig. 1a, the partially correlated and coherent two-qubit states are formed quickly (3) Disappearance of the regularity of the generated oscillatory behaviors’ LQFI and EoF correlations and Jensen–Shannon coherence (due to cavity–qubit intensity dependent interactions), as well as the sudden birth and death phenomena windows of the entanglement of formation, as soon as the two-qubit interaction is switched on.

In Fig. 2, the LQFI and EoF correlations and Jensen–Shannon coherence are identified by $L(t)$, $J(t)$, and $F(t)$ are depicted when the cavity is initially filled with an even coherent field with $|\alpha|^2 = 25$. The difference in results between the initial coherent and even-coherent fields is due to the non-classicality of the even coherent field state at large field intensities [52]. Therefore, our results show that:

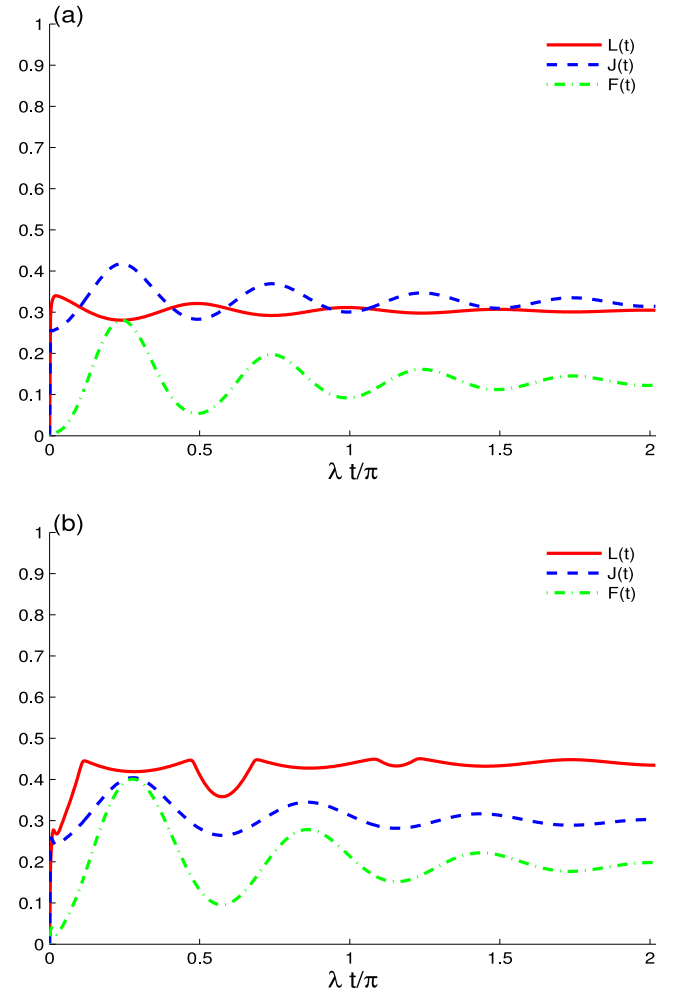


Fig. 3. The dynamics of the functions $L(t)$, $J(t)$, and $F(t)$ of Fig. 1 is shown under the intrinsic decoherence $\gamma = 0.03\lambda$ for different values of the qubit–qubit coupling: $J = 0$ in (a) and $J = 60\lambda$ in (b).

The LQFI correlation, EoF, and Jensen–Shannon coherence is strongly generated with different periodical oscillatory behaviors. The oscillation period of both the $\sqrt{JSD}$ and the EoF reduce to be $\frac{1}{2}\pi$, while of the LQFI is still $\frac{1}{2}\pi$. The increase of the amplitudes and the oscillations of the non-local correlations and coherence functions $L(t)$, $J(t)$, and $F(t)$ refers to the ability of the initial coherent and even-coherent fields to enhance the LQFI and EoF correlations and Jensen–Shannon coherence (due to the cavity–qubit intensity dependent interactions). The non-classicality of the even coherent field state maintains the regularity of the produced oscillatory behaviors’ LQFI and EoF correlations and Jensen–Shannon coherence, while increases the LQFI sudden birth–death entanglement windows.

The effect of increasing the intrinsic decoherence in the absence of the two-qubit interaction on the dynamics of the produced LQFI and EoF correlations and Jensen–Shannon coherence owing to cavity–qubit intensity dependent interactions is shown in Fig. 3a. The oscillating characteristics of the LQFI and EoF non-local correlations, as well as the Jensen–Shannon coherence, disappear. As time evolves, the maxima of the LQFI and EoF correlations and Jensen–Shannon coherence diminish while their minima grow, indicating that the maxima and minima of $L(t)$, $J(t)$, and $F(t)$ converge to a stable value. The two-qubit state in this case exhibits stationary LQFI and EoF correlations, as well as Jensen–Shannon coherence. Moreover, the oscillatory behaviors of the LQFI, $\sqrt{JSD}$, and EoF quantifiers are especially vulnerable to intrinsic decoherence, which leads to the disappearance of the LQFI sudden

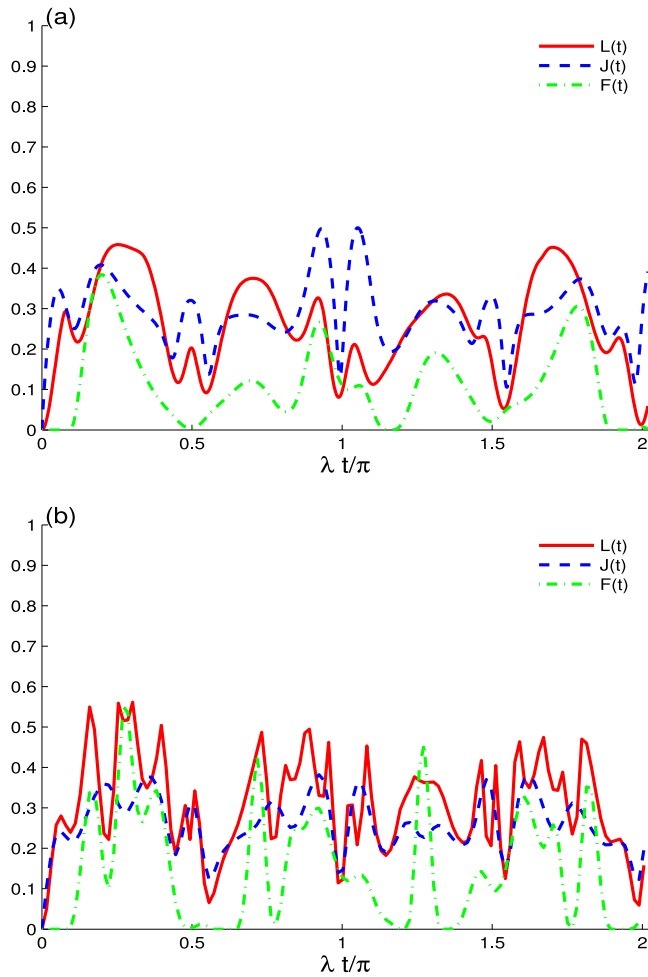


Fig. 4. The dynamics of the functions $L(t)$, $J(t)$, and $F(t)$ under the small initial coherent field intensity $|\alpha|^2 = 2$ when the cavity is initially filled with a coherent field in (a) and when the cavity is initially filled with an even coherent field in (b).

change as well as the sudden birth–death entanglement. When the cavity is initially filled with an even coherent field (see Fig. 3b), the dynamics of the local Fisher information, Jensen–Shannon information, and entanglement of formation are illustrated in the presence of the dipole two-qubit interaction. The maxima of all quantifiers are greater than those shown in Fig. 3a. However, the intrinsic decoherence leads to delays. As a result, we can conclude that the generated two-qubit state has stationary LQFI and EoF correlations, and Jensen–Shannon coherence which depend on the qubit–qubit coupling as well as intrinsic decoherence.

Fig. 4 displays the ability of the cavity–qubit intensity dependent interactions to generate two-qubit correlations as well as Jensen–Shannon coherence when the cavity is initially filled with a coherent and an even coherent fields with a small initial coherent field intensity $|\alpha|^2 = 2$. By comparing Fig. 1a and Fig. 4a, we find that the effect of the mean photon number is clearly visible. For small values of the mean photon number $|\alpha|^2$, the minima of LQFI correlation and Jensen–Shannon coherence, except EoF, increases as the non-classicality of the initial coherent field grows. Fig. 4a shows that the LQFI and EoF non-local correlations and Jensen–Shannon coherence grow with irregular oscillations. The upper and the lower bounds of the LQFI, $\sqrt{JSD}$, and EoF rise. The sudden birth–death entanglement phenomena are more abundant when the cavity is initially filled with an even coherent field, as shown in Fig. 4b. In addition, for the case of the initial even coherent state, Fig. 4a shows that the initial even coherent state leads to: (1) the

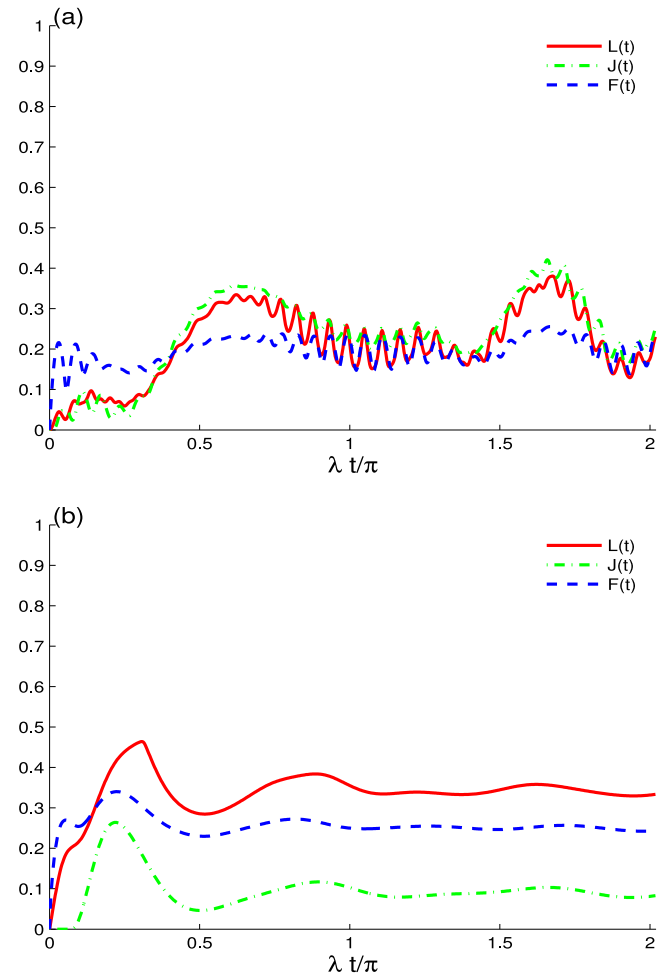


Fig. 5. The dynamics of the functions $L(t)$, $J(t)$, and $F(t)$ of Fig. 3a is shown for different values of the intrinsic decoherence and the qubit–qubit couplings: $(\gamma, J) = (0.30, 0)\lambda$ in (a) and $(\gamma, J) = (0.03, 0)\lambda$ in (b).

LQFI sudden changes (which does not appear in the case of the initial coherent state) appear clear. (2) the amplitudes and the frequency of the generated non-local correlations and coherence oscillations are enhanced.

In Fig. 5a, the effect of a small initial coherent field intensity is clearly displayed in the absence of the decoherence and in the presence of the qubit–qubit interaction coupling J . Due to coupling the qubit–qubit and cavity–qubit interactions with a low initial coherent field intensity, the generated LQFI and EoF non-local correlations are enhanced while the generated two-qubit Jensen–Shannon coherence degrades. The effect of the intrinsic decoherence is considerable for a small initial coherent field intensity; in the absence of the qubit–qubit interaction, the stationary LQFI and EoF correlations and Jensen–Shannon coherence are quickly attained. Finally, our results indicate that the stationary LQFI and EoF correlations, as well as Jensen–Shannon coherence, are affected by the qubit–qubit interaction coupling, initial coherent field intensity, and intrinsic decoherence.

Conclusions

In this paper, we have investigated the generated two-qubit correlations namely; the local quantum Fisher information, entanglement of formation, and two-qubit Jensen–Shannon coherence. The cavity is initially filled with coherent and an even coherent fields while the two-qubit state is initially in a pure state. The cavity–qubit intensity

dependent interactions produce two-qubit Jensen–Shannon coherence more rapidly and strongly than the generated local quantum Fisher information and entanglement of formation. The phenomena of sudden birth and death for Fisher information and entanglement of formation emerge in this system. These effects may be amplified by the mean photon number and the non-classicality of the initial coherent field state. The intrinsic decoherence eliminates the sudden birth–death entanglement. The dipole qubit–qubit interaction increases the local Fisher information, Jensen–Shannon coherence, and entanglement while decreasing the sudden birth–death entanglement. Due to combining the qubit–qubit and cavity–qubit interactions with a low initial coherent field intensity, the LQFI and EoF non-local correlations improve while the generated two-qubit Jensen–Shannon coherence degrades. The two-qubit LQFI and EoF correlations, as well as Jensen–Shannon coherence, increase as the non-classicality of the initial coherent field grows. The non-classicality of the even coherent field state enhances the amplitudes and frequency of non-local correlations and coherence, as well as the sudden birth–death entanglement. Our results further reveal that the qubit–qubit interaction, initial coherent field intensity, and intrinsic decoherence affect the stationary LQFI and EoF, as well as the Jensen–Shannon coherence. According to our LQFI, EoF, and Jensen–Shannon coherence results, (i) the cavity–qubit intensity dependent interaction and qubit–qubit interactions have a large ability to produce two-qubit correlation and coherence. The stationary LQFI, EoF, and Jensen–Shannon coherence are all impacted by the intrinsic decoherence. (ii) The generated two-qubit correlation and coherence are improved by the even coherent cavity field state. (iii) The intrinsic decoherence affect has high ability to generate stationary LQFI and EoF, as well as the Jensen–Shannon coherence. The generated two-qubit correlation and coherence have a significant impact on the precision of predicted measurements in quantum information processing. The two-qubit correlation and coherence were used to realize quantum computations [53,54], quantum tomography [55] as well as to implement large cat states in finite-temperature reservoir [56].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgments

The authors extend their appreciation to the Deputyship for Research & Innovation, Ministry of Education in Saudi Arabia for funding this research work through the project number (IF-PSAU-2021/01/17712). The authors are very grateful to the referees for their important remarks which have helped to improve the manuscript.

References

- [1] Horodecki R, Horodecki P, Horodecki M, Horodecki K. *Rev Modern Phys* 2009;81:865.
- [2] Chitambar E, Gour G. *Rev Modern Phys* 2019;91:025001.
- [3] Sete EA, Eleuch H. *Phys Rev A* 2014;89:013841.
- [4] Bennett CH, Brassard G, Crepeau C, Jozsa R, Peres A, Wootters WK. *Phys Rev Lett* 1993;70:1895.
- [5] Werner RF. *J Phys A: Math Gen* 2001;34:7081.
- [6] Bennett CH, Brassard G, Mermin ND. *Phys Rev Lett* 1992;68:557; Ekert AK. *Phys Rev Lett* 1991;67:661.
- [7] Calabrese P, Cardy J. *J Stat Mech* 2004;P06002.
- [8] Mohamed A-BA, Eleuch H, Raymond Ooi CH. *Phys Lett A* 2019;383:125905.
- [9] Mohamed A-BA, Eleuch H, Raymond Ooi CH. *Sci Rep* 2019;9:19632.
- [10] Bennett CH, DiVincenzo DP, Smolin JA, Wootters WK. *Phys Rev A* 1996;54:3824.
- [11] Vidal G, Werner RF. *Phys Rev A* 2002;65:032314.
- [12] Wootters WK. *Phys Rev Lett* 1998;80:2245.
- [13] Ollivier H, Zurek WH. *Phys Rev Lett* 2001;88:017901.
- [14] Luo S. *Phys Rev A* 2008;77:022301; Mohamed A-BA. *Phys Scr* 2012;85: 055013.
- [15] Luo S, Fu S. *Phys Rev Lett* 2011;106:120401.
- [16] Girolami D, Souza AM, Giovannetti V, Tufarelli T, Filgueiras JG, Sarthour RS, et al. *Phys Rev Lett* 2014;112:210401.
- [17] Dhar HS, Bera MN, Adesso G. *Phys Rev A* 2015;99:032115.
- [18] Matera JM, Egloff D, Killoran N, Plenio MB. *Quantum Sci Technol* 2016;1:01LT01.
- [19] Narasimhachar V, Gour G. *Nature Commun* 2015;6:7689.
- [20] Lloyd S. *J Phys Conf Ser* 2011;302:012037.
- [21] Obada A-SF, Hessian HA, Mohamed A-BA. *Phys Lett A* 2008;372:3699.
- [22] Obada A-SF, Hessian HA, Mohamed A-BA. *J Phys B: At Mol Opt Phys* 2008;41:135503.
- [23] Giovannetti V, Lloyd S, Maccone L. *Phys Rev Lett* 2006;96:010401.
- [24] Hu M-L, Fan H. *Phys Rev A* 2018;98:022312.
- [25] Hu M-L, Wang X-M, Fan H. *Phys Rev A* 2018;98:032317.
- [26] Hu Ming-Liang, Fan Heng. *Phys Rev A* 2017;95:052106.
- [27] Hu Ming-Liang, Hu Xueyuan, Wang Jieci, Peng Yi, Zhang Yu-Ran, Fan Heng. *Phys Rep* 2018;762-764:1–100.
- [28] Radhakrishnan C, Parthasarathy M, Jambulingam S, Byrnes T. *Phys Rev Lett* 2016;116:150504.
- [29] Radhakrishnan C, Parthasarathy M, Jambulingam S, Byrnes T. *Sci Rep* 2017;7:13865.
- [30] Zhang JQ, Xiong W, Zhang S, Li Y, Feng M. *Ann Phys* 2015;527:180.
- [31] Mohamed A-BA, Hashem M, Eleuch H. *Entropy* 2019;21:672.
- [32] Simeonov LS, Vitanov NV, Ivanov PA. *Sci Rep* 2019;9:7340.
- [33] Harlander M, Lechner R, Brownnutt M, Blatt R, Hänsel W. *Nature* 2011;471:200.
- [34] Mohamed A-BA, Eleuch H, Obada A-SF. *Results Phys* 2020;17:103019.
- [35] Sadiek G, Al-Dress W, Shagel S, Elhag H. *Entropy* 2021;23:629.
- [36] Milburn GJ. *Phys Rev A* 1991;44:5401.
- [37] Wallraff A, Schuster DI, Blais A, Frunzio L, Huang R-S, Majer J, et al. *Nature* 2004;431:162.
- [38] Blatt R, Wineland DJ. *Nature* 2008;453:1008; Eleuch H, Ben Nessib N, Bennaceur R. *Eur Phys J D* 2004;29:391.
- [39] Sete EA, Eleuch H. *Phys Rev A* 2010;82:043810.
- [40] Yin Z, Li T, Zhang X, Duan LM. *Phys Rev A* 2013;88:033614.
- [41] Buck B, Sukumar CV. *Phys Lett A* 1981;81:132.
- [42] Valverde C, Goncalves VG, Baseia B. *Physica A* 2016;446:171.
- [43] You JQ, Nori F. *Phys Rev B* 2003;68:064509.
- [44] Mohamed A-BA. *Eur Phys J D* 2017;71:261; *Rep Math Phys* 2013;72:121.
- [45] Kuang L-M, Tong Z-Y, Ouyang Z-W, Zeng H-S. *Phys Rev A* 1999;61:013608.
- [46] Breuer H-P, Petruccione F. *The theory of open quantum systems*. Oxford University Press; 2002.
- [47] Radhakrishnan C, Ding Z, Shi F, Du J, Byrnes T. *Ann Phys* 2019;409:167906.
- [48] Maziero J, Celeri LC, Serra RM, Vedral V. *Phys Rev A* 2009;80:044102.
- [49] Xu J-S, Xu X-Y, Li C-F, Zhang C-J, Zou X-B, Guo G-C. *Nat Commun* 2010;1:7.
- [50] Yu T, Eberly JH. *Phys Rev Lett* 2004;93:140404.
- [51] Mohamed A-BA, Hessian HA, Obada A-SF. *Physica A* 2011;390:519.
- [52] Rohith M, Sudheesh C. *J Opt Soc Amer B* 2016;33:126.
- [53] Strandberg I, Lu Y, Quijandria F, Johansson G. *Phys Rev A* 2019;100:063808.
- [54] Raussendorf R, Bermejo-Vega J, Tyhurst E, Okay C, Zurek M. *Phys Rev A* 2020;101:012350.
- [55] Botelho LAS, Vianna RO. *Eur Phys J D* 2020;74:42.
- [56] Teh RY, Sun F-X, Polkinghorne RES, He QY, Gong Q, Drummond PD, et al. *Phys Rev A* 2020;101:043807.