



Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe



An economic order quantity (EOQ) for items with imperfect quality and inspection errors

Mehmood Khan^a, Mohamad Y. Jaber^{a,*}, Maurice Bonney^b

^a Department of Mechanical and Industrial Engineering, Ryerson University, Toronto, ON, Canada, M5B 2K3

^b Operations Management Division, Nottingham University Business School, Jubilee Campus, Nottingham NG8 1BB, UK

ARTICLE INFO

Article history:

Received 3 September 2008

Accepted 20 January 2010

Available online 2 February 2010

Keywords:

Imperfect process

Misclassification errors

EOQ

ABSTRACT

An approach similar to Salameh and Jaber (2000) has been used in this paper to produce an optimal production/order quantity that takes care of imperfect processes. An imperfect inspection process (Rao et al., 1983) is utilized to describe the defective proportion of the received lot. That is, the inspector may commit errors while screening. The probability of misclassification errors is assumed to be known. The inspection process would consist of three costs: (a) cost of inspection (b) cost of Type I errors and (c) cost of Type II errors. The defective items, classified by the inspector and the buyer would be salvaged as a single batch that is sold at a lower price. A mathematical model is developed to depict this scenario. Numerical examples are provided to illustrate the solution procedure.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

Traditional inventory models tend to obtain an economic (optimal) order quantity (EOQ) or economic production quantity (EPQ) based on the ordering/setup cost and the inventory carrying cost. They make many assumptions while coming up with a closed form solution for the most economical batch size in a stock or a production facility. One assumption is that the items produced by the facility are all of a perfect quality. Another is that the screening process that identifies the defective items in a lot is error-free. That is, the defective items from a lot can be screened out through 100% inspection. This is an idealistic approach. In practice the production lot may contain a substantial number of defective items, possibly because of weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit (Rahim and Ben-Daya, 2001). Also, the screening process, e.g. at the end of an assembly line, is never perfect. Hence, there is a need to determine an optimal order quantity when the inspection process is prone to making errors while screening a defective lot.

A number of researchers have not used the perfect quality assumption above. Porteus (1986) studied the effect of defective items on the basic EOQ model. He assumed that there is a fixed probability that the process would go out-of-control. Rosenblatt and Lee (1986) assumed that the time between the in-control and the out-of-control state of a process follows an exponential distribution and that the defective items are reworked instantaneously. They suggested producing in smaller lots when the

process is not perfect. In a later paper, Lee and Rosenblatt (1987) studied a joint lot sizing and inspection policy for an EOQ model with a fixed percentage of defective products.

While most of the literature in this area deals with deterministic problems, many researchers have discussed stochastic production yield and demand rates. Gerchak et al. (1988) analyzed a single period production problem and extended it to an n -period problem where the production process has a variable yield and the demand is uncertain. Yano and Lee (1995) reviewed and identified shortcomings in the literature dealing with determining lot sizes where production or procurement yields are stochastic. Grosfeld-Nir and Gerchak (2004) also reviewed the literature on single stage and multistage imperfect production systems that involve random yield and inspection. Inderfurth (2004) determined an optimal production policy for a uniformly distributed demand and yield rate and discussed some managerial aspects of this policy. Rekik et al. (2007) extended the work of Inderfurth (2004) for two cases: (a) an additive errors case where the variability of errors is independent of the order quantity, and (b) a multiplicative errors case where the variability of errors is proportional to the order quantity.

Recently, Salameh and Jaber (2000), has been receiving attention. They studied a joint lot sizing and inspection policy for an EOQ model when a random proportion of the units in a lot are defective. They assumed a 100% screening process with no human error. They suggested that poor-quality items should be salvaged as a single batch at the end of the 100% screening process. This model will be referred to as the S & J model in the rest of the paper. Goyal and Cardenas-Barron (2002) suggested a simpler approach to the S & J model, which Goyal et al. (2003) used to determine an integrated vendor–buyer inventory policy for an item with imperfect quality. They suggested that the order

* Corresponding author. Tel.: +1 416 979 5000; fax: +1 416 979 5265.

E-mail address: mjaber@ryerson.ca (M.Y. Jaber).

size should be delivered in n sublots. Chiu (2003) considered the effect of reworking defective items on the EPQ model with backlogging allowed. He suggested that some of the defective items should be reworked while the rest should be scrapped. Chang (2004) studied the S & J model under a fuzzy defective rate and a fuzzy annual demand. Huang (2004) presented an integrated vendor–buyer inventory model for an unreliable process. He derived an analytic solution for the optimal order quantity and the number of deliveries in each purchase order. Papachristos and Konstantaras (2006) discussed the non-shortages in inventory models where the proportion of defective items is a random variable. They proposed an alternative to the S & J model by speculating on the timing of withdrawing and selling the imperfect lot. Liao (2007) studied imperfect production processes that require maintenance. They considered two states, namely the in-control and the out-of-control state of the production process. The maintenance process would either worsen the production system or bring it to a perfect state. They showed that there exists a unique optimal number of imperfect maintenance processes (N) before the production system is brought to the perfect state. Maddah and Jaber (2008) corrected a flaw in the S & J model by using renewal theory. They came up with simpler expressions for the expected profit and the order quantity.

The S & J model suggested that the imperfect items are not reworked but just withdrawn from the received lot. It is also assumed that there is no human error in the screening process. Raouf et al. (1983) studied human error in inspection planning. They came up with one of the first inspection plans with misclassifications for multi-characteristic critical components. They suggested repeating the cycle of inspection to ensure the product quality and determined an optimal number of inspection cycles based on the cost of inspection and misclassifications. Duffuaa and Khan (2002) suggested an inspection plan for these critical components where an inspector can commit a number of misclassifications. They extended the Raouf et al. (1983) inspection plan for the case of a number of misclassifications. This was a realistic approach where an inspector can classify an item to be nondefective, reworkable or scrap. In a later paper, Duffuaa and Khan (2005) carried out a sensitivity analysis to study the effect of different types of misclassifications on the optimal inspection plan.

There may be many sources of errors in inspection, one of which is inaccuracy in records. Kök and Shang (2007) discussed inaccuracies in inventory records. They proved that an inspection adjusted base-stock policy is optimal for a single period problem, where inspection is performed if the inventory recorded is less than a threshold level. Atali et al. (2009) also modeled the discrepancies between actual and the recorded inventories in retail and distribution environments. They quantified the value of RFID (radio-frequency identification) that reduces the amount of such discrepancies. We leave this issue here for future research.

This paper extends the work of S & J model by assuming that the screening process is not error-free. The Raouf et al. (1983) approach is used to suggest that an inspector could make two classifications while screening, i.e. an item may be classified as defective or nondefective. However, because the inspection process is not error-free, a good item may be classified as defective, i.e. a Type I error, while a defective item may be classified as good, i.e. a Type II error (Jacobson, 1952). The rest of the paper is organized as follows. Section 2 describes the model. Section 3 produces a mathematical model. Section 4 presents a numerical example and discusses the results. Section 5 presents a summary and conclusions.

2. Model description

Consider a lot of size y being delivered to the buyer. It is assumed that each lot contains a fixed proportion p of defective items. An inspector screens out the defective items from the lot with fixed rate of misclassifications. That is, a proportion m_1 of nondefective items are classified to be defective and a proportion m_2 of defective items are classified to be nondefective. It is assumed that the probability density functions, $f(p)$, $f(m_1)$ and $f(m_2)$ are known. It is also assumed that the items that are returned from the market are stored with those that are classified as defective by the inspector. They are all sold as a single batch at the end of each cycle at a discounted price. The behavior of the inventory level is illustrated in Fig. 1, where T is the cycle length, B_1 is the batch classified as defective by the inspector while B_2 is the batch of returned units from the market accumulated over T .

An optimal inventory policy is determined using the total revenues and the total costs. The costs considered in the model

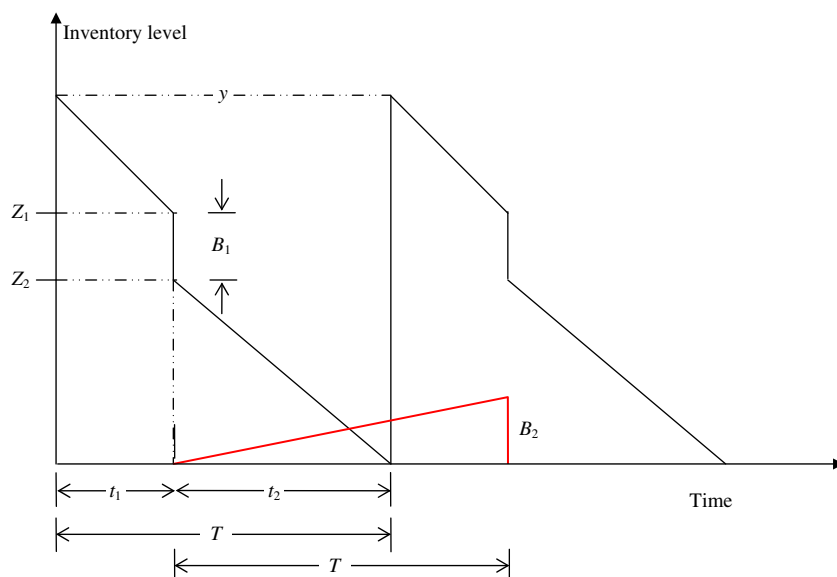


Fig. 1. Behavior of the inventory level over time.

are the procurement cost, screening cost, misclassification cost and the inventory carrying cost. The following nomenclature is used throughout the paper.

D	number of units demanded per year
y	order size
c	unit variable cost
K	fixed ordering cost
A	a parameter used for simplifying the holding cost in Eq. (5)
s	unit selling price of a nondefective item
v	unit selling price of a defective item
x	screening rate
d	unit screening cost
h	unit holding cost
T	cycle length
m_1	probability of Type I error (classifying a nondefective item as defective)
m_2	probability of Type II error (classifying a defective item as nondefective)
p	probability that an item is defective
t_1	inspection time in a cycle
t_2	the remaining time in a cycle, after the defective items are screened out
$f(p)$	probability density function of p
$f(m_1)$	probability density function of m_1
$f(m_2)$	probability density function of m_2
B_1	number of items that are classified as defective in one cycle
B_2	number of defective items that are returned from the market in one cycle
c_a	cost of accepting a defective item
c_r	cost of rejecting a nondefective item

3. Mathematical model

Fig. 1 shows how the inventory behaves with a buyer or a retailer. It should be noted here that this behavior was suggested by S & J (2000). The screening and consumption of the inventory continues until time t_1 , after which all the defectives (B_1) are withdrawn from inventory as a single batch and are sold to the secondary market. The consumption process continues at the demand rate until the end of cycle time T . Due to inspection error, some of the items used to fulfill the demand would be defective. These defective items are later returned to the inventory and are shown in Fig. 1 as B_2 . To avoid shortages, it is assumed that the number of nondefective items is at least equal to the adjusted

demand, that is the sum of the actual demand and items that are replaced for the ones returned (pm_2y) from the market over T . Thus

$$y - y(1-p)m_1 - yp(1-m_2) \geq DT + pm_2y$$

$$y(1-p) - ym_1(1-p) \geq DT$$

$$y(1-p)(1-m_1) \geq DT$$

So, for the limiting case, the cycle length can be written as

$$T = \frac{y(1-p)(1-m_1)}{D} \quad (1)$$

It should be noted that the above expression is unaffected by the Type II error and reduces to the cycle length in S & J if the Type I error becomes zero.

Consider now the different cases of misclassifications that an inspection process can have. There are four possibilities in such an inspection process. Those are: Case (1) A nondefective item is classified as nondefective; Case (2) A nondefective item is classified as defective; Case (3) A defective item is classified as nondefective; and Case (4) A defective item is classified as defective. This scenario is depicted in Fig. 2. The number of items going into different categories following these cases is given by:

$$\text{Case (1): } y(1-p)(1-m_1)$$

$$\text{Case (2): } y(1-p)m_1$$

$$\text{Case (3): } ypm_2$$

$$\text{Case (4): } yp(1-m_2)$$

Now B_1 and B_2 are given by

$$B_1 = y(1-p)m_1 + yp(1-m_2)$$

$$B_2 = ypm_2$$

The items in batch B_2 are returned from the market at the rate ypm_2/T and are taken from the inventory with batch B_1 . Therefore the revenue from salvaging $B = B_1 + B_2$ items is given by

$$R_1 = vB = v(B_1 + B_2) = vy(1-p)m_1 + vyp(1-m_2) + vypm_2$$

or

$$R_1 = vy(1-p)m_1 + vyp$$

The revenue from selling the good items is computed as

$$R_2 = sy(1-p)(1-m_1) + sypm_2$$

So, the total revenue is given as

$$R = R_1 + R_2$$

$$R = vy(1-p)m_1 + vyp + sy(1-p)(1-m_1) + sypm_2 \quad (2)$$

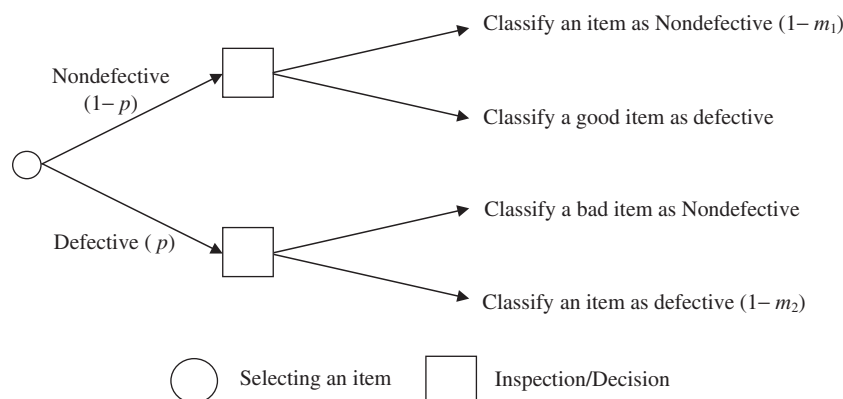


Fig. 2. Four possibilities in the inspection process.

Consider now the different costs of the inventory system. The procurement cost per cycle is

$$PC = K + cy \quad (3)$$

where c is the variable cost. The screening cost per cycle is the sum of the cost of inspection and misclassifications which is given by

$$IC = dy + c_r(1-p)ym_1 + c_a p y m_2 \quad (4)$$

The holding cost per cycle is the cost of carrying the (i) nondefective lot, (ii) defective lot and (iii) returned lot. So, from Fig. 1 the holding cost for a cycle can be written as

$$HC = h \left\{ \frac{(y-Z_1)t_1}{2} + Z_1 t_1 + \frac{Z_2 t_2}{2} \right\} + h \left(\frac{B_2 T}{2} \right)$$

where Z , t represent the stock level and the time spent respectively (before and after separating the defective items). Replacing B_2 with ypm_2 , Z_2 with $Z_1 - B_1$, Z_1 with $y - Dt_1$ and t_1 with D/x , it can be written as

$$HC = \frac{hy^2}{2} \left(\frac{2}{x} - \frac{D}{x^2} + \frac{A^2}{D} \right) + \frac{h}{2} (ypm_2 T) \quad (5)$$

where $A = 1 - D/x - (m_1 + p) + p(m_1 + m_2)$

Therefore the total cost per cycle is given by summing up Eq. (3–5), i.e. $C = PC + IC + HC$ or

$$C = K + cy + dy + c_r(1-p)ym_1 + c_a p y m_2 + \frac{h}{2} \left\{ \left(\frac{2}{x} - \frac{D}{x^2} + \frac{A^2}{D} \right) y^2 + ypm_2 T \right\}$$

Fig. 1 depicts the behavior of different types of inventory in the order cycle. The (red) triangle at the bottom represents the defective lot that is returned by the market and is accumulated into the salvaged lot.

The total profit per cycle can now be written as the difference between the total revenue and total cost per cycle, that is

$$TP(y) = sy(1-p)(1-m_1) + sypm_2 + vy(1-p)m_1 + vyp - [K + cy + dy + c_r(1-p)ym_1 + c_a p y m_2 + \frac{h}{2} \left\{ \left(\frac{2}{x} - \frac{D}{x^2} + \frac{A^2}{D} \right) y^2 + ypm_2 T \right\}] \quad (6)$$

Since p , m_1 and m_2 are random variables with probability density functions $f(p)$, $f(m_1)$ and $f(m_2)$, the expected total profit can be written as

$$E[TP(y)] = sy(1-E[p])(1-E[m_1]) + syE[p]E[m_2] + vy(1-E[p])E[m_1] + vyE[p] - [K - cy - dy - c_r y(1-E[p])E[m_1] - c_a yE[p]E[m_2] - \frac{h}{2} \left\{ \left(\frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2]}{D} \right) y^2 + yE[p]E[m_2]E[T] \right\}] \quad (7)$$

Now from Eq. (1), the expected cycle length would be

$$E[T] = \frac{y(1-E[p])(1-E[m_1])}{D} \quad (8)$$

Maddah and Jaber (2008) corrected the approach in the S & J model to determine the annual profit. They suggested using renewal reward theorem. Using this new approach, the expected annual profit for our model, is written as

$$E[TPU(y)] = \frac{E[TP(y)]}{E[T]}$$

or

$$E[TPU(y)] = sD + \frac{sDE[p]E[m_2]}{(1-E[p])(1-E[m_1])} + \frac{vDE[m_1]}{(1-E[m_1])} + \frac{vDE[p]}{(1-E[p])(1-E[m_1])} - \frac{D \left[\frac{K}{y} + c + d + c_r(1-E[p])E[m_1] + c_a E[p]E[m_2] + \frac{h}{2} \left\{ \left(\frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2]}{D} \right) y \right\} \right]}{(1-E[p])(1-E[m_1])} - h \frac{yE[p]E[m_2]}{2} \quad (9)$$

It should be noted that Eq. (9) converges to the expected annual profit equation in Maddah and Jaber (2008) once the value of errors goes to zero. It can be demonstrated that this expected annual profit follows a concave function. The first derivative of Eq. (9) is given by

$$E[TPU'(y)] = - \frac{D \left[-\frac{K}{y^2} + \frac{h}{2} \left(\frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2]}{D} \right) \right]}{(1-E[p])(1-E[m_1])} - h \frac{E[p]E[m_2]}{2}$$

The second derivative of Eq. (9) is

$$ETPU''(y) = - \frac{2KD}{y^3(1-E[p])(1-E[m_1])}$$

Since $K > 0$, $D > 0$, $0 < E[p] < 1$ and $0 < E[m_1] < 1$, then $ETPU''(y) < 0$ for every $y > 0$, suggesting that the annual profit in Eq. (9) is concave. This fact is also demonstrated graphically in the next section. Thus, the optimal order size that represents the maximum annual profit, is determined by setting the first derivative equal to zero and solving for y to get

$$y^* = \sqrt{\frac{2KD}{hE[p]E[m_2](1-E[p])(1-E[m_1]) + hD \left(\frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2]}{D} \right)}} \quad (10)$$

Note that when $p = m_1 = m_2 = 0$, Eq. (10) reduces to the traditional EOQ formula.

4. Numerical analysis

Consider a production system that replenishes the buyer's orders instantly. This system is not perfect, i.e. it produces some defective items. The inspection process that screens out the defective items is also imperfect. The probability density functions for the fraction of defective items and the inspection errors are mostly taken from the history of a supplier/machine and workers. In the case when these values are not known, the fraction of defectives in a lot can be determined by using the lot size (Porteus, 1986; Urban, 1998) or the time at which a process goes out-of-control in a cycle (Rosenblatt and Lee, 1986). Similarly, the parameters for inspection errors can be determined by the methods suggested by Cary et al. (1994) or Jaraiedi (1983). In the following analysis, most of the data is taken from the S & J model.

D	50 000 units/year		
c	\$25/unit		
K	\$100/cycle		
s	\$50/unit		
v	\$20/unit		
x	1 unit/min		
d	\$0.5/unit		
h	\$5/unit		
c_a	\$500/unit		
c_r	\$100/unit		
$f(p)$	$\begin{cases} 25, & 0 \leq p \leq 0.05 \\ 0, & \text{otherwise} \end{cases}$	$\Rightarrow$	$E(p) = 0.02$
$f(m_1)$	$\begin{cases} 25, & 0 \leq m_1 \leq 0.05 \\ 0, & \text{otherwise} \end{cases}$	$\Rightarrow$	$E(m_1) = 0.02$
$f(m_2)$	$\begin{cases} 25, & 0 \leq m_2 \leq 0.05 \\ 0, & \text{otherwise} \end{cases}$	$\Rightarrow$	$E(m_2) = 0.02$

Assuming that the buyer operates for 8 hours per day for 365 days per year, the annual screening rate would be, $x = 1(60)(8)(365) = 175\,200$ units. Substituting above values in Eqs. (10) and (9), respectively, we obtain the optimal values of

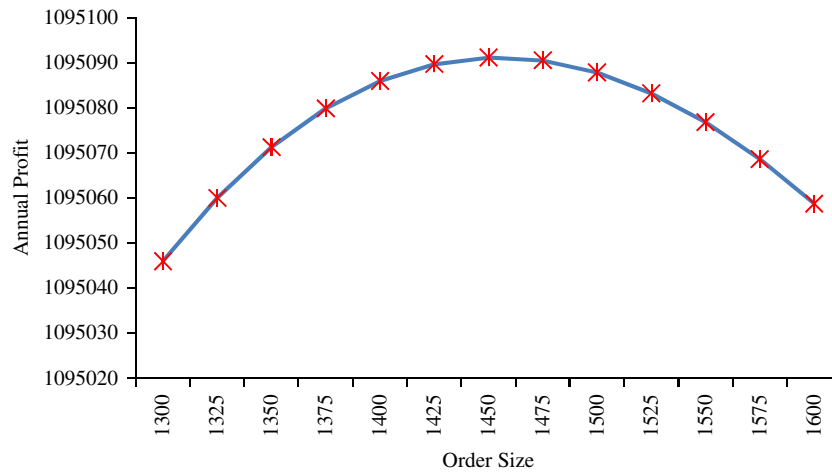


Fig. 3. Expected annual profit is a concave function of the order size.

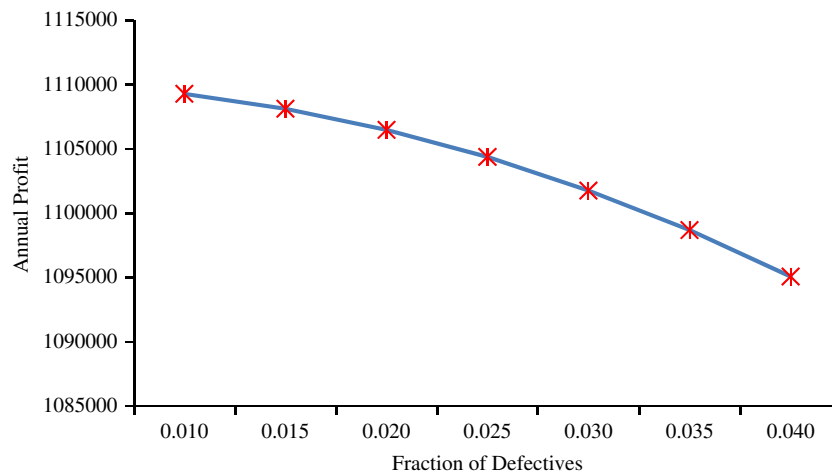


Fig. 4. Relationship between the annual profit and the fraction of defectives.

the order size and the annual profit as

$$y^* = 1455 \text{ units} \\ ETPU = \$1095090/\text{year}$$

Fig. 3 demonstrates the concavity of the annual profit with respect to the order size. That is, there is an optimal order size for the input parameters taken here, with respect to the annual profit. One should notice that although Fig. 3 does not show much variation in annual profit with the order size, there is a noticeable deviation in the costs and profits per cycle as the order size moves away from the optimal one. For example, the cycle profit changes by 31% and 24%, respectively, if the order size per cycle is moved from the optimal value to 1000 and 1800.

The results in our numerical example indicate that the optimal cycle inventory is almost the same as it was in the S & J model but the cost of misclassifications, i.e. false rejection and false acceptance cause a huge drop in the annual profit as compared to that in the S & J model. This identifies the significance of the human errors in inspection. One should observe that this loss in profit has a direct relation to the fraction of defectives in the supplier's lot. Fig. 4 shows this behavior of the annual profit, for the suggested model. This curve is obtained by varying the upper limit of the pdf of the fraction of defectives at a fixed (0.04) level of both inspection errors (Type I and Type II). The order size per cycle is kept at 1439 (Salameh and Jaber, 2000). It is clear that the

annual profit tends to decrease with the fraction of defectives as more and more items would have to be sold at a discounted price.

5. Managerial implications

The model of S & J has recently been extended for a number of practical circumstances such as reworks, shortages and supply chain etc. The model in this paper is an addition to this line of research. The approach adopted here provides the practitioners with a more functional alternative to the S & J model because it incorporates the screening costs more accurately in the economic order sizing decision.

It should be emphasized that the inspection part in the S & J model is suitable for buyers that have an automated screening system where one can expect no errors. On the other hand, if the characteristic of interest cannot be screened through a machine and is inspected by human beings, the screening process is bound to have misclassifications. For example, a superstore would screen the incoming products and sell the nondefective ones to its customers. It would usually sell the items of low quality at a discount price. The returned defective items, that were misclassified, would also be sold at the discount price. The misclassified nondefective items would incur the store a loss. The same course of action applies to a manufacturing firm that utilizes supplier items in its assembly lines and sells the defective items to a secondary market after inspection.

These misclassifications are critical if the parts under inspection are of an aircraft, a space shuttle or a complex gas ignition system (Raouf et al., 1983). The reason is that the results of misclassification in such systems could be fatal. That is why the quality requirements for such components are very tight (Zunzanyika and Drury, 1975). A common practice in the industry is to institute multiple inspections to reduce the effect of errors, for these components (Chandra and Schall, 1988). So, it is vital for a buyer to be aware of not only the accurate parameters of error about his inspectors but also the ways to mitigate these errors.

6. Conclusions

This paper makes use of Salameh and Jaber (2000) and Raouf et al. (1983) models to determine an inventory policy for imperfect items received by a buyer. A realistic approach of screening is adopted. That is, an inspector may classify a nondefective item to be defective (Type I error) and he may also classify a defective item to be nondefective (Type II error). The defective items classified by the inspector and those returned from the market are accumulated and sold at a discounted price at the end of each procurement cycle.

The paper suggests that the annual profit with inspection errors remains concave with respect to the order size. Comparing the results with those in Salameh and Jaber (2000), the optimal order size is almost the same but the annual profit is much smaller. This signifies the effect of inspection errors. The increasing fraction of defectives keeps on reducing the annual profit.

The paper could be extended for the case where demand is uncertain. Furthermore, learning in the inspection rate would also enhance the usefulness of the model presented here.

Acknowledgements

The authors thank the reviewers for their useful comments. M. Khan and M.Y. Jaber thank the Natural Science and Engineering Research Council of Canada (NSERC) for supporting their research.

References

- Atali, A., Lee, H., Ozer, O., 2009. If the Inventory Manager Knew: Value of RFID Under Imperfect Inventory information. Graduate School of Business, Stanford University, Stanford.
- Cary, D.T.J., Torrence, W.D., Schniederjans, M., 1994. Quality assurance in human resource management for computer-integrated manufacturing. *International Journal of Operations & Production Management* 14 (2), 18–30.
- Chandra, J., Schall, S., 1988. The use of repeated measurements to reduce the effect of measurement errors. *IIE Transactions* 20 (1), 83–87.
- Chang, H.C., 2004. An application of fuzzy sets theory to the EOQ model with imperfect quality items. *Computers and Operations Research* 31 (12), 2079–2092.
- Chiu, Y.P., 2003. Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. *Engineering Optimization* 35 (4), 427–437.
- Duffuaa, S.O., Khan, M., 2005. Impact of inspection errors on the performance measures of a general repeat inspection plan. *International Journal of Production Research* 43 (23), 4945–4967.
- Duffuaa, S.O., Khan, M., 2002. An optimal repeat inspection plan with several classifications. *Journal of the Operational Research Society* 53 (9), 1016–1026.
- Gerchak, Y., Vickson, R.G., Parlar, M., 1988. Periodic review production models with variable yield and uncertain demand. *IIE Transactions* 20 (2), 144–150.
- Goyal, S.K., Cárdenas-Barrón, L.E., 2002. Economic production quantity model for items with imperfect quality—a practical approach. *International Journal of Production Economics* 77 (1), 85–87.
- Goyal, S.K., Huang, C.K., Chen, K.C., 2003. A simple integrated production policy of an imperfect item for vendor and buyer. *Production Planning and Control* 14 (7), 596–602.
- Grosfeld-Nir, A., Gerchak, Y., 2004. Multiple lotsizing in production to order with random yields: review of recent advances. *Annals of Operations Research* 126 (1–4), 43–69.
- Huang, C.K., 2004. An optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration. *International Journal of Production Economics* 91 (1), 91–98.
- Inderfurth, K., 2004. Analytical solution for a single-period production-inventory problem with uniformly distributed yield and demand. *Central European Journal of Operations Research* 12 (2), 117–127.
- Jacobson, H.L., 1952. A Study of Inspector Accuracy. *Industrial Quality Control* p. 916–925.
- Jaraiedi, M., 1983. Inspection error modelling and economic design of sampling plans subject to inspection error. Ph.D. Dissertation, University of Michigan, Ann Arbor.
- Kök, A.G., Shang, K.H., 2007. Inspection and replenishment policies for systems with inventory record inaccuracy. *Manufacturing and Service Operations Management* 9 (2), 185–205.
- Lee, H.L., Rosenblatt, M.J., 1987. Simultaneous determination of production cycles and inspection schedules in a production system. *Management Science* 33 (9), 1125–1136.
- Liao, G.L., 2007. Optimal production correction and maintenance policy for imperfect process. *European Journal of Operational Research* 182 (3), 1140–1149.
- Maddah, B., Jaber, M.Y., 2008. Economic order quantity for items with imperfect quality: revisited. *International Journal of Production Economics* 112 (2), 808–815.
- Papachristos, S., Konstantaras, I., 2006. Economic ordering quantity models for items with imperfect quality. *International Journal of Production Economics* 100 (1), 148–154.
- Porteus, E.L., 1986. Optimal lot sizing, process quality improvement and setup cost reduction. *Operations research* 34 (1), 137–144.
- Rahim, M.A., Ben-Daya, M., 2001. Integrated models in production planning, inventory, quality, and maintenance. Kluwer Academic Publishers, Boston.
- Raouf, A., Jain, J.K., Sathe, P.T., 1983. A cost-minimization model for multicharacteristic component inspection. *IIE Transactions* 15 (3), 187–194.
- Rekik, Y., Sahin, E., Dallery, Y., 2007. A comprehensive analysis of the newsvendor model with unreliable supply. *OR Spectrum* 29 (2), 207–233.
- Rosenblatt, M.J., Lee, H.L., 1986. Economic production cycles with imperfect production processes. *IIE Transactions* 18 (1), 48–55.
- Salameh, M.K., Jaber, M.Y., 2000. Economic production quantity model for items with imperfect quality. *International Journal of Production Economics* 64 (1), 59–64.
- Urban, T.L., 1998. Analysis of production systems when run length influences product quality. *International Journal of Production Research* 36 (11), 3085–3094.
- Yano, C.A., Lee, H.L., 1995. Lot sizing with random yields: a review. *Operations Research* 43 (2), 311–334.
- Zunzanyika, X.K., Drury, C.G., 1975. Effects of information on industrial inspection performance. In: Drury, C.G., Fox, J.G. (Eds.), *Human Reliability in Quality Control*. Taylor & Francis Ltd., London.

Update

International Journal of Production Economics

Volume 136, Issue 1, March 2012, Page 253

DOI: <https://doi.org/10.1016/j.ijpe.2011.12.004>



Erratum

Erratum to “An economic order quantity (EOQ) for items with imperfect quality and inspection errors” [Int. J. Prod. Econ. 133 (2011) 113–118]

Lie-Fern Hsu

Baruch College, Department of Management, The City University of New York, New York, NY, USA

ARTICLE INFO

Article history:

Received 15 September 2011

Accepted 4 December 2011

Available online 21 December 2011

Letter to the Editor

In a previously published paper by Khan, M., Jaber, M.Y., and Bonney M., “An economic order quantity (EOQ) for items with imperfect quality and inspection errors”, Int. J. Prod. Econ. 133 (2011) 113–118, I found the following errors:

- 1) On page 116, to obtain Eq. (5), one should replace t_1 with y/x , not D/x .
- 2) For the numerical analysis, they assumed the following probability density functions

$$f(p) = \begin{cases} 25, & 0 \leq p \leq 0.05 \\ 0, & \text{otherwise} \end{cases} \Rightarrow E(p) = 0.02$$

$$f(m_1) = \begin{cases} 25, & 0 \leq m_1 \leq 0.05 \\ 0, & \text{otherwise} \end{cases} \Rightarrow E(m_1) = 0.02$$

$$f(m_2) = \begin{cases} 25, & 0 \leq m_2 \leq 0.05 \\ 0, & \text{otherwise} \end{cases} \Rightarrow E(m_2) = 0.02$$

Note that if the defective rate is uniformly distributed between 0 and 0.05, then its *p.d.f.* should be $1/(0.05-0) = 20$ with an expected value of 0.025. For the *p.d.f.* to be 25 and an expected value of 0.02, the defective rate should be between 0 and 0.04.