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Exploration of new solitons and phase characterization for the extended Gerdjikov–Ivanov equation

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ABSTRACT

In a dense wavelength division multiplexed (DWDM) system, extended Gerdjikov–Ivanov (GI) equation has been explored for kerr law nonlinearity using two different techniques. The employed techniques are effective for identifying exact results including their dynamic characteristics. To explore exact solutions of the governing equation unified method has been applied which is one of the best analytical techniques. These analytical solutions are in form of polynomial as well as rational function. By assigning the specific values to the parameters, dark and bright soliton has been extracted. Afterwards, for phase characterization, the equation has been studied using bifurcation. The system has been first turned into a planer dynamical system, and subsequently into a Hamiltonian system. Then the equilibrium points has been derived. The different phase portrait is plotted by simply giving parameters various values.

Introduction

Nonlinear partial differential equations NLPDEs have a prominent role in a number of engineering disciplines. Lately, many experts were able to get analytical traveling wave solutions for NLPDEs, which is important for learning about rheological features or phenomena. The greatest way to comprehend NLPDEs is to investigate their precise solutions. In particular, the precise solution offers bits of knowledge that, when combined, aid in the modeling of a more suitable and dependable model, the research of higher dimensions soliton propagations, and many other tasks [1–5]. A number of nonlinear evolution equations are used to explain the dynamics of optical solitons in various types of optical fiber [6,7]. The nonlinear Schrödinger's equation is, of perhaps, the most well-known. However, there are a number of additional models that are less well-known and noticeable. The Chen-Lee-Liu equation, the Sasa–Satsuma equation, the Manakov model, the Thirring solitons, the Gabitov–Turitsyn equation, the complex Ginzburg–Landau equation, the Lakshmanan–Porsezian–Daniel model, Boussinesq equation, Helmholtz equation, and many others are among them. The dynamics of soliton development to one such model that has recently gained prominence will be studied in this research that is extended Gerdjikov–Ivanov equation. The Gerdjikov–Ivanov (GI) equation, has been thoroughly studied in polarization-preserving fibers using specific techniques. This model has been investigated by a number of authors, where some of the most latest discoveries may be found in [8–19]. The extended Gerdjikov–Ivanov model is used to describes a variety of physical phenomena in dense wavelength division multiplexed systems, including transmission infrastructure, transatlantic, transboundary lengths, data processing, and telecommunications. The extended simple equation approach [20] and the auxiliary equation method [21] were used to analyze this model first. The employment of DWDM technology in the fiber-optic communication process is critical [22–34]. This model has also been investigated in the light of fractional calculus by many researchers [35–38]. Multiple data streams are transmitted utilizing different wavelengths of light across a single cable in DWDM. Within a specified frequency bound, incoming optical signals are assigned to certain frequencies. When these signals are packed onto one fiber, the capacity of the fiber is increased.

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Many nonlinear media have recently been extensively explored, including power law, parabolic law, dual-power law, log law, exponential, higher-order polynomial-law media, and so on [39–42]. Kerr law [43] is considered in this study. The Kerr-law nonlinearity (cubic nonlinearity) occurs when optical pulse propagates in optical fiber and encounters nonlinear responses due to nonharmonic motion of electrons bound in molecules.

The unified method [44–46] and bifurcation [47–49] are being used in the research to enhance the outcomes of the extended GI model in the DWDM system for kerr law nonlinearity. The suggested analytical technique can produce a wide range of explicit solutions, including periodic soliton as well as rational function solutions. The bifurcation of the system of the stated model is being examined. Using the theory of planner dynamical systems all feasible possibilities for parameter dependency are investigated to present phase portrait of the behavior of the governing model.

The extended GI model in the DWDM system for kerr law nonlinearity has been investigated by many scholars. The bright soliton, singular soliton, Bell-type wave, traveling, and trigonometric waves are extracted for the governing model [50]. The model has been investigated by applying $\exp(-\phi(\zeta))$-expansion method [51]. The optical solitons of the considered model has been retrieved for both kerr law and parabolic law [52]. Its conservation laws are a crucial component in understanding a system's dynamics. In order to gain a more good grasp of such a system, it is necessary to extract these laws. The conservation laws has been extracted previously [53]. To the best of our knowledge, the unified method has not been applied to the considered model for retrieving the exact solutions. The advantage of this method on other methods is that the solutions are in form of polynomial as well as rational functions.

The presented article is organized in seven sections. Section "Description of proposed technique" provides an overview of the utilized techniques. The governing equation is explored in Section "Governing model". Section "Extraction of solitons" shows how to retrieve the exact solution regarding to governing model. Section "Extraction of solitons" contains solutions for polynomial and rational functions. The graphical representation of solutions to the governing equation is discussed in Section "Results and discussion". Section "Bifurcation of governing model" discusses bifurcation using the dynamical planner theory. In Section "Conclusion", the conclusion of the article is presented.

Description of proposed technique

To derive soliton solutions, the preceding assumption implies:

$$U(x,t,u_x,u_t,u_{xt},u_{2x},u_{2xt}\ldots,u_{nxt})=0,\quad p\geq 0, \tag{1}$$

where U is the function. With the use of the traveling wave variable $\alpha = kx-wt$, Eq. (1) could be written as

$$N(u,u',u'',\ldots,u^{n})=0, \tag{2}$$

differentiation involving the new variable α is shown by u'. The following is a discussion of the unified technique, which allows for such exploration of solutions in two separate ways: rational and polynomial functions solutions.

Solution in polynomial form

Let Eq. (2) has solution as

$$Q(\alpha)=\sum_{m=0}^{r} p_m\eta^m(\alpha),\quad a_m\neq 0, \tag{3}$$

p_i are constants to be found also function $\Psi(\alpha)$ is found through utilizing the auxiliary equation as follow:

$$(\eta'(\alpha))^{\lambda}=\sum_{m=0}^{\lambda p} q_m\eta^i(\alpha),\qquad \alpha=kx-wt,\quad \lambda=1,2, \tag{4}$$

where q_m are arbitrary constants. p is find out utilizing the consistency criteria, while value of r is retrieved in terms of p through applying the balance condition among highest derivative term and highest nonlinear term in Eq. (2). The **Unified Method** solves Eq. (3) for $\lambda=1$ and $\lambda=2$.

Rational function solution

This section's key idea is to take into account that Eq. (2) has such a solution in the corresponding rational form

$$F(\alpha)=\frac{\sum_{m=0}^{n} r_m\Psi^m(\alpha)}{\sum_{m=0}^{t} s_m\Psi^m(\alpha)},\quad n\geq t, \tag{5}$$

satisfying auxiliary equation mentioned below,

$$(\Psi'(\alpha))^{\lambda}=\sum_{m=0}^{\lambda v} q_m\Psi^m(\alpha),\qquad \alpha=kx-wt,\quad \lambda=1,2. \tag{6}$$

In Eqs. (5) and (6), r_m, s_m, and q_m are the unidentified constants. The highest order of terms included in Eq. (2) must be balanced according with balancing rule in order to extract the quantitative interpretations of n as well as v. Similar to how we found the unidentified coefficient in Eq. (5), we may determine it by using the consistency criterion. The Eq. (5) will be evaluated using the UM approach. Next, efforts are needed to be made to find solutions for $\lambda=1$ or $\lambda=2$.

Governing model

The extended GI model is presented for Kerr law nonlinearity in DWDM systems by [8,9]

$$i\varphi_t^{(l)}+a_l\varphi_{xx}^{l}+b_{(l)}|\varphi^{(l)}|^4\varphi\varphi^{(l)}+ie_l(\varphi^{(l)})^2(\varphi_x^{(l)})^*+\left\{c_{(l)}\varphi_{xt}^{(l)}+d_l|\varphi^{(l)}|^2\varphi^{(l)}+\sum_{i\neq l}^{K}\alpha_{li}|\varphi^{(i)}|^2\varphi^{(l)}\right\}=0. \tag{7}$$

Group velocity dispersion and spatiotemporal dispersion are characterized by the coefficients a_l and c_l, respectively. Furthermore, the constants d_l signify self-phase modulation, whereas the constants α_{ln} represent the influence of cross-phase modulation. In each single channel where $1<l<N$, the dependent variable $\varphi^{(l)}(x,t)$ indicates soliton.

The following transformation can be used to retrieve the solution of Eq. (7):

$$\varphi^{(l)}(x,t)=\psi_l(\eta)e^{i\phi}, \tag{8}$$

here the function ψ indicates amplitude component of the soliton while ϕ denotes the phase component of the soliton which is written as

$$\eta=x-\mu t,\quad \phi=kx-\lambda t. \tag{9}$$

The soliton's velocity is indicated by μ, where k is the soliton's frequency and λ is the soliton wave number. Substituting Eqs. (8) and (9) in Eq. (7), we obtain

$$\lambda\phi_l-i\mu\phi_l'-k^2a_l\phi_l+2ia_lk\phi_l'+a_l\phi_l''+b_l\phi_l^5+kc_l\lambda\phi_l-ic_l\lambda\phi_l'-ic_lk\mu\phi'-c_l-c_l\mu\phi_l''+d_l\phi_l^3-ke_l\phi_l^3+ie_l\phi_l^2\phi_l'+\left(\sum_{i\neq l}^{K}\alpha_{li}\phi_i^2\right)\phi_l=0, \tag{10}$$

separating the real and imaginary parts, we obtain

$$(\lambda-k^2a_l+kc_l\lambda)\phi_l+b_l\phi_l^5+(a_l-c_l\mu)\phi_l''+(d_l-ke_l+\sum_{i\neq l}^{K}\alpha_{li})\phi_l^3=0, \tag{11}$$

$$(-\mu+2a_lk-c_l\lambda-kc_l\mu+e_l\phi_l^2)\phi_l'=0, \tag{12}$$

using Eq. (12), the soliton's velocity is

$$\mu = \frac{2a_l k - c_l \lambda}{1 + kc_l}, \quad \text{where} \quad kcl \neq 1, \tag{13}$$

here we get the condition $e_l \neq 0$. We obtain the balancing number $N = 1/2$ from Eq. (11). Since, N is not an integer we set $\phi_l = \sqrt{w_l}$. Utilizing into Eq. (11) and multiplying by $4w_l\sqrt{w_l}$, the obtained result is

$$4(\lambda - k^2 a_l + kc_l\lambda)w_l^2 + b_l w_l^4 + (a_l - c_l\mu)(2w_l w_l'' - (w_l')^2) + 4(d_l + \sum_{i\neq l}^{K} \alpha_{li})w_l^3 = 0. \tag{14}$$

The aforementioned equation has balancing number $N = 1$.

Extraction of solitons

This section presents soliton solutions for such propped model Eq. (7) using the suggested methods. Therefore, the unified approach has been used to analyze Eq. (7) as well as produce exact solution.

The solution has the following form

$$w(\eta) = \sum_{m=0}^{1} p_m \zeta^i(\eta), \quad r_1 \neq 0, \tag{15}$$

with the auxiliary equation

$$(\zeta'(\eta))^\varepsilon = \sum_{m=0}^{2\varepsilon} q_m \zeta^m(\eta), \quad \varepsilon = 1, 2. \tag{16}$$

Polynomial function solution

Solitary Wave Solution

In order to obtain solution, firstly we consider $\varepsilon = 1$ in the auxiliary Eq. (16), therefor the obtained outcome is as follow

$$w(\eta) = p_0 + p_1\zeta(\eta), \tag{17}$$

$$\psi'(\eta) = q_0 + c_1\zeta(\eta) + c_2\zeta^2(\eta).$$

By putting Eq. (17) in Eq. (14), non-linear equations are obtained. This system can solved by using Mathematica and Matlab. The results are extracted

$$p_0 = 0, \quad q_0 = 0, \quad q_2 = 2\sqrt{-\frac{b_l}{3a_l - 3c_l\mu}}p_1, \quad \lambda = \frac{c_l\mu q_1^2 + 4a_l k^2 - a_l q_1^2}{c_l k + 1},$$

$$\sum_{i\neq l}^{K} \alpha_{li} = 2\sqrt{-\frac{b_l}{3a_l - 3c_l\mu}}c_l\mu q_1 - 2\sqrt{-\frac{b_l}{3a_l - 3c_l\mu}}a_l q_1 - d_l. \tag{18}$$

Through auxiliary equation $\zeta'(\eta) = q_0 + q_1\zeta(\eta) + q_2\zeta^2(\eta)$ then using Eq. (18), we found the solution of Eq. (14) as:

$$w_l(\eta) = \frac{-q_1 - \tanh\left(\frac{\eta q_1}{2}\right) q_1}{4\sqrt{-\frac{b_l}{3a_l - 3c_l\mu}}}. \tag{19}$$

Then Eq. (7) has the solution

$$\varphi^l(x,t) = \sqrt{\frac{-q_1 - \tanh\left(\frac{(x-\mu t)q_1}{2}\right) q_1}{4\sqrt{-\frac{b_l}{3a_l - 3c_l\mu}}}}e^{i(kx - \lambda t)}. \tag{20}$$

Soliton Solution:

In this case we substitute $\varepsilon = 1$ in auxiliary Eq. (16), we retrieved

$$w(\eta) = p_0 + p_1\zeta(\eta), \tag{21}$$

$$\zeta(\eta) = \zeta(\eta)\sqrt{q_0 + q_1\zeta(\eta) + q_2\zeta^2(\eta)}.$$

By substituting Eq. (21) in Eq. (14), the system of nonlinear equations has been obtained. Maple or Mathematica can be utilized to further solve that system. The following outcomes are retrieved

$$p_0 = 0, \quad q_2 = -\frac{4b_l p_1^2}{3(a_l - c_l\mu)}, \quad \lambda = \frac{4a_l k^2 + c_l\mu q_0 - a_l q_0}{4(c_l k + 1)},$$

$$\sum_{i\neq l}^{K} \alpha_{li} = -\frac{a_l q_1 - c_l\mu q_1 + 2d_l p_1}{p_1}. \tag{22}$$

Utilizing the auxiliary equation $\zeta(\eta) = \zeta(\eta)\sqrt{q_0 + q_1\zeta(\eta) + q_2\zeta^2(\eta)}$ then substituting with Eq. (22), Eq. (14) has the solution in manner below

$$w_l(\eta) = \frac{12p_1 q_0 e^{\eta\sqrt{q_0}}(a_l - c_l\mu)}{16q_0 b_l p_1^2 e^{2\eta\sqrt{q_0}} - 3e^{2\eta\sqrt{q_0}}c_l\mu q_1^2 + 3e^{2\eta\sqrt{q_0}}a_l q_1^2 + 6e^{\eta\sqrt{q_0}}c_l\mu q_1 - 6e^{\eta\sqrt{q_0}}a_l q_1 - 3c_l\mu + 3a_l}. \tag{23}$$

Then Eq. (7) has the solution (see Box I) where $\eta = x - \mu t$.

Rational function solution

In order to find the solution in rational form, we assume that

$$w(\eta) = \frac{p_0 + p_1\zeta(\eta)}{m_0 + m_1\zeta(\eta)}, \tag{25}$$

$$\zeta'(\eta) = \sqrt{q_0 + q_1\zeta(\eta) + q_2\zeta^2(\eta)}. \tag{26}$$

where r_m, s_m, q_i are arbitrary constants. Utilizing Eq. (20) into Eq. (13), system of algebraic equations is obtained. Then, by some symbolic computing softwares, constants are retrieved. The results obtained are as follow:

$$q_0 = \frac{p_0(-3c_l\mu q_2 m_0 m_1^2 + 3a_l q_2 m_0 m_1^2 - 4b_l p_0 p_1 m_1 + 4b_l p_1^2 m_0)}{3(p_1 m_1^3(a_l - c_l\mu))},$$

$$q_1 = \frac{-3c_l\mu q_2 p_0 m_1^3 - 3c_l\mu q_2 p_0 m_0 m_1^2 + 3a_l q_2 p_0 m_1^3 + 3a_l q_2 p_1 m_0 m_1^2 - 4b_l p_0 p_1^2 m_1 + 4b_l p_1^3 m_0}{3(p_1 m_1^3(a_l - c_l\mu))},$$

$$\lambda = \frac{2a_l k^2 m_1^2 - c_l\mu q_2 m_1^2 + a_l q_2 q_1^2 + 2b_l p_1^2}{2m_1^2(c_l k + 1)},$$

$$\sum_{i\neq l}^{K} \alpha_{li} = -\frac{-c_l\mu q_2 m_1^2 + a_l q_2 m_1^2 + 4b_l p_1^2 + 2d_l p_1 m_1}{p_1 m_1}. \tag{27}$$

Utilizing the auxiliary equation $\zeta'(\eta) = \sqrt{q_0 + q_1\zeta(\eta) + q_2\zeta^2(\eta)}$, and substituting the values in Eq. (27). The Eq. (14) has the following solution

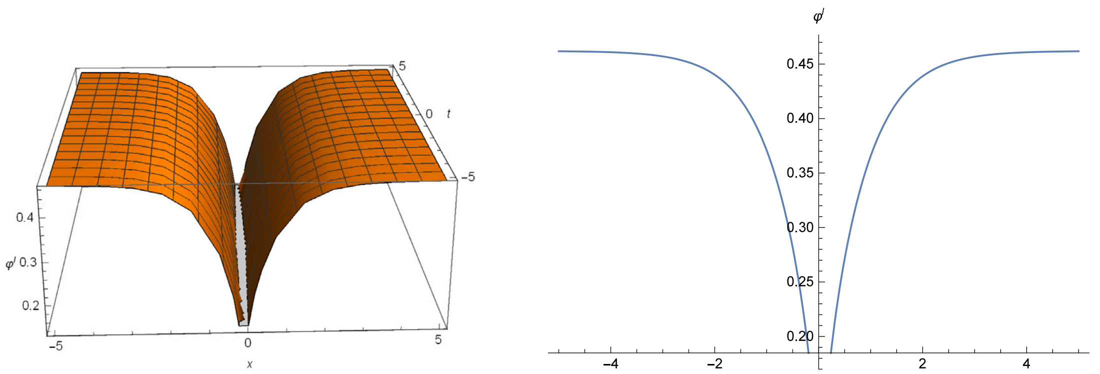
$$\begin{aligned} w(\eta) = {} & -\Big(\Big({m_1}^6 q_2 {p_1}^2 (-c_l\mu + a_l)^2 \left(\mathrm{e}^{\eta\sqrt{q_2}}\right)^2 \\ & + {m_1}^3 \left(\tfrac{4}{3}\sqrt{q_2} b_l {p_1}^2 + {m_1}^2 {q_2}^{3/2} (-c_l\mu + a_l)\right)\Big) \\ & (p_0 m_1 - p_1 m_0)(-c_l\mu + a_l) p_1 \mathrm{e}^{\eta\sqrt{q_2}} \\ & + \frac{(p_0 m_1 - p_1 m_0)^2}{4}\left(q_2(-c_l\mu + a_l){m_1}^2 + \frac{4, b{p_1}^2}{3}\right)^2\Big)p_1\Big) \Big/ \\ & \Big(-{m_1}^6 q_2 {p_1}^2 (-c_l\mu + a_l)^2 \left(\mathrm{e}^{\eta\sqrt{q_2}}\right)^2 \\ & + {m_1}^3 (p_0 m_1 - p_1 m_0)\left(-\frac{4\sqrt{m_2} b{p_1}^2}{3} + {q_1}^2 {m_2}^{3/2}(a_l - c_l\mu)\right) \\ & (a_l - c_l\mu) p_1 \mathrm{e}^{\xi\sqrt{q_2}} - \frac{(p_0 m_1 - p_1 m_0)^2}{4}\left(q_2(a_l - c_l\mu){m_1}^2 + \frac{4b{p_1}^2}{3}\right)^2\Big)m_1. \end{aligned} \tag{28}$$

Hence the solution is as follow

$$\begin{aligned} \varphi^l(x,t) = {} & \Big(-\Big(\Big({m_1}^6 q_2 {p_1}^2 (-c_l\mu + a_l)^2 \left(\mathrm{e}^{\eta\sqrt{q_2}}\right)^2 \\ & + {m_1}^3 \left(\tfrac{4}{3}\sqrt{q_2} b_l {p_1}^2 + {m_1}^2 {q_2}^{3/2} (-c_l\mu + a_l)\right)\Big) \\ & (p_0 m_1 - p_1 m_0)(-c_l\mu + a_l) p_1 \mathrm{e}^{\eta\sqrt{q_2}} \end{aligned}$$

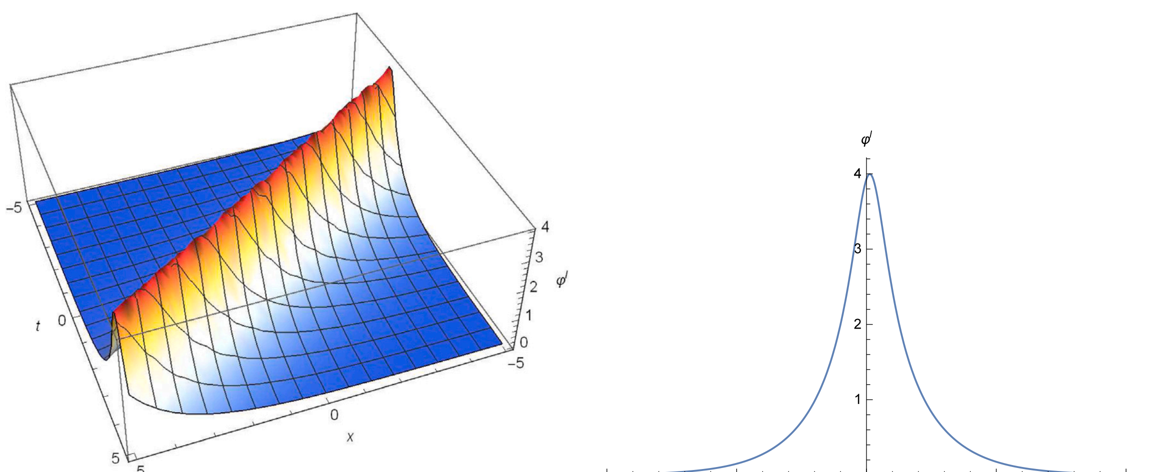
$$\varphi^l(x,t) = \sqrt{\frac{12 p_1 q_0 e^{\eta\sqrt{q_0}}(a_l - c_l\mu)}{16 q_0 b_l p_1^2 e^{2\eta\sqrt{q_0}} - 3e^{2\eta\sqrt{q_0}} c_l \mu q_1^2 + 3e^{2\eta\sqrt{q_0}} a_l q_1^2 + 6e^{\eta\sqrt{q_0}} c_l \mu q_1 - 6e^{\eta\sqrt{q_0}} a_l q_1 - 3c_l\mu + 3a_l}}\, e^{i(kx-\lambda t)}, \tag{24}$$

Box I.



(a) 3D plot of Eq.(20) (b) 2D plot of Eq.(20)

Fig. 1. Bright soliton for the parametric values such as $\mu = 0.02$, $a_l = 0.05$, $b_l = 1$, $c_l = 0.1$, $q_1 = 1.5$, $\lambda = 022$, $k = 0.45$.



(a) 3D plot of Eq.(24) (b) 2D plot of Eq.(24)

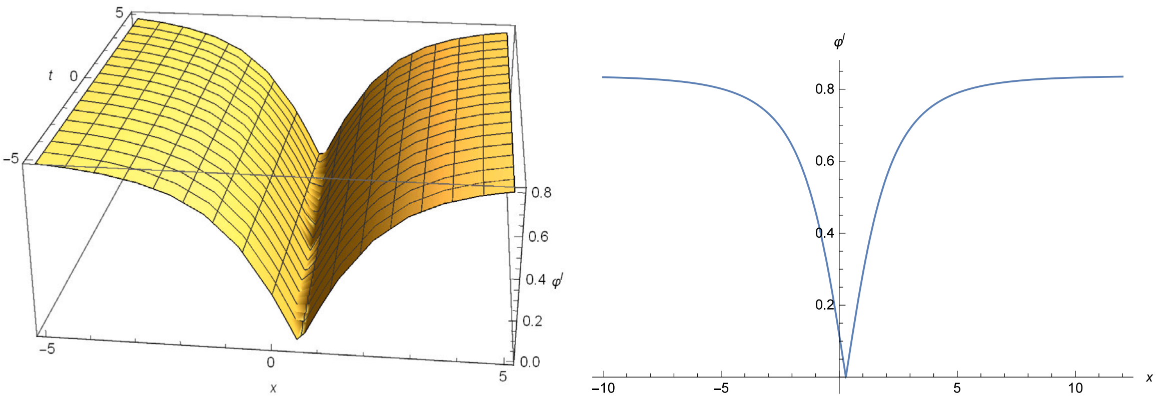
Fig. 2. Dark soliton solution taking parametric values as $\mu = 1.5$, $a_l = 0.05$, $b_l = 0.25$, $c_l = -0.5$, $q_0 = 1.5$, $q_1 = 1.5$, $\lambda = 022$, $k = 0.1$.

$$+ \frac{(p_0 m_1 - p_1 m_0)^2}{4} \left(q_2 \left(-c_l\mu + a_l\right) m_1{}^2 + \frac{4, b p_1{}^2}{3} \right)^2 \Bigg) p_1 \Bigg) \Bigg/$$
$$\Bigg(-m_1{}^6 q_2 p_1{}^2 \left(-c_l\mu + a_l\right)^2 \left(e^{\eta\sqrt{q_2}}\right)^2$$
$$+ m_1{}^3 \left(p_0 m_1 - p_1 m_0\right) \left(-\frac{4\sqrt{m_2}\, b p_1{}^2}{3} + q_1{}^2 m_2{}^{3/2} \left(a_l - c_l\mu\right) \right)$$
$$\left(a_l - c_l\mu\right) p_1 e^{\xi\sqrt{q_2}} - \frac{(p_0 m_1 - p_1 m_0)^2}{4}$$
$$\times \left(q_2 \left(a_l - c_l\mu\right) m_1{}^2 + \frac{4 b p_1{}^2}{3} \right)^2 \Bigg) m_1 \Bigg)^{1/2} e^{i(kx-\lambda t)}, \tag{29}$$

where $\eta = x - \mu t$.

Results and discussion

We show graphical representations of a selection of the calculated results in this section. It is worth noting that the unified method is used to obtain explicit and consistent wave solutions. Numerous researchers have studied the extended GI model for kerr law nonlinearity in the DWDM system. For the governing model [50], the bright soliton, solitary soliton, Bell-type wave, traveling, and trigonometric waves are extracted. The model has been examined using the $\exp(-\phi(\zeta))$-expansion approach, according to [51]. For both kerr law and parabolic law, the optical solitons of the model under consideration have been presented in [52]. To the best of our knowledge, the contemplated model for getting the precise solutions has not been subjected to the unified method. Figs. 1–3 exhibit a visual representation of the derived



(a) 3D plot of Eq.(29) (b) 2D plot of Eq.(29)

Fig. 3. Bright soliton solution for the parametric values of $\mu = -0.05$, $a_l = 0.75$, $b_l = 0.85$, $c_l = -1.5$, $q_0 = 0.75$, $q_1 = 0.03$, $\lambda = 0.52$, $k = 0.2$, $p_1 = 0.35$, $m_0 = 0.65$, $q_2 = 0.2$, $p_0 = 0.1$.

solitary solution in 3D as well as 2D based on various parameters. Bright soliton is retrieved for Eq. (20) in 3D and 2D in Fig. 1. Fig. 2 shows dark soliton that is graphical representation of a soliton solution obtained in Eq. (24), whereas Fig. 3 highlighted rational solution that gives bright soliton for specific parametric values.

In next section we will deal with the qualitative analysis.

Bifurcation of governing model

In the following, the bifurcation of a nonlinear governing equation will be examined. To accomplish this, we transformed the model into an ordinary differential equation, Eq. (14), using the traveling wave solution that was previously discussed.

Through Eq. (14), the following planer system is extracted

$$W' = P,$$

$$z' = \frac{4(\lambda - k^2 a_l = k c_l \lambda)\varphi^2 + 4 b_l \varphi^4 + (c_l \mu - a_l)P^2 + 4(d_l + e_l)\varphi^3}{(c_l \mu - a_l)\varphi}. \tag{30}$$

The system under study is not Hamiltonian, yet. Through Eq. (30), the following equation is obtained

$$\frac{dP^2}{dW} = 2\frac{4(\lambda - k^2 a_l = k c_l \lambda)\varphi^2 + 4 b_l \varphi^4 + (c_l \mu - a_l)P^2 + 4(d_l + e_l)\varphi^3}{(c_l \mu - a_l)\varphi}, \tag{31}$$

And since singular point of a Eq. (31) is $W = 0$, W can have only root under exceptional situations. The solution of Eq. (31) is

$$P^2 = c_1\varphi^2 + \frac{4}{a_l - c_l\mu}(2a_l k^2 \ln\varphi - 2c_l k\lambda \ln\varphi - b_l\varphi^2 - 2\lambda\ln\varphi - 2d_l\varphi + 2e_l\varphi)\varphi^2, \tag{32}$$

applying the expansion of $\ln\varphi$, we get

$$P^2 = -4\frac{a_l k^2 - c_l k\lambda + b_l - \lambda}{a_l - c_l\mu}\varphi^4 - 4\frac{(-4a_l k^2 + 4c_l k\lambda + 2d_l - 2e_l + 4\lambda)}{a_l - c_l\mu}\varphi^3 - 4\frac{3a_l k^2 - 3c_l k^2 - 3kc_l\lambda - 3\lambda}{a_l - c_l\mu}\varphi^2 + c_1\varphi^2, \tag{33}$$

where c_1 is integration constant. Consequently, it is feasible to retrieve the equivalent conserved quantity.

$$H(W,z) = P^2 - \left[-a_0\varphi^4 - a_1\varphi^3 - a_2\varphi^2\right], \tag{34}$$

where $a_0 = 4\frac{a_l k^2 - c_l k\lambda + b_l - \lambda}{a_l - c_l\mu}$, $a_1 = 4\frac{(-4a_l k^2 + 4c_l k\lambda + 2d_l - 2e_l + 4\lambda)}{a_l - c_l\mu}$, $a_2 = 4\frac{3a_l k^2 - 3c_l k^2 - 3kc_l\lambda - 3\lambda}{a_l - c_l\mu} + c_1$. Now, on the basis of values of parameters, we perform a qualitative analysis depending upon the governing model.

As, we have Eq. (34) involving its potential energy as

$$W = -\left[-a_0\varphi^4 - a_1\varphi^3 - a_2\varphi^2\right], \tag{35}$$

moreover

$$W' = -\left[-a_0\varphi^4 - a_1\varphi^3 - a_2\varphi^2\right]. \tag{36}$$

Critical points

It is clear that the critical points for the system (30) for the axis $P = 0$ will be roots for $W'(\varphi) = \left[a_0\varphi^4 + a_1\varphi^3 + a_2\varphi^2\right] = 0$. Thus we get

$$W'(\varphi) = a_0\varphi^4 + a_1\varphi^3 + a_2\varphi^2 = 0, \tag{37}$$

so, the system has the equilibrium points; $U_1 = (0,0)$, $U_2 = \left(\frac{-a_1 + \sqrt{a_1^2 - 4a_0a_1}}{2a_0}, 0\right)$, $U_3 = \left(\frac{-a_1 - \sqrt{a_1^2 - 4a_0a_1}}{2a_0}, 0\right)$. Consider $L(\varphi, P)$ being the matrix(coefficient) related to linearized system on (φ, P). Such matrix is known **Jacobian** for system. The Jacobian corresponding to Eq. (36) takes the form:

$$det[J(\varphi, P)] = -\left(4a_0\varphi^3 + 3a_1\varphi^2 + 2a_2\varphi\right), \tag{38}$$

where $(\varphi, 0)$ is the critical point for obtained dynamical system. Utilizing planar system theory, $(\varphi, 0)$, the equilibrium point for the dynamical system will saddle when $det[J(\varphi,0)] < 0$, a center whenever $det[J(\varphi,0)] > 0$, and it will be cusp point when $det[J(\varphi,0)] = 0$.

Bifurcation regarding to planer dynamical system depends upon the parameters a_0, a_1, a_2, in given model (36). Based on the possibilities for parameters, there are many cases some of them are:

$$\begin{aligned}
R_1 &= [(a_0,a_1,a_2)|a_0 > 0, a_1 > 0, a_2 > 0 \ and \ a_1^2 - 4a_0a_2 = 0],\\
R_2 &= [(a_0,a_1,a_2)|a_0 > 0, a_1 < 0, a_2 > 0 \ and \ a_1^2 - 4a_0a_2 > 0],\\
R_3 &= [(a_0,a_1,a_2)|a_0 > 0, a_1 < 0, a_2 > 0 \ and \ a_1^2 - 4a_0a_2 = 0],\\
R_4 &= [(a_0,a_1,a_2)|a_0 < 0, a_1 = 0, a_2 > 0],\\
R_5 &= [(a_0,a_1,a_2)|a_0 > 0, a_1 = 0, a_2 < 0],\\
R_6 &= [(a_0,a_1,a_2)|a_0 < 0, a_1 < 0, a_2 > 0 \ and \ a_1^2 - 4a_0a_2 > 0],\\
R_7 &= [(a_0,a_1,a_2)|a_0 < 0, a_1 < 0, a_2 < 0 \ and \ a_1^2 - 4a_0a_2 > 0].
\end{aligned} \tag{39}$$

It is possible to create a phase portrait for the system at plane (φ, P) while taking into account the aforementioned parameter limitations also using Matlab for computations. For the cases, where $a_1^2 - 4a_0a_2 < 0$, there is only one equilibrium point $U(0,0)$.

⋆ Case I:

When $(a_0, a_1, a_2) \in R_2$, then the system has two equilibrium points. Both of that equilibrium points are singularity as the determinant of

Jocobi matrix becomes zero. The phase portrait has been plotted for $a_0 = 1$, $a_1 = 2$, and $a_2 = 1$.

⋆ **Case II:**

When $(a_0, a_1, a_2) \in R_1$, then the system has three equilibrium points. Where $U_1 = (0,0)$ is cusp as determinant of Jacobi matrix is zero. Moreover $U_2 = \left(\frac{-a_1+\sqrt{a_1^2-4a_0a_1}}{2a_0}, 0\right)$ is saddle as det(J)<0 whereas $U_3 = \left(\frac{-a_1+\sqrt{a_1^2-4a_0a_1}}{2a_0}, 0\right)$ is center because of positive determinant. The phase portrait is the same as case I.

⋆ **Case III:**

When $(a_0, a_1, a_2) \in R_3$, then the system has two equilibrium points. Both of that equilibrium points are cusp as the determinant of Jocobi matrix becomes zero. The phase portrait is as same as case 1.

⋆ **Case IV:**

When $(a_0, a_1, a_2) \in R_4$, then the system has three equilibrium points. Where $U_1 = (0,0)$ is cusp as determinant of the Jacobian is zero. Furthermore, $U_2 = \left(\frac{-a_1+\sqrt{a_1^2-4a_0a_1}}{2a_0}, 0\right)$ is saddle as det(J)<0 whereas $U_3 = \left(\frac{-a_1+\sqrt{a_1^2-4a_0a_1}}{2a_0}, 0\right)$ is center because of positive determinant. The phase portrait is plotted for $a_0 = 1$, $a_1 = 3$, and $a_2 = 1$.

⋆ **Case V:**

When $(a_0, a_1, a_2) \in R_5$, then the system has two equilibrium points. Where $U_1 = (0,0)$ is cusp because determinant of Jacobian is zero. Also $U_2 = \left(\frac{-a_1+\sqrt{a_1^2-4a_0a_1}}{2a_0}, 0\right)$ is saddle because det(J)<0 while $U_3 = \left(\frac{-a_1-\sqrt{a_1^2-4a_0a_1}}{2a_0}, 0\right)$ is center because of positive determinant. The phase portrait is plotted taking values $a_0 = 1$, $a_1 = 0$, and $a_2 = -1$.

⋆ **Case VI:**

When $(a_0, a_1, a_2) \in R_6$, then the system has two equilibrium points. One of them is cusp and second is center. The phase portrait is plotted for $a_0 = 1$, $a_1 = 2$, and $a_2 = 0$. ⋆ **Case VII:**

When $(a_0, a_1, a_2) \in R_7$, the system has two equilibrium points. Where $U_1 = (0,0)$ is cusp because determinant of Jacobi matrix is zero. Moreover $U_2 = \left(\frac{-a_1+\sqrt{a_1^2-4a_0a_1}}{2a_0}, 0\right)$ is saddle as det(J)<0 while $U_3 = \left(\frac{-a_1-\sqrt{a_1^2-4a_0a_1}}{2a_0}, 0\right)$ is center because of positive determinant. The phase portrait is plotted taking values $a_0 = -1$, $a_1 = -2$, and $a_2 = -1$ (see Figs. 4–9).

Conclusion

In this manuscript, the extended Gerdjikov–Ivanov (GI) equation, in a dense wavelength division multiplexed (DWDM) system is studied. Two methods are applied to generate new results. NLPDEs can be best understood by exploring their exact solutions. To extract the analytical solutions for the discussed model, an effective technique, the unified method is applied. The obtained results are in polynomial as well as rational form. In particular, the exact solution provides pieces of information, which in results helps in modeling a more suitable and reliable model, and to study of higher dimensional soliton propagations and many more. By applying this technique, the obtained solutions are bright and dark solitons. Furthermore, a limiting condition that is graphically shown in 3D has ensured the validity of non-singular solutions. The impact of parameters on the anticipated non-singular results is further illustrated using 2D visual representation. The second technique is used to investigate the model for qualitative analysis. The technique, used for phase characterization of governing model, is the

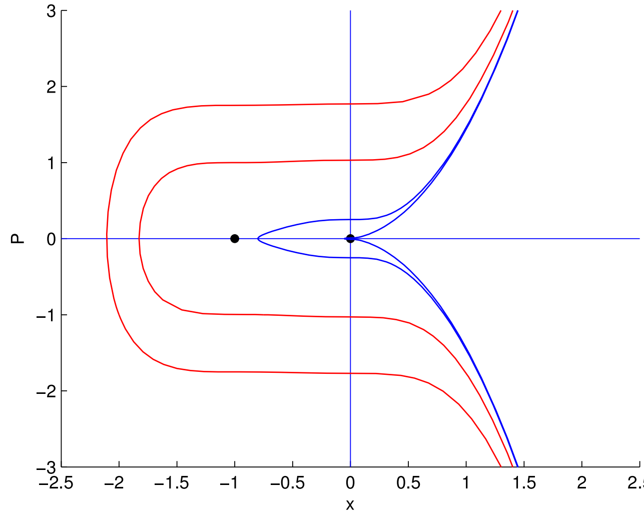


Fig. 4. Global phase portrait for **CaseI**.

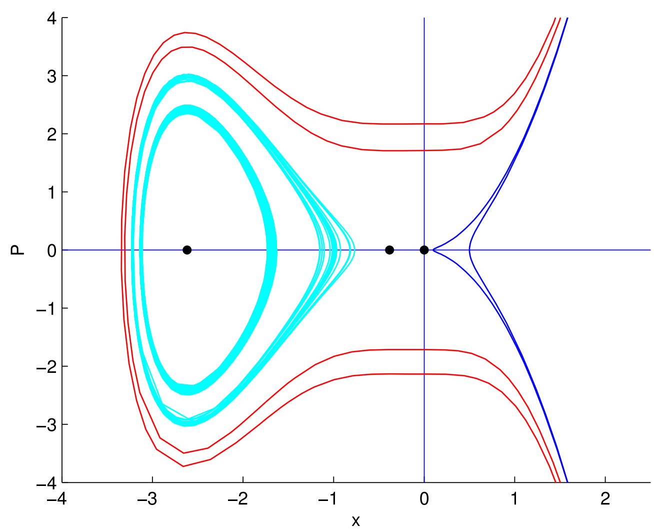


Fig. 5. Global phase portrait for **CaseII**.

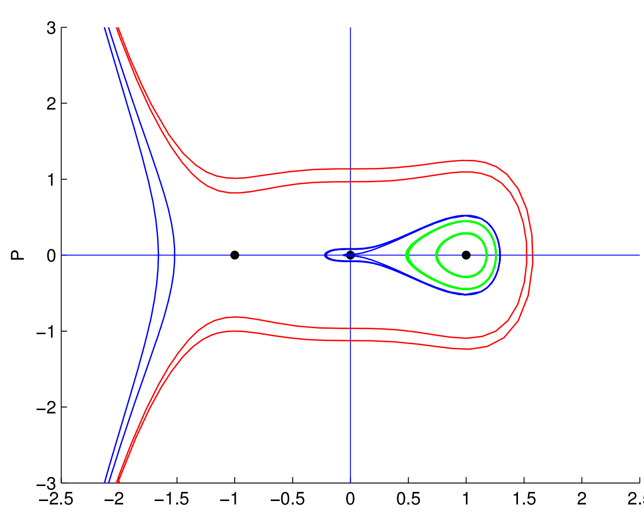


Fig. 6. Global phase portrait for **CaseIV**.

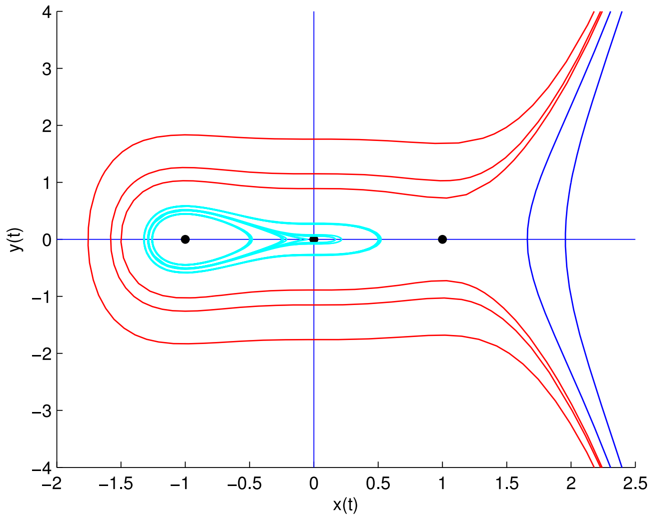


Fig. 7. Global phase portrait for **CaseV**.

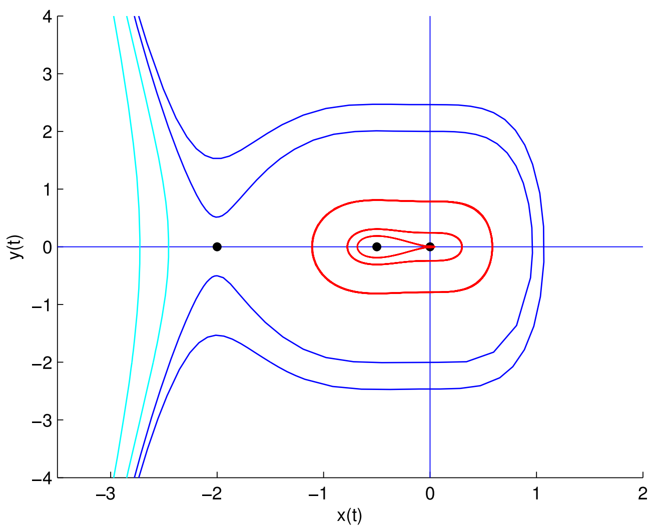


Fig. 8. Global phase portrait for **CaseVI**.

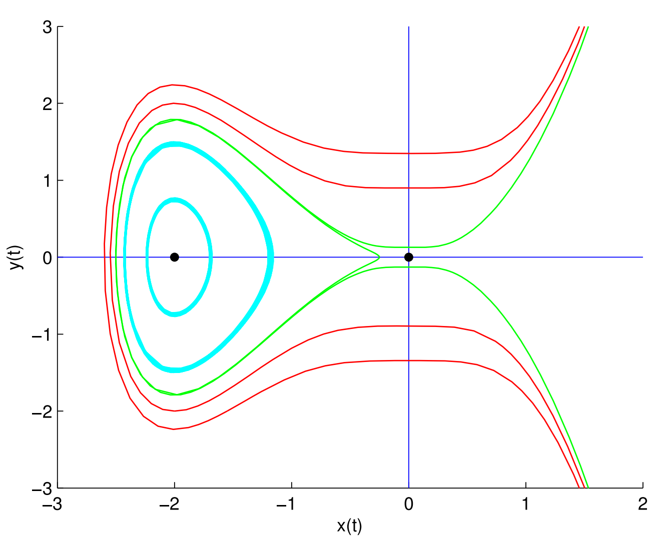


Fig. 9. Global phase portrait for **CaseVII**.

bifurcation. Firstly, the model is turned into the planer system which is a Hamiltonian system. The situations are then anticipated as well as successfully displayed in phase portraits through using equilibrium points. The obtained results are new and never recorded before. These results could have potential applications in a dense wavelength division multiplexed system.

CRediT authorship contribution statement

Tahani A. Alrebdi: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Validation, Writing – original draft, Writing – review & editing. **Nauman Raza:** Conceptualization, Data curation, Investigation, Methodology, Software, Writing – original draft, Writing – review & editing. **Farwa Salman:** Conceptualization, Data curation, Investigation, Methodology, Software, Writing – original draft, Writing – review & editing. **Badriah Alshahrani:** Conceptualization, Formal analysis, Investigation, Validation, Writing – original draft, Writing – review & editing. **Abdel-Haleem Abdel-Aty:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Validation, Writing – original draft, Writing – review & editing. **Hichem Eleuch:** Conceptualization, Formal analysis, Investigation, Validation, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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