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# Economic order quantity model for items with imperfect quality with learning in inspection

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The economic order quantity (EOQ) model is the simplest and earliest inventory model in the literature. Its simple mathematics is attributed to its assumptions, which are rarely met. Salameh and Jaber [2000. Economic production quantity model for items with imperfect quality. Int. J. Prod. Econ. 64, 59–64] addressed one of these assumptions that items received or produced are not of perfect quality. This paper extends Salameh and Jaber's work for the case where there is learning in inspection. The model is realistic in that it considers situations of lost sales and backorders. Mathematical models are developed with numerical examples provided and results discussed for the cases of (i) partial transfer of learning, (ii) total transfer of learning, and (iii) no transfer of learning.



## 1. Introduction

The EOQ is the earliest, simplest and most appreciated inventory model in the literature (Osteryoung et al., 1986). Its popularity amongst academicians and businesspeople has been attributed to the ease of manipulation and calculation (Woolsey, 1990). However, some of its assumptions are never met in practice (e.g., Jaber et al., 2004). One of these assumptions is that items ordered (or produced) are of perfect quality (Cheng, 1991).

There are several studies in the literature that investigated the affect of imperfect quality on the inventory policy. Porteus (1986) and Rosenblatt and Lee (1986) are two of the earliest of these studies. These studies have been the cornerstone for many models, which assumed that defective items are reworked. The defective items cause a manufacturer a rework cost. They also result in the shortage of some of the items. These shortages can be treated as lost sales or backorders by a vendor. Montgomery et al. (1973) studied a periodic review inventory model where lost sales and backorders are caused by the stock-out of inventory in a production system. Nahmias and Smith (1994) discussed a similar model with partial lost sales. They assumed instantaneous deliveries from the warehouse to the retailers. Ouyang et al. (1996) extended a model in the literature for the case where shortages are allowed. They studied a continuous review inventory model where both lead time and order quantity are taken as decision variables. Chiu (2003) studied a manufacturing setting that produces defective items that are either scrap or repairable. He allowed for backorders and formulated expressions for the optimal production and backorder sizes. Liu et al. (2007) discussed a lot sizing problem with limited inventory capacity, which is smaller than production capacity. They did not allow for backlogging while the unmet demand was taken as lost sale. Hill et al. (2007) considered lost sales in a single item two-echelon supply chain where the retailers demand follows Poisson distribution while the warehouse lead time is fixed.

Along the line of research of Porteus (1986) and Rosenblatt and Lee (1986) but with a different treatment of imperfect quality items, Salameh and Jaber (2000) assumed that these items are withdrawn and sold as a single batch by the end of the screening process. The work of Salameh and Jaber (2000) has been receiving increasing attention by researchers in the field. This interest is reflected in the number of citations it has received (e.g., *Scopus* and *Scholar Google*). Following is a brief survey.

Cárdenas-Barrón (2000) observed and corrected a minor error that does not affect the main idea and the remainder of the article. Goyal and Cárdenas-Barrón (2002) presented a simpler approach for determining the economic production quantity for an item with imperfect quality. Rezaei (2005) brought in the concept of shortages and compared the expected profit with that of an EOQ model. Papachristos and Konstantaras (2006) clarified a point related to the condition for preventing stock-outs and extended the model to the case in which withdrawing takes place at the end of the planning horizon. Eroglu and Ozdemir (2007) and Wee et al. (2007) independently extended the work of Salameh and Jaber (2000) to account for backorders. When dealing with imperfect quality items, Konstantaras et al. (2007) examined two options: either to sell them as a single batch at a discounted price or rework them at a cost. Maddah and Jaber (2008) rectified a flaw in Salameh and Jaber's work related to the method of evaluating the expected profit per unit time. Jaber et al. (2008) assumed the

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percentage defective per lot reduces according to a learning curve, which was empirically validated by the data from automotive industry. Recently, Maddah et al. (2009) considered a production/inventory system where items produced/purchased are of two different qualities. Hsu and Yu (2009) investigated the work of Salameh and Jaber (2000) under a one-time-only discount. These works are either direct extensions or modifications to the work of Salameh and Jaber (2000). Others have extended the work of Salameh and Jaber (2000) either by investigating it in a supply chain context (e.g., Huang, 2004; Chung and Huang, 2006, Chung et al., 2009) or by applying fuzzy set theory (e.g., Chang, 2004; Wang et al., 2007; Björk, 2009).

One fertile area in this line of research is that the production facilities do overcome their shortages through a human phenomenon known as learning. That is, the workers tend to produce faster as they spend more time on the same machine in a line. This natural phenomenon was shown by Wright's (1936) learning curve. With an interruption in the production, the workers tend to forget part of their skills. The earliest work in the literature that investigated the lot sizing problem with learning and forgetting is that of Keachie and Fontana (1966). Since there has been some interest in this subject, Jaber and Bonney (1999) provided almost a comprehensive survey (for the period 1966–1998). Some of the works in this line of research are, but not limited to, Balkhi (2003), Chiu et al. (2003), Chiu and Chen (2005), Jaber and Guiffrida (2007), Alamri and Balkhi (2007), and Jaber and Bonney (2003, 2007, 2009). Although there is consensus among these works on the form of the learning curve, it was not so for forgetting (Jaber, 2006). However, Jaber and Bonney (1996) developed a learn-forget curve model (LFCM) that properly represents the learning-forgetting process (Jaber and Bonney, 1997; Jaber et al., 2003; Jaber and Sikström, 2004).

There are situations where the time to inspect defective items follows a learning curve (e.g., Sikström and Jaber, 2002). None of the above surveyed articles and those available in the literature investigated the model of Salameh and Jaber (2000) for learning in inspection. This paper addresses this research gap and extends Salameh and Jaber's model in two directions. First, this paper considers that the screening rate of defectives follows a learning curve where stock-out may occur when the screening rate is slower than the demand rate. These stock-outs can be treated as either lost-sales or backorders. Second, this paper includes the transfer of knowledge in learning when the production moves from one cycle to another in three possible scenarios: (i) no transfer of learning, (ii) total transfer of learning, and (iii) partial transfer of learning.

The remainder of this paper is organized as follows. Section 2, provides a brief introduction to the model of Salameh and Jaber (2000). Section 3 provides a background to the learning and forgetting theory. Section 4 is for mathematical modelling. Section 5 provides numerical examples and discusses results. Finally, this paper concludes in Section 6.

## 2. The model of Salameh and Jaber (2000)

Salameh and Jaber (2000) extended the traditional EOQ model by accounting for imperfect quality items. They assumed a 100% screening and poor-quality items are withdrawn from inventory by the end of the screening period, $\tau$, and are sold as a single batch at a discounted price. The behavior of inventory is as described in Fig. 1, where $y$ is the lot size quantity, $p$ is the fraction of defectives and $f(p)$ is its probability density function, while $T$ is the cycle time.

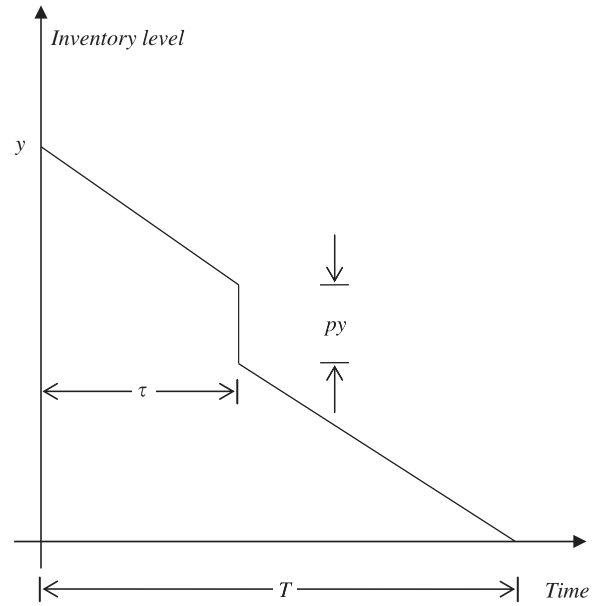


**Fig. 1.** Behavior of the inventory with time for the model of Salameh and Jaber (2000).

The expected per unit time profit function was given as

$$E[TPU](y) = D(s-v+hy/x)+D(v-hy/x-c-d-K/y)E\left[\frac{1}{1-p}\right] - \frac{hy(1-E[\mathrm{p}])}{2} \tag{1}$$

where $p \sim [a, \ b]$, $E[p]=\int_a^b pf(p)\,dp$, $E[1/(1-p)=\int_a^b(1/(1-p))f(p)\,dp$, $D$ is the demand rate, $s$ is the unit selling price of items of good quality, $c$ is the unit purchase cost, $v$ is the unit selling price of defective items ($v<c$), $x$ is the screening rate ($x>D$), $d$ is the unit screening cost, $K$ is the order cost and $h$ is the unit holding cost. To guarantee there are no shortages, Salameh and Jaber (2000) set the condition $p \le D/x$. The optimal *order* quantity, $y^*$, that maximizes (1) was given as (Cárdenas-Barrón, 2000)

$$y^* = \sqrt{\frac{2KDE[1/(1-p)]}{h[1-E[p]-2D(1-E[1/(1-p)])/x]}} \tag{2}$$

## 3. Learning and forgetting

Learning is a natural phenomenon that occurs as a worker performs a task repetitively. The data from a learning curve fits well to the power form of learning suggested by Wright (1936). That is

$$\pi_n = \pi_1 n^{-b} \tag{3}$$

where $\pi_1$ is the time to perform the first repetition $b$ is the learning exponent $0<b<1$, and $n$ is the cumulative number of repetitions. For review of learning curves, reader may refer to Jaber (2006).

Forgetting curve (Jaber and Bonney, 1996) is usually taken to be a mirror image of the learning curve. To determine the forgetting exponent $f_i$ in cycle $i$, it is customary to equate the learning and forgetting time at the instant a worker has inspected $y_i$ units. This determines the value of the intercept of the forgetting curve $\overline{\pi_1}$. The forgetting curve takes the form

$$\overline{\pi}_m = \overline{\pi}_1 m^f \tag{4}$$

where $\overline{\pi}_1 = \pi_1 m^{-(f+b)}$ and $\overline{\pi}_m = \pi_m$, where $m$ is the equivalent number of items that could have been produced. The forgetting exponent in cycle $i$ is determined by Jaber and Bonney (1996) as

$$f_i = \frac{b(1-b)\log(u_i+y_i)}{\log(1+V/\lambda_i)} \tag{5}$$

where $V$ is the time for total forgetting to occur and is assumed to be an input parameter, $u_i$ is the experience remembered in cycle $i$ and $\lambda_i$ is the time to inspect $(u_i+y_i)$ items without interruption. So

$$\lambda_i = \frac{(u_i+y_i)^{1-b}}{x_1(1+b)}$$

$$u_{i+1} = (u_i+y_i)^{(f_i+b)/b} R_i^{-f_i/b} \tag{6}$$

with

$$R_i = [x_1(1-b)(T-\tau_i)+(u_i+y_i)^{(1-b)}]^{1/(1-b)} \tag{7}$$

where $R_i$ is the equivalent number of items that could have been screened if no interruption occurs of length $T-\tau_i$, and $\tau_i$ is the screening time in cycle $i$. In case of breaks in screening in this paper, one should note that $0 < u_i < \sum_{j=1}^{i-1} y_j$ when $T-\tau_i < V$, for partial forgetting; $u_i=0$ when $T-\tau_i \geq V$, for total forgetting; and $u_i = \sum_{j=1}^{i-1} y_j$ when $V$ becomes infinite, for total transfer of learning. Eq. (6) will be used to determine the equivalent experience remembered at the start of each cycle, in case of partial and total transfer of learning, in this paper.

## 4. Mathematical model

Consider a buyer, whose inventory profile is given in Fig. 2, as in Salameh and Jaber (2000) where the inspection time in a cycle is $\tau_i$. Assume that workers screen these items at the rate $x$, which goes up with the passage of time, following learning. Unlike their work, it is assumed that the screening rate is less than the demand $D$, in the beginning, and catches up with time by virtue of learning. The demand is not fulfilled during time $t_{si}$ and there is a shortage of $y_{si}$ items in cycle $i$. These shortages will be dealt with as both lost sales and backorders in this paper. It is important to note that the time $t_{si}$ may become zero in the subsequent cycles due to learning. Three scenarios will be considered for the transfer of learning from cycle to cycle: (i) partial forgetting (partial transfer of learning), (ii) total learning (total transfer of learning) and (iii) total forgetting (no transfer of learning). The optimum cycle time $T_i$ and the order size $y_i$ will be obtained through maximizing the expected annual profit. The following notations will be used throughout the model.

- $D$ demand rate
- $x_1$ initial inspection rate ($x_1 < D$)
- $b$ learning exponent
- $f_i$ forgetting exponent in the $i$th cycle
- $V$ time for total forgetting to occur
- $y_i$ batch size in the $i$th cycle
- $u_i$ experience of screening remembered from the previous $i-1$ cycles
- $Z_i$ lost sales quantity in the $i$th cycle
- $B_i$ backorder quantity in the $i$th cycle
- $p$ percentage of defective items
- $t_{si}$ time when the screening rate is equal to the demand in the $i$th cycle
- $t_{Bi}$ time to inspect $B_i$ and $Dt_{Bi}$ units in the $i$th cycle
- $\tau_i$ time to inspect $y_i$ items
- $T_i$ cycle length in the $i$th cycle
- $c$ cost per unit
- $s$ selling price per unit of items of good quality
- $v$ selling price per unit of defective items
- $d$ screening cost per year
- $x_{si}$ inspection rate by time $t_{si}$ in the $i$th cycle
- $y_{si}$ units inspected and consumed by $t_{si}$, where $x_{si} = x_1(y_{si}+u_i)^b$
- $y_{Bi}$ units inspected and consumed by $t_{Bi}$ in the $i$th cycle
- $I(t)$ inventory level at time $t$
- $c_L$ cost of lost sales per unit
- $c_B$ cost of backorders per unit per unit time

Inspection is usually a manual task where an inspector tests incoming units for specific quality characteristics to determine if the units conform to the quality requirements. Time to inspect (or screen) each unit reduces as the number of inspected units increase as given in Eq. (3), and is represented as $x_n = x_1 n^b$, where $x_1 = 1/\pi_1$ and $x_n = 1/\pi_n$; whereas Salameh and Jaber (2000) assumed $x_1 = x_n > D$. Here, it is assumed that $x_1 < D$ for $y_S < y$. It is also assumed that items are subject to 100% screening and defective units will never make it to customers, meaning that if $x_n < D$, then the demand met is $x_n$ and the rate at which units lost or backordered is $D - x_n$, and $D$ otherwise. The length of the stock-out period or the period over which backorders are accumulated is given as

$$t_{si} = \int_{u_i}^{y_i+u_i} \frac{1}{x_1} n^{-b}\, dn - \int_{y_{si}+u_i}^{y_i+u_i} \frac{1}{x_1} n^{-b}\, dn = \frac{(y_{si}+u_i)^{1-b} - u_i^{1-b}}{(1-b)x_1} \tag{8}$$

where $y_{si} = [(1-b)x_1 t_s + u_i^{1-b}]^{1/1-b} - u_i$ and $y_{si} = (D/x_1)^{1/b} - u_i$. Now the time to screen $y_i$ items in a cycle

$$\tau_i = \frac{\{(y_i+u_i)^{1-b} - u_i^{1-b}\}}{(1-b)x_1} \tag{9}$$

where $u_i$ is computed from Eq. (6). In case of no transfer of learning, that is, a worker does not retain any knowledge from earlier cycles ($u_i = 0$), it will be taken as

$$y_s = y_{si} = (D/x_1)^{1/b} \tag{10}$$

$$t_s = t_{si} = \frac{D^{1-b/b}}{(1-b)x_1^{1/b}} \tag{11}$$

$$\tau = \tau_i = \frac{y^{1-b}}{(1-b)x_1} \tag{12}$$

Similarly, in the case of total transfer of learning, Eqs. (8) and (9) are used with $u_i = \sum_{n=1}^{i} y_n$ rather than Eq. (6).

Two cases (lost sales and backorders) will be considered now to deal with the shortages, in each of the three scenarios for the transfer of learning from one cycle to another. These models are a direct extension to the work of Salameh and Jaber (2000).

### 4.1. Lost sales

In this case, the demand that cannot be fulfilled due to slow screening will be taken as lost sale. By virtue of learning, the screening will become equal to or more than the demand and there will not be any lost sales after some cycles. Three scenarios of learning will be considered now to develop a mathematical model for the expected annual profit of a buyer.

#### 4.1.1. Lost sales for partial transfer of learning

The behavior of inventory in the lost sales case is shown in Fig. 2, which is duplicated in subsequent cycles. In the beginning, the screening rate is slower than the demand rate. Thus, the demand is lost till the time $t_{si}$. At this point, the screening rate and the demand rate become equal. In the rest of the cycle, inventory behaves as it does in Salameh and Jaber (2000). In case of partial transfer of learning (i.e. partial forgetting), a worker loses part of

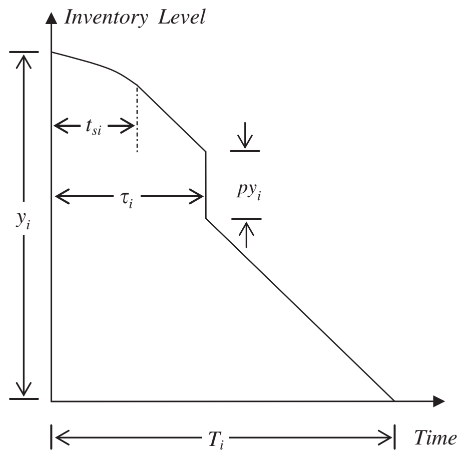


**Fig. 2.** Learning in inspection with lost sales.

his/her knowledge while he/she is not screening during the break which is $T_i$–$\tau_i$. It should be noted that the inspection time keeps on decreasing due to transfer of learning. This affects the overall cycle length and the annual profit.

This model differs from that of Salameh and Jaber (2000) in two ways: the calculation of the holding cost per cycle and the stock-out cost. The inventory level in a cycle, at time $t$ in Fig. 2 is represented as

$$I_l(t) = \begin{cases} y_i - [(1-b)x_1 t]^{1/(1-b)}, & 0 \le t < t_{si} \\ y_i - y_{si} - D(t - t_{si}), & t_{si} \le t < \tau_i \\ (1-p)y_i - y_{si} - D(t - t_{si}), & \tau_i < t \le T_i \end{cases} \tag{13}$$

At time $t$=$T_i$, the inventory level is zero, i.e. $(1-p)y_i - y_{si} - D(T_i - t_{si}) = 0$, and the cycle time is given as

$$T_i = \frac{(1-p)y_i}{D} - \frac{y_{si}}{D} + t_{si} \tag{14}$$

The holding costs for the three different behaviors of inventory shown in Fig. 2 are determined, respectively, from (13) as

$$HC_{1l} = h\int_0^{t_{si}} [y_i - [(1-b)x_1 t]^{1/(1-b)}]\,dt$$
$$= hy_i t_{si} - h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{2-b/1-b} \tag{15}$$

$$HC_{2l} = h\int_{t_s}^{\tau_i} [y_i - y_{si} - D(t - t_{si})]\,dt$$
$$= h(y_i - y_{si} + Dt_{si})(\tau_i - t_{si}) - \frac{hD}{2}(\tau_i^2 - t_{si}^2) \tag{16}$$

$$HC_{3l} = h\int_{\tau_i}^{T_i} [(1-p)y_i - y_{si} - D(t - t_{si})]\,dt$$
$$= h(y_i - y_{si} + Dt_{si})(T_i - \tau_i) - hpy_i(T_i - \tau_i) - \frac{hD}{2}(T_i^2 - \tau_i^2) \tag{17}$$

Adding the costs for three different behaviors, we can get the total holding cost for the lost sales case as

$$HC_L = h(1-p)y_i T_i + hpy_i\tau_i - hT_i(y_{si} - Dt_{si}) + hy_{si}t_{si}$$
$$-\frac{hD}{2}(T_i^2 + t_{si}^2) - h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{2-b/1-b} \tag{18}$$

Substituting $\tau_i$ and $T_i$ in terms of $y_i$ from Eqs. (9) and (14) respectively, and replacing $Z_i$ for ($D_{ts}$–$y_{si}$), the above expression can be simplified as

$$HC_L = \frac{h}{2D}[y_i^2(1-p)^2 + 2y_i Z_i(1-p) + Z_i^2]$$
$$-\frac{hD}{2}t_{si}^2 + hy_{si}t_{si} + \frac{hpy_i\{(y_i+u_i)^{1-b} - u_i^{1-b}\}}{x_1(1-b)}$$
$$-h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{\frac{2-b}{1-b}} \tag{19}$$

It should be noted that the above holding cost reduces to the one in Salameh and Jaber (2000) once $b$, $t_{si}$, $y_{si}$ and $u_i$ become zero. Now,

$$\text{Cost of the lost sales} = c_L(Dt_{si} - y_{si}) = c_L Z_i$$

$$\text{Cost of inspection} = d\tau_i = \frac{d\{(y_i+u_i)^{1-b} - u_i^{1-b}\}}{(1-b)x_1}$$

The total profit per cycle for the lost sales case with partial transfer of learning, is

$$TP_L = [s(1-p) + vp - c]y_i - K - c_L Z_i$$
$$-\frac{h}{2D}[y_i^2(1-p)^2 + 2y_i Z_i(1-p) + Z_i^2]$$
$$+\frac{hD}{2}t_{si}^2 - \frac{(hpy_i + d)\{(y_i+u_i)^{1-b} - u_i^{1-b}\}}{x_1(1-b)}$$
$$-hy_{si}t_{si} + h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{2-b/1-b}$$

The expected total profit per cycle is

$$E[TP_{iL}] = \{s(1-E[p]) + vE[p] - c\}y_i - K - c_L Z_i$$
$$-\frac{h}{2D}\left\{y_i^2 E[(1-p)^2] + 2y_i Z_i(1-E[p]) + Z_i^2\right\} + \frac{hD}{2}t_{si}^2$$
$$-\frac{(hy_i E[p] + d)\{(y_i+u_i)^{1-b} - u_i^{1-b}\}}{x_1(1-b)} - hy_{si}t_{si}$$
$$+h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{2-b/1-b} \tag{20}$$

The expected cycle time $E[T_{iL}]$ can be written using (14) as

$$E[T_{iL}] = \frac{\{1-E[p]\}y_i}{D} - \frac{y_{si}}{D} + t_{si}$$

Salameh and Jaber (2000) determined the expected annual profit from the expected value of the annual profit of the buyer. Maddah and Jaber (2008) corrected this flaw and suggested the use of renewal reward theorem. That is, the expected annual profit should be a ratio of the expected profit per cycle and the expected cycle time. So

$$E[TPU_{iL}] = \frac{E[TP_{iL}]}{E[T_{iL}]} \tag{21}$$

*4.1.2. Lost sales for total transfer of learning*

In case of total transfer of learning, the screening will start in each cycle with a cumulative experience gained from earlier cycles. The total screening time will tend to decrease from one cycle to another. This will bring a change in the subsequent cycle lengths and the costs associated with each cycle. The experience gained in each cycle $i$ will be taken as

$$u_i = \sum_{j=1}^{i-1} y_j \tag{22}$$

This implies that the worker does not lose any knowledge in his break while he is not screening. The holding cost and the expected profit in Eqs. (19) and (20), respectively, will be determined using Eq. (22). This will change the annual profit in (21) accordingly.

### 4.1.3. Lost sales for total forgetting

In case of total forgetting, screening in each cycle starts with no prior knowledge, which means that the worker loses all the experience gained in the earlier cycles. This implies that $u_i$ in cycle $i$ will be taken as zero. Therefore, the order quantity and the screening time in every cycle remains the same This will change the inspection time in (9) and the holding cost in (19), respectively, to

$$\tau = \frac{y^{1-b}}{(1-b)x_1} \tag{23}$$

$$HC_L = \frac{h}{2D}[y^2(1-p)^2 + 2yZ(1-p) + Z^2] - \frac{hD}{2}t_s^2 + \frac{hpy^{2-b}}{x_1(1-b)} + hy_s t_s - h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_s^{2-b/1-b} \tag{24}$$

The expected profit per cycle and the expected annual profit in Eqs. (20) and (21), respectively, will be determined using Eqs. (23) and (24).

### 4.2. Backorders

The behavior of inventory for the backorders case is shown in Fig. 3 where $B_i$ represents the size of backorder in cycle $i$. The screening rate becomes equal to the demand rate at $t_{si}$. The dotted line shows that the backorder that piles up till $t_{si}$ is fulfilled at the time ($t_{si}$+$t_{Bi}$). Inventory in the rest of the cycle, behaves as it does in Salameh and Jaber (2000). This behavior repeats itself in the subsequent cycles. The maximum backorder level in Fig. 3 is

$$B_i = Dt_{si} - y_{si}$$

Now, following its definition in the notations, $y_{Bi}$ can be written as

$$y_{Bi} = Dt_{Bi} + B_i + y_{si} = D(t_{Bi} + t_{si})$$

The time $t_{Bi}$ can be written as

$$t_{Bi} = \int_0^{y_{Bi}} \frac{1}{x_1} n^{-b}\, dn - \int_0^{y_{si}} \frac{1}{x_1} n^{-b}\, dn = \frac{y_{Bi}^{1-b}}{x_1(1-b)} - \frac{y_{si}^{1-b}}{x_1(1-b)}$$

Substituting $y_{Bi}$

$$t_{Bi} = \frac{[D(t_{Bi}+t_{si})]^{1-b}}{x_1(1-b)} - t_{si} \quad \text{or} \quad t_{Bi} + t_{si} = \frac{D^{1-b/b}}{x_1^{1/b}(1-b)^{1/b}}$$

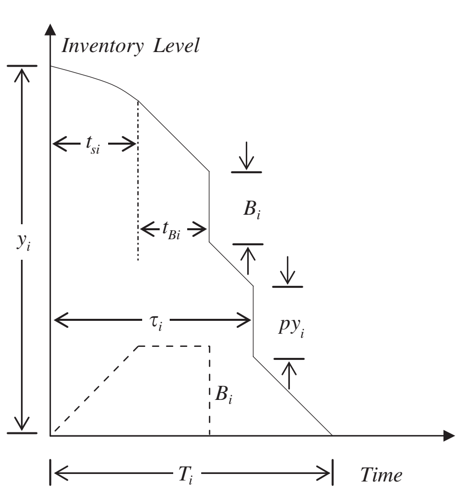


**Fig. 3.** Learning in inspection with backorders.

This time can be taken as

$$t_x = t_{Bi} + t_{si} = \frac{D^{1-b/b}}{x_1^{1/b}(1-b)^{1/b}} \tag{25}$$

Again, with the help of learning, the screening rate will become equal to or more than the demand rate and there will not be any backorders after some cycles. The three scenarios of learning discussed in the lost sales case, will be considered here to develop the expected annual profit of a buyer.

#### 4.2.1. Backorders for partial transfer of learning

The inventory level in Fig. 3 can be represented as

$$I_B(t) = \begin{cases} y_i - [(1-b)x_1 t]^{1/(1-b)}, & 0 \le t < t_{si} \\ y_i - y_{si} - D(t - t_{si}), & t_{si} < t < t_{si} + t_{Bi} \\ y_i - Dt, & t_{si} + t_{Bi} < t < \tau_i \\ (1-p)y_i - Dt, & \tau_i < t < T_i \end{cases} \tag{26}$$

At time $t$=$T_i$, the inventory level is zero, i.e. $(1-p)y_i - DT_i = 0$. The cycle time $T_i$ is

$$T_i = \frac{(1-p)y_i}{D} \tag{27}$$

The holding costs for the four different time intervals shown in Fig. 3 are determined respectively from (26) as

$$HC_{1Bi} = hyt_{si} - h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{2-b/1-b} \tag{28}$$

$$HC_{2Bi} = h(y_i - y_{si} + Dt_{si})t_{Bi} - \frac{hD}{2}[(t_{si}+t_{Bi})^2 - t_{si}^2] \tag{29}$$

$$HC_{3Bi} = h[\tau_i - (t_{si}+t_{Bi})]y_i - \frac{hD}{2}[\tau_i^2 - (t_{si}+t_{Bi})^2] \tag{30}$$

$$HC_{4Bi} = h(1-p)(T_i - \tau_i)y_i - \frac{hD}{2}(T_i^2 - \tau_i^2) \tag{31}$$

Adding Eqs. (28)–(31) to get the holding cost for the backorder case:

$$\begin{aligned} HC_{Bi} = {} & hy_i t_{si} - h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{2-b/1-b} \\ & + h(y_i - y_{si} + Dt_{si})t_{Bi} - \frac{hD}{2}[(t_{si}+t_{Bi})^2 - t_{si}^2] \\ & + h[\tau_i - (t_{si}+t_{Bi})]y_i - \frac{hD}{2}[\tau_i^2 - (t_{si}+t_{Bi})^2] \\ & + h(1-p)(T_i - \tau_i)y_i - \frac{hD}{2}(T_i^2 - \tau_i^2) \end{aligned}$$

Using (25) and simplifying the above expression results in

$$HC_{Bi} = h(1-p)y_i T_i + hpy_i\tau_i + ht_x B + hy_{si}t_{si} - \frac{hD}{2}(T_i^2 + t_{si}^2) - h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{2-b/1-b} \tag{32}$$

Substituting $\tau_i$ and $T_i$ in terms of $y_i$ from (9) and (27) respectively, the above expression can be simplified as

$$HC_{Bi} = \frac{h}{2D}[y_i^2(1-p)^2] - \frac{hD}{2}t_{si}^2 + \frac{hpy_i\{(y_i+u_i)^{1-b} - u_i^{1-b}\}}{x_1(1-b)} + hy_{si}t_{si} + ht_x B - h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{2-b/1-b} \tag{33}$$

It should be noted that the above holding cost reduces to the one in Salameh and Jaber (2000) once $b$, $t_x$ , $t_{si}$, $y_{si}$ and $u_i$ become zero.

Now the backorder cost in a cycle is $= \frac{c_B t_s B_i}{2} + c_B(t_x - t_{si})B_i = c_B\left(t_x - \frac{t_{si}}{2}\right)B_i$

So, the total profit per cycle is given by

$$TP_{iB}=[s(1-p)+vp-c]y_i-K-c_B\left(t_x-\frac{t_{si}}{2}\right)B-\frac{(hpy_i+d)\{(y_i+u_i)^{1-b}-u_i^{1-b}\}}{x_1(1-b)}-\frac{h}{2D}[y_i^2(1-p)^2]+\frac{hD}{2}t_{si}^2-hy_{si}t_{si}-ht_xB+h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)}t_{si}^{2-b/1-b} \quad (34)$$

and the expected total profit per cycle is

$$E[TP_{iB}]=\{s(1-E[p])+vE[p]-c\}y_i-K-c_B\left(t_x-\frac{t_{si}}{2}\right)B-\frac{(hy_iE[p]+d)\{(y_i+u_i)^{1-b}-u_i^{1-b}\}}{x_1(1-b)}-\frac{h}{2D}\left\{y_i^2E[(1-p)^2]\right\}+\frac{hD}{2}t_{si}^2-hy_{si}t_{si}-ht_xB+h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)}t_{si}^{2-b/1-b} \quad (35)$$

The expected cycle time $E[T_{iB}]$ can be written using (27) as

$$E[T_{iB}]=\frac{(1-E[p])y_i}{D}$$

So, the expected annual profit can be written as

$$E[TPU_{iB}]=\frac{E[TP_{iB}]}{E[T_{iB}]} \quad (36)$$

*4.2.2. Backorders for total transfer of learning*

In this case, the worker will retain all the experience gained in the earlier cycles. This experience will be calculated using Eq. (22). The holding cost, the expected profit per cycle, and the expected annual profit in Eqs. (33), (35) and (36), respectively, will be determined using this experience in each cycle.

*4.2.3. Backorders for total forgetting*

In this case, the experience $u_i$ in cycle $i$ becomes zero and the inspection time will be determined by Eq. (23). The holding cost in Eq. (33) will be written as

$$HC_B=\frac{h}{2D}[y^2(1-p)^2]-\frac{hD}{2}t_s^2+\frac{hpy^{2-b}}{x_1(1-b)}+hy_st_s+ht_xB-h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)}t_s^{2-b/1-b} \quad (37)$$

The expected profit per cycle and the expected annual profit in Eqs. (35) and (36) respectively, will be determined using Eqs. (23) and (37).

## 5. Numerical analysis

Consider a buyer who receives a lot of $y$ units with defective items and the lot is screened out by workers. Unlike Salameh and Jaber (2000), at the beginning of the cycle, the screening rate is less than the demand rate and results in lost sales or backorders. As learning takes place in the screening process, the worker catches up with demand. Thus, there will not be any lost sales or backorders after some cycles of procurement.

Learning rates vary from industry to industry. For example, Cunningham (1980) collected learning rates reported in different industries that ranged from 95% ($b$=0.074) in electric power generation to 60% ($b$=0.737) in semiconductor manufacturing. Dutton and Thomas (1984) plotted learning rates found in 108 manufacturing firms, with 55% ($b$=0.862) being the fastest learning rate reported. Dar-El (2000, p. 58) tabulated learning rates reported from previous studies that ranged from 95% ($b$=0.074) to 68% ($b$=0.556). Some studies observed total forgetting to occur in a period ranging from few months to more than a year (e.g., Anderlohr, 1969; McKenna and Glendon, 1985). The learning rate in our example is taken to be 80% ($b$=0.322) as observed in many manufacturing industries (Argote and Epple, 1990).

Besides, Salameh and Jaber (2000) used an inspection cost of \$0.5/unit and obtained an optimal order size of 1439 units per cycle. They assumed a very high value of the inspection rate (175 200 units per year) that represents a plateau in a learning curve. That is, when the screening rate on the learning curve exceeds 175 200, the models developed herein converge to that of Salameh and Jaber (2000) whose closed form solution is given by Eq. (2). The time and the cost of inspection per cycle, using the parameters in Salameh and Jaber (2000), imply that an annual cost of \$87 600 be used in our example. An analysis of how unit inspection cost affects the annual profit will be carried out at the end of this section.

The unit cost of backordering depends on the delay in the supply. Its value is taken to be twice as the unit holding cost (Wee et al., 2007). On the other hand, cost of lost sales is the sum of the lost revenue and the cost of goodwill (Goyal and Giri, 2001; Ouyang et al., 2006). Thus, it should be greater than or equal to the unit selling price. The unit cost of lost sales is taken to be equal to the unit selling price, in this numerical example. The effect of these two costs on the annual profit will be analyzed later in this section. Four percent defective items ($p$) are assumed while the time for total forgetting ($V$) is taken to be 300 days. The sensitivity of the total forgetting time will also be analyzed later. The rest of the data for this numerical analysis is taken from Salameh and Jaber (2000). The probability density function for the percentage of defectives is taken to be

$$f(p)=\begin{cases}25, & 0\le p<0.1\\ 0 & \text{otherwise}\end{cases}$$

Microsoft Excel Solver is used to obtain the optimal annual profit for a certain learning exponent and the percentage of defectives. It is observed that the annual profit plateaus to some extent after 10 cycles, as a result of learning and forgetting. Therefore, in case of partial or total transfer of learning, an average of the results from the first 10 cycles is used to compare them with those from the other scenarios of learning. The input data and the results of the numerical example are presented in Table 1.

It should be noted that the total transfer of learning is a better approach for both lost sales and backorders. The rationale is that while retaining the previous knowledge in screening, the buyer orders less. Total forgetting in both lost sales and the backorders does not result as a profitable option which is an understandable finding as the buyer has to screen with no previous experience.

The difference between lost sales and the backorders is not huge in case of total or partial transfer of learning. The reason is that the reported results are an average of the values in 10 cycles. This minimizes the difference in the two approaches to a great extent. On the other hand, this difference in case of no transfer of learning is a noticeable one.

To enhance the above example, the effect of the change in learning exponent was studied on the three scenarios discussed, for lost sales and backorders, at a fixed percentage of defectives. The average of the expected annual profit from the first 10 cycles of learning was obtained at different values of the learning exponent. The results are shown in Figs. 4 and 5. It can be seen that the annual profit in both the cases, in all the scenarios of

**Table 1**
Input data and results of the numerical examples.

| $D$ | $x_1$ | $K$ | $d$ | $c$ | $s$ | $v$ | $c_B$ | $c_L$ | $h$ | $p$ | $b$ | $V$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 000 units/year | 30 000 units/year | 100 \$/cycle | 87 600 \$/year | 25 \$/unit | 50 \$/unit | 20 \$/unit | 10 \$/unit year | 50 \$/unit | 5 \$/unit year | 0.40 – | 0.32 – | 0.82 year |
| | | | Lost sales | | | | | | Backorders | | | |
| | | | $y^*$ | | | Annual profit | | | $y^*$ | | Annual profit | |
| Partial forgetting | | | 2201 | | | 1 222 394 | | | 2112 | | 1 222 757 | |
| Total learning | | | 1701 | | | 1 228 448 | | | 8085 | | 1 228 516 | |
| Total forgetting | | | 3625 | | | 1 216 192 | | | 2923 | | 1 218 890 | |

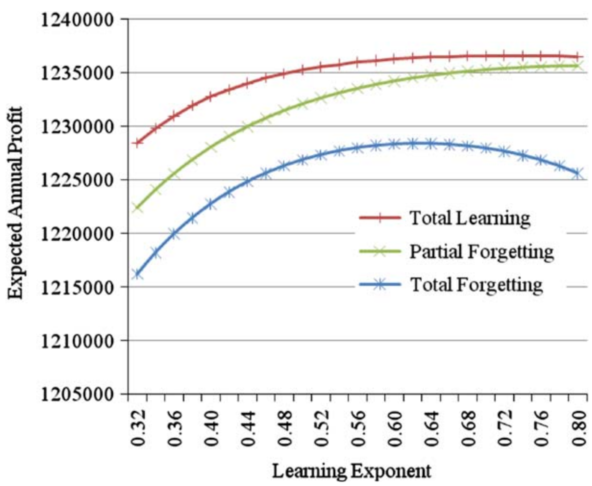


**Fig. 4.** Annual profit w.r.t. $b$ for lost sales at $p$=0.04.

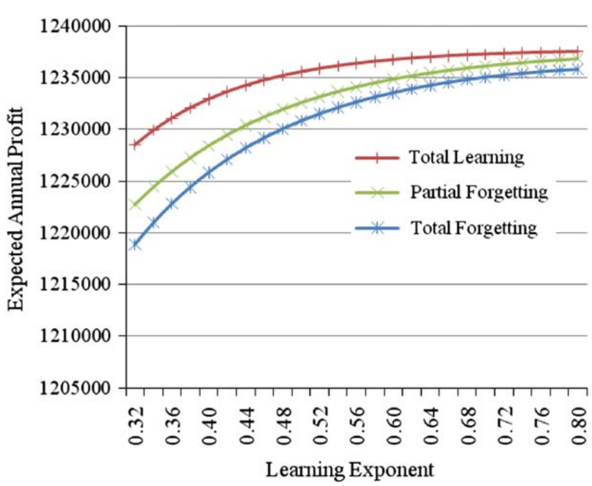


**Fig. 5.** Annual profit w.r.t. $b$ for backorders at $p$=0.04.

learning, tends to be increasing with the learning exponent except only for total forgetting in case of lost sales. That is, learning in screening makes the buyer order less and less. This saves him some of the screening and holding costs, and causes increase in the annual profit. The convexity in case of lost sales with total forgetting tells us that at a very high learning exponent, the buyer tends to pile inventory with him, i.e. an increase in the holding cost which is not there in other scenarios of learning. This counters the increase in the annual profit. In general one can say that the more the knowledge in screening is retained the more is the annual profit. Furthermore, total transfer of learning remains to be the best of the three scenarios discussed, both for lost sales and backorders, in terms of annual profit.

Similarly, to understand the effect of the shortage costs, annual profit was obtained for varying lost sales and backorder costs, for the case of partial transfer of learning. Again, an average of the annual profit from 10 consecutive cycles of learning was obtained at different unit-costs of lost sales, at a fixed value of the learning exponent. Fig. 6 shows that, as intuitively expected, the annual profit tends to decline with the increasing unit lost sales cost. That is, the lost sales cost offsets the annual profit margin of the buyer. On the other hand, the annual profit for the backorder case showed almost no variation when the unit backorder cost was varied from two to four-times the unit holding cost. This indicates that this much change in the unit backorder cost is just not enough to hint a clear difference in the annual profit, especially when the learning rate is fixed. It should be noted that in the backorder case, the demand is eventually met and hence profit is not lost. However, the only additional cost is that of back orders, which is more than the holding cost, for the quantity backordered. In order to investigate the effect of defective items on the annual profit, the sensitivity of the model to $p$, was tested for the lost sales case with partial transfer of learning. Fig. 7 shows that at a fixed learning exponent, the annual profit tends to be decreasing with the percentage of defectives. This is an expected result as the imperfect items are likely to slash the profit obtained from the non-defective items.

To investigate the sensitivity of the time for total forgetting, the response of the model to varying $V$ was tested for the lost sales case with partial transfer of learning. The three curves in Fig. 8 indicate that as the worker retains his/her skills for a longer period, the increase in the profit by virtue of learning ends up at a higher level than for the case where he loses his experience in shorter spells of time.

Finally, the sensitivity of the model to the unit inspection cost is explored. The response of the annual profit for the lost sales case, with total transfer of learning, is plotted in Fig. 9, at different levels of learning. That is, the unit inspection cost is varied from \$0.5 to \$2.5 for $b$ between 0.05 and 0.4. It should be noted here that the unit inspection cost in all the above analyses was \$0.5. Fig. 9 indicates that the annual profit drops by a great extent by varying the unit inspection cost at lower levels of $b$ (i.e. when the learning is slow). This difference in the annual profit starts diminishing as learning becomes faster (higher values of $b$). This indicates that the unit inspection cost affects the annual profit more at the higher values of screening time. One should notice that the screening time per cycle gets shorter and shorter with learning which tends to increase the annual profit. Besides, this screening time has an inverse relation to the learning exponent $b$.

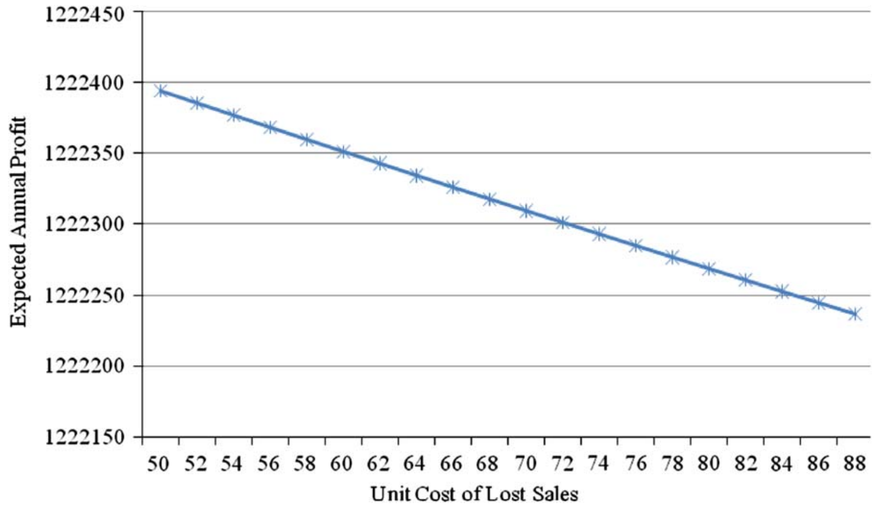


**Fig. 6.** Annual profit w.r.t. $c_L$ for lost sales with partial forgetting.

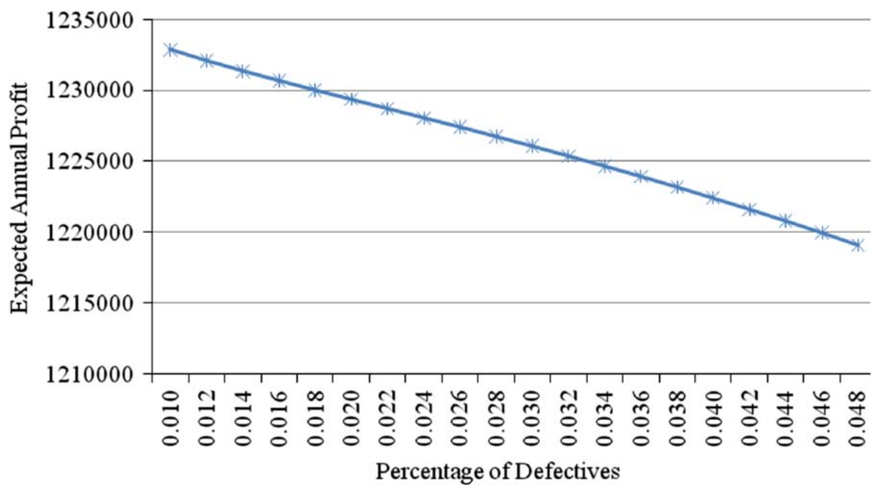


**Fig. 7.** Annual profit w.r.t. $p$ for lost sales with partial forgetting.

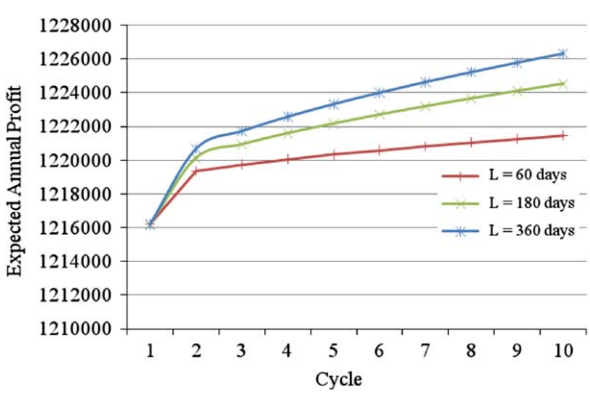


**Fig. 8.** Annual profit w.r.t. $L$ for lost sales with partial forgetting.

## 6. Summary and conclusions

This paper is an extension of Salameh and Jaber (2000) for the case where the buyer's inspection process undergoes learning while screening for defective items in a lot. It is assumed that a 100% inspection is carried out with an error free screening and the rate of screening tends to increase by virtue of learning. This counters an assumption in Salameh and Jaber (2000) that the inspection rate is fixed and is always greater than demand. This paper makes a realistic move and contributes to the area by giving some insights to a practical scenario. Having a screening rate lesser than demand rate in the beginning of the screening process, incurs shortages which are tackled in the paper as both lost sales and backorders. Three scenarios of learning, available in the literature, are compared for the above set-up. These scenarios are (i) total forgetting, where an inspector starts in every cycle with no prior experience, (ii) total transfer of learning, where the inspector does not lose any knowledge or skills in the breaks and the learning curve continues as if there were no interruptions, and (iii) partial transfer of learning, where an inspector carries part of his experience to the subsequent cycles. The last situation is the most generalized and the realistic one. The results indicate that total transfer of learning remains better for both the lost sales and the backorders set-up. The reason is that the buyer orders less and pays less in inventory and screening cost, by virtue of learning. The sensitivity of the model was tested for a number of parameters. The results indicated that the annual profit tends to increase with the learning exponent in screening. That is, the faster the learning in screening the lesser is the screening time. A similar finding of the research is that the annual profit can be increased by retaining more and more knowledge in screening

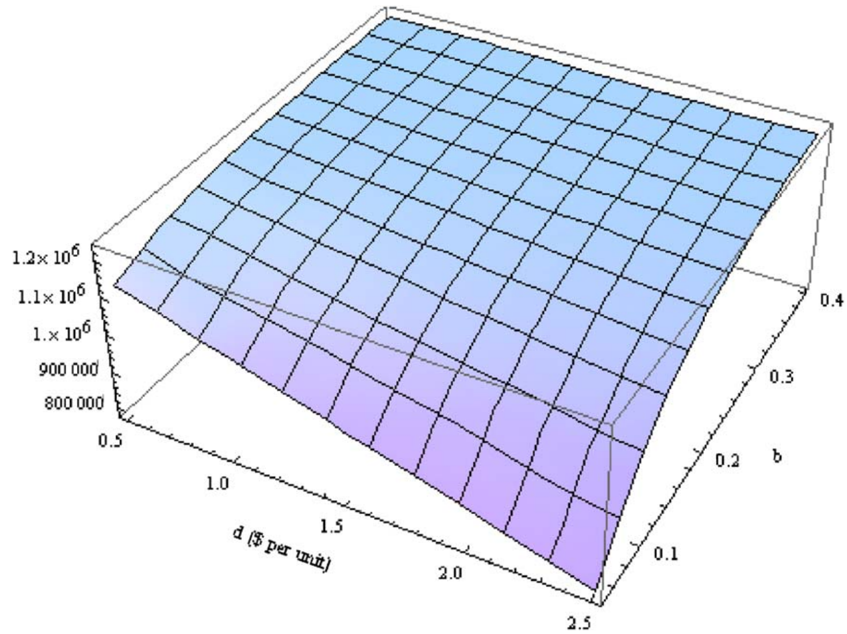


**Fig. 9.** Annual profit w.r.t. $d$ and $b$ for lost sales with total transfer of learning.

process. This was pointed out by experimenting different levels of time for total forgetting, at fixed values of the learning exponent.

It was noticed that an increase in the percentage of defectives decreases the annual profit at a fixed exponent of learning. The unit cost of lost sales is also shown to have a similar effect on the annual profit. It was shown that an increase in the unit screening cost reduces the annual profit to a great extent at the slower rates of learning.

The study could be enhanced in a number of ways. For example, one could study the effect of learning in suppliers' proportion of defectives. The buyer and the supplier can agree on a fixed proportion of defective items, and look for a coordinated order size per cycle.

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