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A computational model for hybrid nanofluid flow on a rotating surface in the existence of convective condition

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ABSTRACT

The current work aims to study heat transfer properties within the flow. A newly developed idea regarding hybrid nanofluid is utilized to obtain the desired outcomes. This study focused on hybrid base nano liquid flow past an exponentially stretching rotating surface in the presence of the convective condition. Ethylene glycol-water (50%–50%) mixture is taken as base fluid. Thermophysical properties are discussed. The probe of emerging parameters indicates that hybrid base nanofluid gives a preferable rating as compared to nanofluid. Appropriate choice of composition plays a vital role. Physical characteristics of essential parameters such as Prandtl number, convective parameter, stretching ratio, rotation parameter, and temperature exponent are evaluated. Graphical depiction reveals that the velocity curve decline by increasing rotation parameter for CuO-hybrid base fluid and TiO_2 hybrid base fluid. By expanding the values of the stretching ratio the velocity profile $p^{'}(\eta)$ is decreasing while the velocity curve $q^{'}(\eta)$ increased for CuO as well as TiO_2 hybrid base fluid. Concentration boundary layer thickness growing by enlarging rotation parameters for both types of nanofluid. Moreover, the influence of various parameters on skin friction factor and Nusselt numbers are also evaluated.

1. Introduction

Motivation in rotating flows derives from different geophysics and engineering applications like rotor-stator parts, rotational machinery, anti-cyclonic stream circulation, earth magma flow, geological tectonic plate spreading below the rotating ocean, centrifugal filtration, food processing, viscometry, hurricane and tornado dynamics. Wang examined the rotating flow nanofluid over a stretched sheet [1]. Kumari et al. [2] investigated the power-law rotating fluids flow past a stretched surface. Takhar and Nath [3]

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Nomenclature

Nc	convective parameter
A	temperature exponent parameter
p, q	dimensionless components of velocity
q_w	wall heat flux $\left[\frac{w}{m^2}\right]$
c_{fx}, c_{fy}	Skin friction along the x-direction and y-direction
u_0, v_0	rates of stretching
u_w, v_w	exponentially stretching velocities at the surface
Pr	Prandtl number
Nu_x	Nusselt number
Re_x	local Reynolds number
u, v, w	components of velocity $\left[\frac{m}{s}\right]$ in x, y, z-direction
T, T_∞, T_w	the temperature of the fluid, ambient temperature, wall temperature $[K]$

Greek Symbols

φ	nano-particles volume fraction
α	stretching ratio parameter
θ	dimensionless temperature component
γ	rotation parameter
Ω	constant angular velocity $[s^{-1}]$
η	dimensionless space variable
ν_f, ν_{nf}	kinematic viscosities $\left[\frac{m^2}{s}\right]$ of the base fluid and nanofluid respectively
μ_f, μ_{nf}	dynamic viscosities $\left[\frac{Nc}{m^2}\right]$ of the base fluid and nanofluid respectively
$\rho_s, \rho_f, \rho_{nf}$	density $\left[\frac{kg}{m^3}\right]$ of solid nanoparticles, base fluid, and nanofluid respectively
α_{nf}	thermal diffusivity $\left[\frac{m^2}{s}\right]$ of nanofluid
τ_w	viscous stress at the surface $[Nm^{-2}]$
k_s, k_f, k_{nf}	thermal conductivities $\left[\frac{W}{mK}\right]$ of solid nanoparticles, base fluid, and nanofluid
$(c_\rho)_{nf}, (c_\rho)_f$	volumetric heat capacity $\left[\frac{J}{kgK}\right]$ of nanofluid and base fluid respectively Subscripts
w	condition at wall
∞	condition at free steam
s	solid nano-particles
f, nf	base fluid and nanofluid respectively Superscripts
$' \; '' \; '''$	1st, 2nd and 3rd derivative for η

numerically studied rotating flow on a stretching unstable surface with Magnetohydrodynamics (MHD). Javed et al. [4] offered the viscous rotational flow of comparable solutions over an exponentially stretched sheet. Rotating flow in the existence of suction on an exponentially shrinking sheet was investigated by Rosali et al. [5]. Zaimi et al. [6] introduced the rotating flow of viscoelastic over a stretched surface.

In recent years, many researchers have studied rotating nanofluid flow behavior and transfer of heat phenomena of steady, incompressible, and real fluid flow past a stretching surface because of its several applications in industrial procedures such as paper drying, glass blasting, etc. Choi was the first who give the concept of nanofluids. He firstly utilized this concept to raise thermal conductivity and rate of heat transfer by using nanoparticles. The thermal conductivity of fluids enhances with nanoparticles [7]. Wong and Leon [8] have studied many applications of nanofluids. For engineering, in complex fluid, nanofluid is used for producing nanostructured materials [9]. Nadeem et al. [10–12] explored the flow of nanofluid on stretching/shrinking and curve surfaces. Many researchers studied the flow of nanofluids and heat transfer on different surfaces [13–33].

The concept of the boundary layer was first given by Ludwig Prandtl. He categorized the fluid forms into two classes, one inner the boundary layer where the viscosity influence is maximal and the second outer the boundary where the boundary influence can be neglected. In fluid dynamics, considerable collections of natural flowing in engineering applications are generally like as, glass-blowing, plastic sheets, paper producing, ship and air automobiles, polymers, revolving of fibers, protrusion cooling of elastic sheets, the earth's atmosphere. Khan and Sanjayanand [34] explored the heat transfer and viscoelastic flow over an exponentially stretching

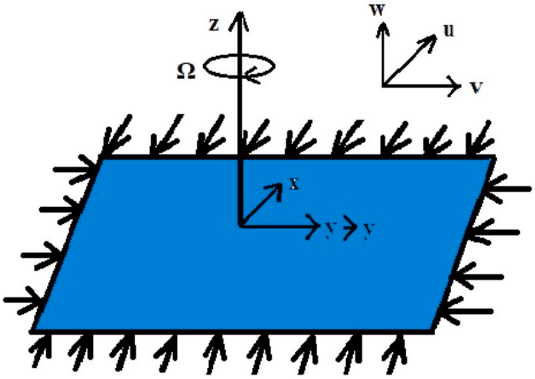

Fig. 1. Geometry of the problem.

surface. Rana and Bhargava [35] examined heat transfer and flow of nanofluid on a nonlinearly stretching sheet. An exponentially stretching surface has substantial application such as the state of copper wires of hardening and softening. Sanjayanand and Khan [36] evaluated the heat transfer effects on an exponentially stretching sheet. Mustafaa et al. [37] explored the convective boundary condition and flow of a nanofluid on an exponentially stretched surface. Khan and Pop [38] investigated the boundary layer over a stretching surface with the nanofluid flow. Rout et al. [39] examined Cu-kerosene and Cu-water nano liquids of axisymmetric squeezing flow.

Keeping in mind the above-mentioned applications, current work aims to study enhancement in the heat transfer properties within the flow. We consider the three-dimensional, steady, incompressible, rotating flow of an electrically conducting nanoparticle lying on an exponentially stretching surface. The partial differential equations are minimized to a simpler one. Numerical results are found with the MATLAB built-in solver bvp4c. Physical characteristics of essential parameters such as convective parameter Nc, stretching ratio α, rotation parameter γ , and temperature exponent A are evaluated. The output of velocity and temperature field according to various values of substantial parameters are demonstrated graphically. So, our purpose here is to refine the features of nanoparticles by implementing hybrid base nanofluids that greatly enhance boundary layer thickness effects.

2. Explanation of the mathematical problem

The coordinate system and the physical model of the problem are shown in Fig. 1.

Let us consider the three-dimensional, steady, incompressible, rotating flow of viscous and electrically conducting nanofluid lying on a stretchable surface in the region $z \geq 0$, where fluid is rotating with an angular velocity around the z-axis. To stretch the wall exponentially with speed u_w and v_w. Consider the CuO and TiO_2 nanoparticles with ethylene glycol-water (50%–50%) mixture as a base fluid. The nanofluid imitates the exponentially expanding layer just slightly. By applying boundary layer approximation and the pressure gradient, as well as viscous dissipation, are considered negligible, the governing three-dimensional equations become:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \tag{1}$$

Momentum equations

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-2\Omega v=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial z^2}, \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+2\Omega u=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 v}{\partial z^2}. \tag{3}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_{nf}\frac{\partial^2 T}{\partial z^2}. \tag{4}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}\ ,\ \rho_{nf}=\rho_f(1-\phi)+\phi\rho_s\ ,\ \alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}}\ ,$$

$$(\rho C_p)_{nf}=(\rho C_p)_f(1-\phi)+\phi(\rho C_p)_s. \tag{5}$$

Here, μ_{nf} stands for dynamic viscosity, ρ_{nf} shows density, α_{nf} indicates thermal diffusivity, k_{nf} denotes thermal conductivity and

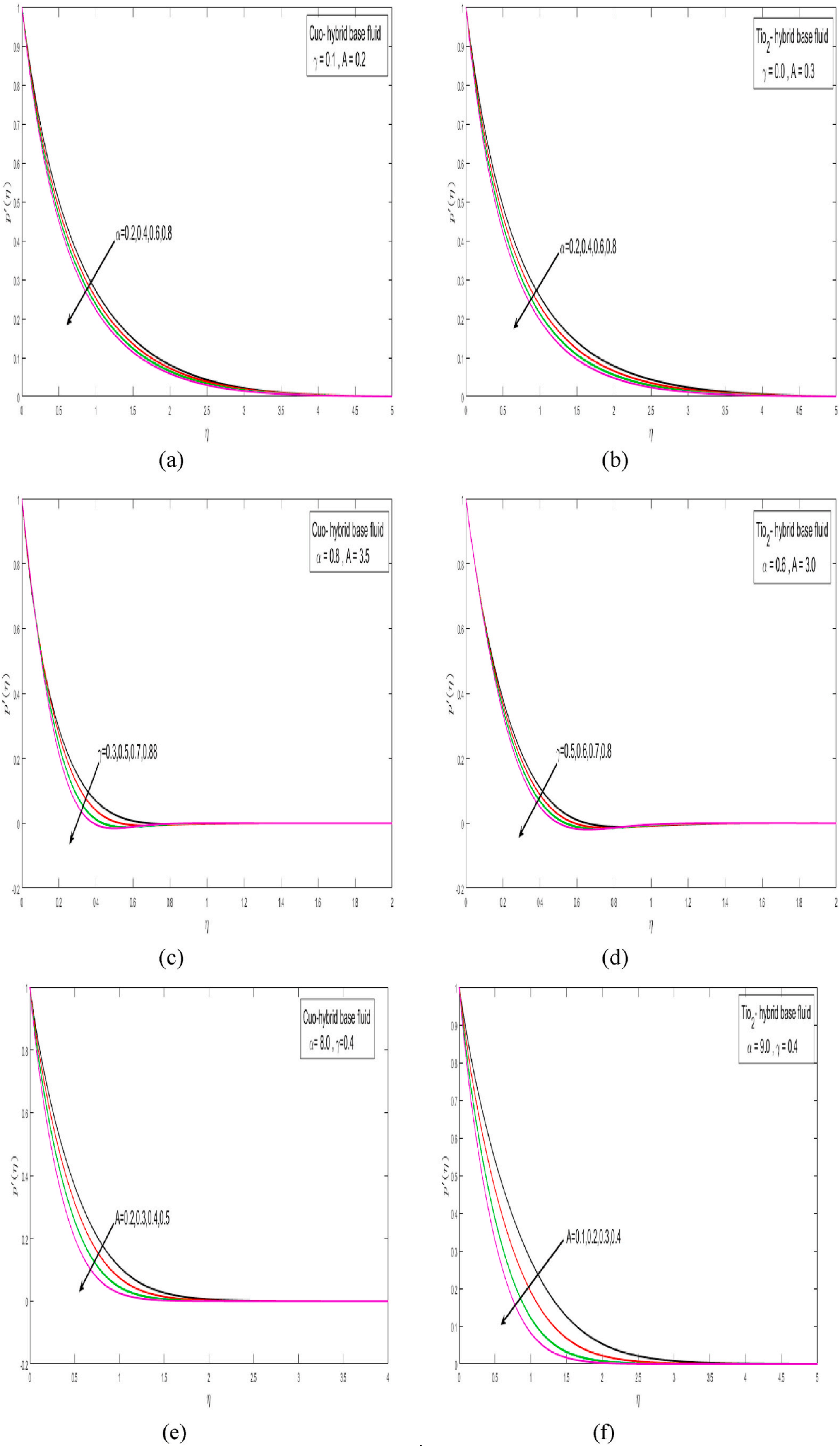

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Fig. 2. Influence of stretching ratio parameter α $(a-b)$, rotation parameter γ $(c-d)$, and temperature exponent A $(e-f)$, on velocity $p'(\eta)$ for CuO-hybrid base fluid and TiO_2 hybrid base fluid of nanoparticles.

$(\rho C_p)_{nf}$ is heat ability of nanofluid.

The boundaries are

$$u = u_w,\ v = v_w,\ -k_{nf}\frac{\partial T}{\partial z} = h_f(T_w - T),\ at\ \ z = 0, \tag{6(a)}$$

$$u \to 0,\ v \to 0,\ T \to T_\infty,\ as\ \ z \to \infty. \tag{6(b)}$$

Exponentially stretching velocities at the surface and wall temperature is defined as follows

$$u_w = u_0 e^{\frac{x+y}{L}},\ v_w = v_0 e^{\frac{x+y}{L}},\ T_w = T_\infty + T_0 e^{\frac{A(x+y)}{2L}}. \tag{7}$$

3. Transformation methodology

The suitable similarity transformations are

$$u = u_0 e^{\frac{x+y}{L}} p'(\eta),\ v = u_0 e^{\frac{x+y}{L}} q'(\eta),\ w = -\left(\frac{\nu u_o}{2L}\right)^{\frac{1}{2}} e^{\frac{x+y}{2L}}\{p + \eta p' + q + \eta q'\}, \tag{8}$$

$$T = T_\infty + T_0 e^{\frac{A(x+y)}{2L}}\theta(\eta), \quad \eta = \left(\frac{u_0}{2\nu L}\right)^{\frac{1}{2}} e^{\frac{x+y}{2L}} z. \tag{9}$$

Where η is the similarity space parameter, T_w shows the temperature at the wall,T_∞ is the free stream temperature.

After applying the similarity transformations on Eqs. (1)-(6b), the continuity equation is identically satisfied while momentum and energy equations take the following form

$$\frac{\rho_f}{\rho_{nf}}(A_1(\phi))p''' + p''(p+q) - 2p'(p'+q') + 4\gamma q' = 0, \tag{10}$$

$$\frac{\rho_f}{\rho_{nf}}(A_1(\phi))q''' + q''(p+q) - 2q'(p'+q') - 4\gamma p' = 0, \tag{11}$$

$$\frac{1}{\Pr}\frac{\left(\frac{K_{nf}}{K_f}\right)}{\left(1 - \phi + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}\theta'' - A(p'+q')\theta + (p+q)\theta' = 0, \tag{12}$$

$$A_1(\phi) = \frac{\mu_{nf}}{\mu_f},\ \gamma = \frac{\Omega_0 L}{u_0},\ \Pr = \frac{(\mu C_p)_f}{k_f},\ \frac{K_{nf}}{K_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}. \tag{13}$$

here.

The boundary conditions are transformed

$$p(0) = 0,\ p'(0) = 1,\ q(0) = 0,\ q'(0) = \alpha,\ \theta'(0) = -Nc\left(\frac{k_f}{k_{nf}}\right)(1 - \theta(0)),\ as\ \ \eta \to 0, \tag{14(a)}$$

$$p' \to 0,\ q' \to 0,\ \theta \to 0,\ as\ \ \eta \to \infty. \tag{14(b)}$$

Where

Table 1
Thermo-physical properties of hybrid base fluid and nanoparticles.

Physical Properties	$\rho\left(\frac{kg}{m^3}\right)$	$c_p\left(\frac{J}{kg.K}\right)$	$K\left(\frac{W}{m.K}\right)$
$C_2H_6O_2 - H_2O$	1063.8	3630	0.387
CuO	6500	540	18
TiO_2	4250	686.2	8.9538

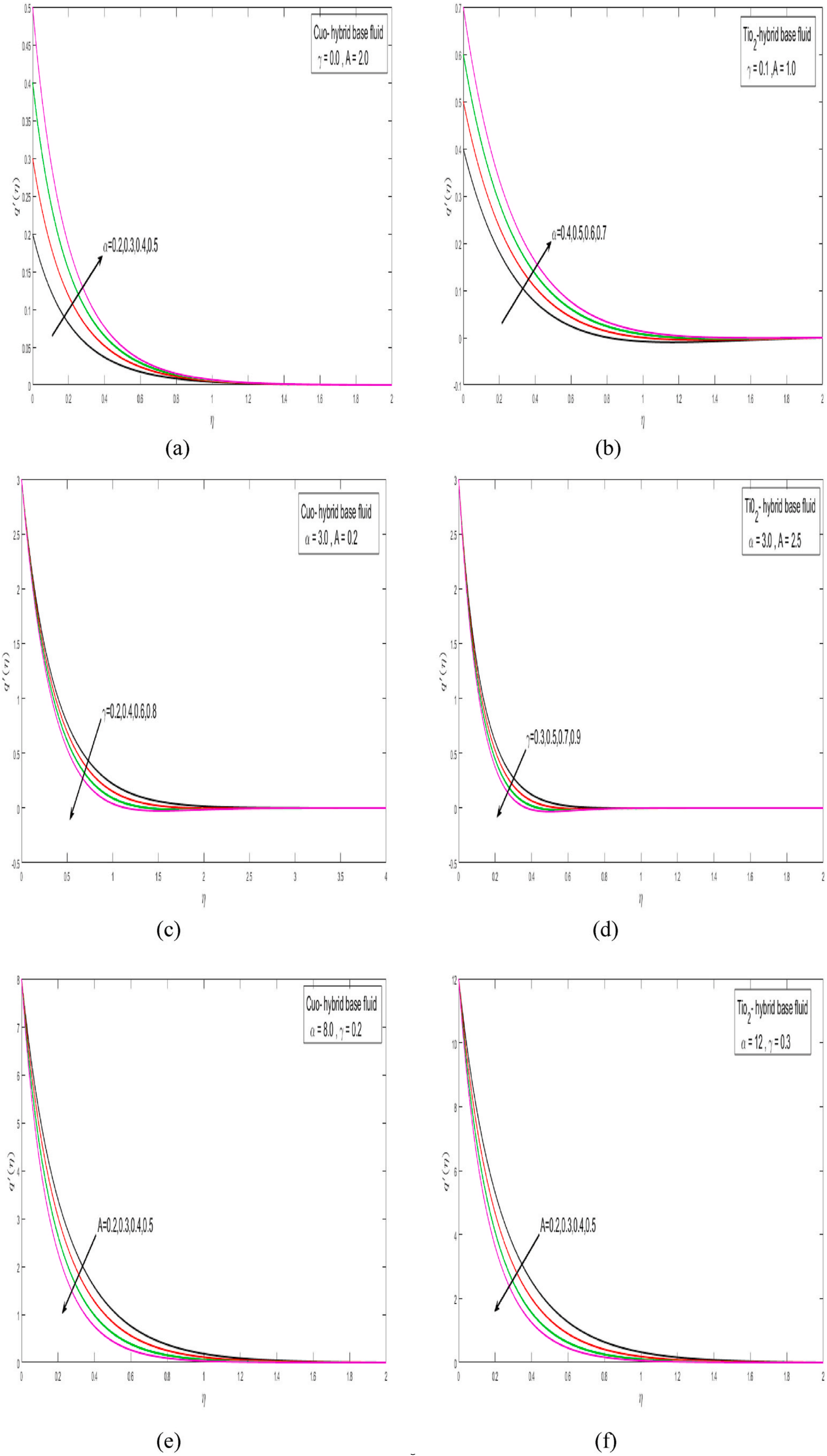

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Fig. 3. Influence of stretching ratio parameter α $(a-b)$, rotation parameter γ $(c-d)$, and temperature exponent A $(e-f)$, on the velocity profile $q'(\eta)$ for CuO-hybrid base fluid and TiO_2 hybrid base fluid of nanoparticles.

$$\alpha=\frac{v_0}{u_0},\ Nc=\frac{h_f}{k_f}\sqrt{\frac{2\nu_f L}{u_0}}e^{-\left(\frac{x+y}{2L}\right)}. \tag{15}$$

In addition, the coefficient of skin friction and the Nusselt number are described as:

$$C_{fx}=\frac{\tau_{wx}}{\frac{1}{2}\rho_f u_w^2},\ C_{fy}=\frac{\tau_{wy}}{\frac{1}{2}\rho_f u_w^2},\ Nu_x=\frac{xq_w}{k_f(T_w-T_\infty)}. \tag{16}$$

The wall shear stresses $\tau_{wx},\ \tau_{wy}$ and heat flux q_w expressions are

$$\tau_{wx}=\mu_{nf}\left(\frac{\partial u}{\partial z}\right)_{z=0},\ \tau_{wy}=\mu_{nf}\left(\frac{\partial v}{\partial z}\right)_{z=0},\ q_w=-k_{nf}\left(\frac{\partial T}{\partial z}\right)_{z=0}. \tag{17}$$

Now, using Eqs. $(8-9)$ and (17) in (16), we get

$$\frac{1}{\sqrt{2}}C_{fx}(\mathrm{Re}_x)^{\frac{1}{2}}=\frac{1}{(1-\phi)^{2.5}}p''(0),\ \frac{1}{\sqrt{2}}C_{fy}(\mathrm{Re}_x)^{\frac{1}{2}}=\frac{1}{(1-\phi)^{2.5}}q''(0), \tag{18a}$$

$$\sqrt{2}\ \frac{L}{x}Nu_x(\mathrm{Re}_x)^{\frac{-1}{2}}=-\frac{k_{nf}}{k_f}\theta'(0). \tag{18(b)}$$

4. Numerical technique

For the sake of numerical solution, the bvp4c approach is utilized on Eqs. 10–(14b). Assume

$$y_1=p,\ y_2=p',\ y_3=p'',\ y_4=q,\ y_5=q',\ y_6=q'',\ y_7=\theta,\ y_8=\theta'. \tag{19}$$

Therefore, the corresponding equations take the form

$$y_3'=(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)(2y_2(y_2+y_5)-y_3(y_1+y_4)-4\gamma y_5), \tag{20}$$

$$y_6'=(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)(2y_5(y_2+y_5)-y_6(y_1+y_4)+4\gamma y_2), \tag{21}$$

$$y_8'=\left(\frac{k_f}{k_{nf}}\right)\frac{(pC_p)_{nf}}{(pC_p)_f}\mathrm{Pr}A(y_1+y_4)y_7-(y_2+y_5)y_8, \tag{22}$$

with conditions:

$$y_1(a)=0,\ y_2(a)=1,\ y_4(a)=0,\ y_5(a)=\alpha, \tag{23(a)}$$

Table 2
Impact of temperature exponent (A), stretching ratio parameter (α), rotation parameter (γ) for the convective parameter (Nc) on coefficients of skin friction (C_{fx}, C_{fy}) and Nusselt number.(Nu_x).

				CuO-hybrid base fluid			TiO_2-hybrid base fluid		
A	α	γ	Nc	$-C_{fx}$	$-C_{fy}$	$-Nu_x$	$-C_{fx}$	$-C_{fy}$	$-Nu_x$
0.2	1.5	0.2	3.0	3.75355	7.35911	2.67158	3.07124	5.95009	2.42455
0.3				4.56987	9.02321	2.67612	3.71876	7.28771	2.43836
0.4				5.26560	10.42751	2.67995	4.27355	8.42148	2.45003
0.5				5.88207	11.66397	2.68332	4.76678	9.42180	2.46022
0.2	1.6	0.2	3.0	3.80407	7.94798	2.67201	3.11156	6.42710	2.42583
0.3				4.63218	9.74360	2.67666	3.76877	7.87093	2.43997
0.4				5.33773	11.25883	2.68059	4.33162	9.09435	2.45192
0.5				5.88972	12.59308	2.68404	4.83189	10.17368	2.46234
0.2	1.6	0.3	3.0	3.45300	8.27614	2.67192	2.83024	6.67860	2.42571
0.3				4.20414	10.15745	2.67645	3.42112	8.19529	2.43964
0.4				4.84714	11.74099	2.68025	3.93092	9.47784	2.45131
0.5				5.41762	13.13297	2.68357	4.38596	10.60726	2.46145

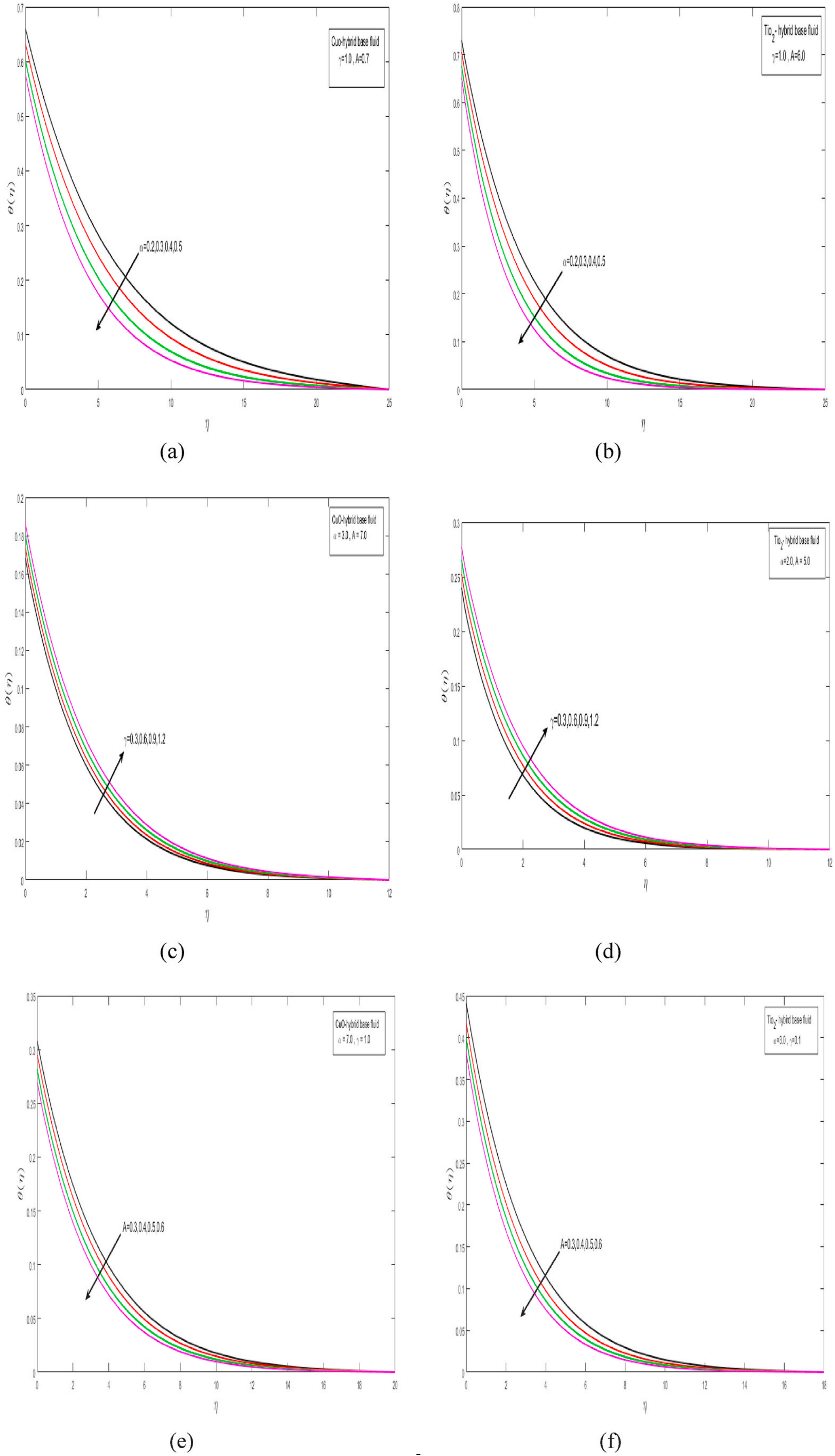

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Fig. 4. Influence of stretching ratio parameter α $(a-b)$, rotating parameter γ $(c-d)$, temperature exponent A $(e-f)$, on temperature curve $\theta(\eta)$ for CuO-hybrid base fluid and TiO_2 hybrid base fluid of nanoparticles.

$$y_8(a) = -Nc\frac{k_f}{k_{nf}}(1-y_7(0)),\ y_7(0) = c_0, \tag{23(b)}$$

$$y_2(b)=0,\ y_6(b)=0,\ y_7(b)=0. \tag{23(c)}$$

5. Graphical results and discussion

This section examines the influence of velocity in two lateral directions $(p'(\eta),\ q'(\eta))$ and temperature distribution $\theta(\eta)$ for some relevant parameters like as stretching ratio α, rotation parameter γ, and temperature exponent A for nanoparticles of CuO as well as TiO_2. Graphical results acquire in the existence of nanoparticles at the same time in sequence to find their real impact and effect on these parameters. Figs. (2) to (4) are presented to fulfill the motive. Fig. 2(a–f) are plotted to locate the impact of stretching ratio parameter α, rotation parameter γ and temperature exponent A on the velocity profile $p'(\eta)$ in horizontal for CuO and TiO_2 hybrid base fluid. It can be recognized from Fig. 2(a–b) that increasing the stretching ratio α velocity is decreased for CuO and TiO_2 hybrid base fluid respectively. Similarly, it can be noted from Fig. 2(c–d), increasing the rotation parameter γ the velocity $p'(\eta)$ in both types of nanoparticles decreases. From Fig. 2(e–f), it is clear that increasing the temperature exponent A the velocity profile $p'(\eta)$ decreased for two types of nanoparticles. It has been noted that the horizontal velocity field reveals reducing mode while growing the amount of α, γ, A respectively for CuO hybrid base fluid and TiO_2 hybrid base fluid. The explanation for this is because increasing values of temperature exponent A with the variation of distinct values of α and γ for CuO hybrid base fluid as well as TiO_2 hybrid base fluid amplifies and for this reason the two lateral directions velocities and temperature curve decreases. But rotation parameter (γ) has the exact opposite effect on momentum and temperature curves for two types of nanofluids. The higher values of γ the liquid velocity in two lateral directions decreases however enhances the boundary layer thickness $\theta(\eta)$ for both nanofluids. From Fig. 3(a–f), demonstrate the impact of stretching ratio α, rotation parameter γ, temperature exponent A on the vertical velocity component $q'(\eta)$ for CuO and TiO_2 hybrid base fluid. It can shows that, Fig. 3(a–b) when stretching ratio α is rising the velocity $q'(\eta)$ is also rises for both types of nanoparticles. The influence of α on the y-axis the fluid velocity increases in this case but the fluid velocity along the x-axis is divergent for CuO-hybrid base fluid and TiO_2 hybrid base fluid. Velocity profile $q'(\eta)$ is decreased for the value of γ is depicted in Fig. 3(c–d). It enhanced for expending value of γ. Fig. 3(e–f) indicates the results of A on the velocity profile. So increasing the values of A then the velocity distribution decreased for CuO and TiO_2 hybrid base fluid. Dissimilarity in temperature distribution $\theta(\eta)$ against α, γ, A for both types of nanoparticles are presented through Fig. 4(a–f). From Fig. 4(a–b), it can be recognized that the temperature profile $\theta(\eta)$ is decreased for both hybrid base fluid nanoparticles when the value of α is increased for both types of nanoparticles. Fig. 4(c–d), elaborate the effect of rotation parameter γ on the temperature profile. This amplifies that by rising the value of γ, the temperature profile $\theta(\eta)$ of the CuO and TiO_2 hybrid base fluid is increased. Temperature field $\theta(\eta)$ growing upward for any value that is being considered α with A for two types of nanofluids. Fig. 4(e–f) represent the diminishing performance of temperature profile for increasing values of A for CuO-hybrid base fluid and TiO_2 hybrid base fluid. Now, we analyze the behavior of different parameters for both nanoparticles CuO and TiO_2 hybrid base fluids on the coefficients of skin friction $(C_{fx},\ C_{fy})$ and the Nusselt number Nu_x through Fig. 5 (a–b), 6(a-b) and 7(a-b). Fig. 5(a–b) elaborate the impact of the skin friction coefficient C_{fx} (along the x-axis) with a stretching ratio α and volume fraction ϕ for both types of nanoparticles. Here, C_{fx} is decreased for CuO and TiO_2 hybrid base fluids with the increases of α. Fig. 6(a–b), show that the skin friction along the y-axis (C_{fy}) is decreased with growing the stretching ratio α and ϕ for CuO-hybrid base fluid as well as TiO_2 hybrid base fluid. Because the diminution in coefficients of skin friction $(C_{fx},\ C_{fy})$ are reduced the force which acts as a barrier to the fluid's movement. As a consequence, the fluid's viscosity declined, and skin friction factors decrease as well as

Table 3
Impact of rotation parameter (γ), stretching ratio parameter (α), temperature exponent (A) for the convective parameter (Nc) on coefficients of skin friction (C_{fx}, C_{fy}) and Nusselt number.(Nu_x).

				CuO-hybrid base fluid			TiO_2-hybrid base fluid		
γ	α	A	Nc	$-C_{fx}$	$-C_{fy}$	$-Nu_x$	$-C_{fx}$	$-C_{fy}$	$-Nu_x$
0.2	1.5	0.2	3.0	9.38387	19.21879	2.67158	7.67811	14.87524	2.42455
0.3				8.55336	19.21879	2.67147	7.01223	15.50381	2.42440
0.4				7.79397	20.05785	2.67130	6.39177	16.15117	2.42413
0.5				7.10563	20.90154	2.67107	5.81812	16.81028	2.42374
0.2	1.6	0.2	3.0	9.51018	19.86995	2.67201	7.77892	16.06777	2.42583
0.3				8.63252	20.69037	2.67192	7.07562	16.69651	2.42571
0.4				7.82536	21.53002	2.67176	6.41750	17.34479	2.42547
0.5				7.08873	22.37599	2.67155	5.80597	18.00576	2.42511
0.2	1.6	0.3	3.0	7.61645	15.03868	2.67612	6.19793	12.14619	2.43836
0.3				6.94217	15.72942	2.67588	5.64962	12.68717	2.43797
0.4				6.33561	16.42652	2.67553	5.14782	13.24031	2.43734
0.5				5.79232	17.11620	2.67512	4.69261	13.79690	2.43652

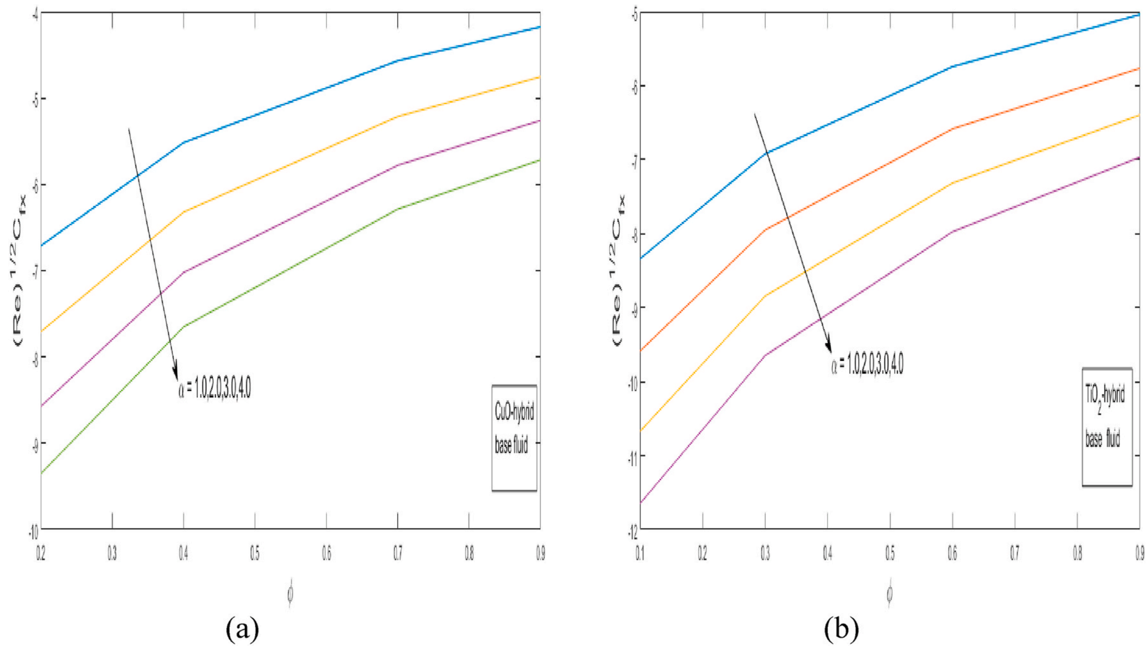

Fig. 5. (a–b) Influence on skin friction of α and ϕ along the x-axis for CuO-hybrid base fluid and TiO_2 hybrid base fluid of nanoparticles.

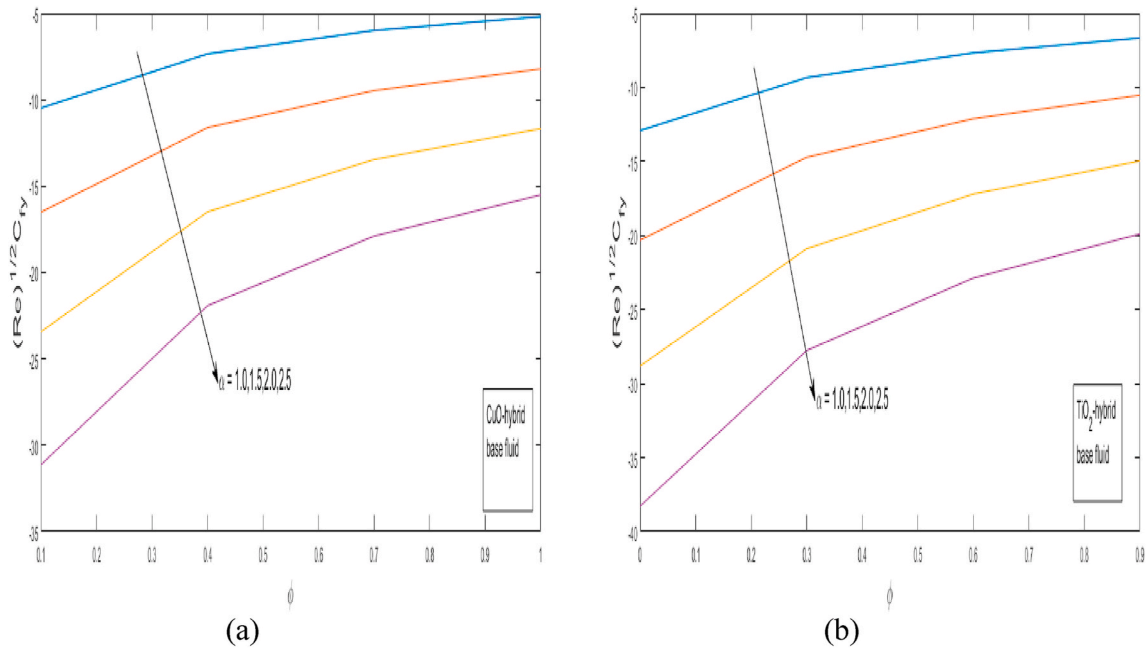

Fig. 6. (a–b) Impact on skin friction along the y-axis of α and ϕ for CuO-hybrid base fluid and TiO_2 hybrid base fluid of nanoparticles.

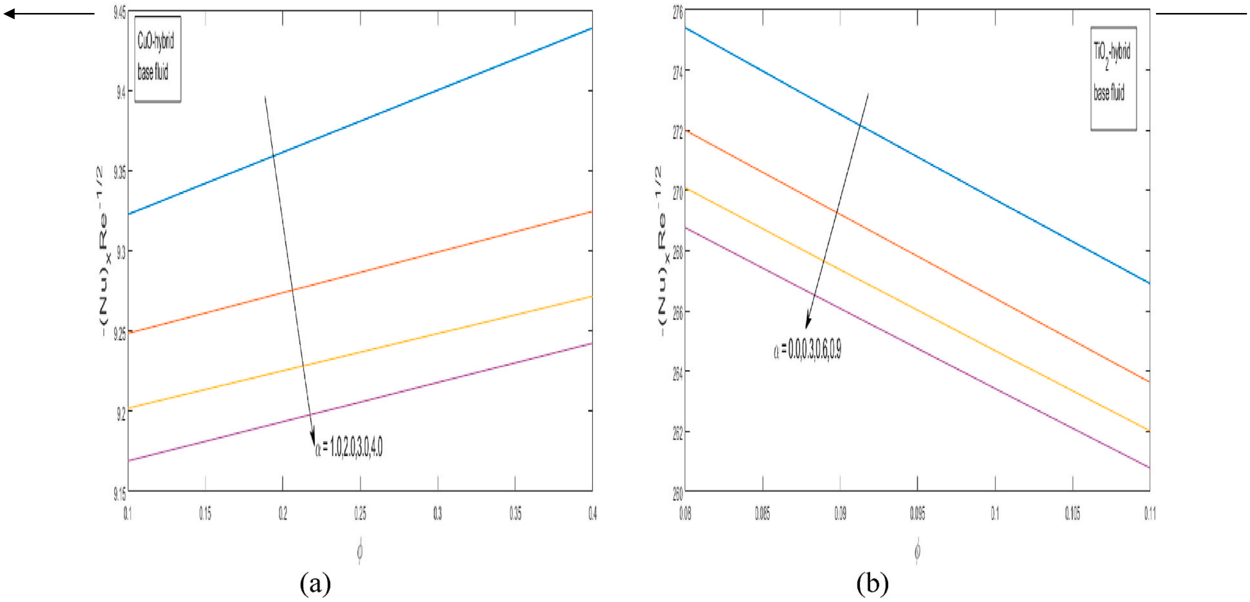

Fig. 7. (a–b) Influence on Nusselt number of α and ϕ for CuO-hybrid base fluid and TiO_2 hybrid base fluids of nanoparticles.

for CuO-hybrid base fluid and TiO_2 hybrid base fluid. Fig. 7(a–b) represent the variation of ϕ and the stretching ratio α on the rate of heat transfer. Here, one can observe that the rate of heat transfer is decreased with the rises of α and ϕ for CuO and TiO_2 hybrid base fluids. Table 1 shows the thermophysical properties of solute nanoparticles. Tables 2 and 3 demonstrate the skin friction (C_{fx}, C_{fy}) values and heat flux with the rising manner for various values of parameters for CuO-hybrid base fluid as well as TiO_2 hybrid base fluid.

6. Concluding remarks

This paper elaborates the consequences of the three-dimensional, steady flow of a rotating nanofluid with stretching ratio and temperature exponent over an exponentially stretching surface. The impact of physical parameters on two lateral velocities, temperature profile, friction factors, and heat flux are solved in a comprehensive form. The main findings are:

- Temperature profile $\theta(\eta)$ and two lateral velocities ($p'(\eta)$, $q'(\eta)$) exhibit diminishing performance for extended values of A.
- By enhancing the value of rotation parameter γ, two lateral direction velocities decrease but the temperature profile increases.
- Velocity curve $p'(\eta)$ shows decline behavior when growing the amount of stretching ratio for both types of nanofluids.
- The vertical velocity profile $q'(\eta)$ is increasing with increasing for higher values of the stretching ratio parameter α for CuO-hybrid base fluid as well as TiO_2 hybrid base fluid.
- By increasing the stretching ratio α with ϕ the friction factors (C_{fx}, C_{fy}) and the rate of heat transfer is decreased respectively.

Authorship contributions

All authors have an equal contribution.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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