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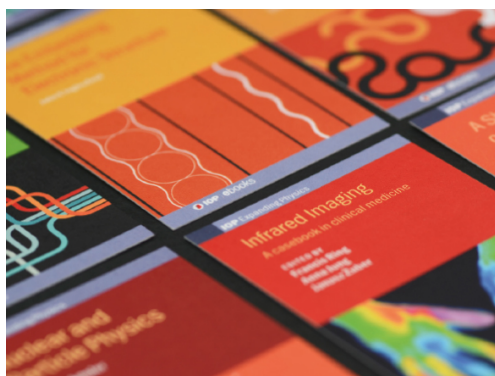
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# Stability analysis of fluid film bearings under laminar and turbulent regimes

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**Abstract.** Stability of short journal bearings is investigated in both the linear and nonlinear regimes under laminar and turbulent operating conditions with consideration of drag force effects. Exerted forces on fluid film bearings comprises of two different components. (1) Pressure force components in radial and tangential directions, and (2) Drag force (shear/friction force) components in radial and tangential directions. In most existing literature, related to rotor bearing system supported on journal bearing stability analysis, the effect of fluid film drag force has been neglected. Utilizing Ng-Pan-Elrod model and the modified Reynold's equation, detailed calculation of the turbulent pressure distribution is carried out. Local stability of periodic solutions was studied using Hopf bifurcation theory. The shaft stiffness and inclusion of drag force and turbulent effects were found to play an important role in stability regions and bifurcation profiles of a flexible rotor bearing system.

It was found that for shafts supported on short journal bearings, consideration of drag force effect could lead to expansion of un-stable region at high Sommerfeld numbers (low shaft static loads). Dynamic coefficients of short length journal bearing were analytically calculated under laminar and turbulent regimes based on Ng-Pan-Elrod model. Linear stability charts of a flexible rotor supported on laminar and turbulent journal bearings are found by calculating the threshold speed of instability associated to the start of unstable oil whirl phenomenon. Local journal trajectories of the rotor-bearing system were found at different operating conditions solely based on the calculated dynamic coefficients in laminar and turbulent flow. Results show discrepancy between laminar and turbulent models, when shaft stiffness is great than 10, for the entire range of Sommerfeld number by increasing the Reynolds number in turbulent models.

## 1. Introduction

Multistage compressors are used extensively in high pressure natural gas operations and liquid natural gas industries. These compressors frequently suffer from high vibrations and sub-synchronous whirl. This large vibration sub-synchronous conditions may be detrimental to the rotor-bearing system and it is normally evident at frequencies different from the shaft rotating frequency. This phenomenon is crucial for the safe operation of these multistage compressors as well as other turbomachinery. For these machines, there exists a "threshold speed of instability" after which the rotor bearing system becomes unstable in oil whirl/whip that is characterized by sub-synchronous large vibrations [1, 2]. For such system, traditionally linearized stiffness and damping coefficients (dynamic coefficients) are used as basis for stability analysis of the rotor bearing systems.



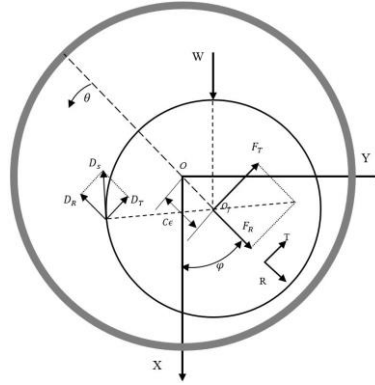
Local instabilities of a journal system is characterized by an operating condition that is close-enough to an operating equilibrium point. Although the load-displacement response of a journal system is inherently nonlinear, such local instabilities may be described by linearized bearing coefficients and parameters. If, however, the displacements are sufficiently far from an equilibrium condition, nonlinear behavior prevails [1, 3]. This phenomena was investigated by various researchers, see for example Choy et al. [3] and Meruane and Pascual [4]. On the other hand, experimental results showed that high loaded slow-speed journals with high viscosity laminar lubricant would improve bearing stability [5]. Lahmar et al. and Singh et al. [6, 7] provided pressure distribution and bearing coefficients for thrust and compliant journal bearings under laminar flow assumption but no discussion were provided in turbulent regime. Unfortunately, conditions of high rotating speed and low viscosity produces turbulent conditions and such results cannot be used because of the assumptions of laminar flow and constant viscosity [8].

The purpose of this paper is first to derive an analytical expression to accurately estimate the turbulent short bearing forces and turbulent dynamic coefficients for the whole range of operating conditions considering shear force effect. For linear and nonlinear stability analysis, Hopf bifurcation theory is used to find the local stability of periodic solutions near bifurcating operating points. Local stability near bifurcation operating points is investigated by Hopf bifurcation analysis and it was shown that for high Sommerfeld number and a dimensionless shaft stiffness less than 10, there is little difference between laminar and turbulent stability boundaries, while significant reduction in the size of the stable region was observed at low Sommerfeld numbers. This reduction in the size of stable region is further increased at high Reynolds numbers. The shaft stiffness was found to play an important role in bifurcating regions on the stable boundaries. State of the art argued that shear force magnitude is in the order of journal bearing clearance over radius ( $C/R$ ) times pressure force; however, provided numerical solutions of bearing force parameters based on finite bearing assumption show that at small eccentricity ratios ( $\varepsilon \leq 0.1$ ), considering short bearing theory ( $L/D \leq 0.5$ ), journal bearings shear force exceeds pressure force [9]. Akers et al. [10] concluded that, for the majority of a journal bearing operating region, inclusion of friction force in fluid film bearing stability analysis expands safe operating region. Utilizing nonlinear stability analysis, Wang and Khonsari [11] showed that the drag force has significant effects on the threshold speed, bifurcation profile, and the size and shape of periodic solutions of a rigid rotor symmetrically supported by two identical laminar journal bearings. They did not expand their analysis for the entire operating region of journal bearings and limited their study based on one operating condition. This paper provides turbulent and drag force effects on the threshold speed of instability of a flexible rotor bearing system under turbulent flow assumption for the entire operating condition. Nonlinear stability analysis, using Hopf bifurcation theory, has been utilized to obtain bifurcation profiles for short journal bearings.

## 2. Dynamic fluid film forces of a rotor-bearing system considering shear effect

Figure 1 demonstrates radial and tangential components of friction forces applied on fluid film bearings. Assuming constant oil viscosity throughout the fluid film, using Ng-Pan-Elrod turbulent model and based on short bearing theory with half-Sommerfeld boundary conditions equations of motion can be derived based on the following steps. Turbulent short bearing pressure forces can be obtained from [12]. Viscous shear stress at journal surface of short fluid film bearings, under turbulent flow condition, can be obtained as follow [13, 14],

$$D_s = \left(\frac{k_\theta}{12}\right) \frac{R\mu\omega}{h} \quad (2.1)$$



**Figure 1.** Fluid film bearing friction force components in radial and tangential directions.

where  $h$  is the fluid film thickness and  $k_\theta$  is turbulent shear coefficient in circumferential direction which can be obtained from [12]. Under laminar flow assumption,  $k_\theta$  approaches 12 ( $Re \rightarrow 0$ ). In this section Ng-Pan-Elrod model is utilized as it is proved to provide more conservative results in rotor bearing system design applications (stability analysis). Under continuity assumption, considering circumferential flow rate remains constant in cavitation region ( $\pi \leq \theta < 2\pi$  for Gumbel boundary condition), the following statement can be obtained [15],

$$q_{cav} = q \text{ (at } h = h_{min}) \quad (2.2)$$

Hence, effective length of fluid film bearing within cavitation region can be calculated as,

$$h \times L_{eff} = h_{min} \times L \rightarrow L_{eff} = \frac{L(1 - \epsilon)}{1 + \epsilon \cos(\theta)} \quad (2.3)$$

Considering a rotating coordinate system and integrating Eqn. (2.1) over the area of the journal, the resulting turbulent shear forces acting on journal bearing in radial and tangential directions may be obtained considering Gumbel ( $\pi$  film) boundary condition which is provided as follow,

$$\begin{aligned} D_R &= -R \left( \int_0^\pi \int_{-\frac{L}{2}}^{\frac{L}{2}} D_s \sin(\theta) dz d\theta + \int_\pi^{2\pi} \int_{-\frac{L_{eff}}{2}}^{\frac{L_{eff}}{2}} D_s \sin(\theta) dz d\theta \right) \\ D_t &= R \left( \int_0^\pi \int_{-\frac{L}{2}}^{\frac{L}{2}} D_s \cos(\theta) dz d\theta + \int_\pi^{2\pi} \int_{-\frac{L_{eff}}{2}}^{\frac{L_{eff}}{2}} D_s \cos(\theta) dz d\theta \right) \end{aligned} \quad (2.4)$$

In order to be able to integrate the above equation to find an analytical expression for shear force in tangential direction, it is assumed  $((1 + \epsilon \cos(\theta)))^{\hat{a}_2} \approx (1 + \hat{a}_3 \epsilon \cos(\theta))$ . The constant  $\hat{a}_3$  may be found by curve fitting via optimization, which is calculated to be 0.8437, and 0.91 for Constantinescu and Ng-Pan-Elrod turbulent models, respectively. By applying the aforementioned assumption along with applying Gumbel boundary condition, the non-dimensional form of the turbulent short bearing shear forces can be derived as,

$$\begin{aligned} D_R &= -\frac{R^2 \mu \omega L}{12C} \left( \frac{H((1 + \epsilon)^{\hat{a}_2} - (1 - \epsilon)^{\hat{a}_2}) + 12\hat{a}_2 \ln\left(\frac{1 + \epsilon}{1 - \epsilon}\right)}{\epsilon \hat{a}_2} \right. \\ &\quad \left. + \frac{H((1 + \epsilon)(1 - \epsilon)^{\hat{a}_2} + (\epsilon - 1)(1 + \epsilon)^{\hat{a}_2}) + 24\epsilon(1 - \hat{a}_2)}{\epsilon(1 + \epsilon)(\hat{a}_2 - 1)} \right) \end{aligned} \quad (2.5)$$

$$D_t = \frac{R^2 \mu \omega L}{12C} \left( \frac{\pi}{\epsilon} \left( (12 + H(1 - \hat{a}_3)) \left( 1 - \frac{1}{\sqrt{1 - \epsilon^2}} \right) + \frac{H \hat{a}_3 (\sqrt{1 - \epsilon^2} - 1) - \epsilon^2 (12 + H + H \hat{a}_3 (\sqrt{1 - \epsilon^2} - 2))}{(1 + \epsilon) \sqrt{1 - \epsilon^2}} \right) \right)$$

where  $H = \hat{a}_1 (\overline{Re})^{\hat{a}_2}$  and  $M = a_1 (\overline{Re})^{a_2}$ . The total force acting on the journal bearing can be calculated as follow,

$$\begin{aligned} F_R^t &= F_R + D_R \\ F_t^t &= F_t + D_t \end{aligned} \quad (2.6)$$

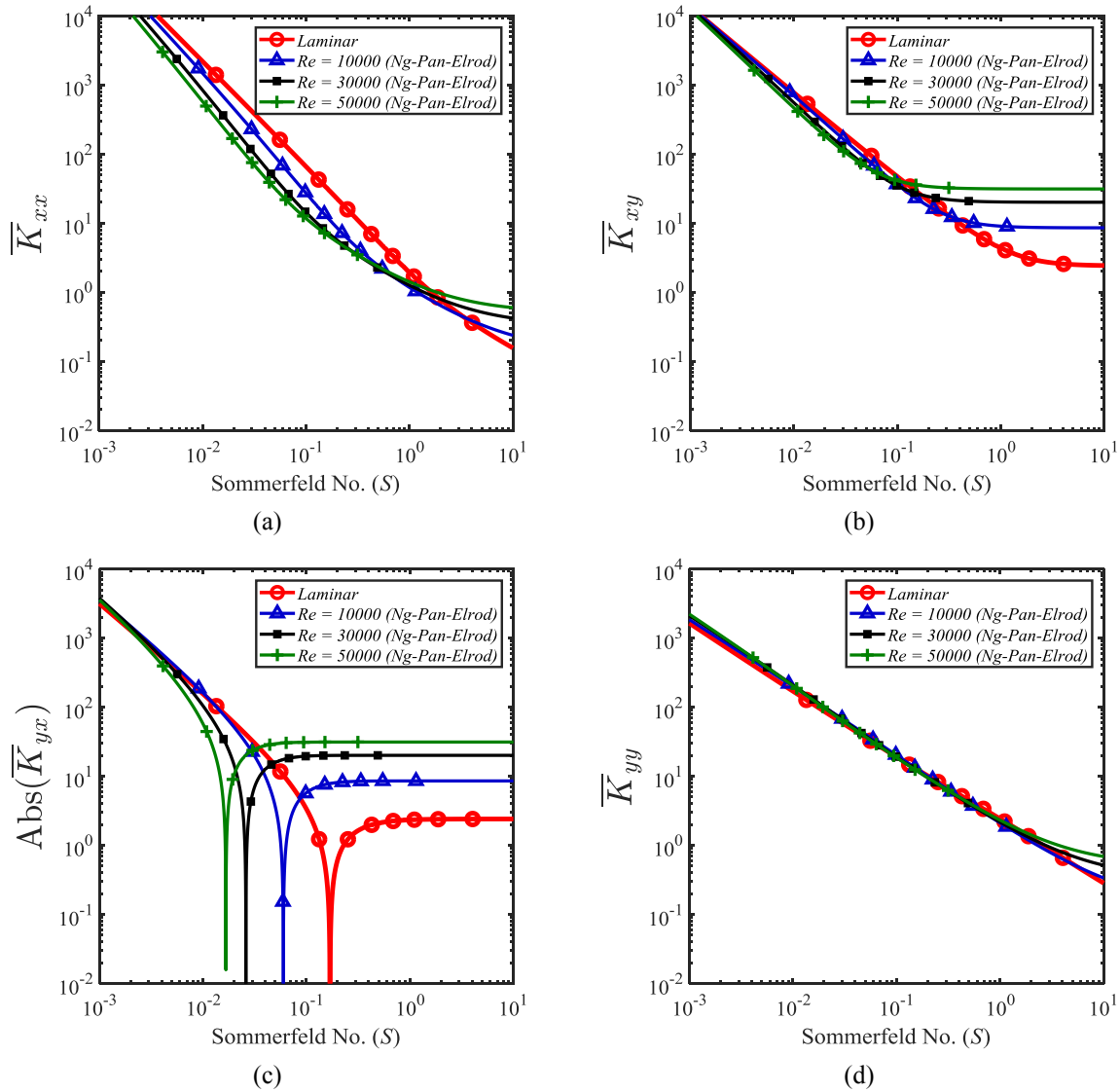
where  $F_R$  and  $F_t$  are analytical short bearing turbulent forces from [12]. Having the force components in  $R$  and  $T$  directions by means of Equation (2.6), the non-dimensional bearing stiffness and damping coefficients in  $R$ - $T$  coordinate system are calculated using [12]. The dimensionless analytically calculated values for stiffness and damping of a short journal bearing coefficients in both laminar and turbulent regimes (considering shear effect) at different Reynolds numbers, for a rotor bearing setup mentioned in [12], are plotted against their calculated Sommerfeld number ( $S = \frac{\mu \omega L R^3}{\pi W C^2}$ ) and are shown in figure 2 and figure 3, respectively. By comparing the results of direct stiffness coefficients, Figure 2 and [12], it can be concluded that stiffness coefficients at high Sommerfeld number increase by increasing Reynolds number considering fluid shear effect; however, opposite trend was observed through ignoring shear force effect at high Sommerfeld numbers. Figure 3 and [12] demonstrate minor changes on damping coefficients when shear effect is included in total force components. By making use of the derivations in [12], the stability margins of a rotor bearing system, for different non-dimensional shaft stiffness coefficients ( $S_z = \frac{C K_s}{W}$ ) are calculated. Stability parameter  $\bar{\Gamma} = \frac{C}{W} M \omega_s^2 = \left( \frac{C K_s}{W} \right) \bar{\omega}_s^2 = S_z \bar{\omega}_s^2$  is plotted vs the Sommerfeld number as shown in figure 4. By comparing [12] and figure 4, it can be seen that as the non-dimensional shaft stiffness increases, by increasing the Reynolds number, the stable operating margin expands towards higher threshold speeds at high Sommerfeld numbers ( $S \geq 0.6$ ) and hence the rotor-bearing system becomes stable at higher rotating spin speeds of the shaft; however, by ignoring the shear force effect, stability margin decreases by increasing the Reynolds number.

### 3. Equations of motion of a flexible rotor bearing system: balanced response

The dimensionless equations of motion for a rotor-bearing system consisting of a flexible rotor supported by two identical fluid-film journal bearings are presented by [12] which can be rewritten in the following dimensionless format,

$$\begin{aligned} \bar{x}_2'' + \frac{S_z}{\bar{F}} (\bar{x} - \epsilon \cos \varphi) - \frac{2}{\bar{F}} &= 0 \\ \bar{y}_2'' + \frac{S_z}{\bar{F}} (\bar{y} - \epsilon \sin \varphi) &= 0 \end{aligned} \quad (3.1)$$

$$\begin{aligned} 2\bar{F}_R^t &= \frac{S_z}{S} (\epsilon - \bar{x} \cos \varphi - \bar{y} \sin \varphi) \\ 2\bar{F}_T^t &= \frac{S_z}{S} (\bar{x} \sin \varphi - \bar{y} \cos \varphi) \end{aligned} \quad (3.2)$$



**Figure 2.** Stiffness coefficients comparison of laminar and turbulent short journal bearings considering shear effect ( $L/D = 0.5$ ), (a)  $K_{xx}$ , (b)  $K_{xy}$ , (c)  $\text{abs}(K_{yx})$ , and (d)  $K_{yy}$ .

where “ $t$ ” represents  $d/(\omega dt)$ .  $\bar{x} = \frac{x}{c}$  and  $\bar{y} = \frac{y}{c}$  provide dimensionless shaft center location in Cartesian coordinate.  $\bar{F}_R^t$  and  $\bar{F}_T^t$  are the total dimensionless force components in radial and tangential directions, respectively. Assume  $x_1 = \epsilon$ ,  $x_2 = \varphi$ ,  $x_3 = \bar{x}$ ,  $x_4 = \bar{x}'$ ,  $x_5 = \bar{y}$ ,  $x_6 = \bar{y}'$ . To solve the above nonlinear equations of motion, two second-order nonlinear equations of motion Equation (3.1), are converted into four first-order Equations (3.5)-(3.8). Through simultaneously solving two terms from Equation (3.2); Equations (3.3) and (3.4) can be calculated in terms of  $(x_1, x_2, x_3, x_5)$ .

The above system of equations as give in Equations (3.3)-(3.8) have the suitable form of  $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \bar{f})$  and possess a steady state equilibrium position  $\mathbf{x}_s$  for the application of the Hopf bifurcation theory. The steady state equilibrium position  $\mathbf{x}_s$  in terms of  $(x_{1s} = \epsilon_s, x_{2s} = \varphi_s, x_{3s} = \bar{x}_s, x_{4s} = \bar{x}'_s, x_{5s} = \bar{y}_s, x_{6s} = \bar{y}'_s)$  can be found analytically by forcing  $\mathbf{f}(\mathbf{x}, \bar{f}) = 0$

$$x_1' = \text{Func}[x_1, x_2, x_3, x_5] \quad (3.3)$$

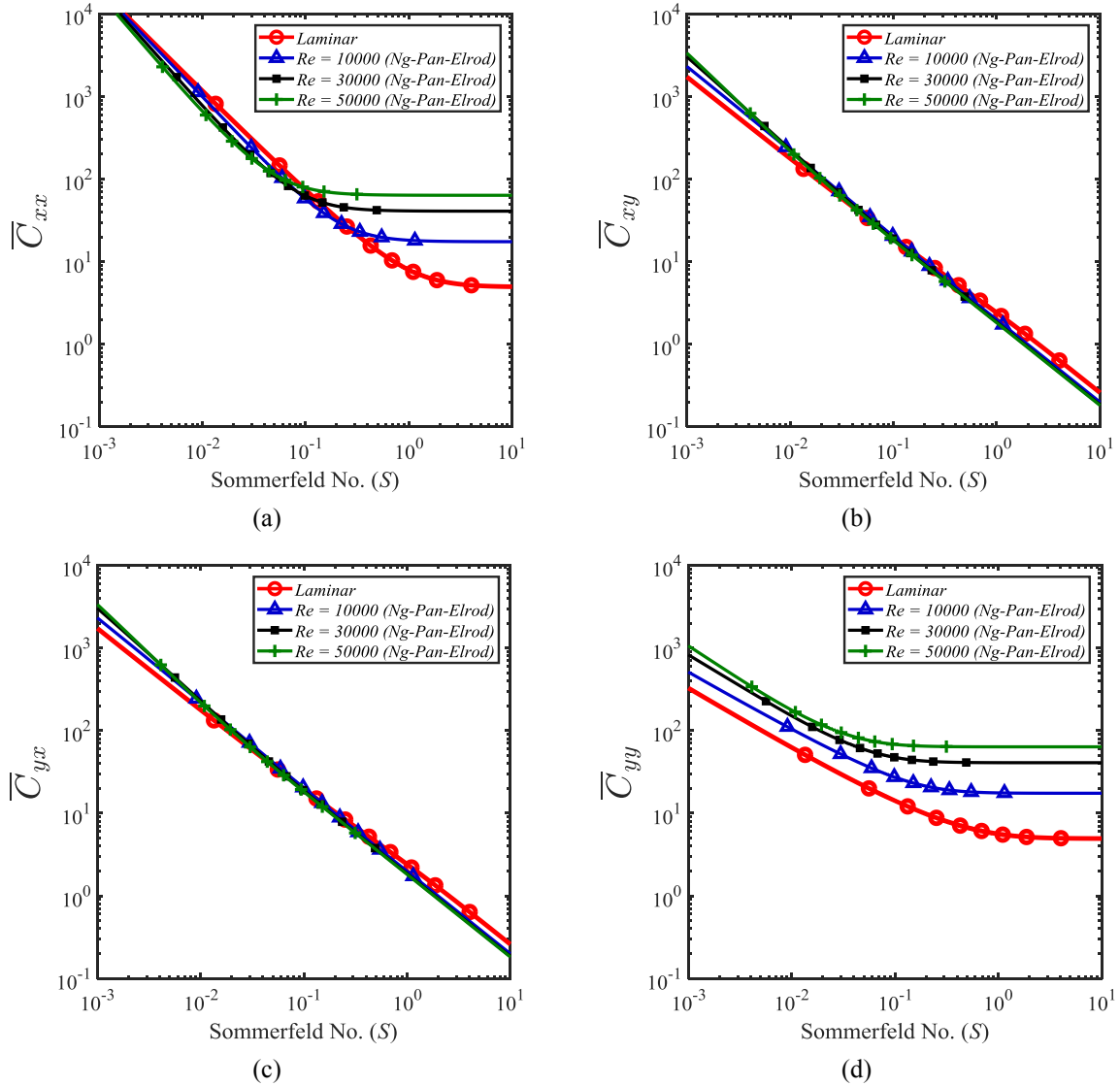
$$x_2' = \text{Func}[x_1, x_2, x_3, x_5] \quad (3.4)$$

$$\dot{x}_3 = x_4 \quad (3.5)$$

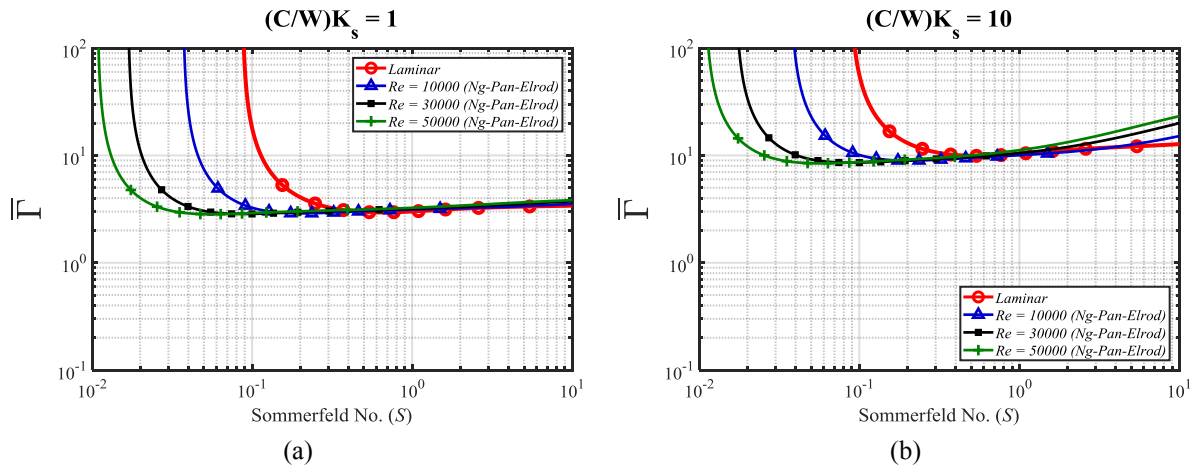
$$\dot{x}_4 = -\frac{S_z}{\bar{F}}(x_3 - x_1 \cos x_2) + \frac{2}{\bar{F}} \quad (3.6)$$

$$\dot{x}_5 = x_6 \quad (3.7)$$

$$\dot{x}_6 = -\frac{S_z}{\bar{F}}(x_5 - x_1 \sin x_2) \quad (3.8)$$



**Figure 3.** Damping coefficients comparison of laminar and turbulent short journal bearings considering shear effect ( $L/D = 0.5$ ), (a)  $C_{xx}$ , (b)  $C_{xy}$ , (c)  $C_{yx}$ , and (d)  $C_{yy}$ .



**Figure 4.** Turbulent and shear effects on the stability parameter  $\Gamma$  of a flexible shaft supported on short length journal bearings ( $L/D = 0.5$ ) at non-dimensional shaft stiffness coefficients of, (a)  $CK_s/W = 1$ , (b)  $CK_s/W = 10$ .

#### 4. Application of hopf bifurcation theory in flexible rotor bearing systems considering shear force effect

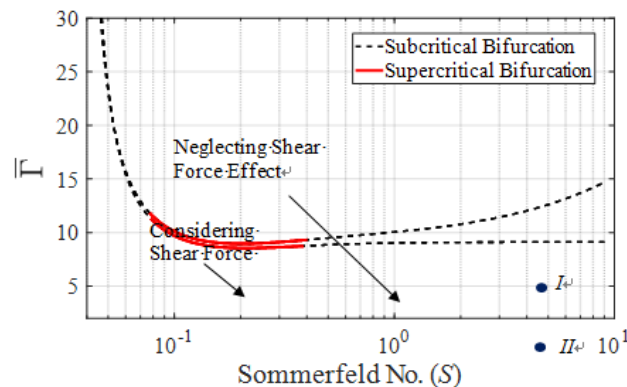
Consider the rotor-bearing system whose specifications are listed in table 1. This perfectly balanced rotor-bearing system consists of a flexible shaft symmetrically supported by two identical fluid-film journal bearings. Based on the Hopf bifurcation theory explained in [12], the nonlinear stability analysis of a rotor bearing system tabulated in table 1, considering shear force effect, is presented in figure 5.

Figure 5 demonstrates significant discrepancies between two stability regions (with and without the consideration of shear effect) at high Sommerfeld numbers ( $S \geq 0.4$ ). The difference between two bifurcation profiles grows in size by increasing the Sommerfeld number.

**Table 1.** Specification of the rotor-bearing system.

|                                             |                         |
|---------------------------------------------|-------------------------|
| Journal bearing diameter ( $D$ )            | 0.0254 m                |
| Journal bearing length ( $L$ )              | 0.0127 m                |
| Nondimensionalize shaft stiffness ( $S_z$ ) | 10                      |
| Average Reynolds number $\overline{Re}$     | 10000                   |
| Journal bearing clearance ( $C$ )           | $50.8 \times 10^{-6}$ m |

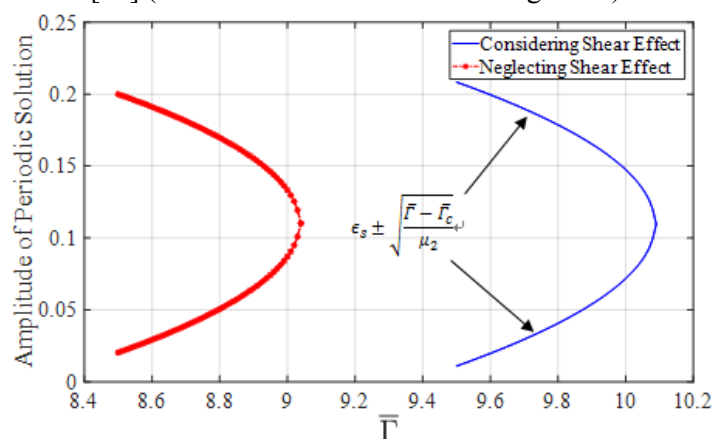
Bifurcation profile results at Sommerfeld ( $S = 1$ ) are also demonstrated in figure 6 to compare dimensionless instability threshold speed with and without consideration of shear force effect. Figure 6 shows that the periodic solutions of the journal orbit shrinks to a single point as the running speed approaches the critical value  $\bar{\Gamma}_c$ . It is shown that the dimensionless instability threshold speed  $\bar{\Gamma}_{cr}$  increases from 9.04 to 10.09 (11.6% increase) through considering shear force components in the equations of motion. It should be noted that by converting dimensionless instability threshold speed to a dimensional form ( $\omega_{cr} = \sqrt{\frac{g\bar{\Gamma}_{cr}}{2C}}$ ), the instability threshold speed with consideration of the drag force is 9,881.54 rpm and the instability threshold speed neglecting the drag force becomes 8,921.61 rpm (959.93 rpm difference).



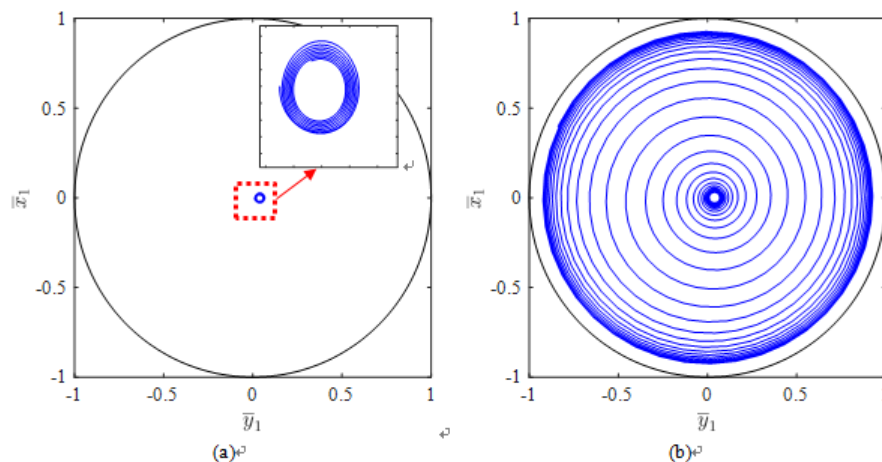
**Figure 5.** Nonlinear stability analysis comparison with and without consideration of shear force effect.

The amplitude of the periodic solution of the journal orbit corresponding to a specific running speed  $\bar{r}$  is symmetrical at  $\bar{r}_c$  and is bounded by  $\epsilon_s \pm \sqrt{\frac{\bar{r} - \bar{r}_c}{\mu_2}}$ . Where  $\epsilon_s$  is the static equilibrium position of journal center at speed  $\bar{r}_c$ . Trajectories of operating point  $I$  ( $S = 3$ ,  $\bar{r} = 10$ ), shown in figure 5, are calculated for two different cases with and without consideration of the shear force effect and demonstrated in figure 7. As it can be seen from figure 7, stable region obtained based on consideration of shear force effect can become unstable by neglecting the drag force effect in equations of motion of a flexible rotor-bearing system.

The shear force effect results on the stability of a flexible rotor-bearing system presented above are consistent with the outcomes provided by [11, 16]. Newkirk and Grobel [16] concluded that shear component applying opposite to the direction of rotating speed of the shaft tends to stabilize the rotor bearing system. It shall be noted that not only the difference between two bifurcation profiles (with and without consideration of shear effect) grows by increasing the Sommerfeld number, but also the discrepancy becomes significant by increasing the Reynolds number. Thus, inclusion the shear force effect in the rotor bearing equations of motion always causes the system to become more stable, especially at high Sommerfeld numbers, by increasing the Reynolds number which is in contradiction with the results obtained in [12] (where shear force effect was neglected).



**Figure 6.** Bifurcation profile comparison with and without consideration of shear effect at Sommerfeld ( $S$ ) = 1.



**Figure 7.** Journal bearing trajectories of a balanced rotor-bearing system at operating point  $I$ , (a) considering shear effect, (b) neglecting shear effect.

## 5. Conclusions and rotor bearing system design guidelines

In this paper, the stability of flexible shaft supported on end journal bearings was studied for a range of operating conditions in turbulent flow bearings considering shear force effect. Utilizing Gumbel boundary condition, analytical expressions for eight spring and damping constants were derived for the dynamic turbulent oil film. This was obtained by linearizing the oil film forces around the steady-state equilibrium position of the journal center and by incorporating the shear effects for Ng-Pan-Elrod model. The calculated dynamic coefficients were then utilized to obtain the whirl onset velocity for a flexible rotor supported by two identical journal bearings at various Reynolds numbers. Stable operating region of flexible shafts supported on both turbulent and laminar journal bearings were shown to grow in size by increasing the shaft non-dimensional stiffness parameter. At high shaft stiffness numbers ( $S_z \geq 5$ ), it was found that at low load and high Sommerfeld regions  $S \geq 0.7$  the stability will be affected by the type of flow within the bearing chamber and stable operating region increase by increasing the Reynolds number, a region where the laminar based stability analysis would consider as un-safe operating condition. Neglecting the shear force effect would result in opposite trend at high Sommerfeld numbers (see [12]). On the other hand, as the static load on shaft increases in magnitude (low Sommerfeld numbers), the unstable operating region grows in size and will result in the unstable oil whirl to happen at even higher static loads.

The Hopf bifurcation theory was used to study the orbital stability of periodic solutions near bifurcating points of the rotor bearing system, i.e. when the operating spin speed of the shaft is close to its threshold speed of instability. It was shown that, considering shear force effect, would increase stable operating region, especially when rotor bearing system stability type undergoes subcritical bifurcation. The instability threshold speed and its predicted bifurcation type using the proposed method could be beneficial at the design stage as well as for troubleshooting purposes of an unstable rotor-bearing system. If the bifurcation type of a rotor bearing system, operating at  $S^*$  and  $\bar{\Gamma}^*$ , is in the subcritical bifurcation region, effort should be made on changing  $S^*$  through controlling the oil temperature or even oil grade to shift bifurcation type from subcritical to supercritical.

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