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Thermal Fisher information and entropy squeezing for superconducting qubits

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ABSTRACT

We investigate the effects of temperature on quantum Fisher information (QFI), fidelity, and entropy squeezing in two interacting superconducting qubits (SCQs). We show how the variations in these quantum quantifiers may be controlled by carefully selecting the model's physical parameters (PPM). We obtain a trapping phenomena for Fisher information, allowing us to improve the QFI and retain the amount of information while also preserving it from information loss due to temperature impacts. We evaluate the fidelity of the SCQ to explain the dynamic aspects of the QFI in relation to the PPM. Furthermore, we examine the influence of the PPM on the variation in the squeezing entropy for SCQs. The results indicate how, in terms of quantum measurement theory, the system under consideration might be valuable in enabling the realization of experiments under optimal conditions.

Introduction

Since Feynman first proposed the quantum computer idea in 1982, the race to move on from classical computer to quantum computer has been ongoing. To realize this type of computer, the classical bit has been replaced with the qubit. These quantum systems are attracting research attention not only because of their high speed and effectiveness, but also because of the diverse nature of the controllable quantum states in these systems [1–9]. Physical systems that are used to process and transmit quantum information should obey the laws of quantum mechanics, e.g., superposition and nonlocal correlation. The simplest composite systems that can display behavior that follows these laws consist of two-level systems. SCQs are regarded as the most convenient candidate devices for performing quantum information processing. Therefore, preparation of SCQs and the processing of quantum information using these two-level systems are the main tasks that must be addressed. In addition, it is also intersting to explore the effects of environmental noise and temperatures on the operation of SCQs [10–14]. The quantum coherence of SCQs can be enhanced by diminishing their sensitivity to charge noise through the addition of a large shunt capacitance to the Josephson junction. At present, the major goal of scientific research in this field is to protect the quantum states from temperature and decoherence effects by considering the advantages of efficient coupling, which can be enabled between SCQs through interactions with a common tunable coupler [15].

Recently, a great deal of work has been performed on analyzing the precision of parameter estimation (PE) [16,17] by examining the effects of particle loss, state generation, and environmental noise sources [18–22]. QFI plays a vital role in detecting the sensitivity of the quantum states based on changes in the PE. QFI was first used by Fisher to study PE [23]. For each given measurement, the QFI is used to detect the maximum information about the PE. The estimation of unknown parameters aids in the development of various fields for different tasks in quantum technology by improving the methods used to measure the sensitivity of these parameters [17,24–27]. The essential task of estimation theory in this case is to determine the unknown parameter rate coming from physical effects in a quantum system with high accuracy. In this context, the lower bound of the Cramér-Rao inequality can be

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characterized using the QFI [28]. Therefore, the QFI can ultimately be regarded as the primary problem to be solved. Recently, the dynamics of the QFI have been studied widely in the literature by considering the fluctuation and decoherence phenomena that result from interactions between the quantum system and its own environment [29].

Over the past two decades, squeezed states have been discussed in the context of quantized electromagnetic fields [30] and atomic systems [31], with many important and diverse applications being discovered [32]. The spin-squeezed model, which is dependent on the Raman scattering that occurs in the presence of strong field pulses, was implemented to transfer the correlations between atomic systems and light-based systems [33]. The squeezing effect in systems of spins with optimal and nonlinear features has been investigated in detail [34]. Investigation of the correlations between atomic spin squeezing and bosonic quadrature has also been considered [35]. In addition, various experimental realizations were achieved by considering a set of V-type atoms [36]. In the aforementioned studie, the Heisenberg uncertainty relation (HUR) has been introduced for analyzing the atomic squeezing. However, the HUR can only exhibit limited squeezing information about quantum systems, particularly in the case where the atomic inversion has a zero value [30], where the HUR limitations have recently been overcome by application of the entropic uncertainty relation (EUR) [31].

Motivated by the considerations described above, the main aim of this work is to study the variations in the QFI, the fidelity, and entropy squeezing in a system of two SCQs under the influence of an externally applied temperature. We demonstrate the influences of the PPM on the variations in the quantum quantifiers and extract the various quantum phenomena. We find a trapping phenomenon for the Fisher information, which makes it possible to enhance the QFI and preserve the amount of information, while also protecting it from loss due to the temperature effect. We evaluate the fidelity of the SCQs to explain the dynamic features of the QFI with respect to the PPM. Furthermore, we study the effects of the PPM on the variations in the squeezing entropy for the SCQs and present the relationship between the PE precision and the squeezing effect in the proposed model. The results obtained show how the system studied in the context of quantum measurement theory here could be useful in enabling implementation of experiments using optimal conditions.

The remainder of this manuscript is structured as follows. Section 2 presents the physical model of the two SCQs as well as the formalism that demonstrates the model's dependency on the physical parameters. Section 3 discusses the quantum quantifiers used in this study.

Section 4 presents and analyzes the acquired results. Finally, in Section 5, conclusions are drawn.

Hamiltonian and dynamics

Here, we focus on two interacting SCQs connected by a fixed capacitor. This capacitor is utilized in experiments to regulate and manipulate the interactions between the two SCQs. This physical model has attracted a lot of interest since it has a variety of possible applications [37–40]. The Hamiltonian of the two SCQs can be written as [37]:

$$H = \left[-2E_{C1}\left(\frac{1}{2}-n_{g1}\right)-E_m\left(\frac{1}{2}-n_{g2}\right)\right]\sigma_{z1}+\left[-2E_{C2}\left(\frac{1}{2}-n_{g2}\right)-E_m\left(\frac{1}{2}-n_{g1}\right)\right]\sigma_{z2}$$

$$-\frac{1}{2}(E_{J1}\sigma_{x1}+E_{J2}\sigma_{x2}-2E_m\sigma_{zz}), \tag{1}$$

where E_{Ci} and E_{Ji} design the charging and Josephson energies of the i^{th} SCQ, respectively. E_m describes the mutual coupling between the SCQs. $\sigma_{x1} = \sigma_x \otimes I$, $\sigma_{x2} = I \otimes \sigma_x$, and $\sigma_{zz} = \sigma_z \otimes \sigma_z$, where $\sigma_{x,z}$ are the x and z Pauli operators, and I represents the identity operator. The parameter $n_{gi} = V_{gi}C_{gi}/2e$ denotes the normalized SCQ, where V_{gi} and C_{gi} are, respectively, the voltage and the control gate capacitance.

For the sake of simplicity, we take the situation when $n_{g1} = n_{g2} = 1/2$, i.e., the degenerate point, which corresponds to the condition of noise insensitivity. In this case, the Hamiltonian in Eq. (1) becomes.

$$H = -\frac{1}{2}(E_{J1}\sigma_{x1}+E_{J2}\sigma_{x2}-2E_m\sigma_{zz}), \tag{2}$$

which is independent of E_{Ci}.

We highlight how the QFI, entropy squeezing, and fidelity are dependent on the external temperature. The thermal density matrix $\rho(T)$ at equilibrium for the two-SCQ system can be given in terms of eigenvalues and eigenvectors as.

$$\rho(T) = \frac{1}{Z}e^{-\beta H} = \frac{1}{Z}\sum_n e^{-\beta\epsilon_n}|\psi_n\rangle\langle\psi_n|, \tag{3}$$

where Z designs the partition function, and $\beta = 1/k_BT$. In order to evaluate the measures of quantumness, we consider numerical evaluation for obtaining the eigenvectors $|\psi_n\rangle$ and eigenvalues ϵ_n of the Hamiltonian.

Quantum Fisher information and entropy squeezing

Here, we provide a brief overview of the system's physical parameters, such as the QFI, fidelity, and squeezing entropy.

The QFI may be used to determine how precisely a quantum state can detect an unknown parameter, indicated by ϕ. The QFI is expressed as:

$$Q_F = \text{Tr}\left[\rho(\phi)L^2\right], \tag{4}$$

where $\rho(\phi)$ represents the density operator of the system, and L defines the symmetric logarithmic derivative that satisfies.

$$\partial_\phi\rho(\phi) = \frac{1}{2}[\rho(\phi)L + L\rho(\phi)]. \tag{5}$$

The Cramér-Rao inequality is using to describe the smallest possible uncertainty in the PE.

$$\Delta\phi \geq 1\sqrt{\mu Q_F},$$

where $(\Delta\phi)^2$ represents the mean square error, and μ is the number of repeated trials.

We investigate the thermal density operator of the state of the two SCQs in the representation spanned by the product states $|0\rangle_1 \otimes |0\rangle_2$, $|2\rangle = |0\rangle_1 \otimes |1\rangle_2$, $|3\rangle = |1\rangle_1 \otimes |0\rangle_3$, and $|4\rangle = |1\rangle_1 \otimes |1\rangle_2$ to demonstrate the reliance of the Fisher information on both the Josephson energies and the temperature. We specify the singlet-SCQ phase gate using.

$$U(\phi) := e^{i\phi}|1\rangle\langle 1| + |0\rangle\langle 0|, \tag{7}$$

and the output thermal state is given by $\rho_{\text{out}} = U\rho_{\text{int}}U^+$. After the phase operation is performed, the SCQs will be subject to an external temperature effect.

Another intriguing variable to be investigated in this study is fidelity. The fidelity is provided by:

$$F(T) = \left(tr\sqrt{\sqrt{\rho(0)}\,\rho(T)\sqrt{\rho(0)}}\right)^2 \tag{8}$$

which follows the inequality $0 \leq F \leq 1$. The states become less distinct from one another as fidelity grows. Because of these characteristics, the function F is seen as an appealing measure of how effectively a ket state may be preserved under the PPM effect.

Finally, we evaluate the squeezing information entropy using the HUR. It has been demonstrated in some circumstances that the HUR cannot offer appropriate information concerning atomic squeezing [59].

For a two-level system, which is described by the spin operators $\widehat{S}_x$, $\widehat{S}_y$, and $\widehat{S}_z$, and verified by the relation $[\widehat{S}_x, \widehat{S}_y] = i\widehat{S}_z$, the uncertainty relation caused by the commutation relation mentioned above is.

$$\Delta\widehat{S}_x\Delta\widehat{S}_y \geq \frac{1}{2}|\langle\widehat{S}_z\rangle|. \tag{9}$$

where

$$\Delta\widehat{S}_\alpha = \sqrt{\langle\widehat{S}_\alpha^2\rangle - \langle\widehat{S}_\alpha\rangle^2}. \tag{10}$$

The optimal entropic uncertainty relation for sets of observables operating in even N-dimensional Hilbert space has been established using quantum entropy [60]. This uncertainty relationship is denoted by:

$$\sum_{\alpha=1}^{N+1} H(\widehat{S}_\alpha) \geq \frac{N}{2}\ln\left(\frac{N}{2}\right) + \left(1+\frac{N}{2}\right)\ln\left(1+\frac{N}{2}\right), \tag{11}$$

$H(\widehat{S}_\alpha)$ denotes the information entropy that corrsponds to the variable $\widehat{S}_\alpha$. The Shannon entropy is then given by:

$$H(\widehat{S}_\alpha) = -\sum_{i=1}^{N} P_i(\widehat{S}_\alpha)\ln P_i(\widehat{S}_\alpha), \alpha = x, y, z. \tag{12}$$

Here, $P_i(\widehat{S}_\alpha) = \langle\Psi_{\alpha i}|\rho|\Psi_{\alpha i}\rangle$. $|\Psi_{\alpha i}\rangle$ is the eigenvector of $\widehat{S}_\alpha$ $(\widehat{S}_\alpha|\Psi_{\alpha i}\rangle = \lambda_{\alpha i}|\Psi_{\alpha i}\rangle)$ for $i = 1, 2, ..., N$. For a two-level atom system (where $N = 2$), we obtain.

$$H(\widehat{S}_\alpha) = -\frac{1}{2}(\rho_\alpha(t)+1)\ln\left(\frac{1}{2}[\rho_\alpha(t)+1]\right)$$

$$-\frac{1}{2}(1-\rho_\alpha(t))\ln\left(\frac{1}{2}[1-\rho_\alpha(t)]\right), \forall\alpha = x, y, z.$$

where $\leq H(\widehat{S}_\alpha)$ is composed of values between 0 and ln2 with Shannon entropies corresponding to the spin operators $\widehat{S}_x, \widehat{S}_y, \widehat{S}_z$ that verify the following relation:

$$H(\widehat{S}_x) + H(\widehat{S}_y) + H(\widehat{S}_z) \geq 2\ln 2. \tag{13}$$

By defining $\delta H(\widehat{S}_\alpha) = e^{[H(\widehat{S}_\alpha)]}$, the inequality above becomes.

$$\delta H(\widehat{S}_x)\delta H(\widehat{S}_y)\delta H(\widehat{S}_z) \geq 4. \tag{14}$$

The EUR defined in Eq. (14) illustrates the impossibility of extracting the entirety of the information simultaneously for $\widehat{S}_x(t), \widehat{S}_y(t)$, and $\widehat{S}_z(t)$, where $\delta H(\widehat{S}_x)$, $\delta H(\widehat{S}_y)$, and $\delta H(\widehat{S}_z)$ correspond to the uncertainty measures of the polarization components $\widehat{S}_x$, $\widehat{S}_y$ and $\widehat{S}_z$, respectively. The relation given in Eq. (14) determines the squeezing entropy [60], and $\widehat{S}_\alpha(\alpha = x$ or $y)$ is "squeezed in entropy" if.

$$E(\widehat{S}_\alpha) = \delta H(\widehat{S}_\alpha) - \frac{2}{(\delta H(\widehat{S}_z))^{12}} < 0, \alpha = x, y. \tag{15}$$

Results and discussion

Based on the equation for the QFI, we can examine the precision of the PE for the SCQ while considering the effects of the Josephson energies and the temperature. In Fig. 1, we show the variations in the function Q_F of the SCQ as a function of the Josephson energy E_J for numerous values of temperature T. In general, Fig. 1 displays some interesting features of the QFI relative to different values of E_J, where the PE precision is broadly depends on the Josephson energies of the SCQs, thus exhibiting the trapping phenomenon of the QFI. Indeed, for each value of T, the QFI increases from a minimal initial value at E_J, and then tends to reach a steady value as the value of the parameter E_J increases significantly. Furthermore, the function Q_F reaches a steady value rapidly over small temperature intervals. The increase in the amount of QFI is accompanied by an improvement in the estimation precision for the different temperature ranges. This phenomenon is regarded as an outcome of the increase in coherence within these temperature limits. In Fig. 2, we show the variations in the function Q_F of the SCQ as a function of temperature T for numerous values of Josephson energy E_J. We find that the Q_F of the SCQ decays exponentially and then the PE precision will be suppressed with increasing temperature. Furthermore, we can see that retardation of the QFI loss may be achieved by regularizing the parameter E_J. With increasing energy E_J for the SCQs, the QFI will decrease slowly with increases in T. In summary, the results obtained here show that control and enhancement of the PE precision can be achieved by considered choice of the PPM.

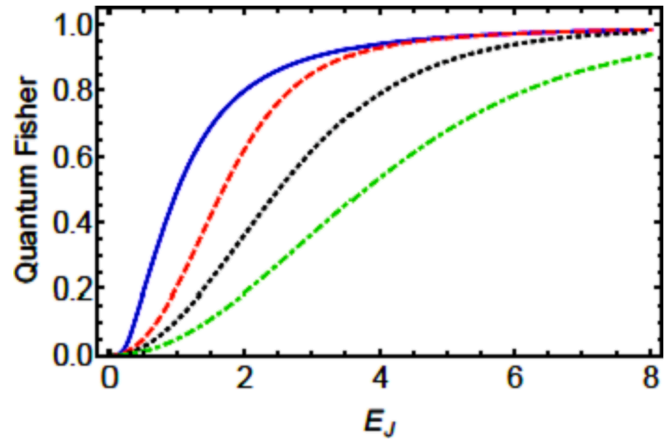

Fig. 1. Thermal quantum Fisher information parameter Q_F of the qubit system as a function of Josephson energy E_J for different values of temperature T with $E_m = 1$. The solid blue line exhibits the case where $T = 0.02$, the red dashed line exhibits the case where $T = 0.6$, the black dotted line exhibits the case where $T = 1.2$, and the green dot-dashed line exhibits the case where $T = 2$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

In Figs. 3 and 4, we have plotted the variations in the fidelity versus the Josephson energy E_J and the temperature T, respectively. From these figures, we see that the fidelity is affected in a similar manner to the QFI. We can see that the functions F_Q and Q_F both exhibit comparable behavior in relation to different parameter values of E_J and T. These

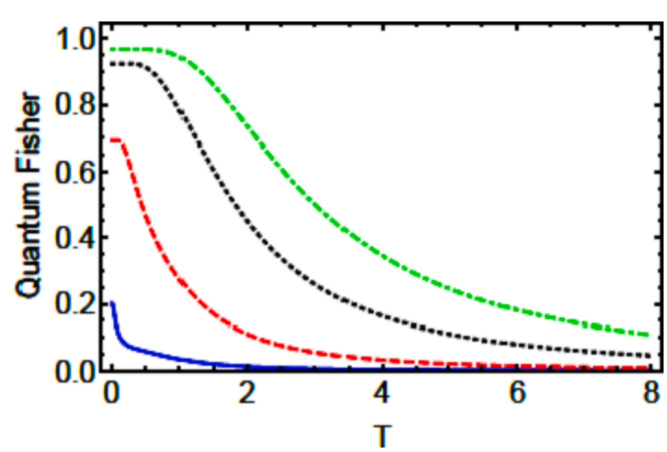

Fig. 2. Thermal quantum Fisher information parameter Q_F of the qubit system as a function of temperature T for various values of Josephson energy E_J with $E_m = 1$. The solid blue curve exhibits the case where $E_J = 0.5$, the red dashed curve exhibits the case where $E_J = 1.5$, the black dotted curve exhibits the case where $E_J = 3.5$, and the green dot-dashed curve exhibits the case where $T = 5.5$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

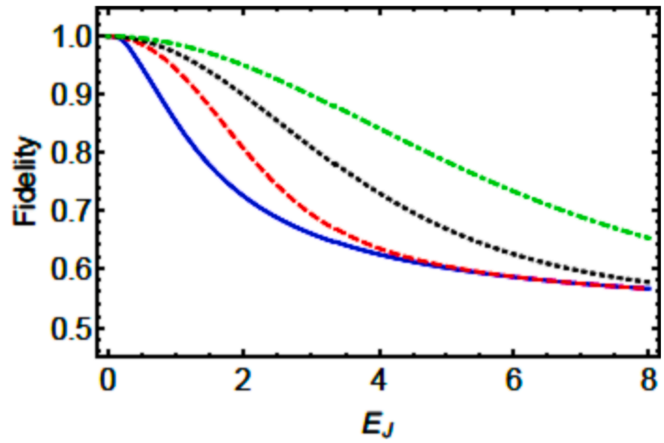

Fig. 3. Fidelity F of the qubit system as a function of Josephson energy E_J for different values of temperature T with $E_m = 1$. The solid blue line exhibits the case where $T = 0.02$, the red dashed line exhibits the case where $T = 0.6$, the black dotted line exhibits the case where $T = 1.2$, and the green dot-dashed line exhibits the case where $T = 2$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

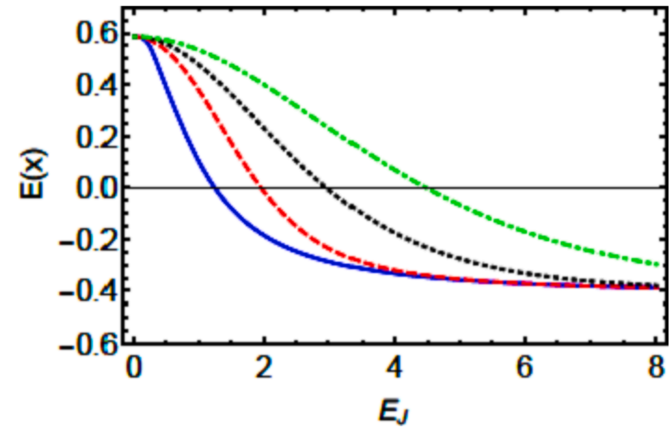

Fig. 5. Entropy squeezing $E(x)$ of the SC-qubit system as a function of Josephson energy E_J for different values of temperature T with $E_m = 1$. The solid blue line exhibits the case where $T = 0.02$, the red dashed line exhibits the case where $T = 0.6$, the black dotted line exhibits the case where $T = 1.2$, and the green dot-dashed line exhibits the case where $T = 2$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

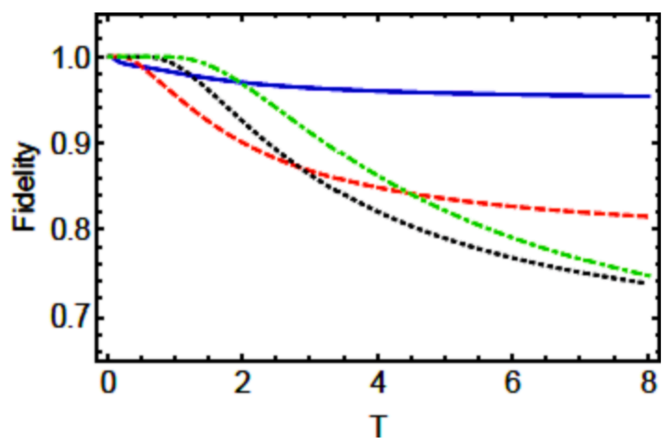

Fig. 4. Fidelity F of the qubit system as a function of temperature T for various values of Josephson energy E_J with $E_m = 1$. The solid blue curve exhibits the case where $E_J = 0.5$, the red dashed curve exhibits the case where $E_J = 1.5$, the black dotted curve exhibits the case where $E_J = 3.5$, and the green dot-dashed curve exhibits the case where $T = 5.5$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

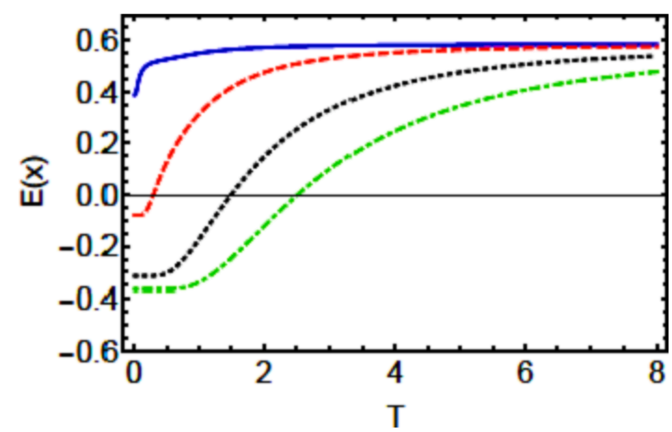

Fig. 6. Entropy squeezing $E(x)$ of the qubit system as a function of temperature T for various values of Josephson energy E_J with $E_m = 1$. The solid blue curve exhibits the case where $E_J = 0.5$, the red dashed curve exhibits the case where $E_J = 1.5$, the black dotted curve exhibits the case where $E_J = 3.5$, and the green dot-dashed curve exhibits the case where $T = 5.5$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

features show that fidelity is a suitable measure for describing this sort of information in the current model.

To illustrate the effect of the PPM on the squeezing of the SCQs, Fig. 5 shows the variation in the entropy squeezing $E(\sigma_x)$ as a function of energy E_J for various temperature values. From the figure, we can see that the squeezing only occurs in the variables σ_x in larger energy ranges. Interestingly, the value at which the function $E(x)$ reaches a negative value is dependent on the value of the temperature T. When T is reduced, $E(x)$ reaches a negative value rapidly, and then enhances the squeezing effect with variations in the energy E_J. In Fig. 6, we show the variations in $E(x)$ versus the temperature T for different values of E_J. We can see that $E(x)$ increases with increasing temperature, and then suppresses the squeezing effect. In this context, the squeezing behavior occurs within the limits of small values of T and large values of E_J. Additionally, we find that the function $E(x)$ is affected in a similar manner to the QFI. We can see that the functions $E(x)$ and Q_F show an inverse monotonic relation according to the values of the parameters E_J and T. These features also mean that the function $E(x)$ is suitable as an indicator to describe the variations in the QFI in the present model.

Conclusion

We have explored the relationships among the QFI, the fidelity, and entropy squeezing in a system of two interacting SCQs. We have considered the effects of both the Josephson energy and the external temperature. We have explained how the variations in the quantum quantifiers can be controlled by making a considered selection of the physical parameters of the model (PPM). We have also discussed the PE precision for the SCQs with respect to the PPM. We have shown that the variations in the QFI for the SCQs are broadly dependent on the PPM. We have obtained a trapping phenomenon for the Fisher information, which makes it possible to enhance the QFI and preserve the amount of information, while also protecting it from information loss as a result of the temperature effect. We have evaluated the fidelity of the SCQs to explain the dynamic features of the QFI with respect to the PPM. Additionally, we have studied the effect of the PPM on the variations in the squeezing entropy for the SCQs. Furthermore, we have shown that the QFI and the entropy squeezing behavior exhibit an inverse monotonic relationship according to the values of the PPM. The results obtained here show how the system studied in terms of quantum measurement theory could be useful in enabling implementation of experiments under optimal conditions.

CRediT authorship contribution statement

Zainab M.H. El-Qahtani: Visualization, Supervision, Project administration. **K. Berrada:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing. **S. Abdel-Khalek:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing. **H. Eleuch:** Validation, Funding acquisition, Investigation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Makhlin Y, Schon G, Shnirman A. Rev Mod Phys 2001;73:357.
[2] You JQ, Nori F. Phys Today 2005;58:42.
[3] Clarke J, Wilhelm FK. Nature 2008;453:1031.
[4] Schoelkopf RJ, Girvin SM. Nature 2008;664:664.
[5] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature **398**, 786 (1999) O. Astafiev, Y. A. Pashkin, T. Yamamoto, Y. Nakamura, and J. S. Tsai, Phys. Rev. B **69**, 180507(R) (2004).
[6] Berrada K, Fanchini FF, Abdel-Khalek S. Phy Rev A 2012;85:052315.
[7] Ventura D, Kak S. Inf Sci 2000;128:147–8.
[8] Ladd TD, Jelezko F, Laflamme R, Nakamura Y, Monroe C, O'Brien JL. Nature 2010; 464:45–53.
[9] Kelly J, Barends R, Fowler AG, Megrant A, Jeffrey E, White TC, et al. Nature 2015; 519:66–9.
[10] Song C, Xu K, Liu W, Yang C-P, Zheng S-B, Deng H, et al. Phys Rev Lett 2017;119: 180511.
[11] Bernien H, Schwartz S, Keesling A, Levine H, Omran A, Pichler H, et al. Nature 2017;551:579–84.
[12] Zhang J, Pagano G, Hess PW, Kyprianidis A, Becker P, Kaplan H, et al. Nature 2017; 551:601–4.
[13] Sung KJ, Yao J, Harrigan MP, Rubin NC, Jiang Z, Lin L, et al. Quantum Sci Technol 2020;5:044008.
[14] Xiang-Ping L, Mao-Fa F, Xiao-Juan Z, Jian-Wu C. Chin Phys Lett 2006;23: 123138–41.
[15] An J-H, Wang S-J, Luo H-G. Physica A 2007;382:753–64.
[16] Giovannetti V, Lloyd S, Maccone L. Nat Photonics 2011;5:222.
[17] Dowling J. Contemp Phys 2008;49:125.
[18] Modi K, Cable H, Williamson M, Vedral V. Phys Rev X 2011;1:021022.
[19] Pires DP, Silva IA, deAzevedo ER, Soares-Pinto DO, Filgueiras JG. Phys Rev A 2018; 98:032101.
[20] Fiderer LJ, Braun D. Nat Commun 2018;9:1351.
[21] Joo J, Munro WJ, Spiller TP. Phys Rev Lett 2011;107:083601.
[22] Berrada K, Abdel Khalek S, Raymond Ooi CH. Phys Rev A 2012;86:033823.
[23] R. A. Fisher, Proc. Cambridge Phil. Soc. **22**, 700 (1929) reprinted in Collected Papers of R. A. Fisher, edited by J. H. Bennett (Univ. of Adelaide Press, South Australia, 1972), pp. 15-40].
[24] Giovannetti V, Lloyd S, Maccone L. Science 2004;306:1330.
[25] Simmons S, Jones JA, Karlen SD, Ardavan A, Morton JJL. Phys Rev A 2010;82: 022330.
[26] Higgins BL, Berry DW, Bartlett SD, Wiseman HM, Pryde GJ. Nature 2007;450:393.
[27] Dorner U, Smith BJ, Lundeen JS, Wasilewski W, Banaszek K, Walmsley IA. Phys Rev A 2009;80:013825.
[28] Cramer H. Mathematical Methods of Statistics. Princeton, NJ: Princeton University Press; 1946.
[29] Chin AW, Huelga SF, Plenio MB. Phys Rev Lett 2012;109:233601.
[30] Song H, Luo S, Hong Y. Phys Rev A 2015;91:042110.
[31] Xiao-Ming Lu, Wang X, Sun CP. Phys Rev A 2010;82:042103.
[32] Berrada K. J Opt Soc Am B 2017;34:1912.
[33] Li Y-L, Xiao X, Yao Y. Phys Rev A 2015;91:052105.
[34] Mogilevtsev D, Garusov E, Korolkov MV, Shatokhin VN, Cavalcanti SB. Phys Rev A 2018;98:042116.
[35] Drummond PD, Ficek Z. Quantum Squeezing. Berlin: Springer; 2004.
[36] Wodkiewicz K. Phys Rev B 1981;32:4750.
[37] Shaw MD, Schneiderman JF, Bueno J, Palmer BS, Delsing P, Echternach PM. Phys Rev B 2009;79:014516.
[38] Paladino E, Mastellone A, DArrigo A, Falci G. Phys Rev B 2010;81:052502.
[39] Paraoanu GS. Phys Rev B 2006;74:140504(R).
[40] Li J, Chalapat K, Paraoanu GS. Phys Rev B 2008;78:064503.