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Do benchmark African equity indices exhibit the stylized facts?

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ABSTRACT

This paper investigates if benchmark African equity indices exhibit the stylized facts reported for financial time series returns. The returns distributions of the Africa All-Share, Large, Medium and Small Company Indices were found to be leptokurtotic, had fat-tails, over time experienced volatility clustering and exhibited long memory in volatility. Both the All-Share and Large Company Indices were found to exhibit leverage effects. In contrast, positive shocks had a greater impact on future volatility for the Small Company Index which implies a reverse leverage effect. This finding could reflect a bull/bubble market for small capitalisation stocks in Africa.

1. Introduction

Benchmark indices are an important aspect of professional finance. They are used to evaluate active portfolio manager's performance, contribute to price discovery in financial markets and act as barometers of market performance and stability, *inter alia* (Dunne, Moore, & Portes, 2007; Yuan, 2005). Financial time series data share common characteristics which are commonly referred to as the ‘stylized facts’ (Pagan, 1996; Cochrane, 2001). The parameter estimates which make up the stylized facts provide the building blocks for relatively simple and more complex quantitative activities in finance which includes passive and active investment strategies, risk management techniques such as VaR, and pricing derivative securities. The empirical finance literature documents the stylized facts for developed, and many emerging, capital markets. A notable exception is the dearth of empirical facts for African capital markets. Specifically in the context of African markets understanding benchmark equity market indices is important for financial

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innovation to expand investors' opportunity sets. By way of contrast, Hasbrouck (2003) demonstrates that even in a highly developed financial environment like the US the introduction of Exchange–Traded Funds (ETFs) helped complete the market as evidenced by contributing to substantial price discovery. Given the nascent stage of development of many African markets, documenting the stylized facts for African equity indices is a necessary first step for financial innovation to help complete the market.

The aim of this paper is to fill this void by comprehensively investigating the extent to which the Africa All-Share Index and its small, medium and large company sub-indices exhibit the stylized facts.

In the empirical finance literature, symptoms of the stylized facts include asset returns distributions which have 'fat-tails', are skewed, experience volatility clustering and long memory in volatility (often characterized by slow decay of autocorrelations of squared and absolute returns, see Ding, Granger, and Engle (1993)), and that we observe a leverage effect in equity markets where the magnitude of the response of future volatility differs depending on whether past news is positive or negative (Black, 1976; Pagan, 1996). Recent theoretical work adopts an agent-based modelling framework, which with some success, constructs simple markets where the interactions of participants with heterogeneous beliefs produces data with statistical properties consistent with the stylized facts (He & Li, 2007; He, Hamill, & Li, 2008; Amilon, 2008). In contrast, studies on the behaviour of asset prices using African data are few mainly due to the difficulty of obtaining data of sufficient frequency and duration for any meaningful empirical analysis. The earlier weak-form market efficiency tests using African data reported conflicting results (Samuels & Yacout, 1981; Dickinson & Muragu, 1994). Recent studies suggest that monthly price behaviour of a number of countries including Botswana, Cote d'Ivoire, Kenya, Mauritius and South Africa are weak-form efficient (Magnusson & Wydick, 2002). However, studies using multiple variance-ratios tests and GARCH model respectively reject the notion of weak-form efficiency in all the markets examined except South Africa (Smith, Jefferis, & Ryoo, 2002; Jefferis & Smith, 2005). Recent studies lend further support to the weak-form inefficiency of the securities market in Egypt, Ghana, Mauritius and South Africa from 1990 to 2003 (Appiah-Kusi & Menyah, 2003; Simons & Laryea, 2006).

From a policy perspective, benchmark indices provide important information for regulators and policy makers on the performance and stability of capital markets. A cornerstone of economic policy in many emerging markets involves financial sector reform which includes developing capital markets. The rationale being that well developed capital markets promote higher economic growth, attract international investments, mobilise domestic savings and provides liquidity. As a result, a well developed capital market can facilitate the efficient allocation of scarce economic resources thereby promoting economic development (Schumpeter, 1911; McKinnon, 1973; Shaw, 1973; Levine & Zervos, 1996; Levine, 1997). The belief that capital markets contribute to economic development in Africa is reflected by the fact that the number of stock exchanges had increased from 8 in 1980 to 26 by 2008. Proposals have been put forward to establish new stock markets by a number of countries including Congo D.R., Equatorial Guinea, Ethiopia, the Gambia, Lesotho, Madagascar, Mauritania and Sierra Leone (UNDP, 2003; Moin, 2007; Databank, 2008). Tables A1–A6 in Appendix A report, for 26 African Stock Markets, principal governing laws, type of trading system, the extent of foreign ownership, class of securities traded on each exchange and their taxation treatment, the number of listed firms, and market capitalisations. The data was compiled from various sources: the African Stock Markets Handbook published by the United Nations Development Programme (UNDP) in 2003[1], the African Securities Exchanges Association (ASEA) Yearbook 2007 published by ASEA in 2008[2], and the 2007 Global Stock Markets Factbook published by Standard & Poor's in 2007. Column 3 in Table A1 reports the year each stock exchange was established. Opened in 1887, Johannesburg Stock Exchange is the oldest in Africa followed by the Cairo and Alexandria Stock Exchanges which were established in 1888. Table A2 highlights that most of the 26 stock markets in Africa are automated with 15 having electronic clearing and settlement systems. Out of the 21 countries where settlement information is available, only 6 had a manual clearing and settlement system. Table A2 also shows that the majority of the exchanges trade on all days during the week. The majority of exchanges have no restrictions on foreign participation on their stock market as shown in Table A3. Most of the exchanges recommend the use of International Accounting Standard for reporting financial performance. The type of securities traded on the various exchanges is reported in Table A4 and, with the exception of

[1] Available at http://emergingafrica.com/files/UNDPafricanstockmarkets.pdf, accessed on 11th June 2009.
[2] Available at http://www.africansea.org/ASEA/Library/ASEA_Yearbook2007.pdf, accessed on 11th June 2009.

South Africa, none of the exchanges trade in options. The number of firms listed on the various exchanges in reported in Table A5. Egypt and South Africa have by far the largest number of listed firms — 354 and 348 as of 2008, respectively. A number of countries had no firms listed on their exchanges: Angola, Cameroon, Cape Verde, Gabon and Rwanda. The Johannesburg Stock Exchange is by far the largest in Africa with a market capitalisation in 2008 of US$ 468,650 million. The Cairo and Alexandria Stock Exchange and the Casablanca Stock Exchange follow second and third respectively with market capitalisations of US$ 140,230 and US$62,240.

In Africa financial sector reforms have largely been driven by the IMF, the World Bank and the African Development Bank. From an investment perspective, African equity markets have the potential to offer significant portfolio diversification opportunities for international investors given their low correlation with global equity markets reported by Moin (2007). Current research (e.g., MSCI/ABRI, 2007) shows, for example, that the recent sub-prime crisis in global equity markets has had a minimal impact on emerging African markets excluding South Africa.

Data published by the World Federation of Exchanges (WFEs) (2008) suggest that stock markets in Africa pale into insignificance in comparison to other emerging markets for a number of indicators including size, market capitalisation, percentage of GDP, securities traded and operational efficiency. They are small in size compared to either developed markets or to new emerging markets in Asia and Latin America. The total value of African stocks outside of South Africa was only 0.62% of world stock market capitalisation, and 1.55% of all emerging markets stocks at the end of 2007 (WFEs, 2008). Similarly, African markets–excluding South Africa accounted for only 2.50% of the total global equity listings in contrast to 10.51% by India for instance (WFEs, 2008). Stock markets in Africa are also small in relation to their own economies. For instance, the capitalisation of the stock market in Mozambique is only 3.20% of nominal GDP, while Nigeria, Uganda and Tunisia's capitalisations are between 25 and 52% of GDP (WFEs, 2008). The comparative figures for developed markets such as Hong Kong are 284.10%, UK 138.90% and USA 113.10%. With regards to emerging markets, the comparative figures for some Asia and Latin American countries such as Malaysia is 174.40%, India 165.60% and Brazil 104.3% (WFEs, 2008). Some authors have argued that the small sizes of African markets make them vulnerable to speculation and manipulation by insiders at the expense of other investors (see Magnusson & Wydick, 2002). Mlambo and Biekpe (2005) argue that because of their size African stock markets are extremely illiquid and thinly traded and this severely affects their informational efficiencies.

We investigate the statistical properties of the Africa All-Share Index and its constituent size indices: Africa Large Company Index, Africa Medium Company Index and the Africa Small Company Index from its inception on 1st January 1999 to 22nd February 2008 using daily data. This study is distinct from the extant literature in that these indices are a composite measure of the average performance of all stocks exchanges in Africa, excluding South Africa, and can therefore be regarded as benchmarks for the performance of African markets and African large, medium and small capitalisation stocks. Given their status, it is of considerable academic and practical importance to establish the extent to which each index exhibits the 'stylized facts'. We employ a range of methodologies to achieve this goal. After our exploratory data analysis we investigate the distributional properties of returns by implementing non-parametric Kernel estimators (Pagan & Ullah, 1999) of the Probability Density Function (PDF) for each index. Next, we draw upon Extreme Value Theory (EVT) to examine the relative extremes of each index's distribution and employ a range of GARCH models to characterize possible volatility clustering, examine each index's information structure, and test for a leverage effect. Finally, we employ the Geweke and Poter-Hudak (1983) and Robinson and Henry (1999) semi-parametric estimators of the memory parameter to test for long memory in volatility.

The econometric analysis indicates that each index's distributional properties are consistent with the stylized facts. The four indices are non-normally distributed, have more density around the mean of the distribution than the normal distribution (leptokurtotic) and experienced extreme daily returns fluctuations ranging from −12.34% for the Medium Company Index to 13.89% for the Small Company Index. Using the Coefficient of Variation as a simple measure of risk-adjusted return, the Small Firm Index provides the highest return per unit of risk indicating the existence of a small firm effect. All of the indices exhibited volatility clustering and long memory in volatility. The All-Share and Large Company Indices both exhibited a leverage effect. The Small Company Index exhibited what could be termed a 'reverse' leverage effect. While a reverse leverage effect is inconsistent with the stylized facts for developed markets, this finding is, when viewed in light of our entire empirical analysis, suggestive of small company bull market, or possibly a speculative bubble.

This paper is organized as follows: Section 2 introduces the data and outlines the methodologies employed to investigate the stylized facts; Section 3 reports the empirical results, while Section 4 concludes the paper.

2. Data and methodology

2.1. Data

We analyze daily data, supplied by African Business Research Limited[3], for the Africa All-Share (ex South Africa) Index (ABRA) and its constituent size indices: Africa Large Company Index (ABRL), Africa Medium Company Index (ABRM), and the Africa Small Company Index (ABRS) from 1st January 1999 to 22nd February 2008. To be eligible for inclusion in the ABRA Index a country must allow non-nationals to invest in the stock market, the repatriation of dividends or capital should not be significantly inhibited by exchange controls, all securities must have a minimum market value of $1 million at the quarterly index review date and must achieve a traded turnover of at least 0.1% of its market capitalisation in the quarter preceding the index review date in at least two of the four quarters prior to the quarterly index review date. The ABRL Index comprises the largest 50 companies; the ABRM Index comprises the next 100 largest companies, while the ABRS Index covers companies below the top 150 with the number of constituents varying. Our primary interest is the analysis of returns. We use p_t to denote the price index at time $t=0,...,2387$ and log returns r_t are defined as $r_t=\ln p_t-\ln p_t-1$.

2.2. Methodology

Our empirical investigation of the stylized facts begins with the descriptive statistics. This is followed by a formal assessment of the distributional properties of returns which involves testing if the sub-indices are significantly different from the African All-Share Index by estimating the non-parametric Kernel estimators of the probability density function (PDF) for each index (Pagan and Ullah, 1999). A natural extension of this analysis is to focus on the tails of the distribution given our prior expectation that if the African equity indices exhibit the stylized facts the distribution will be leptokurtotic. Extreme Value Theory (EVT) (see (Beirlant, Goegebeur, Segers, & Teugels, 2004) for a detailed account) provides estimators for modelling the behaviour of the relative extremes of the PDFs. Consequently, we estimate the Hill, Pickand and moment tail-index estimators. To investigate the time-varying nature of volatility, we first employ a range of GARCH methodologies. This allows for a formal test of volatility clustering, characterises each index information structure and tests for the possible presence of a leverage effect. We then apply the Geweke and Poter-Hudak (1983) and Robinson and Henry (1999) semi-parametric estimators of the memory parameter to study long memory in volatility. Collectively, the methodologies outlined to investigate the stylized facts are extensive and well established in the literature. Consequently, we have included in Appendix B a detailed exposition and associated references for each methodology.

3. Empirical results

3.1. Descriptive statistics and distribution of returns

Table 1 reports the summary statistics of the daily returns (r_t) for the African All-Share Index (ABRA) and its three constituent indices: African Large Company Index (ABRL), African Medium Company Index (ABRM) and the African Small Company Index (ABRS). As would be expected, the mean daily returns for each series is close to zero (ABRA = 0.0005; ABRL = 0.0005; ABRM = 0.0006; ABRS = 0.0007). Annualizing the daily log returns assuming 252 trading days for each series implies average annual returns of approximately 12.6%, 12.6%, 15.1% and 17.9% for the ABRA, ABRL, ABRN and ABRS Indices respectively. This is an interesting result when we examine the standard deviations. Not only do the small and medium sized companies yield the highest returns, but also have lower levels of risk than large companies. This implies a risk-return trade off where the Coefficient of Variation (CV) for small firms yields the highest return per unit of risk, followed by medium sized

[3] Visit: www.africanbusinessresearch.com.

Table 1
Descriptive statistics.

	Mean	Std	Skewness	Kurtosis	Min	Max	Studentized range	Jarque–Bera
ABRA	0.0005	0.0094	0.0857	11.22	−0.0747	0.0735	15.83	6723**
ABRL	0.0005	0.0114	0.0178	16.86	−0.1075	0.1020	18.46	19,117**
ABRM	0.0006	0.0087	−0.0192	47.01	−0.1234	0.1255	28.59	192,643**
ABRS	0.0007	0.0070	3.2274	70.23	−0.0445	0.1389	26.35	453,696**

Notes: **indicates significance at 1% level, or less. Africa All-Share Index (ABRA) and its three constituent indices: Africa Large Company Index (ABRL), Africa Medium Company Index (ABRM) and the Africa Small Company Index (ABRS).

firms with large firms providing the lowest CV. While this finding is inconsistent with the risk-return trade off predicted by portfolio theory or the Capital Asset Pricing Model, it is consistent with the documented small-firm-effect (Banz, 1981; Cochrane, 2001; Dimson, Marsh, & Staunton, 2001). Estimates of skewness and the kurtosis coefficient for each index provide insights into the shape of each distribution relative to the normal. The normal distribution has a kurtosis coefficient of 3 (mesokurtic); Kurtosis<3 is indicative of low peaked distribution (platykurtic); Kurtosis>3 reflects a high peaked distribution and 'fat-tails' (leptokurtic). The large and small firm indices are positively skewed whereas the medium sized firms' index is negatively skewed. The associated kurtosis figures for each index are in excess of three highlighting the leptokurtotic nature of the returns data. Examination of the minimum and maximum value indicates the range of possible returns on any given day, *ex post*. From a downside risk perspective, the large and medium firm indices experienced declines of −10.75% (ABRL) and −12.34% (ABRM). The small firm index's most pronounced fall was −4.45% (ABRS). In terms of daily gains, small firms' maximum daily return was 13.89%, which was the highest relative to medium firms (12.55%) and large firms (10.20%). The studentized range statistics (which is the range divided by the standard deviation) are consistent with fat-tailed behaviour compared with a normal distribution. Recalling that ±3 standard deviation should account for 99.73% of the observations, which is equivalent to a studentized range statistic of 6; statistics ranging from 15.83 for the ABRA Index to 28.59 for the ABRM Index confirms the presence of extreme values and possibly fat-tails. The Jarque–Bera Test confirms statistically that the distribution of each index is significantly different from the normal distribution.

The descriptive statistics provide a static analysis for each returns distribution. In reality, returns distributions are time variant. Fig. 1 provides intuitive insights as to how prices, and by implication returns, have changed. It is obvious from this plot that the differences in the descriptive statistics reported in Table 1 for the small firm index are driven by returns from late 2006 onward. The time-varying nature of returns is depicted in Fig. 2. The pattern of returns is similar for the ABRA, ABRL and ABRM Indices. For the early sample period each series experienced extreme daily fluctuations in returns reflecting the minimum and maximum returns reported in Table 1. In contrast the large changes in daily returns for small firms occurred during the latter sample period around 2006. It appears from each plot that returns for each series became more volatile in the latter period. Overall, our exploratory data analysis provides evidence that the returns for the African indices are consistent with the stylized facts reported in the empirical finance literature.

3.2. Tests for differences in Probability Density Functions (PDF)

We extend our introductory statistical analysis to characterize the PDFs for each index and to test for differences in PDFs for the sub-indices in relation to the Africa All-Share Index. While the descriptive statistics suggest the indices have different returns properties, this allows us to formally test if the sub-indices are significantly different from the African All-Share Index (ABRA). Fig. 3 reports non-parametric Kernel estimates of the PDFs for the large Company Index (ABRL), the normal distribution and the Africa All-Share Index. The figure on the left reports 95% bootstrap confidence intervals for the All-Share Index constructed using the procedure suggested by Hall (1992) whereas for the figure on the rights reports a 95% uniform confidence band constructed using the procedure suggested by Hall (1993). Consistent with the results reported for the Jarque–Bera Test, the normal plot is outside the 95% confidence interval for the All-Share Index for both estimators. In hypothesis testing terminology this is equivalent to rejecting the null hypothesis that the PDFs are statistically normally distributed. When we compare the PDF for the Large Company Index it is also outside the All-Share Index 95% confidence interval which confirms statistically that it is also significantly different from the All-Share Index. This result is consistent for both of Hall's estimators.

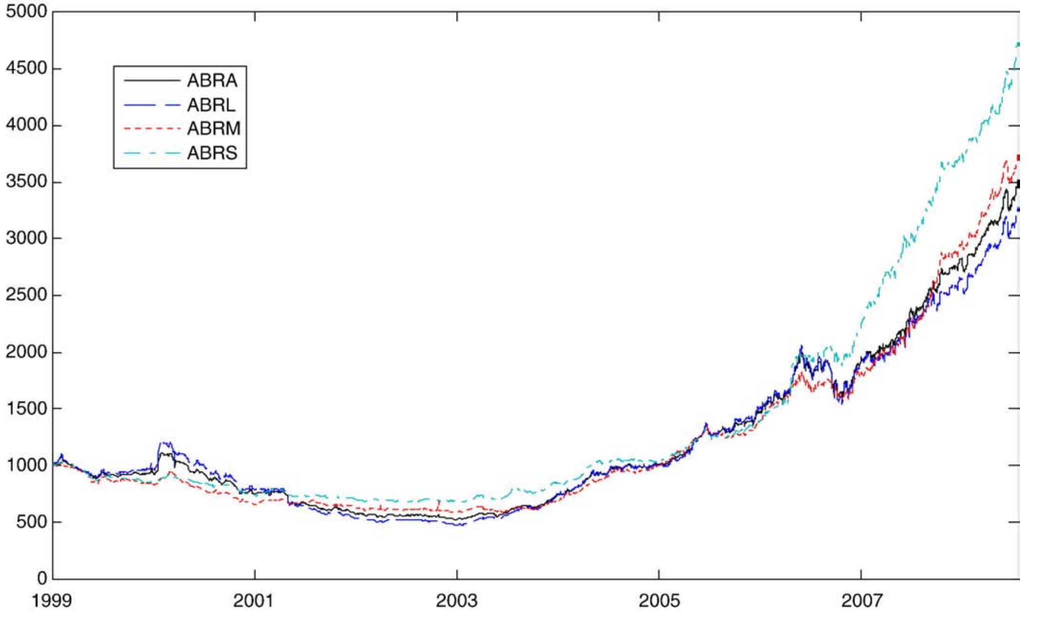

Fig. 1. Plot of prices for the Africa All-Share Index (ABRA), Africa Large Company Index (ABRL), Africa Medium Company Index (ABRM) and the Africa Small Company Index (ABRS) from January 1999 to February 2008.

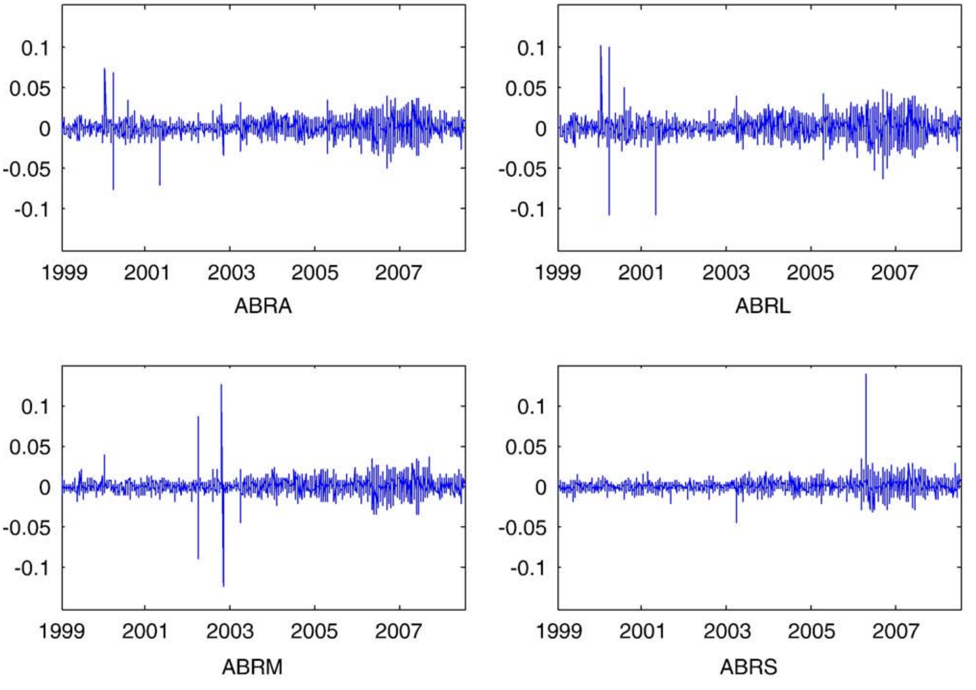

Fig. 2. Plot of log daily returns for the Africa All-Share Index (ABRA), Africa Large Company Index (ABRL), Africa Medium Company Index (ABRM) and the Africa Small Company Index (ABRS) from January 1999 to February 2008.

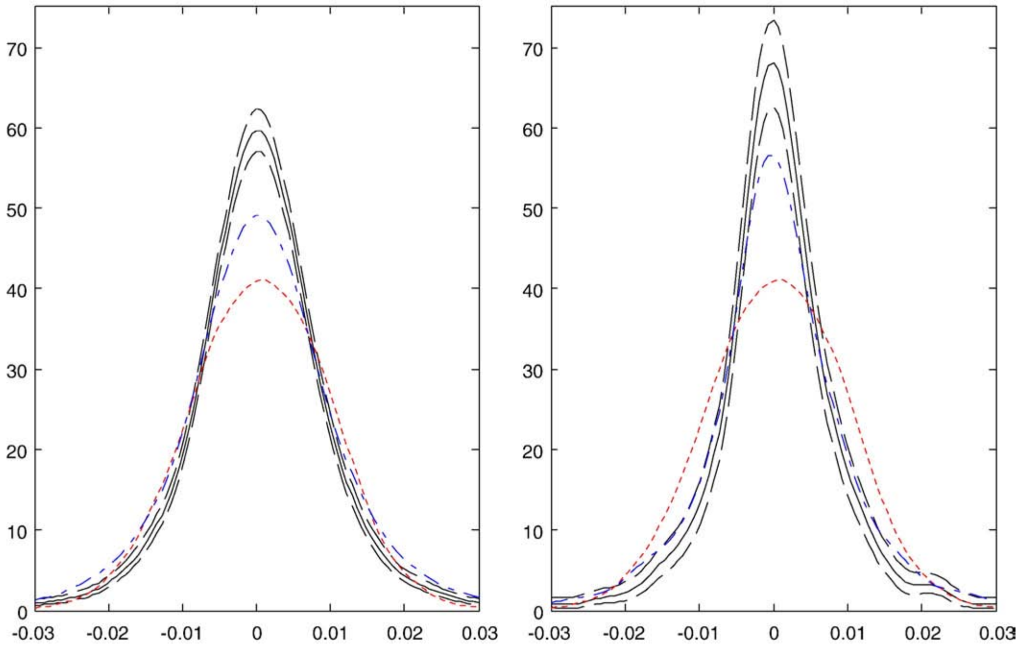

Fig. 3. The estimated probability density functions for the Africa All-Share Index (ABRA) with confidence intervals (left) and confidence band (right), of the Africa Large Company Index (ABRL) (dash–dot line), and of normal distribution (dot line).

Figs. C1 and C2 in Appendix C report the equivalent plots for the medium company and small company Indices respectively. Again, the normal PDF and both indices PDFs are outside the 95% confidence interval for the All-Share Index for both methods. From a portfolio management perspective this type of analysis is important. Active management of equity portfolios is defined in terms of size (small, medium and large stocks) and style (value versus growth). Assessing whether the sub-indices are significantly different identifies a pool of securities which are clearly defined in terms of the size dimension of an equity investment strategy. If a fund manager had a mandate for actively managing a value-small-cap African portfolio for example (s)he could focus his/her attention on the ABRS constituents and direct research activity to identifying stocks on the value versus growth spectrum.

3.3. Extreme values analysis

Extreme Value Theory (EVT) provides techniques for modelling the behaviour of the relative extremes, or 'tails', of PDFs. Intuitively, fat-tails simply reflect the empirical fact that we observe more frequently extreme observation than would be predicted by the normal distribution. A fat left-tail reflects stock market declines/crashes, while a fat right-tail can reflect a bull/bubble market.

Results for the tail-index estimates are reported in Fig. 4 and Figs. C3–C5 in Appendix C. For each figure the Hill, Pickand and moment tail-index estimates are plotted on the first, second and third rows respectively for cut-off points ranging from 50 ($k=50$) to 350 ($k=350$). The columns of graphs on the left of each figure are for the left-hand tail of the distribution, while the columns of graphs on the right are for the right-hand tail of the distribution. The x-axis for each box is the cut-off points, while the y-axis is the tail-index estimators for each cut-off point. The cut-off point is a choice variable which influences the tail-index estimator. For example, a cut-off point of 100 estimates the tail-index for the largest 100 positive observations (right-tail) and vice versa. A tail-index of zero is equivalent to the tail density of the normal distribution. The results for the Hill and Pickand estimators are mainly for the purpose of comparison as the moment estimator is superior. It is a generalization of Hill whereas the Pickand approach can be volatile which makes inference difficult. This is reflected in Fig. 4 which plots each estimator for the four indices. It

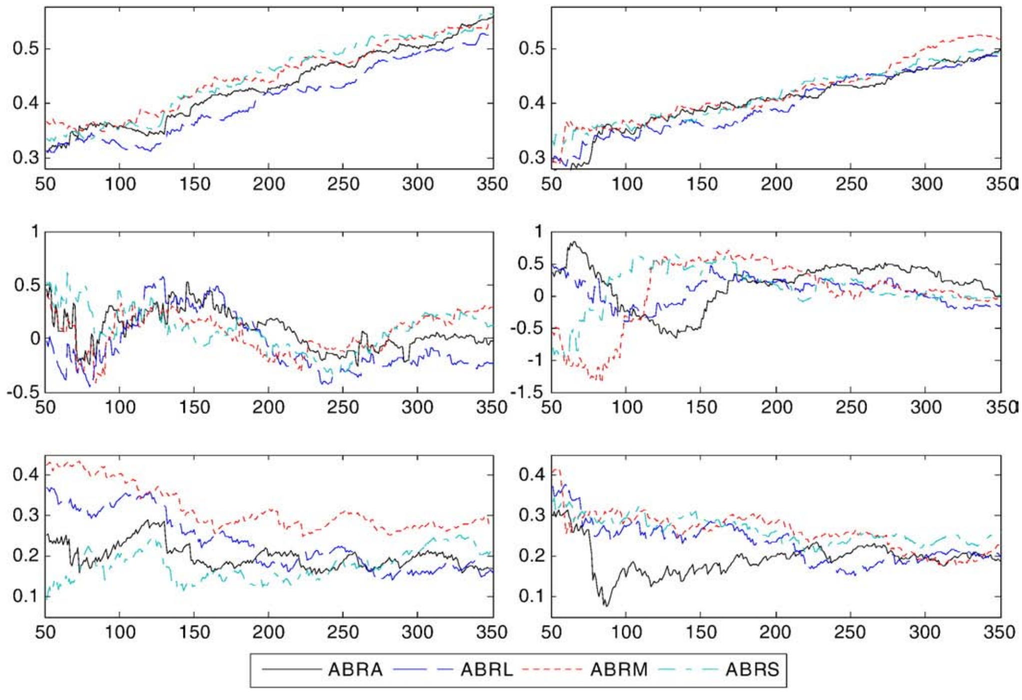

Fig. 4. The Hill (first row), the Pickand (second row), and the moment (the third row) estimates plots of the negative tail (left column) and the positive tail (right column) of the Africa All-Share Index (ABRA), Africa Large Company Index (ABRL), Africa Medium Company Index (ABRM) and the Africa Small Company Index (ABRS).

is obvious from examining the plots that the Pickand estimator is considerably more volatile than the other two. The Hill estimator exhibits a positive trend in both tails as the cut-off point for the four indices increases. The plots for the moment estimator indicate that each index has additional density in both tails of their PDF. In terms of the left-tail, the ABRM has the most density across the range of cut-off points whereas the ABRS has the least, but is still 'fat', up to a cut-off point of 240. For the right-hand tail, the ABRA appears to have the least density, relative to the sub-indices, up to a cut-off point of 220.

Figs. C3–C5 repeat this exercise, but include the Africa All-Share Index for comparison. If the sub-index tail estimator is above the 95% confidence band for the ARBA Index the interpretation would be that the sub-index has a fat-tail and that it is significantly more fat-tailed, greater density, than the benchmark. We can see from Figs. C3–C5 that for the moment estimator the three sub-indices are all in the ARBA Index's 95% confidence interval across the range of cut-off points. Overall, this indicates that while the sub-indices have fat-tails, none have significantly more density in either tail than the ARBA Index. When we look at the tail-index estimates for each index with cut-off points 100, 200 and 300, we will find that small firms have less density than larger firms in their left-tail, which, in practical terms, highlights they experienced fewer relatively extreme negative returns. When we examine the corresponding right-tail estimates, it appears that each sub-index has similar density in the right-tail.

3.4. GARCH estimates

Table 2 reports the results for the ABRA Index from implementing a range of GARCH models designed to describe the volatility process of an asset's return. The first two rows in the table report the point estimates for the AR (1) model which was adopted as the mean equation for each model. The remaining rows report the coefficients for the conditional variance equation for each GARCH model. The results for the benchmark GARCH (1, 1) model are consistent from what was inferred from the volatility plots. Both the ARCH (α_1) and GARCH (β_1) coefficients are significant at the 1% level, or less. This provides evidence of volatility

Table 2
Estimates of GARCH class of models for the Africa All-Share Index (ABRA).

	GARCH	EGARCH	GJR	APARCH	FIGARCH	HYGARCH
α	0.0004* (0.0001)	0.0001 (0.0001)	0.0002 (0.0002)	0.0002 (0.0002)	0.0003** (0.0001)	0.0003** (0.0001)
b	−0.0863** (0.0227)	−0.0735** (0.0160)	−0.0695** (0.0200)	−0.0724** (0.0195)	−0.0851** (0.0214)	−0.0811** (0.0221)
$\omega \times 10^4$	0.0179* (0.0070)	0.0000 (59,332)	0.0265** (0.0036)	0.2674 (0.1570)	0.6328** (0.0889)	0.0349** (0.0079)
α_1	0.1063** (0.0243)	1.0001** (0.1776)	0.0897** (0.0099)	0.1324** (0.0105)	0.4610** (0.0419)	0.4896** (0.0620)
β_1	0.8829** (0.0270)	0.9961** (0.0023)	0.8498** (0.0083)	0.8726** (0.0080)	0.6615** (0.0343)	0.7071** (0.0366)
θ_1		−0.0155** (0.0033)				
θ_2		0.1797** (0.0183)				
γ_1			0.0910** (0.0113)	0.1583** (0.0242)		
δ				1.4709** (0.1178)		
d					0.3673** (0.0319)	0.3955** (0.0616)
log α						0.0725 (0.0418)

Notes: *indicates significance at 5%; **indicates significance at 1% level, or less.

clustering where positive price changes tend to be followed by positive price changes, and vice versa, which reflects the time-varying nature of volatility for financial assets. The point estimates ($\alpha_1 = 0.1063$; $\beta_1 = 0.8829$) are sensible and consistent with values reported in the empirical finance literature. EGARCH and GJR–GARCH capture the asymmetric response of good and bad ‘news’ to future volatility. Whereas volatility in the standard GARCH (*p, q*) model responds to positive and negative return shocks equally, asymmetric models allow ‘good’ and ‘bad’ news surprises to have different impacts on future volatility. Since negative shocks tend to have larger impacts, we expect θ_1 to be negative. The absolute value of the coefficients $\theta_1 + \theta_2$ in the EGARCH model reflects the magnitude of positive shocks ($\varepsilon_{t-1} \geq 0$) and $\theta_1 - \theta_2$ reflects the magnitude of negative shocks ($\varepsilon_{t-1} < 0$). Therefore, evidence of a leverage effect would be a negative sign on θ_1. The magnitude of a positive shock is 0.164 ($-0.0155 + 0.1797$) whereas it is 0.195 ($-0.0155 - 0.1797$) for a positive shock which indicates a leverage effect for the All-Share Index. In terms of the GJR–GARCH model the leverage effect enters the conditional variance equation as an indicator variable taking the value of ‘1’ when $\varepsilon_{t-1} < 0$ and zero otherwise. A significant positive γ_1 is consistent with a leverage effect. A significant point estimate of 0.0910 for γ_1 in the GJR–GARCH model confirms the existence of the leverage effect.

The flexible APARCH model also confirms the leverage effect with a significant $\gamma_1 = 0.1583$. The δ coefficient simply reflects the empirical estimate of the appropriate power for the conditional variance equation (σ_t^δ) which is set at 2 for the GARCH conditional variance function (σ_t^2). FIGARCH and HYGARCH incorporate the stylised fact referred to as the long memory in volatility. Long memory has been evidenced by the empirical finding that the absolute value or powers, particularly squares, of return's autocorrelations tend to decay slowly. The significant point estimates for the memory parameter $d = 0.3673$ for FIGARCH and $d = 0.3955$ for HYGARCH confirms long memory for the ABRA Index. HYGARCH nests the FIGARCH specification. The log α statistic tests for the appropriate long memory model. The null hypothesis is accepted to nest the FIGARCH specification.

Table C1 in Appendix C reports the results for the GARCH modelling for the ARBL Index. The ARCH ($\alpha_1 = 0.0624$) and GARCH coefficients ($\beta_1 = 0.9372$) for the ARBL Index are similar to those reported for the ABRA Index which indicates that the information structure for both the All-Share Index and the Large Company Index is similar. Both asymmetry parameters ($\theta_1 = -0.0253$; $\theta_2 = 0.1797$) in the EGARCH model are significant, have signs consistent with our prior expectations and are of a magnitude consistent with the leverage effect. The significant leverage coefficients for the GJR–GARCH ($\gamma_1 = 0.1347$) and APARCH ($\gamma_1 = 0.2316$) models confirms the leverage effect for large firms. The ABRL Index also exhibits long memory with significant point estimates for the memory parameter for FIGARCH ($d = 0.1132$) and HYGARCH ($d = 0.3121$). The only significant difference between the All-Share and Large Company Indices is that log α is statistically significant which allows us to reject FIGARCH in favour of the HYGARCH specification.

The findings for the ABRM Index reported in Table C2 suggest that medium firms have different properties relative to large companies. Though they do exhibit the empirical property of volatility clustering as evidenced by the significant ARCH and GARCH coefficients, the information structure implied by the coefficients vary. The ARCH coefficient ($\alpha_1 = 0.1085$) is higher whereas the GARCH coefficient is lower ($\beta_1 = 0.6588$). In economic terms, this implies that recent news is more important whereas past news is less important in determining the evolution of volatility compared to larger companies. The findings from the asymmetric models do not support the existence of a leverage effect although θ_1 in the EGARCH model is negative but it is insignificant. The negative sign on the point estimate for the asymmetry coefficient in the APARCH model ($\gamma_1 = -0.3519$) is also inconsistent with a leverage effect, but implies that positive news significantly reduced future volatility which is consistent with empirical evidence. The story from the long memory in volatility supports FIGARCH as log α is insignificant, but HYGARCH provides evidence of significant long memory ($d = 0.8599$).

Table C3 reports the results for the ABRS Index. The point estimates for the ARCH ($\alpha_1 = 0.0740$) and GARCH coefficients ($\beta_1 = 0.9272$) are most akin to the ABRL and ABRA Indices. This would suggest they have similar information structure. The results for the leverage models suggest that small firms have interesting asymmetry coefficients. For the EGARCH model $\theta_1 = 0.0213$ and $\theta_2 = 0.1575$, both of which are significant, does not support a leverage effect. The magnitude of a negative news shock is -0.1362 ($\theta_1 - \theta_2$: $0.0213 - 0.1575$), whereas for a positive innovation it is 0.1788 ($\theta_1 + \theta_2$: $0.0213 + 0.1575$). This implies that for small firms positive news increases volatility. This is consistent with the point estimates for the γ_1 coefficient for both the GJR–GARCH and APARCH models both of which are significantly negative. The memory parameters are significant for the FIGARCH ($d = 0.5939$) and HYGARCH ($d = 0.8325$) models, with FIGARCH again preferred as log α is insignificant. This implies a 'reverse' leverage effect for small firms.

3.5. Semi-parametric estimates of memory parameter

Table 3, and Tables C4–C6 in Appendix C report the Geweke and Poter-Hudak (GPH) and the Robinson and Henry (RH) estimators of the memory parameter (d) for returns, squared returns, and absolute returns for ABRA, ARBL, ARBM and ARBS indices, respectively. A well known empirical feature of high frequency (daily) financial time series is that the returns themselves contain little autocorrelation, but the squared returns and absolute returns have a significant positive autocorrelation over long lags (Ding et al., 1993). Estimates of $0 < d < 1/2$ indicates long memory, $d \geq 1/2$ implies the process is not covariance stationary, while for $-1/2 < d < 0$ indicates short memory — also referred to as negative dependence or anti-persistence. Notice that the autocorrelations can then be described as $\rho_j \approx G_1 j^{2d-1}$, where G_1 is a constant and $\mu = 2d - 1$ correspond to the hyperbolic decay index. Consistent with the approach adopted in the literature, we report the estimated results for a range of bandwidths: $m = 50$, 100, 150, 200, and 250 (Geweke, 1998; Lobato & Savin, 1998).

Table 3
Estimates of the memory parameter (d) for the Africa All-Share Index (ABRA).

	$\hat{d}_{GPH}$	t	p-value	95% CI	$\hat{d}_{RH}$	t	p-value	95% CI
r_t	0.2919	2.833	0.0046	[0.0900, 0.4939]	0.2486	3.516	0.0004	[0.1100, 0.3872]
	0.1670	2.403	0.0162	[0.0308, 0.3031]	0.1952	3.905	0.0000	[0.0972, 0.2930]
	0.1461	2.625	0.0087	[0.0370, 0.2551]	0.1656	4.056	0.0000	[0.0856, 0.2456]
	0.0592	1.243	0.2140	[−0.0342, 0.1527]	0.0996	2.818	0.0048	[0.0303, 0.1689]
	0.1023	2.416	0.0157	[0.0193, 0.1854]	0.1191	3.767	0.0002	[0.0571, 0.1811]
r_t^2	0.5490	5.329	0.0000	[0.3471, 0.7510]	0.5302	7.498	0.0000	[0.3916, 0.6688]
	0.2691	3.874	0.0001	[0.1330, 0.4053]	0.3113	6.226	0.0000	[0.2133, 0.4093]
	0.2225	3.999	0.0001	[0.1135, 0.3315]	0.2883	7.061	0.0000	[0.2083, 0.3683]
	0.1769	3.711	0.0002	[0.0835, 0.2704]	0.2346	6.637	0.0000	[0.1653, 0.3039]
	0.1914	4.518	0.0000	[0.1084, 0.2744]	0.2334	7.382	0.0000	[0.1715, 0.2954]
$\lvert r_t \rvert$	0.5741	5.572	0.0000	[0.3722, 0.7761]	0.6498	9.190	0.0000	[0.5112, 0.7884]
	0.4530	6.521	0.0000	[0.3169, 0.5892]	0.5336	10.67	0.0000	[0.4356, 0.6316]
	0.4325	7.773	0.0000	[0.3234, 0.5415]	0.4927	12.07	0.0000	[0.4127, 0.5728]
	0.3466	7.270	0.0000	[0.2532, 0.4400]	0.4307	12.18	0.0000	[0.3614, 0.4999]
	0.3459	8.166	0.0000	[0.2629, 0.4289]	0.4250	13.44	0.0000	[0.3630, 0.4870]

Notes: d_{GPH} is the Geweke and Poter-Hudak (1983) estimator of the memory parameter; d_{RH} is the Robinson and Henry (1999) estimator of the memory parameter. Estimates are reported for bandwidths (m) (50, 100, 150, 200, and 250). r_t is the daily log return, r_t^2 is the squared return and $|r_t|$ is the absolute return.

Table 3 reports the memory parameter for the ARBA Index. The panel of r_t in the first row reports the results from the GPH and the RH estimation for a bandwidth of 50 ($m=50$, the second row reports the results of the GPH and the RH estimation with $m=100$, and so on. This also holds for the panels of r_t^2 and $|r_t|$, and for Tables C4–C6. The reported t statistic tests the null hypothesis that the memory parameter is equal to zero (H_0: $d=0$). We see that most of the estimated d_{GPH} for the returns are significant at 5% level but not at 1% while those for the squared returns, and the absolute returns are significant, and also the estimated d for the absolute returns is bigger than that of squared returns. This highlights that absolute returns are more persistent than the squared return, which is consistent with the findings reported in the empirical finance literature (Ding et al., 1993). We also see that all of the estimated d_{RH} are significantly positive. Therefore, for the ABRA Index there is clear evidence of long memory in volatility. Tables C4–C6 report the equivalent results for ABRL, ABRM and ABRS Indices, respectively. We see that there is clear evidence of long memory in volatility (significant estimates of d for the squared and absolute return except for the ABRM), and absolute return are more persistent than the squared return. In terms of squared return, when we compare the degree of persistence among the four indices, we see the degree of persistence decreases from ABRA, ABRL, and ABRS to ABRM. But for the absolute returns, we see that the ABRS is the most persistent and the degree of persistence decreases for ABRS, ABRA, ABRL, and ABRM. Overall, all the indices show evidence of long memory in volatility and absolute return are more persistent than the squared return, although the magnitudes for them are slightly different.

4. Conclusion

The aim of this paper was to comprehensively investigate if benchmark African equity indices exhibited the stylized facts which have been reported for financial time series for developed and emerging markets. A major difference between this study and prior studies that have used African data is that they examined data on national exchanges rather than continent wide indices. Benchmark indices have many uses and are an important aspect of professional finance. Understanding their statistical properties is of significant practical importance to economic policy makers, regulators and investors alike.

Overall, our results for the All-Share, Medium and Large Company Indices are broadly consistent with the stylized facts. We observe extreme return fluctuations which vary over time. The estimated PDFs have more density in the centre of the distribution and at the tails than would be predicted by the normal distribution. The more frequently than expected extreme returns is reflected in the tail-index estimates which confirm the presence of fat-tails. All of the indices experience volatility clustering which is captured by the ARCH and GARCH coefficients. These coefficients reflect the information structure which was broadly similar for the ARBA, ARBL and ARBS Indices. Every index exhibited long memory in volatility. The All-Share and Large Company Indices both exhibited significant leverage effects. Collectively these findings are of considerable importance to regulators and policy makers. Volatile markets diminish investor confidence and the ability to attract capital. This has been vividly highlight since the onset of the global banking crisis since 2007 and the subsequent impact on asset values worldwide. Given the importance of financial innovation as a tool to aid economic development across the continent of Africa we tentatively suggest that our findings could inform policy makers.

When we consider our findings for small firms collectively what could be described as a 'small-firm-effect' emerges. At the outset our descriptive statistics implied superior performance by small companies on a risk-adjusted basis. Our analysis highlights the importance of investigating returns over time. While the other indices exhibited patterns consistent with the stylized facts over the entire sample period, small firm's out-performance of the remaining size indices and the All-Share Index, and their volatility structure, is driven from 2006 onward. This would also account for the additional density in the right-tail of the Small Company Index. The existence of a 'reverse' leverage effect, which literally interpreted implies that positive shocks–good news–increases future volatility, taken with a fat right-tail is indicative of a bull market or possibly a bubble. While this conclusion is speculative at this point, our findings should provide impetus for further investigation of African small company effects.

Finally, it could be argued that simply knowing the stylized facts for a market demystifies it and could be a catalyst for capital inflows into the index and further financial innovation. New securities based on the indices analyzed in this paper, such as Exchange–Traded Funds (EFTs) and derivatives contracts, is capable of significantly expanding investors' opportunity sets. One specific extension of this work could be to investigate diversification possibilities within the data and with international markets.

Acknowledgement

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Appendix A. Institutional features of African stock markets

Table A1
List of African stock markets.

Country	Stock market name	Year established	Principal governing law
Algeria	Algiers Stock Exchange–Bourse des Valeurs Mobilieres d'Alger	1993	Act 93-10 (1993)
Angola	Angolan Stock Exchange and derivatives	2008	N/A
Botswana	Botswana Stock Exchange	1989	Botswana Stock Exchange Act (1994)
Cameroon	Bourse de Doula (Doula Stock Exchange)	2001	N/A
Cape Verde	Cape Verde Stock Exchange–Bolsa de Valores de Cabo Verde	2005	N/A
Cote D'Ivoire	Abidjan Stock Exchange–Bourse Regionale des Valeurs Mobilieres	1973	N/A
Egypt	Cairo & Alexandria Stock Exchange	1888	Capital Market Law
Gabon	Gabon Stock Exchange	2001	N/A
Ghana	Ghana Stock Exchange	1989	Stock Exchange Act (1971)
Kenya	Nairobi Stock Exchange	1954	Capital Market Act
Libya	Libyan Stock Market	2006	General Peoples' Assembly Decision No. 134
Malawi	Malawi Stock Exchange	1995	Capital Market Development Act (1990) and Companies Act (1984)
Mauritius	Stock Exchange of Mauritius	1988	Stock Exchange Act (1988)
Morocco	Casablanca Stock Exchange	1929	N/A
Mozambique	Mozambique Stock Exchange–Bolsa de Valores de Mozambique	1999	N/A
Namibia	Namibian Stock Exchange	1992	N/A
Nigeria	Nigerian Stock Exchange	1960	Lagos Stock Exchange Act
Rwanda	Rwanda Stock Exchange	2008	N/A
South Africa	JSE Ltd	1887	Stock Exchange Control Act (1985)
Sudan	Khartoum Stock Exchange	1994	Stock Exchange Act (1982)
Swaziland	Swaziland Stock Exchange	1990	Financial Institutions Order (1975)
Tanzania	Dar-es-Salaam Stock Exchange	1996	Capital Markets and Securities Act (1994), Amended in 1997
Tunisia	Tunisian Stock Exchange–Bourse de Tunis	1969	Financial Markets Re-organisation Law, 1994
Uganda	Uganda Securities Exchange	1997	N/A
Zambia	Lusaka Stock Exchange	1993	Securities Act (1993)
Zimbabwe	Zimbabwe Stock Exchange	1896	Zimbabwe Stock Exchange Act (1973)

Table A2
Trading system on African stock markets.

Country	Trading system	Clearing & settlement system	Settlement cycle	Trading days
Algeria	ATS	Electronic — since 2002	$T+4$	Mondays
Angola	N/A	N/A	N/A	N/A
Botswana	Call over with presiding officer (open outcry)	Manual	$T+4$	Monday–Friday
Cameroon	N/A	Manual	N/A	N/A
Cape Verde	N/A	N/A	N/A	Monday–Friday

Table A2 (*continued*)

Country	Trading system	Clearing & settlement system	Settlement cycle	Trading days
Cote D'Ivoire	Automated trading system — since 1998	Electronic — since 1998	$T+5$	Monday–Friday
Egypt	Automated trading system — since 2001	Electronic — since 2001	$T+2$	Sunday–Thursday
Gabon	N/A	N/A	N/A	N/A
Ghana	Automated trading system — since March 2009	Electronic — since March 2009	$T+3$	Monday–Friday
Kenya	Automated trading system — since 2006	Electronic — since 2006	$T+5$	Monday–Friday
Libya	Automated trading — since April 2008	Electronic — since April 2008	$T+3$	Sunday–Thursday
Malawi	Floor-based, call over	Manual	$T+7$	Monday–Friday
Mauritius	Automated trading system, strict delivery versus payment	Electronic — since 1998	$T+3$	Monday–Friday
Morocco	Automated trading system — since March 1997	Electronic — since 1997	$T+3$	Monday–Friday
Mozambique	Automated trading system — since 2007	Electronic — since 2007	$T+3$	Tuesdays, Thursdays and Fridays
Namibia	Automated trading system — since 2005	Electronic — since 2005	$T+5$	Monday–Friday
Nigeria	Automated trading system — since April 1999	Electronic — since April 1999	$T+3$	Monday–Friday
Rwanda	N/A	N/A	N/A	N/A
South Africa	Automated trading system — since 1996	Electronic — since 1999	$T+5$	Monday–Friday
Sudan	Manual trading system	Manual	$T+0$	Sunday–Thursday
Swaziland	Floor-based, call over	Manual	$T+7$	Monday–Friday
Tanzania	Automated trading system —since 1999	Electronic — since 1999	$T+5$	Monday–Friday
Tunisia	Automate trading system — since 1996	Electronic — since 1996	$T+3$	Monday–Friday
Uganda	Floor-based, call over	Manual	$T+5$	Monday, Tuesday and Thursday
Zambia	Automated trading system — since 2002	Electronic — Since 2002	$T+3$	Monday–Friday
Zimbabwe	Floor-based, call over	Manual	$T+7$	Monday–Friday

Table A3
Foreign participation.

Country	Foreign participation — foreign investment ceiling for listed stocks	Recommended accounting standard
Algeria	No restrictions — all listed stocks are freely available to foreign investors.	Local accounting standard
Angola	N/A	N/A
Botswana	No restrictions — all listed stocks are freely available to foreign investors.	Local accounting standard
Cameroon	N/A	N/A
Cape Verde	N/A	N/A
Cote D'Ivoire	No restrictions — all listed stocks are freely available to foreign investors	Local accounting standard
Egypt	No restrictions — all listed stocks are freely available to foreign investors.	International accounting standard
Gabon	N/A	N/A
Ghana	74% general foreign ownership cap and 10% foreign	International accounting standard
Kenya	75% general foreign ownership cap	International accounting standard
Libya	N/A	N/A
Malawi	49% general foreign ownership cap	International accounting standard
Mauritius	100% in general foreign ownership; Financial Services Commission approval required.	International accounting standard
Morocco	No restrictions — all listed stocks are freely available to foreign investors	International accounting standard
Mozambique	No restrictions — all listed stocks are freely available to foreign investors.	Local accounting standard
Namibia	100% in general foreign ownership; Central Bank approval required	Local accounting standard
Nigeria	No restrictions — all listed stocks are freely available to foreign investors.	International accounting standard
Rwanda	N/A	N/A
South Africa	No restrictions — all listed stocks are freely available to foreign investors.	International accounting standard
Sudan	N/A	N/A

(*continued on next page*)

Table A3 (*continued*)

Country	Foreign participation — foreign investment ceiling for listed stocks	Recommended accounting standard
Swaziland	No restrictions — all listed stocks are freely available to foreign investors.	International accounting standard
Tanzania	Foreign investors are allowed to invest up to 60% in aggregate of share capital. Furthermore, once invested there is a lock-in period of six months before foreigners are allowed to invest 100% in Corporate Bonds	International accounting standard
Tunisia	49.9% general foreign ownership cap; 0% for retailing sector	Local accounting standard
Uganda	No restrictions — all listed stocks are freely available to foreign investors.	International accounting standard
Zambia	No restrictions — all listed stocks are freely available to foreign investors.	Local accounting standard
Zimbabwe	40% general foreign ownership cap; 10% for a single shareholder	International accounting standard

Table A4
Traded securities and taxation.

Country securities	Traded	Tax rates — withholding taxes
Algeria	Ordinary shares and corporate bonds	No taxes on dividends: 0%; no taxes on interests
Angola	N/A	N/A
Botswana	Ordinary shares, mutual funds and treasury bonds	Dividends: 15%; interests: 15%; and no capital gains tax
Cameroon	N/A	N/A
Cape Verde	Equities, and treasury bonds	N/A
Cote D'Ivoire	Equities and bonds	Dividends: 10%; interests: 18%; and no capital gains tax
Egypt	Ordinary shares, mutual funds, treasury bonds and debentures	Tax on dividends 0: interests: 15%; and no capital gains tax
Gabon	N/A	N/A
Ghana	Ordinary shares, corporate and government bonds	Dividends: 10%; interests: 10%; and no capital gains tax
Kenya	Ordinary shares, preference shares, treasury bonds, and debentures	For non-residents: dividends: 10%; interests: 15%; and no capital gains tax
Libya	N/A	No taxes on dividends: 0%; no taxes on interests: 0%
Malawi	Ordinary shares, preference shares, corporate bonds and treasury bonds	For non-residents — no tax on dividends: 0%; interests: 15%; and capital gains
Mauritius	Ordinary shares, preferred shares, and corporate bonds	No taxes on dividends: 0%; interests: 0%; short- and long-term capital gains
Morocco	Stocks, government bonds and other securities	Dividends: 10%; interests: 10%; and capital gains: 35%
Mozambique	Ordinary shares and corporate bonds	Dividends: 10%; interests: 10% and no tax on capital gains
Namibia	Equities, preference shares and bonds	No tax on dividends: 0%; interests: 10%; and no tax on capital gains
Nigeria	Ordinary shares, preference shares, corporate bonds and treasury bonds	Dividends: 10%; interests: 10% and capital gains: 10%
Rwanda	N/A	N/A
South Africa	Equities, preference shares, corporate debentures, and traditional options on equities	No taxes on dividends: 0%; no taxes on interests
Sudan	N/A	No taxes on dividends: 0%; no taxes on interests
Swaziland	Equities, government guaranteed bonds, and debentures	Dividends: 15%; interests: 10% and no tax on capital gains
Tanzania	Equities, treasury bonds, and corporate bonds	Dividends: 5%; interests: 5% and no tax on capital gains
Tunisia	Ordinary shares, unit trusts, corporate bonds, and gov't bonds	No tax on dividends: 0%; interests: 15%; and no capital gains tax
Uganda	Corporate bonds, ordinary shares, treasury bills, commercial paper	Dividends: 15%; interests: 15%; and no capital gains tax
Zambia	Equities and government debt instruments	Dividends: 15%; interests: 15%; and no capital gains tax
Zimbabwe	Ordinary, preference shares, warrants, corporate and treasury bonds	Dividends: 20%; interests: 10%; and no capital gains tax

Table A5
Number of listed firms.

Country	Year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Algeria	1993	N/A	N/A	N/A	N/A	N/A	N/A	N/A	2	3	3	3	3	3	3	3	3	3
Angola	2008	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Botswana	1989	11	11	11	12	12	12	14	15	16	16	19	25	25	28	31	31	32
Cameroon	2001	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Cape Verde	2005	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Cote D'Ivoire	1973	27	24	27	31	31	35	35	38	41	38	38	38	39	39	40	38	38
Egypt	1888	656	674	700	746	649	654	861	1033	1076	1110	1151	967	792	744	603	435	354
Gabon	2001	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Ghana	1989	15	15	17	19	21	21	21	22	22	22	24	25	29	30	32	32	36
Kenya	1954	57	56	56	56	56	58	58	57	57	55	57	51	47	47	51	54	47
Libya	2007	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	5	6	N/A
Malawi	1995	N/A	N/A	N/A	N/A	1	3	6	6	7	7	9	8	8	10	11	12	15
Mauritius	1988	22	30	35	40	40	40	40	41	40	40	40	40	41	42	41	41	40
Morocco	1929	62	65	51	44	47	49	53	55	53	55	56	53	52	56	65	73	77
Mozambique	1999	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	5	7	9	8	10	12	10	13	N/A
Namibia	1992	3	4	8	10	12	13	15	14	13	13	13	13	13	13	9	27	24
Nigeria	1960	153	174	177	181	183	182	186	194	195	194	195	200	207	214	202	212	214
Rwanda	2008	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
South Africa	1887	683	647	640	640	626	642	668	668	616	542	472	426	403	388	402	411	348
Sudan	1994	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	49	51	53	N/A
Swaziland	1990	3	4	4	4	6	4	5	7	6	5	5	5	5	5	5	6	5
Tanzania	1996	N/A	N/A	N/A	N/A	N/A	N/A	2	4	4	4	5	5	6	8	10	10	14
Tunisia	1969	17	19	21	26	30	34	38	44	44	46	46	46	44	46	48	51	49
Uganda	1997	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	2	2	3	3	3	7	8	9	10
Zambia	1993	N/A	N/A	N/A	2	6	7	9	9	9	9	11	11	11	13	15	16	21
Zimbabwe	1896	62	62	64	64	64	64	67	70	69	72	76	81	79	79	80	79	79

Table A6
Stock market capitalisation.

18 stock markets with complete data on market capitalisation in US$ mm from 2000 to 2008										
Stock market name	Year established	2000	2001	2002	2003	2004	2005	2006	2007	2008
Botswana Stock Exchange	1989	978	1269	1723	2131	2548	2437	3947	5445	3460
Abidjan Stock Exchange	1973	1185	1165	1329	1650	2083	2327	4155	8305	8390
Cairo & Alexandria Stock Exchange	1888	28,741	24,335	26,245	27,073	38,516	79,672	93,477	139,273	140,230
Ghana Stock Exchange	1989	502	528	740	1426	2644	1375	1729	13,710	14,660
Nairobi Stock Exchange	1954	1283	1050	1676	4178	3891	6384	11,378	13,345	10,170
Malawi Stock Exchange	1995	212	152	107	84	98	110	112	1260	1770
Stock Exchange of Mauritius	1988	1335	1061	1324	1955	2379	2617	3598	7919	3240
Casablanca Stock Exchange	1929	10,899	9087	8319	13,152	25,064	27,220	49,360	75,494	62,240
Mozambique Stock Exchange	1999	100	102	85	114	102	98	93	242	N/A
Namibian Stock Exchange	1992	311	151	201	308	442	415	542	1590	590
Nigerian Stock Exchange	1960	4237	5404	5989	9494	14,464	19,356	32,819	87,371	49,320
JSE Ltd	1887	204,952	139,750	182,616	267,745	455,536	565,408	715,025	828,185	468,650
Swaziland Stock Exchange	1990	73	127	146	98	96	99	140	234	N/A
Dar-es-Salaam Stock Exchange	1996	233	398	695	189	262	122	540	2786	2630
Tunisian Stock Exchange - Bourse de Tunis	1969	2828	2303	2131	2464	2641	2876	4446	10,830	5300
Uganda Securities Exchange	1997	37	34	52	86	120	159	254	3160	3020
Lusaka Stock Exchange	1993	236	217	231	280	350	560	945	4827	4930
Zimbabwe Stock Exchange	1896	2432	7792	15,632	4975	1941	2402	26,557	N/A	N/A

Appendix B. Methodologies

Appendix B1. The probability density function

With observations $\{x_t\}_{t=1}^{T}$ the marginal one-period probability density function f evaluated at x (a, b) can be estimated consistently (under appropriate regularity conditions, see, for example, Pagan & Ullah, 1999) by the Nadaraya–Watson kernel estimator

$$\hat{f}(x) = \frac{1}{Th}\sum_{t} K\left(\frac{x-x_t}{h}\right) \tag{1}$$

for each x in the interval (a, b), where K is a r-th order kernel function and h is the smoothing parameter.

Under some regularity conditions (see Pagan & Ullah, 1999, for instance), this non-parametric estimator has an asymptotic normal distribution, when $Th \to \infty$ $Th \to \infty$, and $h \to 0$

$$\sqrt{Th}\left(\hat{f}(x)-f(x)\right) \to N\left(0, f(x)\int K^2(u)du\right), \, a<x<b, \tag{2}$$

where K is an appropriate kernel function, and where the asymptotic bias vanishes when .$\sqrt{Th}h^2 \to \infty$. On the basis of this limit distribution, one can easily construct point wise confidence intervals. However, rather than relying on the asymptotic limit distribution, we apply the bootstrap (see, for instance, Horowitz (2001, 2003), for general discussions) to construct such point wise confidence intervals, where we follow Hall (1992). As discussed by Hall (1992), there are two ways to deal with the asymptotic bias. One is the method of explicit bias removal; the other method is under-smoothing, which relies on choosing the bandwidth h so small that the asymptotic bias becomes negligible. Hall (1992) compares these two methods in terms of the errors in the coverage probabilities of bootstrap confidence intervals. The conclusion seems to be that under-smoothing is the better method for handling the asymptotic bias when the aim is to minimize the differences between the true and nominal rejection and coverage probabilities of the bootstrap-based confidence intervals. To get this result a fourth order kernel, such as $K(z) = (15/32)(7z^4 - 10z^2 + 3)$ if $|z|<1$, and $K(z) = 0$ otherwise, needs to be used. Accordingly, this kernel and Hall's suggestion on under-smoothing will be used for the bias removal in our study.

Obviously, a set of $(1-\alpha)$ pointwise confidence intervals constructed for a discretized finite interval will not achieve $(1-\alpha)$ joint coverage probability. So, we also consider a uniform confidence band. Hall (1993) suggests a bootstrap procedure to construct a simultaneous confidence bands which we will use in our study. Hall (1993) suggests removing the asymptotic bias by the so-called explicit method, because it is difficult to determine the appropriate amount of under-smoothing. Hall (1993) uses the standard normal kernel to estimate f, while the second order approximation of the bias is estimated by estimating the second order derivative of f, appearing in the bias approximation, by using the second derivative of the standard normal kernel. We will follow these suggestions in our study.

Appendix B2. Power–law tail behaviour

Since the work of Mandelbrot (1963), power–law tail behaviour was found in a wide range of financial time series, it has become one of the salient features of financial markets. If f_{Normal} is the probability density function of a normal distribution with mean μ and variance σ^2, then we have $log f_{\text{Normal}}(x) \sim -\frac{1}{2\sigma^2}x^2$ as $x \to \pm\infty$. A random variable X is said to follow a power–law or Pareto distribution with shape parameter $\alpha>0$ and scale parameter $\beta>0$ if $\Pr[X>x] = (x/\beta)^{-\alpha}$, for $x \geq \beta$, then we have $\log f_{\text{Pareto}}(x) \sim -(\alpha+1)\log(x)$ as $x \to +\infty$. The difference of the tail behaviour between the normal and Pareto distribution is huge.

Estimations of the tail-index have been studied in great detail in Extreme Value Theory (EVT). Let X_1, X_2,...,X_n be a sequence of observations from some distribution function F, with its order statistics $X_{1,n} \leq X_{2,n} \leq ... \leq X_{n,n}$. As an analogue to the Central Limit Theorem for averages, we know that if maximum $X_{n,n}$, suitably centered and scaled, converges to a non-degenerate random variable, then sequences $\{\alpha_n\}(\alpha_n>0)$ and $\{b_n\}$ exist such that

$$\lim_{n\to\infty} \Pr\left(\frac{X_{n,n}-b_n}{a_n} \leq x\right) = G_\gamma(x), \tag{3}$$

where

$$G_y(x) := exp\left(-(1+\gamma x)^{-1/\gamma}\right) \tag{4}$$

for some $\gamma \in R$ with x such that $1+\gamma x > 0$; Note that for $\gamma = 0$, $-(1+\gamma x)^{-1/\gamma} = e^{-x}$. If the above equation holds, then we say that F is in the max-domain of attraction of G_γ and γ is called the extreme value index. In Pareto distribution, the extreme value index $\gamma = 1/\alpha$ measures the thickness of the tail distribution. The bigger the γ the heavier the tail. The estimation of γ has been thoroughly studied (see Beirlant et al. (2004) for a detailed account). We outline three major estimators, the Hill estimator, the Pickand estimator, and the moment estimator by Dekkers, Einmahl, and de Haan (1989). The Hill Index relies on the average distance between extreme observations and the tail cut-off point to extrapolate the behaviour of the tails into the broader part of the distribution. The Hill Index is defined by

$$H_{k,n} = \frac{1}{k}\sum_{j=1}^{k} logX_{n-j+1,n} - logX_{n-k,n}. \tag{5}$$

This estimator is consistent for $k \to \infty$, $k/n \to 0$ as $n \to \infty$, and under extra conditions, $\sqrt{k}(H_{k,n} - \gamma)$ is asymptotically normal with mean 0 and variance γ^2. Pickand's estimator is defined as

$$\hat{\gamma}_{P,k} = \frac{1}{log\,2} log\left(\frac{X_{n-[k/4]+1,n} - X_{n-[k/2]+1,n}}{X_{n-[k/2]+1,n} - X_{n-k+1,n}}\right) \tag{6}$$

The simplicity of the Pickand's estimator is appealing but offset by large asymptotic variance, equal to $\gamma^2(2^{2\gamma+1}+1)\{(2^{2\gamma}-1)\log 2\}^{-2}$. Dekkers et al. (1989) introduced a moment estimator, which is a direct extension of Hill Index,

$$M_{k,n} = H_{k,n} + 1 - \frac{1}{2}\left(1 - \frac{H_{k,n}^2}{H_{k,n}^{(2)}}\right)^{-1}, \tag{7}$$

where

$$H_{k,n}^{(2)} = \frac{1}{k}\sum_{j=1}^{k}\left(logX_{n-j+1,n} - logX_{n-k,n}\right)^2. \tag{8}$$

They also proved the consistency and asymptotic normality.

When estimating tail indices the key decision variable is the tail cut-off point. This choice involves a trade off between bias and variance. If k is chosen conservatively with few order statistics in the tail then the tail estimate will be sensitive to outliers in the distribution and have a high variance. On the other hand if the tail includes observations in the central part of the distribution, the variance is reduced but the estimate is biased upward. So we plot these estimates over a range of tail sizes.

Appendix B3. GARCH class models

Another striking feature of the return series is *volatility clustering*. A number of econometric models of changing conditional variance have been developed to test and measure volatility clustering. The most widely used one is the one introduced by Engle (1982) and its generalization, the GARCH model, introduced by Bollerslev (1986). Following their specification, for instance, if we model the returns as an AR (1) process, then a GARCH (p, q) model is defined by:

$$\begin{cases} r_t = a + br_{t-1} + \varepsilon_t,\ \varepsilon_t = \sigma_t z_t, \\ \sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2,\ z_t \sim N(0,1), \end{cases} \tag{9}$$

where L is the lag operator, $\alpha(L)=\Sigma_{t=1}^{q}\alpha_i L^i$ and $\beta(L)=\Sigma_{j=1}^{p}\beta_j L^j$. Defining $\upsilon_t=\varepsilon_t^2-\sigma_t^2$, the process can be written as an ARMA (m, p) process

$$[1-\alpha(L)-\beta(L)]\varepsilon_t^2 = \omega + [1-\beta(L)]v_t, \tag{10}$$

with $m=\max\{p, q\}$.

As one of the stylized facts in empirical finance literature, for the estimated parameters one often see that a small influence of the most recent innovation (small α_1) is accompanied by a strong persistence of the variance coefficient (large β_1). It is also interesting to observe that the sum of the coefficients $\alpha_1+\beta_1$ is close to one, which indicates that the process is close to an integrated GARCH (IGARCH) process. Such parameter estimates are rather common when considering returns from high frequency daily financial data of both share and foreign exchange markets (Pagan, 1996).

We extend the standard GARCH models, which weights positive and negative shocks equally, to incorporate two asymmetric GARCH models: exponential GARCH, or EGARCH (1, 1) specification (see Nelson, 1991) and the GJR–GARCH model (see Glosten, Jagannathan, & Runkle, 1993). We have:

$$log\sigma_t^2 = \omega + [1-\beta(L)]^{-1}[1 + \alpha(L)]g(z_{t-1}), \tag{11}$$

The value of $g(z_t)$ depends on several elements to accommodate the asymmetric relation between stock return and volatility changes,

$$g(z_t) = \theta_1 z_t + \theta_2[|z_t|-E|z_t|] \tag{12}$$

$E|z_t|$ depends on the assumption made on the unconditional density of z_t. Indeed, for the normal distribution, $E|z_t| = \sqrt{2/\pi}$. The absolute values of $\theta_1+\theta_2$ and $\theta_1-\theta_2$ capture the asymmetry in response to positive and negative z_t. A leverage effect is present if θ_1 is negative.

Asymmetry is captured in the GJR–GARCH model by the inclusion of a dummy variable in the conditional variance function which takes the value '1' for negative shocks and '0' otherwise. If leverage is present, we expect the coefficient on this dummy variable to be positive to reflect the fact that a negative shock, bad news, increases volatility. We have the GJR–GARCH conditional variance function including the dummy variable:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\left(\alpha_i\varepsilon_{t-i}^2 + \gamma_i S_{t-i}^{-}\varepsilon_{t-i}^2\right) + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2, \tag{13}$$

where s_{t-i}^{-} is the dummy.

Ding et al. (1993) proposed APARCH model as:

$$\sigma_t^{\gamma} = \omega + \sum_{i=1}^{q}\alpha_i(|\varepsilon_{t-i}|-\gamma_i\varepsilon_{t-i})^{\delta} + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^{\delta}, \tag{14}$$

where $\delta>0$ and $-1<\gamma_i<1(i=1,...,q)$.

The standard GARCH model implies that shocks to the conditional variance decay exponentially. However IGARCH implies that the shocks to the conditional variance persist indefinitely. In response to the finding that most of the financial time series are long memory volatility process, Baillie, Bollerslev, and Mikkelsen (1996) consider the Fractional Integrated GARCH (FIGARCH) process, where a shock to the conditional variance dies out at a slow hyperbolic rate. Later on, Chung (1999) suggests a slightly different parameterization of the model:

$$[1-\alpha(L)](1-L)^d(\varepsilon_t^2-\sigma^2) = \omega + [1-\beta(L)]v_t, \tag{15}$$

where $\omega=[1-\alpha(L)](1-L)^{d}\sigma^2$, and σ^2 is the unconditional variance of the corresponding GARCH model, with the fractional differencing parameter $0\le d\le 1$.

Davidson (2004) constructed the hyperbolic GARCH (HYGARCH):

$$[1-\alpha(L)][(1-\alpha) + \alpha(1-L)^d]\varepsilon_t^2 = \omega + [1-\beta(L)]v_t. \tag{16}$$

The HYGARCH nests FIGARCH when $\alpha=1$ or equivalently when log $\alpha=0$.

Appendix B4. Semi-parametric estimation of memory parameters

While the FIGARCH and HYGARCH approaches model memory parameters within the GARCH parameterization framework, the semi-parametric approach is more general and imposes fewer restrictions on the distributional properties of the data generating process. There are several possible definitions of the property of “long memory”. Following Baillie (1996), given a series x_t, $t=0,\pm 1,...$, with autocorrelation function ρ_j at lag j, we say that the process possesses long memory if ρ_j decays “slowly”, i.e., if the quantity

$$\lim_{n\to\infty} \sum_{j=-n}^{j=n} |\rho_j| \tag{17}$$

is non-finite. Equivalently, the spectral density $s(\omega)$ will be unbounded at low frequencies. We semiparametrically model long memory in a covariance stationary series x_t, $t=0,\pm 1,...$, by

$$s(\omega)\approx G\omega^{-2d},\ \omega\to 0^+, \tag{18}$$

where $0<G<\infty$. Under the above equation, $s(\omega)$ has a pole at $\omega=0$ for $0<d<1/2$ (when there is long memory in x_t), while $d\geq 1/2$ implies the process is not covariance stationary; $s(\omega)$ is positive and finite for $d=0$; for $-1/2<d<0$, we have short memory, negative dependence, or anti-persistence. Notice that the autocorrelations can then be described as $\rho_j\approx G_1 j^{2d-1}$, where G_1 is a constant and $\mu=2d-1$ correspond to the hyperbolic decay index.

Geweke and Poter-Hudak (1983), henceforth GPH, suggested a semi-parametric estimator of the fractional differencing parameter, d, that is based on a regression of the ordinates of the log spectral density. The estimator exploits the theory of linear filters to write the process $(1-L)^d y_t=u_t$, where $u_t \sim I(0)$ i.e., the process $\{u_t\}$ is stationary. Let $s_y(\omega)$ and $s_u(\omega)$ be the spectral densities of y_t and u_t, respectively. Then

$$s_y(\omega) = |1-e^{-i\omega}|^{-2d} s_u(\omega). \tag{19}$$

This can be expressed as

$$\log s_y(\omega) = \log\left(4\sin^2\left(\frac{\omega}{2}\right)\right)^{-d} + \log s_u(\omega). \tag{20}$$

Given spectral ordinates $\omega_1, \omega_2,...,\omega_m$, this becomes

$$\log s_y(\omega_j) = \log s_u(0) - d\log\left(4\sin^2\left(\frac{\omega_j}{2}\right)\right) + \log\left(s_u(\omega_j)/s_u(0)\right). \tag{21}$$

GPH suggest estimating d from a regression of the ordinates from the periodogram of y_t, that is $I_y(\omega_j)$.

Hence, for $j=1,2,...,m$,

$$\log I_y(\omega_j) = c - d\log\left(4\sin^2\left(\frac{\omega_j}{2}\right)\right) + v_j, \tag{22}$$

where

$$v_j = \log\left(s_u(\omega_j)/s_u(0)\right) \tag{23}$$

and v_j are assumed to be i.i.d. with zero mean and variance $\pi^2/6$. When u_t is white noise, then the above regression should provide a good estimate of d. When u_t is autocorrelated, GPH show that the above regression holds approximately for frequencies in the neighbourhood of zero. If this neighbourhood shrinks at an appropriate rate with the sample size, then the GPH procedure should realize a consistent estimator of d. If the number of ordinates m is chosen such that $m=g(T)$, where $g(T)$ is such that $\lim_{T\to\infty} g(T)=\infty$, $\lim_{T\to\infty} g(T)/T=0$, and $\lim_{T\to\infty}(\log(T)^2)/g(T)=0$, then the OLS estimator of d will have the limiting distribution

$$\sqrt{m}\left(\hat{d}_{\mathrm{GPH}}-d\right) \xrightarrow{d} N\left(0, \frac{\pi^2}{24}\right). \tag{24}$$

Robinson (1995) provided the formal proof for $-1/2<d<1/2$, Velasco (1999) proved the consistency of $\hat{d}_{\mathrm{GPH}}$ in the case $1/2\le d<1$ and its asymptotic normality in the case $1/2\le d<3/4$. The variance of this estimator can be obtained from the usual OLS regression formula. It is clear from this result that the GPH estimator is not $\sqrt{T}$-consistent and will converge at a slower rate.

We also employ Robinson and Henry's (1999) estimator of d, henceforth RH. They suggest a semi-parametric Gaussian estimate of the memory parameter d, by considering

$$\hat{d}_{\mathrm{RH}} = arg\min_{d} R(d) \tag{25}$$

where $R(d)$ is

$$R(d) = log\left\{\frac{1}{m}\sum_{j=1}^{m} \omega_j^{2d} I\left(\omega_j\right)\right\} - 2\frac{d}{m}\sum_{j=1}^{m} log\omega_j, \tag{26}$$

in which $m\in(0,[T/2]$ and $\omega_j=2\pi j/T$. They proved that when $m<[T/2]$ such that, as $T\to\infty$,

$$\frac{1}{m}+\frac{m}{T}\to 0 \tag{27}$$

and under some further conditions (see Robinson & Henry, 1999), we have

$$\sqrt{m}\left(\hat{d}_{\mathrm{RH}}-d\right)\xrightarrow{d} N\left(0,\frac{1}{4}\right) \tag{28}$$

as $T\to\infty$.An issue in the application of the GPH and the RH estimators is the choice of the number of spectral ordinates m. For both the GPH and the RH estimators of d we report the corresponding estimates for $m=50$, 100, 150, 200, and 250, which is consistent with the insights and recommendations from the available literature (see, Geweke, 1998; Lobato & Savin, 1998; Robinson, 1995).

Appendix C. Statistical results

Table C1
Estimates of GARCH class of models for the Africa Large Company Index (ABRL).

	GARCH	EGARCH	GJR	APARCH	FIGARCH	HYGARCH
a	0.0004* (0.0002)	0.0004* (0.0002)	0.0003 (0.0002)	0.0002 (0.0002)	0.0004** (0.0002)	0.0005** (0.0002)
b	−0.0759** (0.0206)	−0.0494** (0.0174)	−0.0624** (0.0210)	−0.0635** (0.0201)	−0.0827** (0.0212)	−0.0836** (0.0229)
$\omega\times10^4$	0.0085** (0.0014)	0.0000 (58,575)	0.0335** (0.0052)	0.5203 (0.2769)	0.8869** (0.1261)	0.0365** (0.0101)
a_1	0.0624** (0.0039)	1.0001** (0.2484)	0.0834** (0.0084)	0.1437** (0.0103)	0.9926** (0.0015)	0.4990** (0.0720)
β_1	0.9372** (0.0028)	0.9959** (0.0025)	0.8474** (0.0075)	0.8709** (0.0076)	0.9728** (0.0027)	0.6838** (0.0446)
θ_1		−0.0253** (0.0045)				
θ_2		0.1797** (0.0225)				
γ_1			0.1347** (0.0125)	0.2316** (0.0232)		
δ				1.3650** (0.1134)		
d					0.1132** (0.0146)	0.3121** (0.0544)
log a						0.2088** (0.0676)

Notes: *indicates significance at 5%; **indicates significance at 1% level, or less.

Table C2
Estimates of GARCH class of models for the Africa Medium Company Index (ABRM).

	GARCH	EGARCH	GJR	APARCH	FIGARCH	HYGARCH
a	0.0004* (0.0002)	0.0005** (0.0001)	0.0004* (0.0002)	0.0004* (0.0002)	0.0004* (0.0002)	0.0005** (0.0002)
b	−0.1266**	−0.2175**	−0.1333**	−0.1435**	−0.1266**	−0.1111**

(continued on next page)

Table C2 (*continued*)

	GARCH	EGARCH	GJR	APARCH	FIGARCH	HYGARCH
	(0.0265)	(0.0218)	(0.0258)	(0.0247)	(0.0266)	(0.0279)
$\omega\times 10^4$	0.1610** (0.0185)	0.0000 (33,918)	0.1405** (0.0180)	0.9146 (0.6616)	0.6921** (0.0258)	0.1402 (0.1524)
a_1	0.1085** (0.0180)	1.0001** (0.1166)	0.1221** (0.0242)	0.0855** (0.0147)	0.7673** (0.0273)	0.2102 (0.1906)
β_1	0.6588** (0.0395)	0.9952** (0.0014)	0.6958** (0.0393)	0.7992** (0.0258)	0.6588** (0.0395)	0.7056* (0.3214)
θ_1		−0.0004 (0.0059)				
θ_2		0.1217** (0.0088)				
γ_1			−0.0409 (0.0257)	−0.3519** (0.0805)		
δ				1.5122** (0.1401)		
d					0.0000 (0.0077)	0.8599** (0.2908)
log a						−0.3071 (0.4709)

Notes: *indicates significance at 5%; **indicates significance at 1% level, or less.

Table C3
Estimates of GARCH class of models for the Africa Small Company Index (ABRS).

	GARCH	EGARCH	GJR	APARCH	FIGARCH	HYGARCH
a	0.0001 (0.0001)	0.0000 (0.0001)	0.0001 (0.0001)	0.0001 (0.0001)	0.0000 (0.0001)	0.0000 (0.0000)
b	−0.0201 (0.0199)	−0.0427* (0.0165)	−0.0394* (0.0197)	−0.0424* (0.0192)	0.0048 (0.0183)	−0.0073 (0.0000)
$\omega\times 10^4$	0.0036** (0.0006)	0.0000 (55,304)	0.0042** (0.0007)	0.1387 (0.1003)	0.4062** (0.0976)	0.0046* (0.0019)
a_1	0.0740** (0.0066)	1.0001** (0.2417)	0.0882** (0.0084)	0.0790** (0.0062)	0.3459** (0.0172)	0.1890* (0.0986)
β_1	0.9272** (0.0064)	0.9974** (0.0013)	0.9297** (0.0065)	0.9334** (0.0061)	0.8444** (0.0150)	0.9025** (0.0381)
θ_1		0.0213** (0.0051)				
θ_2		0.1575** (0.0188)				
γ_1			−0.0420** (0.0092)	−0.1784** (0.0405)		
δ				1.2822** (0.1558)		
d					0.5939** (0.0307)	0.8325** (0.1336)
log a						0.0044 (0.0075)

Notes: *indicates significance at 5%; **indicates significance at 1% level, or less.

Table C4
Estimates of the memory parameter (d) for the Africa Large Company Index (ABRL).

	$\hat{d}_{\mathrm{GPH}}$	t	p-value	95% CI	$\hat{d}_{\mathrm{RH}}$	t	p-value	95% CI
r_t	0.2595	2.519	0.0118	[0.0576, 0.4615]	0.2280	3.224	0.0013	[0.1100, 0.3872]
	0.1130	1.627	0.1038	[−0.0232, 0.2491]	0.1427	2.854	0.0043	[0.0447, 0.2407]
	0.1012	1.820	0.0688	[−0.0078, 0.2103]	0.1307	3.202	0.0014	[0.0507, 0.2108]
	0.0278	0.583	0.5600	[−0.0657, 0.1212]	0.0637	1.803	0.0714	[−0.0056, 0.1330]
	0.0788	1.859	0.0630	[−0.0043, 0.1618]	0.0883	2.791	0.0053	[0.0263, 0.1502]
r_t^2	0.5270	5.114	0.0000	[0.3250, 0.7289]	0.4896	6.924	0.0000	[0.3510, 0.6282]
	0.2291	3.298	0.0010	[0.0929, 0.3652]	0.2443	4.885	0.0000	[0.1463, 0.3423]
	0.1888	3.394	0.0007	[0.0798, 0.2979]	0.2310	5.658	0.0000	[0.1510, 0.3110]
	0.1305	2.738	0.0062	[0.0371, 0.2240]	0.1849	5.230	0.0000	[0.1156, 0.2542]
	0.1523	3.595	0.0003	[0.0692, 0.2353]	0.1818	5.750	0.0000	[0.1198, 0.2438]
$\|r_t\|$	0.5761	5.591	0.0000	[0.3742, 0.7781]	0.6528	9.231	0.0000	[0.5142, 0.7913]
	0.4365	6.284	0.0000	[0.3003,0.5726]	0.5081	10.16	0.0000	[0.4101,0.6061]

Table C4 (*continued*)

	$\hat{d}_{GPH}$	t	p-value	95% CI	$\hat{d}_{RH}$	t	p-value	95% CI
	0.4242	7.624	0.0000	[0.3151, 0.5332]	0.4848	11.87	0.0000	[0.4048, 0.5648]
	0.3333	6.991	0.0000	[0.2399, 0.4267]	0.4213	11.92	0.0000	[0.3520, 0.4906]
	0.3402	8.031	0.0000	[0.2571, 0.4232]	0.4175	13.20	0.0000	[0.3555, 0.4795]

Notes: d_{GPH} is the Geweke and Poter-Hudak (1983) estimator of the memory parameter; d_{RH} is the Robinson and Henry (1999) estimator of the memory parameter. Estimates are reported for bandwidths (m) (50, 100, 150, 200, and 250). r_t is the daily log return, r_t^2 is the squared return and $|r_t|$ is the absolute return.

Table C5
Estimates of the memory parameter (d) for the Africa Medium Company Index (ABRM).

	$\hat{d}_{GPH}$	t	p-value	95% CI	$\hat{d}_{RH}$	t	p-value	95% CI
r_t	0.3100	3.008	0.0026	[0.1080, 0.5119]	0.3106	4.393	0.0000	[0.1721, 0.4492]
	0.3114	4.484	0.0000	[0.1753, 0.4476]	0.2832	5.664	0.0000	[0.1852, 0.3812]
	0.2230	4.008	0.0001	[0.1139, 0.3320]	0.2196	5.380	0.0000	[0.1396, 0.2997]
	0.1542	3.234	0.0012	[0.0607, 0.2476]	0.1754	4.962	0.0000	[0.1061, 0.2447]
	0.1508	3.561	0.0004	[0.0678, 0.2339]	0.1633	5.163	0.0053	[0.1013, 0.2252]
r_t^2	0.1083	1.051	0.2932	[−0.0936, 0.3103]	0.2036	2.879	0.0040	[0.0650, 0.3422]
	0.0128	0.185	0.8534	[−0.1233, 0.1490]	0.1128	2.257	0.0240	[0.0148, 0.2108]
	0.0014	0.026	0.9794	[−0.1076, 0.1105]	0.0732	1.793	0.0730	[−0.0068, 0.1532]
	0.0158	0.332	0.7396	[−0.0776, 0.1093]	0.0779	2.204	0.0276	[0.0086, 0.1472]
	0.0173	0.407	0.6837	[−0.0658, 0.1003]	0.0735	2.323	0.0202	[0.0115, 0.1354]
$\lvert r_t \rvert$	0.5978	5.802	0.0000	[0.3958, 0.7997]	0.6099	8.625	0.0000	[0.4713, 0.7485]
	0.4553	6.555	0.0000	[0.3192, 0.5915]	0.5157	10.31	0.0000	[0.4177, 0.6137]
	0.3278	5.891	0.0000	[0.2187, 0.4368]	0.4012	9.828	0.0000	[0.3212, 0.4812]
	0.3137	6.581	0.0000	[0.2203, 0.4072]	0.3889	11.00	0.0000	[0.3196, 0.4582]
	0.2713	6.405	0.0000	[0.1883, 0.3543]	0.3710	11.73	0.0000	[0.3090, 0.4330]

Notes: d_{GPH} is the Geweke and Poter-Hudak (1983) estimator of the memory parameter; d_{RH} is the Robinson and Henry (1999) estimator of the memory parameter. Estimates are reported for bandwidths (m) (50, 100, 150, 200, and 250). r_t is the daily log return, r_t^2 is the squared return and $|r_t|$ is the absolute return.

Table C6
Estimates of the memory parameter (d) for the Africa Small Company Index (ABRS).

	$\hat{d}_{GPH}$	t	p-value	95% CI	$\hat{d}_{RH}$	t	p-value	95% CI
r_t	0.1561	1.515	0.1298	[−0.0459, 0.3581]	0.2344	3.314	0.0009	[0.0958, 0.3729]
	0.1624	2.338	0.0194	[0.0262, 0.2985]	0.2269	4.537	0.0000	[0.1289, 0.3249]
	0.1768	3.178	0.0015	[0.0677, 0.2858]	0.2063	5.052	0.0000	[0.1263, 0.2863]
	0.1651	3.464	0.0005	[0.0717, 0.2586]	0.2050	5.798	0.0000	[0.1357, 0.2743]
	0.1717	4.046	0.0001	[0.0885, 0.2548]	0.2078	6.572	0.0000	[0.1459, 0.2698]
r_t^2	0.2584	2.507	0.0122	[0.0564, 0.4603]	0.2795	3.953	0.0001	[0.1409, 0.4181]
	0.2346	3.377	0.0007	[0.0984, 0.3707]	0.2524	5.047	0.0000	[0.1544, 0.3504]
	0.2053	3.690	0.0002	[0.0962, 0.3143]	0.2183	5.346	0.0000	[0.1382, 0.2983]
	0.1475	3.095	0.0020	[0.0541, 0.2410]	0.1664	4.708	0.0000	[0.0971, 0.2357]
	0.0947	2.236	0.0254	[0.0117, 0.1777]	0.1247	3.943	0.0001	[0.0627, 0.1867]
$\lvert r_t \rvert$	0.6371	6.183	0.0000	[0.4351, 0.8390]	0.6379	9.021	0.0000	[0.4993, 0.7765]
	0.6356	9.149	0.0000	[0.4994, 0.7717]	0.5850	11.70	0.0000	[0.4870, 0.6830]
	0.5141	9.241	0.0000	[0.4051, 0.6232]	0.5109	12.51	0.0000	[0.4309, 0.5909]
	0.4202	8.796	0.0000	[0.3265, 0.5138]	0.4505	12.74	0.0000	[0.3812, 0.5198]
	0.3618	8.535	0.0000	[0.2787, 0.4449]	0.4108	12.99	0.0000	[0.3488, 0.4728]

Notes: d_{GPH} is the Geweke and Poter-Hudak (1983) estimator of the memory parameter; d_{RH} is the Robinson and Henry (1999) estimator of the memory parameter. Estimates are reported for bandwidths (m) (50, 100, 150, 200, and 250). r_t is the daily log return, r_t^2 is the squared return and $|r_t|$ is the absolute return.

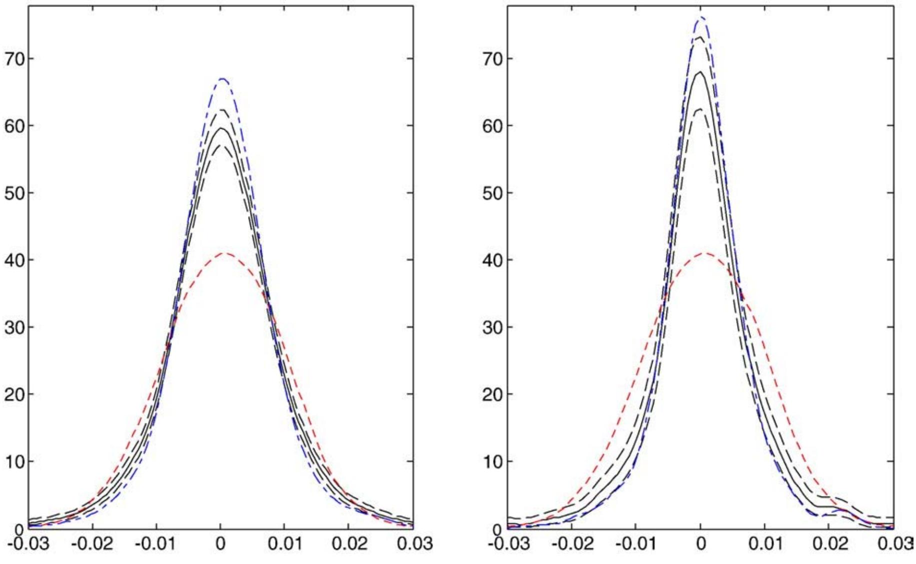

Fig. C1. The estimated probability density functions for the Africa All-Share Index (ABRA) with confidence intervals (left) and confidence band (right), of the Africa Medium Company Index (ABRM) (dash–dot line), and of normal distribution (dot line).

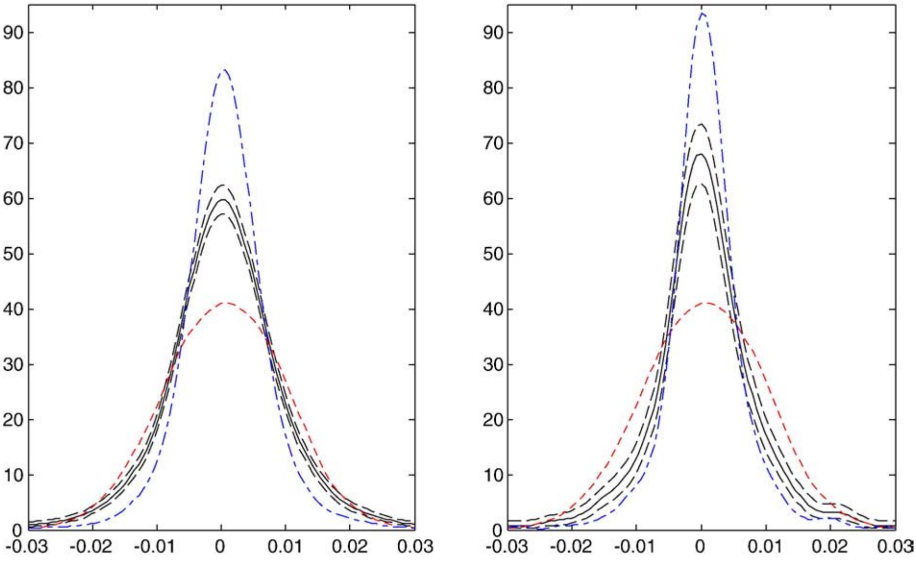

Fig. C2. The estimated probability density functions for the Africa All-Share Index (ABRA) with confidence intervals (left) and confidence band (right), of the Africa Small Company Index (ABRS) (dash–dot line), and of normal distribution (dot line).

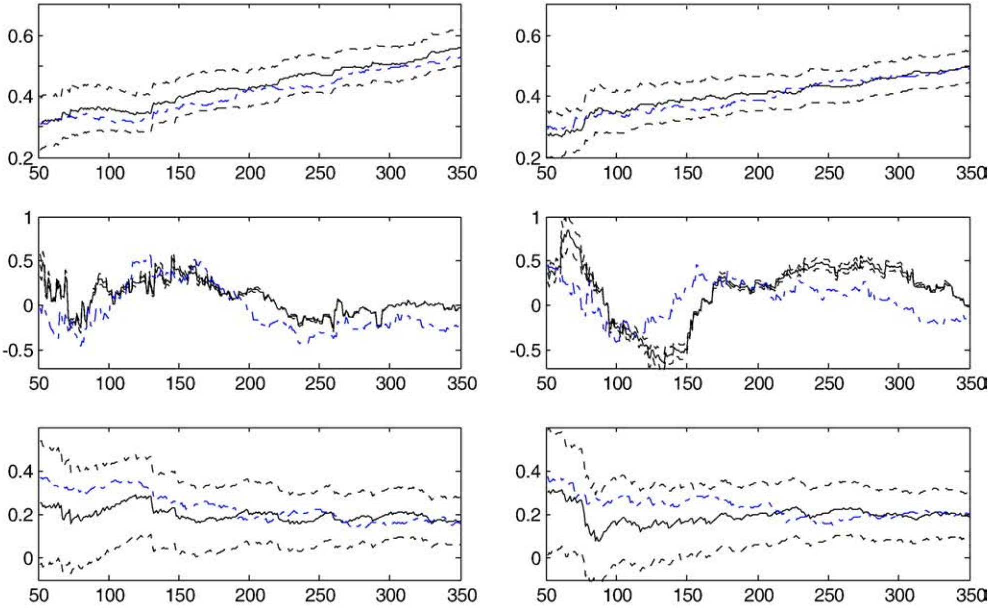

Fig. C3. The Hill (first row), the Pickand (second row), and the moment (the third row) estimates plots of the negative tail (left column) and the positive tail (right column) of the Africa All-Share Index (ABRA), with confidence intervals, compared to Africa Large Company Index (ABRL).

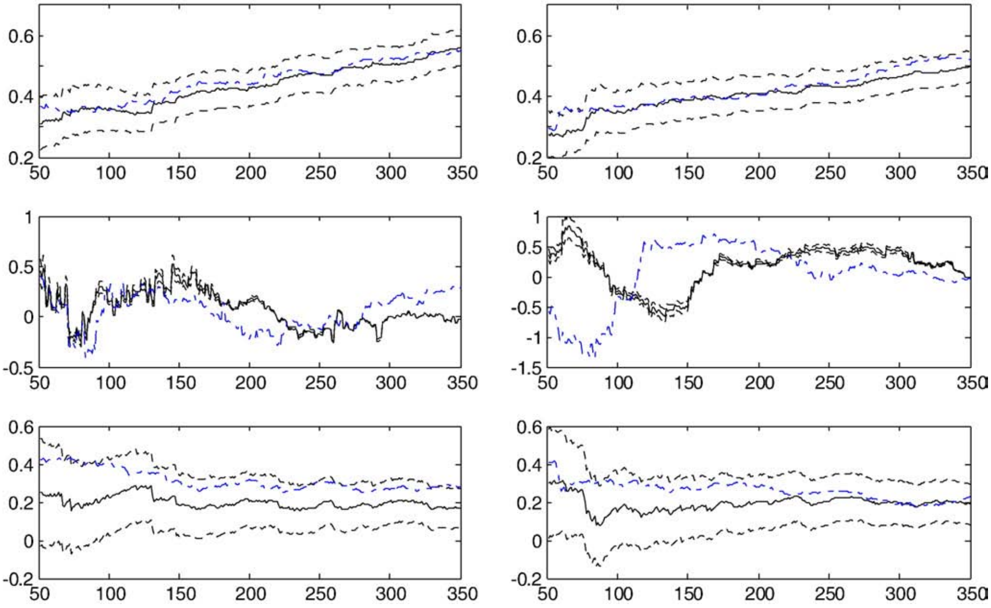

Fig. C4. The Hill (first row), the Pickand (second row), and the moment (the third row) estimates plots of the negative tail (left column) and the positive tail (right column) of the Africa All-Share Index (ABRA), with confidence intervals, compared to Africa Medium Company Index (ABRM).

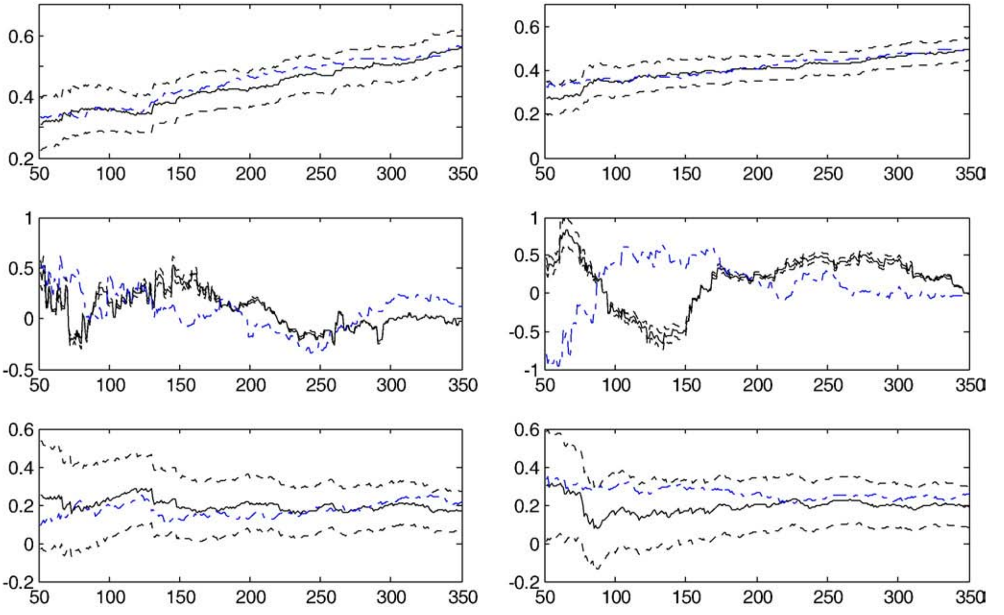

Fig. C5. The Hill (first row), the Pickand (second row), and the moment (the third row) estimates plots of the negative tail (left column) and the positive tail (right column) of the Africa All-Share Index (ABRA), with confidence intervals, compared to Africa Small Company Index (ABRS).

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