



Quantum microwave-to-optical conversion in electrically driven multilayer graphene

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Abstract: In this paper, we propose a novel quantum approach for microwave-to-optical conversion in a multilayer graphene structure. The graphene layers are electrically connected and pumped by an optical field. The physical concept is based on using a driving microwave signal to modulate the optical input pump by controlling graphene conductivity. Consequently, upper and lower optical sidebands are generated. To achieve low noise conversion, the lower sideband is suppressed by the multilayer graphene destruction resonance. A perturbation approach is implemented to model the effective permittivity of the electrically driven multilayer graphene. Subsequently, a quantum mechanical analysis is carried out to describe the evolution of the interacting fields. It is shown that a quantum microwave-to-optical conversion is achieved for multilayer graphene of the proper length (i.e., number of layers). The conversion rate and the number of converted photons are evaluated according to several parameters. These include the microwave signal frequency, the microwave driving voltages, the graphene intrinsic electron density, and the number of graphene layers. Owing to multilayer dispersion and to the properties of graphene, it is shown that a significant number of photons (converted from microwave to optical frequency range) is achieved for microvolt microwave driving voltages. Furthermore, a frequency-tunable operation is achieved using this technique simply by modifying the optical pump frequency.

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1. Introduction

Over the past few decades, merging photonics and microwave systems has been intensively investigated, leading to the development of microwave photonic systems [1, 2]. In these systems, microwave signals are generated [3], detected [4], and processed [5] using photonics techniques, enabling vast bandwidths, unprecedented tunability, and efficient transmissions over long distances [6].

Recently, scientific interests have extended to combine quantum photonics and quantum microwave systems [7-17], as each has its own distinct practical advantages. For instance, quantum microwave systems hold potential for quantum signal processing and computation [18-20]. However, they operate at cryogenic temperatures, and thus, it is practically impossible to transmit signals over long-distances. On the other hand, quantum photonic systems operate at room temperature and are compatible with optical fibers and waveguides [21, 22]. Therefore, several techniques have been proposed to couple quantum photonics and quantum microwave systems. These include atomic interface technique [9], opto-mechanical techniques [11, 15] and electro-optic (i.e., EO)-based techniques [7, 8, 13, 14]. The EO-based techniques offer wide operation bandwidths and are tunable and scalable. Typically, the EO effect is used to modulate an optical input pump by a driving microwave signal. Upper and lower optical sidebands are generated. However, in a quantum limit, the lower side band imposes noise upon the conversion

process. This is because converting a pump photon into a lower side band photon implies generating a microwave photon. Nevertheless, this process can be conducted by means of spontaneous emission even in the absence of the driving microwave signal [23]. Therefore, a single sideband modulation scheme has been proposed for quantum microwave-to-optical conversion, in which the lower side band is suppressed. For example, in [14], a resonant whispering gallery resonator (filled with EO crystal) has been demonstrated to achieved single side band microwave-to-optical conversion, while in [16], coupled nanophotonics resonators (also filled with EO crystal) were proposed. However, the EO coefficients are very small even in best known EO crystals, such as the $LiNbO_3$. Large microwave voltages (i.e., in millivolts) [16] are required to conduct the microwave-to-optical conversion process. High Q-factor resonators were proposed to enhance the EO effect. However, incorporating resonators limits the tunability of the conversion process.

On the other hand, graphene-based-structures have recently been proposed to achieve efficient optical modulation [24]. The principle of operation is based on variations in graphene optical conductivity in response to externally applied voltage or electric field. Interestingly, optical modulation based on graphene structures requires significantly smaller driving microwave voltages, as compared to the EO modulators [25]. This is very beneficial for quantum microwave systems, as signals generated by superconducting systems are essentially in the microvolt range.

In this work, we propose a novel quantum technique for microwave-to-optical conversion based on multilayer graphene structure. We consider a quantum microwave signal generated by an external superconducting circuitry driving the multilayer graphene. The multilayer graphene is electrically connected in an interdigital fashion and subjected to an optical input pump. From an electrical point of view, the structure functions as a capacitor, while from an optical standpoint, the structure functions as a periodic medium. The driving microwave signal modulates the optical input pump by modifying graphene conductivity. Normally, upper and lower side bands are generated. To achieve single side band modulation, the destruction resonance frequency of the multilayer graphene (at which the group velocity is zero) is designed to occur at the lower sideband frequency. The graphene medium is considered at cryogenic temperature and the induced optical and microwave dissipation are characterized by the means of effective optical and microwave time decay rates, Γ and Γ_m respectively. Consequently, a quantum mechanics frame work is developed to describe the fields evolution. Thanks to multilayer dispersion and to the properties of graphene, it is shown that microwave-to-optical conversion is achieved for microvolt driving signals and over a vast microwave frequency range. Frequency-tunable conversion is achieved simply by controlling the frequency of the optical input pump.

The remaining part of the paper is organized as follows. In section 2, the multilayer graphene is modeled. The electrical capacitance is calculated in subsection 2.1. A perturbation approach, describing the modulated optical conductivity, is implemented in subsection 2.2. An effective permittivity expression for the multilayer graphene is obtained in subsection 2.3. In section 3, quantum mechanical analyses are carried out to describe the fields' evolution. In section 4, the quantum microwave-to-optical conversion is evaluated considering realistic parameters. In section 5, concluding remarks are made.

2. Proposed multilayer graphene structure

Consider a multilayer graphene occupying the yz plane, as shown in Fig. 1. The graphene layers are electrically driven by a microwave signal of frequency f_m and subjected to an optical input pump of frequency f_1 . The graphene layers are connected in an interdigital configuration. The optical input pump is applied normally to the graphene layers and propagates in the direction of the x -axis. In the following sections, the microwave-to-optical conversion process will be modeled and evaluated.

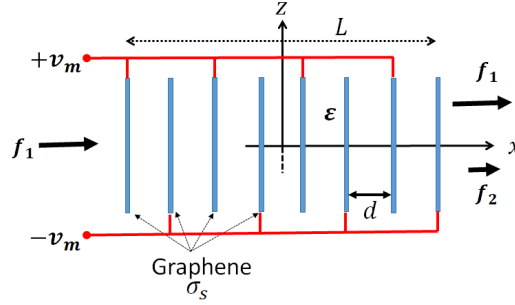


Fig. 1. Multilayer graphene structure connected in an interdigital fashion.

2.1. Equivalent electrical capacitance

The proposed multilayer graphene made of graphene layers which are separated by distance d and connected in an interdigital configuration. From an electrical point of view, this structure can be modeled as a capacitor. Due to symmetry, N identical graphene layers can be conceived as $2N - 2$ shunted identical capacitors (each of capacitance $C = 2 \frac{\epsilon_0 \epsilon}{d}$) [26], as shown in Fig. 2. Here ϵ is the permittivity of the filling material. Hence, the total capacitance (per unit area) is given by $C_T = (2N - 2)C = \frac{4(N-1)\epsilon_0 \epsilon}{d}$.

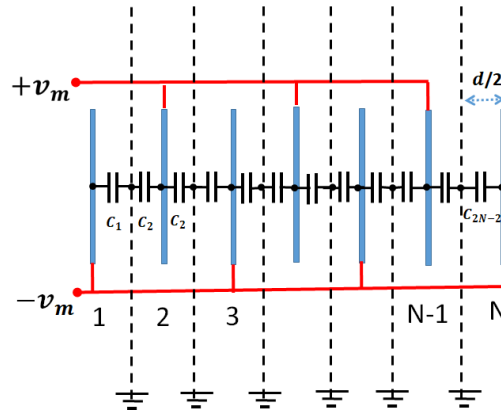


Fig. 2. The equivalent electric capacitance of the multilayer graphene structure.

2.2. Modulated optical conductivity

From an optical standpoint, multilayer graphene can be modeled by the means of effective permittivity. To this end, one may start with the graphene's conductivity; a perturbation approach will then be implemented to describe the effective permittivity. The graphene conductivity is conducted by both interband and intraband mechanisms, given by [27]:

$$\sigma_s = \frac{iq^2}{4\pi\hbar} \ln \left(\frac{2\mu_c - (f + i\tau^{-1})\hbar}{2\mu_c + (f + i\tau^{-1})\hbar} \right) + \frac{iq^2 K_B T}{\pi\hbar^2 (f + i\tau^{-1})} \left(\frac{\mu_c}{K_B T} + 2 \ln \left(e^{-\frac{\mu_c}{K_B T}} + 1 \right) \right), \quad (1)$$

Here, the first term describes the interband conductivity, the second term represents the intraband conductivity, q is the electron charge, $\hbar$ is the plank's constant, τ expresses the scattering relaxation time, K_B represents the Boltzman constant, T is the temperature, f is the frequency, and μ_c

expresses the graphene chemical potential. In this work, the operation temperature is considered at the cryogenic level. The graphene conductivity at this temperature is mainly dominated by the interband mechanism, while the intraband converge to Drude model [28].

The graphene chemical potential is given by:

$$\mu_c = \hbar V_f \sqrt{\pi n_0 + \frac{2C_T}{q} v_m}, \quad (2)$$

where n_0 is the electron density per unit area, $V_f = 10^6$ m/s, which is the Fermi velocity of the Dirac fermions, and v_m is the driving microwave voltage, defined by:

$$v_m = v e^{-i2\pi f_m t} + c.c., \quad (3)$$

f_m is the microwave frequency and c.c. represents the complex conjugate.

On substituting the microwave voltage in Eq. (3) into the chemical potential in Eq. (2) and using the approximation $\sqrt{1+\chi} \approx 1 + \frac{\chi}{2}$ for $\chi \ll 1$, the chemical potential for $2C_T v \ll \pi n_0 q$, can be approximated by:

$$\mu_c = \mu'_c + v \mu''_c e^{-i2\pi f_m t} + c.c., \quad (4)$$

where

$$\mu'_c = \hbar V_f \sqrt{\pi n_0}, \quad \text{and} \quad \mu''_c = \hbar V_f \frac{C_T}{q \sqrt{\pi n_0}}. \quad (5)$$

By substituting the chemical potential in Eq. (4) into the conductivity expression in Eq. (1), and for $v \mu''_c \ll \mu'_c$, the graphene's conductivity can be approximated up to the first order [29], given by:

$$\sigma_s = \sigma'_s + v \sigma''_s e^{-i2\pi f_m t} + c.c., \quad (6)$$

where

$$\sigma'_s = \frac{iq^2}{4\pi\hbar} \ln \left(\frac{2\mu'_c - (f + i\tau^{-1})\hbar}{2\mu'_c + (f + i\tau^{-1})\hbar} \right) + \frac{iq^2 K_B T}{\pi \hbar^2 (f + i\tau^{-1})} \left(\frac{\mu'_c}{K_B T} + 2 \ln \left(e^{-\frac{\mu'_c}{K_B T}} + 1 \right) \right), \quad (7)$$

$$\sigma''_s = \frac{iq^2}{\pi \hbar} \frac{(f + i\tau^{-1})\hbar}{4(\mu'_c)^2 - (f + i\tau^{-1})^2 \hbar^2} \mu''_c + \frac{iq^2 K_B T}{\pi \hbar^2 (f + i\tau^{-1})} \tanh \left(\frac{\mu'_c}{2K_B T} \right) \frac{\mu''_c}{K_B T}. \quad (8)$$

Here, $v \sigma''_s \ll \sigma'_s$.

2.3. Propagating optical fields

The dispersion relation of the multilayer graphene medium, shown in Fig. 1, is given by [30]:

$$\cos(d\beta) = \cos \left(d \sqrt{\epsilon} \frac{2\pi f}{c} \right) - i \frac{Z_0}{2\sqrt{\epsilon}} \sin \left(d \sqrt{\epsilon} \frac{2\pi f}{c} \right) \sigma_s, \quad (9)$$

where β is the propagation constant and Z_0 is the free space impedance.

Inferred from the conductivity expression in Eq. (6), the propagation constant can be presented as:

$$\beta = \beta' + v \beta'' e^{-i2\pi f_m t} + c.c., \quad (10)$$

Consequently, on substituting the propagating constant in Eq. (10) into the dispersion relation in Eq. (9), and expanding the nonlinear terms, two outcomes can be obtained. First, it can be shown that β' satisfies the dispersion relation in Eq. (9) with σ'_s in lieu of σ_s . Second, it can be shown that β'' is given by:

$$\beta'' = i \frac{Z_0}{2d\sqrt{\varepsilon}} \frac{\sin\left(d \frac{2\pi f \sqrt{\varepsilon}}{c}\right)}{\sin(d\beta')} \sigma_s''. \quad (11)$$

Hence, the effective permittivity of the graphene medium is:

$$\varepsilon_{effj} = \varepsilon'_{effj} + \nu \varepsilon''_{effj} e^{-i2\pi f_m t} + c.c., \quad (12)$$

where

$$\varepsilon'_{effj} = \left(\frac{\beta'_j}{k_{0j}}\right)^2, \quad \text{and} \quad \varepsilon''_{effj} = 2 \frac{\beta'_j \beta''_j}{k_{0j}^2}. \quad (13)$$

As can be observed from Eq. (12), the microwave signal modulates the effective permittivity of the multilayer graphene. Thus, upper and lower side bands, with frequencies $f_2 = f_1 + f_m$ and $f_3 = f_1 - f_m$, respectively, are generated.

In this work, we design the destruction resonance of the multilayer graphene to occur at f_3 so that the lower side band is maximally suppressed. It then follows that the spontaneous process is minimized, as discussed previously. Here, it is important to note that the group velocity at the destruction resonance frequency is zero, similar to the reflection resonance for externally incident optical waves. However, in the current scenario there are no reflected waves as the lower side band is generated within the graphene layers and the layered medium is reciprocal. It then follows that the lower sideband is suppressed. This can be achieved by setting $d = \frac{c}{f_3 \sqrt{\varepsilon}}$. The medium transmittance can be calculated to quantify the suppression of the lower side band (as shown next in the results section). Hence, the optical fields existing in the multilayer graphene are given by:

$$\vec{E}_j = u_j \left(e^{-i2\pi f_j t + i\beta'_j x} + c.c. \right) \hat{e}_y, \quad (14)$$

where u_j is the slow varying amplitude and $j \in \{1, 2\}$.

The classical Hamiltonian can be written as:

$$\mathcal{H} = \frac{1}{2} \int_{\mathcal{V}} \left(\varepsilon_0 \varepsilon_{eff} |\vec{E}_t|^2 + \mu_0 |\vec{H}_t|^2 \right) \partial \mathcal{V}, \quad (15)$$

where $\vec{E}_t$ is the total electric field, $\vec{H}_t$ represents the total magnetic field, and $\mathcal{V} = \mathcal{A}L$ is the volume. This hamiltonian in Eq. (15) describes the total electromagnetic energy of the system. The first part represents the total electric field taking into account the effective permittivity, presented in Eq. (12). The second part take into account the magnetic energy, while the system of the graphene multilayers has zero magnetic susceptibility. The effective permittivity is approximated by implementing a perturbation approach considering a weak driving microwave voltage. Hence, the chemical potential (represented by its expansion) can be approximated up to the first order. This is validated by imposing the condition $2C_T \nu \ll \pi n_0 q$. Consequently, the graphene conductivity, and the effective permittivity can be approximated up to the first order. This approach is verified when numerical calculations are carried out.

On substituting the expressions of the propagating fields in Eq. (14) into the Hamiltonian expression in Eq. (15), and using the effective permittivity in Eq. (12), the Hamiltonian expression can be rewritten as:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1, \quad (16)$$

where

$$\mathcal{H}_0 = \mathcal{V} \sum_{j=1}^2 \varepsilon'_{effj} \varepsilon_0 u_j^* u_j + c.c., \quad (17)$$

and

$$\mathcal{H}_1 = \mathcal{V} \varepsilon''_{eff} \varepsilon_0 u_1^* v^* u_2 + c.c. \quad (18)$$

Here, $\mathcal{H}_0$ represents the classical free fields Hamiltonian, and $\mathcal{H}_1$ is the classical interaction Hamiltonian. These expressions are used in the next section to describe the quantum evolution of the interacting fields.

3. Quantum mechanics analysis

The optical and microwave fields can be quantized through the following relations:

$$u_j = \left(\frac{\hbar f_j}{\varepsilon'_{eff} \varepsilon_0 \mathcal{V}} \right)^{\frac{1}{2}} \hat{a}_j, \quad \text{and} \quad v = \left(\frac{\hbar f_m}{C_T \mathcal{A}} \right)^{\frac{1}{2}} \hat{b}, \quad (19)$$

where $\hat{a}_j$ and $\hat{b}$ are the annihilation operators of the j^{th} optical mode and the microwave mode, respectively.

The quantum Hamiltonian can be obtained by substituting the annihilation (and creation) operators, defined above, into the classical Hamiltonian in Eq. (16), yielding:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1, \quad (20)$$

where

$$\hat{\mathcal{H}}_0 = \hbar f_m \hat{b}^\dagger \hat{b} + \hbar f_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar f_2 \hat{a}_2^\dagger \hat{a}_2, \quad (21)$$

and

$$\hat{\mathcal{H}}_1 = \hbar g (\hat{a}_2^\dagger \hat{b} \hat{a}_1 + h.c.), \quad (22)$$

where $h.c.$ is the Hermitian conjugate and g is the conversion rate given by:

$$g = \varepsilon''_{eff} \sqrt{\frac{f_1 f_2}{\varepsilon'_{eff} \varepsilon_0 \mathcal{V}}} \sqrt{\frac{\hbar f_m}{C_T \mathcal{A}}}. \quad (23)$$

On substituting the quantum Hamiltonian expression of Eq. (20) into the Heisenberg equations of motion, that is $\frac{\partial \hat{x}}{\partial t} = \frac{i}{\hbar} [\hat{\mathcal{H}}, \hat{x}]$, one yields [31]:

$$\frac{\partial \hat{a}_1}{\partial t} = -i f_1 \hat{a}_1 - i g \hat{b}^\dagger \hat{a}_2, \quad (24)$$

$$\frac{\partial \hat{a}_2}{\partial t} = -i f_2 \hat{a}_2 - i g \hat{b} \hat{a}_1, \quad (25)$$

$$\frac{\partial \hat{b}}{\partial t} = -i f_m \hat{b} - i g \hat{a}_1^\dagger \hat{a}_2. \quad (26)$$

In this work, the optical pump input $\hat{a}_1$ is considered intensive and treated classically. Consequently, on using the rotation approximation ($\hat{a}_j = \hat{A}_j e^{-i\omega_j t}$ and $\hat{b} = \hat{B} e^{-i\omega_M t}$), the equations of motion are given by:

$$\frac{\partial \hat{A}_2}{\partial t} = -i g A_1 \hat{B}, \quad (27)$$

$$\frac{\partial \hat{B}}{\partial t} = -i g A_1^* \hat{A}_2. \quad (28)$$

The solution of Eqs. (27) and (28) for $\hat{A}_2$, is expressed as:

$$\hat{A}_2(t) = \hat{A}_2(0) \cos(g|A_1|t) - ie^{-i\phi_0} \hat{B}(0) \sin(g|A_1|t), \quad (29)$$

where ϕ_0 is the phase of the optical input A_1 . We can deduce from Eq.(29) that, if the interaction time satisfies

$$g|A_1|t = \frac{\pi}{2}, \quad (30)$$

then the quantum state of the optical upper sided band $\hat{A}_2$ depends only on the microwave signal $\hat{B}$ state, thus achieving a quantum microwave-to-optical conversion. In the following section, more analysis is provided, considering realistic parameters.

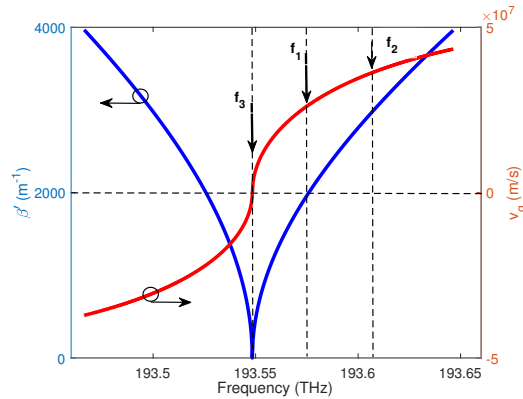


Fig. 3. Propagation constant and the group velocity versus optical frequency. The optical pump (i.e., f_1) and the upper and lower side bands (i.e., f_2 and f_3 , respectively) are shown for $f_m = 50GHz$.

4. Results and discussion

In this section, numerical evaluations are presented, considering practical parameters. In the following analysis, we consider air as the filling material, the optical lower sideband frequency is fixed at $f_3 = 193.5484$ THz, and the separation distance between graphene layers is given by $d = \frac{c}{f_3} = 1.55\mu m$. The frequency of the optical pump f_1 and the microwave signal frequency f_m are varied accordingly. The area of the graphene layers is $\mathcal{A} = 1mm^2$. It is important to note that this work does not address the practical challenges of fabricating a large number of stacked graphene layers. Multilayer graphene films with few (up to tens) graphene layers have been reported [32], however, large number of graphene layers that are electrically connected is yet to be realized. We note here that the main scope of this work is to investigate the viability of the physical concept behind the proposed scheme. Thus, an ideal case of periodic and symmetric graphene layers with no imperfections is considered.

4.1. Propagation constant and transmittance

In this subsection, the propagation constant (and the group velocity) are calculated for the considered multilayer graphene medium. Also, the transmittance T of the medium is simulated (using the transfer matrix method [33]) to quantify the suppression of the lower sideband and an extraction ratio is calculated.

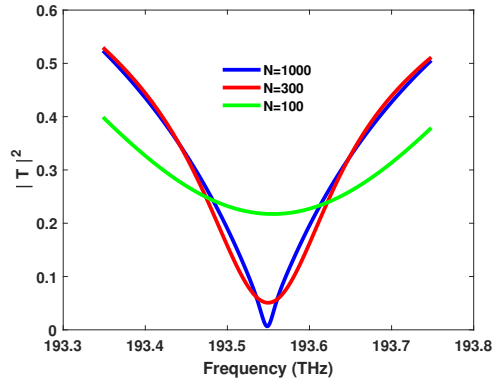


Fig. 4. Transmittance versus optical frequency. Different numbers of layers, i.e., N , are considered.

In Fig. 3, The propagation constant and the group velocity (i.e., v_g) are shown versus optical frequency. An example of f_1 , f_2 , and f_3 are shown for $f_m = 50\text{GHz}$. Here, v_g is defined by $v_g = \frac{\partial f}{\partial \beta}$.

In Fig. 4, the transmittance is displayed versus the optical frequency, considering different numbers of graphene layers. The extraction ratio between the upper and lower side bands can be defined as $\eta_E = \frac{T^2(N)|_{f=f_3}}{T^2(N)|_{f=f_2}}$. As can be observed in Fig. 4, a large number of layers is needed to reach a reasonable extraction ratio. For example, for $f_m = 50\text{GHz}$, the extraction ratio equals $\eta_E = 1.1$ for $N = 100$, $\eta_E = 3$ for $N = 300$, and $\eta_E = 32$ for $N = 1000$.

4.2. Conversion rate

In this subsection, the conversion rate is characterized versus different parameters including the drive microwave voltage, the microwave frequency, the electron densities, and the medium length.

In Fig. 5, the conversion rate g is evaluated versus the microwave frequency. Here, $N = 1000$, $v = 1\mu\text{V}$, and different electron densities n_0 are considered. Higher conversion rates can be achieved for smaller electron densities. However, the electron density must satisfy a threshold value given by $2C_T v \ll \pi n_0 q$.

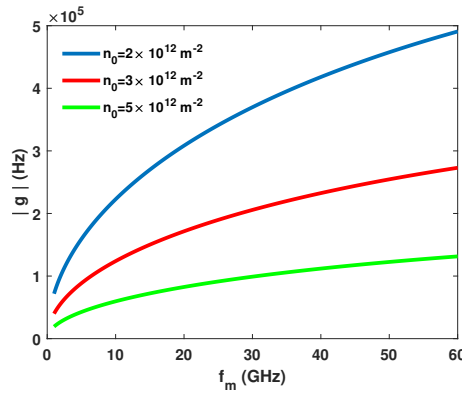


Fig. 5. The conversion rate g versus microwave signal frequency. Different intrinsic electron densities n_0 are considered.

In Fig. 6, the conversion rate is shown versus the driving microwave voltage. Here, $N = 1000$, and $n_0 = 2 \times 10^{12} m^{-2}$. Significant conversion rates can be achieved for microvolt ranges. This is owing to the dispersion property of the multilayer graphene and to the significant variation of the graphene conductivity in response to even very small driving electric voltages.

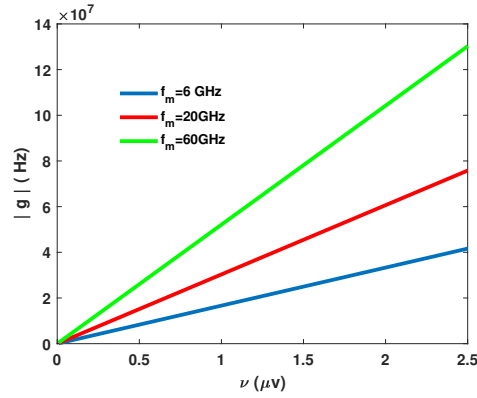


Fig. 6. The conversion rate g versus the voltage of the microwave signal. Different microwave signal frequencies are considered

The length of the multilayer graphene medium is given by $L = (N - 1)d$. Consequently, by using $t = L \frac{1}{v_g}$ in Eq. (30), the required optical pump amplitude is:

$$|A_1| = \frac{\pi v_g}{2L|g|}. \quad (31)$$

In Fig. 7, the conversion rate g is displayed versus the graphene medium length L and the pump amplitude A_1 . As can be seen, significant conversion rates can be achieved for few propagated millimeters, yet with reasonable pump amplitudes.

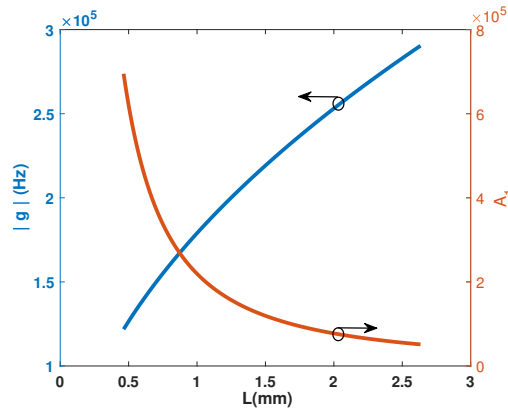


Fig. 7. The conversion rate g versus the medium length and the optical pump operator.

4.3. Open system

The equations of motion in Eqs. (27) and (28) describe a closed-quantum system. However, both the optical and the microwave fields decay with time.

The optical decay is attributed to attenuation and reflection of the multilayer graphene, modeled by including the time decay parameter Γ in the equations of motion. The attenuation optical time decay rate is defined by $\Gamma_A = 2v_g \text{Im}(\beta)$. The reflection of the multilayer graphene is also modeled by an equivalent decay coefficient Γ_R . This decay coefficient (we called it reflection decay coefficient) can be defined by setting $\exp(-t_0 \Gamma_R) = T_0^2$. Here, T_0 is transmittance of a single block of the multilayer graphene (composed of d filling material and a single graphene layer), and $t_0 = \frac{d}{v_g}$ is the total flight time over a single block.

On the other hand, given that v and $\hat{b}$ are linearly related as shown in Eq. (19), we obey the following approach to model the microwave decay rate. First, the microwave rms power losses is calculated, given by $\frac{v^2}{2R_g}$, where $R_g = \text{Re}\left(\frac{1}{\sigma_s}\right)$ is the graphene resistance for a square layer. Here, the graphene conductivity is calculated at the microwave frequency and $T = 3mK$ [28]. Second, the microwave energy at a time, let us say t_0 , is approximated as the initial energy at time $t = 0$ minus the rms dissipated energy, that is $qv - \frac{v^2}{2R_g}t_0$. Finally, the effective microwave decay rate Γ_m is introduced to calculate the microwave energy at the same time t_0 , yielding, $qv - \frac{v^2}{2R_g}t_0 = qve^{-\frac{\Gamma_m}{2}t_0}$. It then follows that $\Gamma_m = -\frac{2}{t_0} \ln\left(1 - \frac{vt_0}{2qR_g}\right)$. We note here that Γ_m depends on the applied voltage amplitude as the electrical dissipation is a nonlinear process.

In Fig. 8, the optical and the microwave decay time rates are presented versus the microwave frequency. Here, f_3 is fixed at the destruction resonance, and f_1 is adjusted in accordance to f_m , as $f_1 = f_3 + f_m$, $n_0 = 2 \times 10^{12} m^{-2}$, and $v = 1\mu v$.

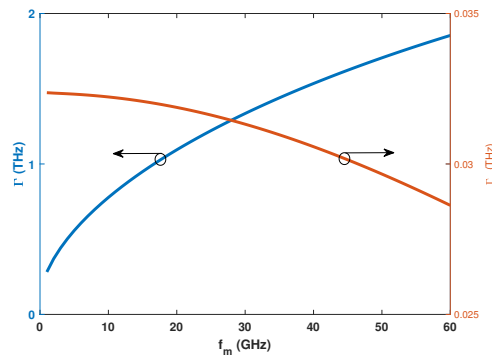


Fig. 8. The optical and microwave decay coefficients versus microwave frequency.

The equations of motion are then:

$$\frac{\partial \hat{A}_2}{\partial t} = -\frac{\Gamma}{2} \hat{A}_2 - ig A_1 \hat{B} + \sqrt{\Gamma} N_2, \quad (32)$$

and

$$\frac{\partial \hat{B}}{\partial t} = -\frac{\Gamma_m}{2} \hat{B} - ig A_1^* \hat{A}_2 + \sqrt{\Gamma_m} N_m, \quad (33)$$

where $\Gamma = \Gamma_A + \Gamma_R$ is the total optical decay coefficient, Γ_m represents the microwave decay coefficient, and $\Gamma_R = \frac{v_g}{d} \ln\left(\frac{1}{T_0^2}\right)$. Here, N_2 and N_m are the quantum Langevin noise operators, obeying $[N(t_1), N(t_2)^\dagger] = \delta(t_1 - t_2)$ and $\langle N(t_1)^\dagger N(t_2) \rangle = \frac{1}{\exp(\hbar f/k_B T)} \delta(t_1 - t_2)$. The dissipation characterized by the time decay rates Γ and Γ_m are included in the equation of motion Eqs. (32) and (33). Hence, according to the fluctuation-dissipation theorem, the Langevin forces are included. The langevin forces represent the noise in the microwave and optical frequencies as

the feed-back of the environment to the system. On the other hand, we note that the dissipation usually destroys squeezing. Furthermore, the spontaneous down conversion process in this work is minimized by the means of lower side band suppression. Thus, squeezing between different frequency fields due to spontaneous process is ignored.

For the seek of completeness, it worth mentioning that the reflected optical pump is also modulated by the microwave signal. Here, given that the layered structure is reciprocal, the reflected optical pump obey the same dynamics as the transmitted optical pump. While, on other hand, the modulation of the side bands is neglected.

To evaluate the number of converted photons, one may write the evolution equations for the mean optical and microwave fields:

$$\frac{\partial \langle \hat{A}_2 \rangle}{\partial t} = -\frac{\Gamma}{2} \langle \hat{A}_2 \rangle - ig A_1 \langle \hat{B} \rangle, \quad (34)$$

$$\frac{\partial \langle \hat{B} \rangle}{\partial t} = -\frac{\Gamma_m}{2} \langle \hat{B} \rangle - ig A_1^* \langle \hat{A}_2 \rangle. \quad (35)$$

Using the quantum regression theorem [31], one can obtain a complete system of differential equations describing the number photons evolution (See Appendix A).

On considering Heaviside step pump switching function $H(t)$, the system of the differential equations can be solved and the solution for $\langle \hat{A}_2^\dagger \hat{A}_2 \rangle$ and $\langle \hat{B}^\dagger \hat{B} \rangle$ can be obtained (see Appendix B). The $\langle \hat{A}_2^\dagger \hat{A}_2 \rangle$ solution contains terms that correlate with the microwave state and others that decorrelated with microwave state. The signal to noise ratio (i.e., SNR) (also presented in the Appendix B) is defined as the ratio of the terms correlates with microwave state to those that decorrelated with microwave state.

On imposing the condition of $A = \left(\frac{\Gamma - \Gamma_m}{4g} \right)$, the parameter α approaches zero (i.e., $\alpha \rightarrow 0$), which implies large SNR and, thus, the decorrelate terms can be ignored (as shown in Appendix B). Hence, the solution of $\langle \hat{A}_2^\dagger \hat{A}_2 \rangle$ can given by:

$$\langle \hat{A}_2^\dagger \hat{A}_2 \rangle = Agte^{-\frac{t(\Gamma + \Gamma_m)}{4}} \langle \hat{A}_2^\dagger \hat{B} \rangle|_{t=0}, \quad (36)$$

where $t = L/v_g$ is the interaction time.

It is important to note that the optical and the microwave fields can be considered decorrelated at $t = 0$ (See Appendix C), which implies:

$$\langle \hat{B}^\dagger \hat{A}_2 \rangle|_{t=0} = \langle \hat{B}^\dagger \rangle|_{t=0} \langle \hat{A}_2 \rangle|_{t=0} \simeq \sqrt{\langle \hat{A}_2^\dagger \hat{A}_2 \rangle|_{t=0} \langle \hat{B}^\dagger \hat{B} \rangle|_{t=0}}. \quad (37)$$

The SNR in this case simplifies to:

$$SNR = \frac{\langle \hat{B} \rangle|_{t=0}}{\langle \hat{A}_2 \rangle|_{t=0}}, \quad (38)$$

It is apparent that the SNR in Eq. (38) is large, given that $\langle \hat{B} \rangle|_{t=0}$ is the initial microwave expectation value of the annihilation operator, while the $\langle \hat{A}_2 \rangle|_{t=0}$ is initially at the noise level. For example, for a microwave voltage signal of $v = 1 \mu V$, the SNR is greater than 30 dB.

In Fig. 9, the number of converted photons $\langle \hat{A}_2^\dagger \hat{A}_2 \rangle$ is presented versus the frequency of the microwave signal. As can be seen, a significant number of optical photons are converted over a wide microwave frequency range. Here different lengths of multilayer graphene (i.e., number

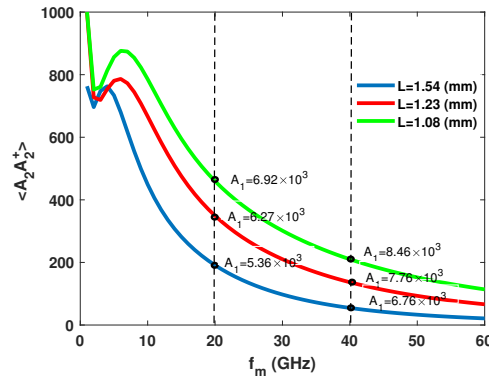


Fig. 9. The number of converted photons, i.e., $\langle \hat{A}_2^\dagger \hat{A}_2 \rangle$, versus the frequency microwave signal.

of layers N) are considered. These include $L = 1.54\text{mm}$ (i.e., $N = 1000$), $L = 1.23\text{mm}$ (i.e., $N = 800$), and $L = 1.08$ (i.e., $N = 700$). Also, we observe that a larger number of converted photons can be achieved for shorter lengths of multilayer graphene. However, in this case, larger optical pump amplitudes are required and smaller extraction ratios result. For example, for $f_m = 20\text{GHz}$, the number of converted photons is $\langle \hat{A}_2^\dagger \hat{A}_2 \rangle = 191, 350$ and 460 for $L = 1.54\text{mm}$, 1.23mm and 1.08mm , respectively. The corresponding pump amplitudes are $A_1 = 5.36 \times 10^3$, 6.27×10^3 , and 6.92×10^3 , while the extraction ratios are $\eta_E = 31, 20$, and 16 , respectively. Here, the classical slow varying field amplitude of the optical pump field, i.e., u_1 , can be calculated from the pump operator, i.e., A_1 , by Eq. (19). Importantly, as required by the developed model, u_1 values are of moderate level. For example, in Fig. 9, $A_1 = 6.92 \times 10^3$ for $f_m = 20\text{GHz}$ and $L = 1.08\text{mm}$ medium length. The corresponding electric optical pump field intensity is $u_1 = 71 \times 10^4\text{ V/m}$. Thus, the optical pump can be safely treated classically and in the same time its intensity is below the damage threshold of graphene. On other hand, we note that the peak of the number of converted photons in Fig. 9 is attributed to the transmittance response of the layered media. This can be explained by recalling that the optical decay coefficient, Γ , is composed of the absorption decay coefficient Γ_A and the reflection decay coefficient Γ_R . The transmittance (or equivalently the reflection) of the layer medium is dispersive and depends on the frequency of the converted photons (which is $f_3 = f_1 + f_m$). This can be verified by comparing the transmittance response, versus f_m , with the number of photons in Fig. 9.

5. Conclusion

A novel approach for quantum microwave-to-optical conversion is proposed and thoroughly investigated. The proposed scheme includes a multilayer graphene pumped by an optical field. The graphene layers are connected in interdigital configuration and driven by a microwave signal. Electrically, the multilayer graphene functions as a capacitor, while optically, the multilayer graphene functions as a periodic medium. A perturbation approach is implemented to model the graphene's optical conductivity in the presence of the driving microwave signal. A quantum mechanical analysis is carried out to describe the fields' evolution. Numerical evaluations are presented, considering realistic parameters. The destruction resonance of the medium is fixed, by setting d , while the optical pump frequency and the microwave signal frequency are varied so that the lower side frequency is at the destruction resonance. The numerical evaluations show significant conversion achieved for low driving voltages, reasonable optical pumping, and

over a vast frequency bandwidth. This, in principle, is due to the dispersion properties of the proposed multilayer structure and to the unique properties of graphene. First, the conversion rate is evaluated versus the microwave frequency for different electron densities. It is found that a significant conversion rate is achieved for a vast microwave frequency range. Here, greater conversion rates are achieved for smaller electron densities. However, electron densities have minimum thresholds imposed by the derived model. Second, the conversion rate is evaluated versus the microwave driving voltages, considering different microwave frequencies. It is found that significant conversion rates are achieved for microvolt microwave voltages. Higher conversion rates are achieved for higher microwave frequencies. Third, the conversion rate is evaluated versus the optical pump amplitude and the multilayer graphene length. It is illustrated that a significant conversion rate is achieved for a few millimeter lengths with reasonable optical pump amplitudes. Smaller pump amplitudes are required for longer multilayer graphene lengths. Fourth, an open quantum system is considered, with decaying optical and microwave fields. The mean number of the converted photons is evaluated versus microwave frequency. It is shown that, a significant number of photons is converted for a vast microwave frequency range.

Appendix A

The quantum regression theorem allows one to from an equation for the expectation of the product of two operators from the equation of the expectation of one operator [31]. Consequently, on using the quantum regression theorem for Eq.(34) and (35), one obtains:

$$\frac{\partial \langle \hat{A}_2^\dagger \hat{A}_2 \rangle}{\partial t} = -\frac{\Gamma}{2} \langle \hat{A}_2^\dagger \hat{A}_2 \rangle + gA \langle \hat{A}_2^\dagger \hat{B} \rangle. \quad (39)$$

$$\frac{\partial \langle \hat{B}^\dagger \hat{B} \rangle}{\partial t} = -\frac{\Gamma_m}{2} \langle \hat{B}^\dagger \hat{B} \rangle - gA \langle \hat{B}^\dagger \hat{A}_2 \rangle. \quad (40)$$

Here, the pump field amplitude A_1 is considered with $\frac{\pi}{2}$ phase ($A_1 = A \exp(\frac{i\pi}{2}) = iA$) for seek of simplicity.

Also, in order to have complete system of differential equations, the quantum regression theorem can be used again, yielding:

$$\frac{\partial \langle \hat{A}_2^\dagger \hat{B} \rangle}{\partial t} = -\frac{\Gamma_m}{2} \langle \hat{A}_2^\dagger \hat{B} \rangle - gA \langle \hat{A}_2^\dagger \hat{A}_2 \rangle, \quad (41)$$

$$\frac{\partial \langle \hat{B}^\dagger \hat{A}_2 \rangle}{\partial t} = -\frac{\Gamma}{2} \langle \hat{B}^\dagger \hat{A}_2 \rangle + gA \langle \hat{B}^\dagger \hat{B} \rangle. \quad (42)$$

These four differential equations form two systems of two differential equations. The pump field A usually depends on the time due to the fact that the pump switches on at $t = 0$. One may model the pump switch by:

$$A(t) = Cs(t), \quad (43)$$

where C is the constant amplitude and s is the switching on function.

Appendix B

On considering Heaviside step pump switching function, where $s(t) = H(t)$, the solution of the two systems of differential equations in Eqs. (39)-(42), can be given by:

$$\begin{aligned} \langle \hat{A}_2^\dagger \hat{A}_2 \rangle = e^{-\frac{(\Gamma+\Gamma_m)}{4}t} & \left[Ag \langle \hat{A}_2^\dagger \hat{B} \rangle|_{t=0} \frac{\sin(\alpha t)}{\alpha} + \langle \hat{A}_2^\dagger \hat{A}_2 \rangle|_{t=0} \cos(\alpha t) \right. \\ & \left. - \frac{(\Gamma-\Gamma_m)}{4} \langle \hat{A}_2^\dagger \hat{A}_2 \rangle|_{t=0} \frac{\sin(\alpha t)}{\alpha} \right], \end{aligned} \quad (44)$$

$$\begin{aligned} \langle \hat{B}^\dagger \hat{B} \rangle = e^{-\frac{(\Gamma+\Gamma_m)}{4}t} & \left[-Ag \langle \hat{B}^\dagger \hat{A}_2 \rangle|_{t=0} \frac{\sin(\alpha t)}{\alpha} + \langle \hat{B}^\dagger \hat{B} \rangle|_{t=0} \cos(\alpha t) \right. \\ & \left. + \frac{(\Gamma-\Gamma_m)}{4} \langle \hat{B}^\dagger \hat{B} \rangle|_{t=0} \frac{\sin(\alpha t)}{\alpha} \right], \end{aligned} \quad (45)$$

where

$$\alpha = \sqrt{g^2 A^2 - \left(\frac{\Gamma - \Gamma_m}{4} \right)^2}. \quad (46)$$

As can be seen from Eq. (44), the the amplitude of the optical signal $\hat{A}_2$ is proportional to the the amplitude of the microwave signal $\hat{B}$. The first term between brackets in Eq. (44) is the desired term, which is proportional to the microwave intensity. However, the remaining terms are decorrelated with the microwave state and conceived as noise. Thus, the signal-to-noise ratio (SNR) is given by:

$$SNR = \frac{Ag \frac{\sin(\alpha t)}{\alpha} \langle \hat{A}_2^\dagger \hat{B} \rangle|_{t=0}}{\cos(\alpha t) - \frac{\Gamma-\Gamma_m}{4} \frac{\sin(\alpha t)}{\alpha} \langle \hat{A}_2^\dagger \hat{A}_2 \rangle|_{t=0}}, \quad (47)$$

On imposing the condition of small α value (i.e., $\alpha \rightarrow 0$) by setting $A = \left(\frac{\Gamma-\Gamma_m}{4g} \right)$, the SNR can be approximated by:

$$SNR = \frac{\langle \hat{A}_2^\dagger \hat{B} \rangle|_{t=0}}{\langle \hat{A}_2^\dagger \hat{A}_2 \rangle|_{t=0}}. \quad (48)$$

It then follows that the SNR is large under this condition since the expectation value of $\hat{A}_2$ is typically at noise level at $t = 0$. It then follows that the decorrelate terms in Eq. (44) are relatively small and can be ignored.

Appendix C

The operator $\hat{A}_2$ can be decomposed in two parts: one is the expectation part $\langle \hat{A}_2 \rangle$ and the other is the fluctuation part $\delta \hat{A}_2$. Hence, $\hat{A}_2 = \langle \hat{A}_2 \rangle + \delta \hat{A}_2$, where $\langle \delta \hat{A}_2 \rangle = 0$. It then follows that $\langle \hat{A}_2^\dagger(t) \hat{A}_2(t) \rangle = \langle \hat{A}_2^\dagger(t) \rangle \langle \hat{A}_2(t) \rangle + \langle \delta \hat{A}_2^\dagger(t) \delta \hat{A}_2(t) \rangle$. Here, the left hand side contains the expectation value of the product of the two operators. While in the right hand side, the first term is the product of the expectation of each operator, and the second term is the mean value of the fluctuation. The average of the fluctuation $\langle \delta \hat{A}_2^\dagger(t) \delta \hat{A}_2(t) \rangle$ present in any time. However, at time $t = 0$ we suppose that the mean value of the fluctuation is small compared to the expectation values of the operators. In other words, $\langle \hat{A}_2^\dagger(0) \rangle \langle \hat{A}_2(0) \rangle \gg \langle \delta \hat{A}_2^\dagger(0) \delta \hat{A}_2(0) \rangle$. It then follows that we can write $\langle \hat{A}_2(0) \rangle \simeq \sqrt{\langle \hat{A}_2^\dagger(0) \hat{A}_2(0) \rangle}$. Same result can be obtained for the operator $\hat{B}$.

33. T. Zhan, X. Shi, Y. Dai, X. Liu and J. Zi, "Transfer matrix method for optics in graphene layers," J. Phys.: Condens. Matter, **25**(21), 215301 (2013).