Wave forces related to waves conditions and structures characteristics

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Rezumat: Valurile marine au forme geometrice neregulate şi amplitudini variabile. . Calculul forțelor pe care le produc asupra construcțiilor hidrotehnice maritime este dificil şi aproximativ. În lucrare se prezintă succint modul de evaluare a forțelor din valuri funcție de caracteristicile impactului dintre val şi structură şi se propune un procedeu de calcul care permite o analiză rapidă şi simplă a acțiunii valurilor sub forma unor spectre dinamice de răspuns. Spectrele dinamice de răspuns se obțin prin rezolvarea numerică a unei integrale de tip Duhamel. Pe baza acestor spectre, se poate calcula apoi răspunsul dinamic maxim, modelând structura ca un sistem echivalent cu un singur gard de libertate sau cu mai multe grade de libertate dinamică. Lucrarea conține exemple numerice rezolvate.

Abstract: Sea waves have irregular geometrical shapes and varying amplitudes. It is very difficult to make an approximation of the forces they produce on the maritime structures. Wave-generated pressures on structures related to the wave conditions and geometry of the structure is presented in this work. Within this work, a calculation procedure is suggested, a procedure that allows a quick and simple analysis of the sea waves loadings under the form of dynamic response spectrums. These can be estimated using the registered oscillogrames of the sea waves, that become oscillogrames of the wave pressure on the maritime structure. The dynamic response spectrums are estimated by solving numerically an integral of Duhamel type. Taking as basis these spectrums, the maximal dynamic response could be calculated, modeling the structure as an equivalent system with a single or multiple degrees of freedom. This work contains the solutions of some numerical examples.

Keywords: wave forces, pressure distribution, arbitrary general dynamic loading, maximal dynamic response

1. Introducere

The wave forces represent an important element in designing the maritime structures, being, at the same time, hard to be calculated and subjected to several uncertainties.

Most of the times, the sea waves are considered, in a simplified way, as regulated waves, having a quite sinusoidal shape, named monochromatic waves (Fig.1a). The visual observations and the measurements show that the sea surface is formed of waves having variable periods and heights and various directions. The term „irregular waves" is used to design the natural state of the sea for which the waves characteristics present statistic variability, as opposed to the monochromatic waves, the properties of which are considered to be constant. In the recordings made in fixed points, there could be noticed non-repetitive wave profiles (Fig.2).

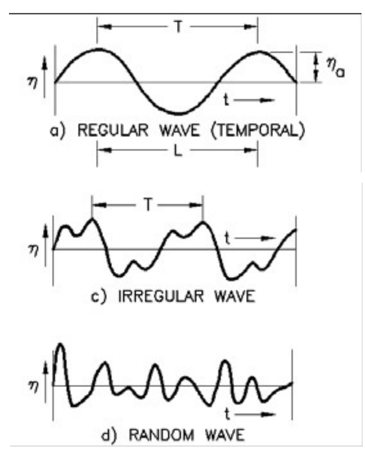


Fig. 1Sea waves representations

ISSN-1584-5990

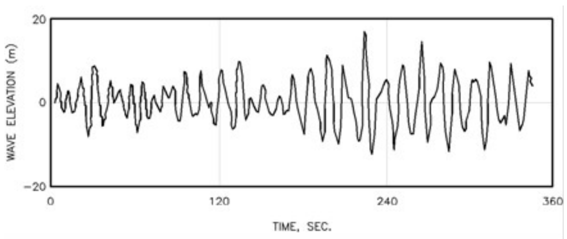


Fig. 2 An irregular wave profile, resulted from records

Even if there could be noticed also individual waves, these ones present significant individual variations of the height and of the period, from one wave to another. The recordings from a certain point represent the superposition of several waves, of different directions, that are intercrossed at one specific moment. The sea state during the storm is always well-represented by the irregular waves (short-crest waves). The long-crest waves (or the monochromatic waves) are suited to describe the swell-state of the sea, when the waves characteristics vary in a restraint range.

Fig. 3 indicates in a simplified way, the dynamic character of the sea wave's loadings, by changing the intensity, direction and vertical position during the wave cycle.

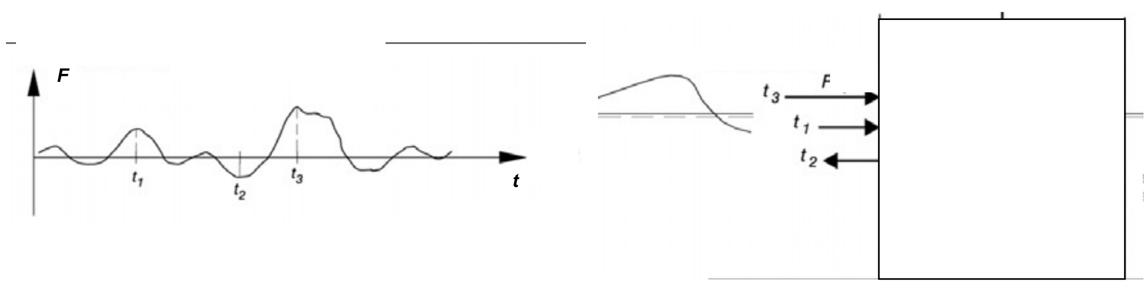


Fig. 3 The dynamic character of the horizontal forces generated by waves

Wave-generated pressures on structures are complicated functions of the wave conditions and geometry of the structure. Fig. 4 show several types of time-varying wave impact loading. Even if the waves' action has a dynamic character, taking into consideration that the fundamental period of the defending structures (0,2…2s) is significantly smaller than the period of the regular waves, the tension state is deduced usually by the static application of the wave loading.

We present hereafter the usual way of determining the value of the pressures due to the waves, for the structures with vertical walls and for rubble mound structures.

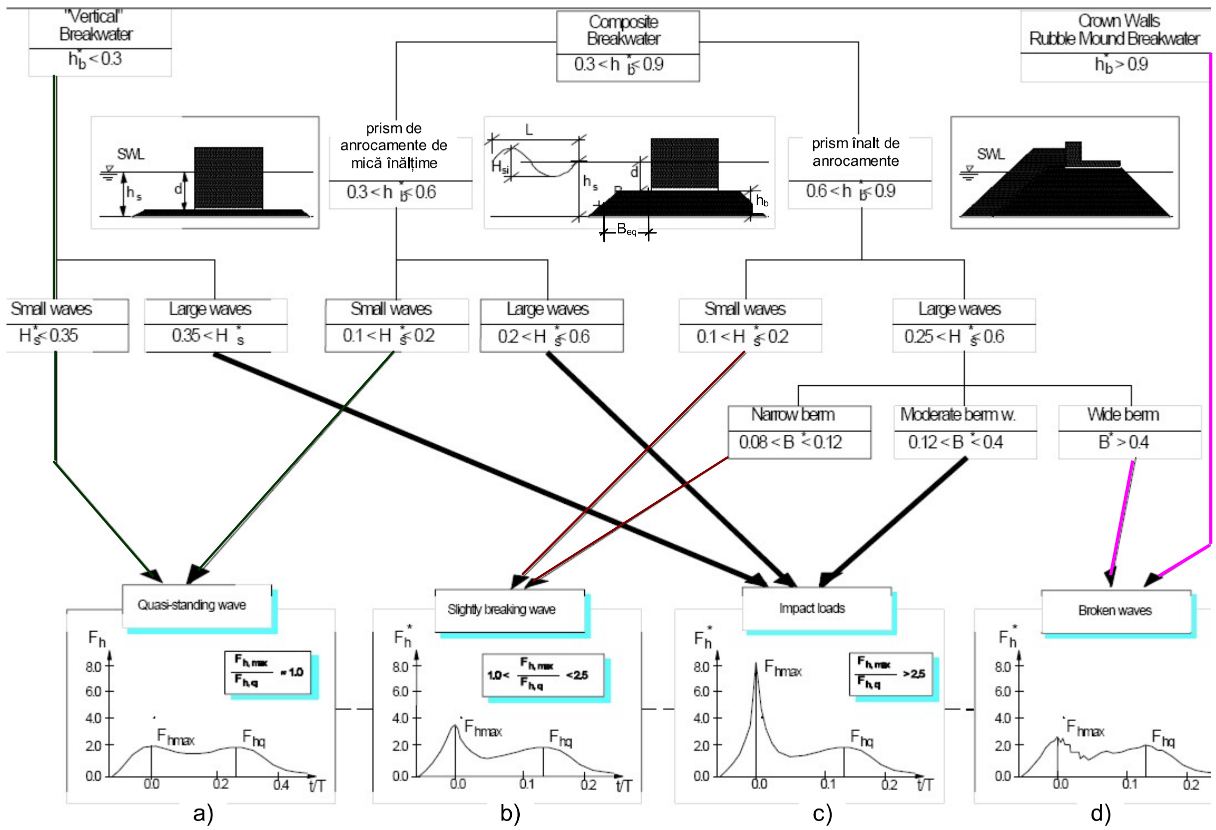
2. The estimation of the pressure force of the waves on the structures with vertical walls

The horizontal force induced by the action of the waves is deduced according to the effect that the waves have on the vertical walls.

The total hydrodynamic pressure distribution on a vertical wall consists of two time-varying components: the hydrostatic pressure component due to instantaneous water depth at the wall, and the dynamic pressure component due to acceleration of the water particles.

There can be identified three types of time-varying force generated by the wave on the vertical walls, that are summarized in Fig. 5. For non-breaking waves, the pressure at the wall has a slight variation in time. This load is called pulsating or quasi-static because the period is much longer than natural period oscillation of the structure. Consequently the wave load can be treated like a static load. A special consideration is required if the wall is placed on fine soils where pore pressure may build up, resulting in significant weakening of the soil.

Sainflou's formula, derived theoretically for regular, non-breaking waves (but applied also for irregular waves) assumes a linear pressure distribution below the water level (Fig.6). This relation cannot be used for breaking or overtopping waves.



T – wave period, H_b - breaking wave height

Adimensional quantity: $h_b^* = \dfrac{h_b}{h_s}$; $H_s^* = \dfrac{H_{si}}{h_s}$; $B^* = \dfrac{B_{eq}}{L}$; $F_h^* = \dfrac{F_h}{\rho g H_b^2}$; $H_s^* = \dfrac{H_{si}}{h_s}$;

Fig. 4 [A. Kortenhaus et.a.]

Waves that break in a plunging mode develop an almost vertical front before they curl (Fig. 5b). If this almost vertical front occurs just prior to the contact with the wall, then a very high pressure is generated. This pressure has extremely short durations. A negligible amount of air is entrapped and result a very large single peak force followed by very small force oscillations). The duration of the pressure peak is of the order of hundredths of a second. Goda's formula assumes trapezoidal shape for pressure distribution along front. (Fig. 7)

If a large amount of air is entrapped in a pocket, a double peaked force is generated followed by pronounced force oscillations (Fig. 5c). The first and largest peak is induced by the wave crest impact on the structure. The second peak is induced by the subsequent maximum compression of air pocket. The force oscillations are due to the pulsation of air pocket. The double peaks have spacing in range of milliseconds to hundredths of seconds, and period of the force oscillations is in the range 0.2-1seconds.(Fig. 8)

Based on the registration of the pressure shocks, Minikin's method leads to overestimated pressures, (15 to 18 times higher), having their maximal level at or near the still water level.

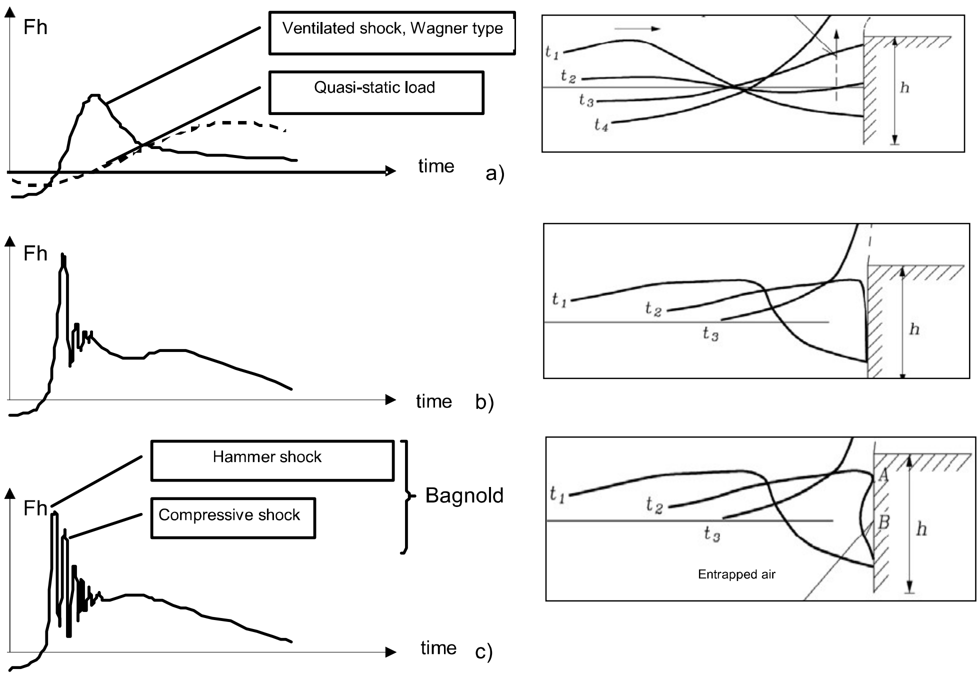
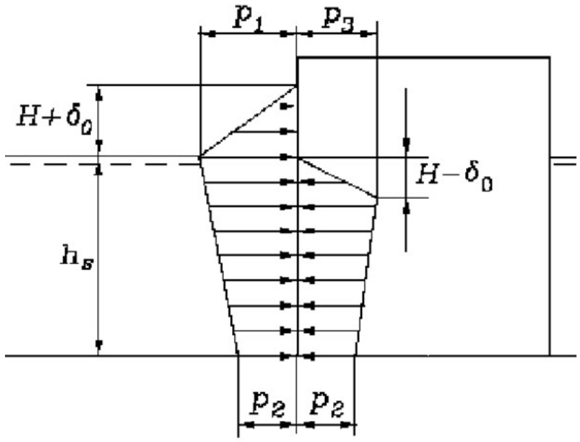


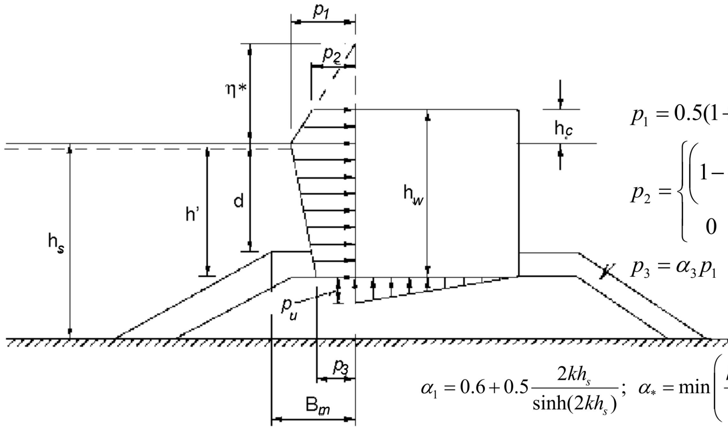
Fig. 5 Three type of wave forces on verical wall: a) non-breaking wave, b) – breaking wave with almost vertical front c) – breaking wave with large air pocket



$$p_1 = \left(p_2 + \rho \cdot g \cdot h_s\right)\frac{H+\delta_0}{h_s + H + \delta_0};\ p_2 = \frac{\rho \cdot g \cdot H}{\cosh(k \cdot h_s)};\ p_3 = \rho \cdot g (H - \delta_0)$$

$$\delta_0 = \frac{\pi H^2}{L}\coth(kh_s)$$

Fig. 6 Pressure distribution within the wave (regardless to the hydrostatic pressure) for the non-breaking waves



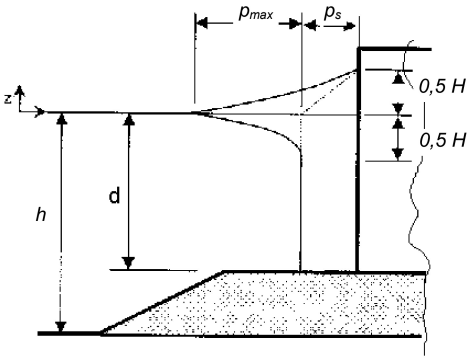
$$p_1 = 0.5(1+\cos\beta)(\alpha_1 + \alpha_* \cos^2\beta)\rho g H_{pr}$$

$$p_2 = \begin{cases} \left(1 - \dfrac{h_c}{\eta^*}\right) \text{p}_1 \text{ pentru } \eta^* > h_c \\ 0 \quad \text{pentru } \eta^* < h_c \end{cases} \qquad \eta^* = 0.75(1+\cos\beta)H_{pr}$$

$$p_3 = \alpha_3 p_1 \qquad p_u = 0.5(1+\cos\beta)\alpha_1\alpha_3\rho g H_{pr}$$

$$\alpha_1 = 0.6 + 0.5\frac{2kh_s}{\sinh(2kh_s)}; \quad \alpha_* = \min\left(\frac{h_b - d}{3h_b}\left(\frac{H_{pr}}{d}\right)^2, \frac{2d}{H_{pr}}\right); \quad \alpha_3 = 1 - \frac{h_w - h_c}{h_s}\left(1 - \frac{1}{\cosh(kh_s)}\right)$$

Fig. 7 The pressure distribution within the wave for the breaking waves (Goda)



$$p_{\max} = 101\rho g d\left(1 + \frac{d}{h}\right)\frac{H_b}{L_h};$$

$$p_s = \begin{cases} \dfrac{1}{2}\rho g H\left(1 - \dfrac{2z}{H_b}\right), \ 0 \le z \le \dfrac{H_b}{2} \\ \dfrac{1}{2}\rho g H_b |z| \ , \ \text{z<0} \end{cases}$$

Fig. 8 The estimation and the distribution of the pressures by Minikin method for the cases in which the breaking wave's crest is bellow the design breaker crest

3 Calculation of the pressure force within the waves against the rubble mound structures

Due to the complex movement of the water during the breaking of the waves on the rubble mound, it was not possible to evaluate, exclusively by theoretical methods, a formula for the force that the waves deploy over the protection structures. It starts, conventionally, from the hypothesis that, after the breaking, the movement is produced following a parabolic trajectory, noted as AB in Fig. .9

Starting from the components of velocity on the crest of the wave ($V_x=V_A$ and $V_y=0$) and from the equation of the rubble mound ($y=x/m$), resulting the coordinates of the impact point, B, that depends mostly on the velocity at the wave crest level.

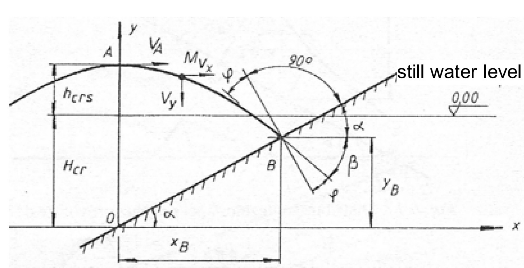
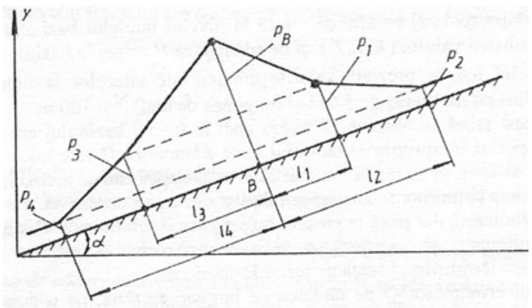


Fig. 9 The waves' impact on a rubble mound

In order to allot the pressures exercised by the wave, figured in Fig. 10 , there are empirical relations that are applied, in which interfere the impact velocity following the normal direction against the rubble mound, V_B, which is obtained also by applying semi-empirical relations.



$$p_B = 1.7\rho\frac{V_B^2}{2}\cos^2\varphi;\; p_1 = p_3 = 0.4p_B;\; p_2 = p_4 = 0.1p_B$$

$$l_1 = 0.025S;\; l_2 = 0.065S;\; l_3 = 0.053S;\; l_4 = 0.135S;\; S = \frac{m\lambda}{2\sqrt[4]{m^2-1}}$$

Fig. 10 The distribution of the pressures on the rubber mound [3]

Another formulation, very similar, is given by the Djunkovski [4] method, and the Iribarren method uses a simplified formula for the maximal pressure in the impact point, without taking in consideration its dependency on the slope.

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As opposed to the usual way of dealing with the loadings within the wave, as forces that are applied statically, in this work the waves are considered as some certain dynamic loads and it is proposed a simplified formula of calculation for the action given by the waves, based upon the concept of the response spectrum. The spectral response forms the most convenient means for to illustrate the maximal response. There will be calculated the spectrums of the displacements, velocities and accelerations. Each of the three spectrums gives a certain quantity, with a different physical significance. The displacement spectrum indicates the maximal displacement of the system, the velocity spectrum indicates the maximal specific energy stored within the system, and the acceleration spectrum refers to the maximal value of the equivalent static force.

4 The calculation procedure

Sea wave loading is considered as **arbitrary general dynamic loading *p(t)***, specifically the intensity of loading $p(\tau)$ acting at time $t=\tau$. The entire loading history may be considered to consist of a succession of short impulses $p(\tau)d\tau$, each producing its own differential response. For linearly elastic system, the total response can be obtained by summing all the differential response, using Duhamel Integral. Concerning a natural frequency ω, and the critical damping coefficient, ν, the instantaneous displacement is estimated following the formula:

$$u(t) = \frac{1}{m\cdot\omega^*}\int_0^t p(\tau)e^{-\nu\omega(t-\tau)}\sin\omega^*(t-\tau)\,\mathrm{d}\tau \qquad (1)$$

where $\omega^* = \omega\sqrt{1-\nu^2}$ represents the own pulsation when damping is present [1].

Considering the trigonometrically identity $\sin(\omega t - \omega\tau) = \sin\omega t\cos\omega\tau - \cos\omega t\sin\omega\tau$, the response in time could be obtained by the following formula:

$$\begin{aligned} u(t) &= \frac{1}{m\cdot\omega^*}\frac{\sin\omega^* t}{e^{\nu\omega t}}\int_0^t p(\tau)e^{\nu\omega\tau}\cos\omega^*\tau\,\mathrm{d}\tau - \\ &-\frac{1}{m\cdot\omega^*}\frac{\cos\omega^* t}{e^{\nu\omega t}}\int_0^t p(\tau)e^{\nu\omega\tau}\sin\omega^*\tau\,\mathrm{d}\tau = \\ &= \frac{1}{m\cdot\omega^*}\frac{\sin\omega^* t}{e^{\nu\omega t}}A(t) - \frac{1}{m\cdot\omega^*}\frac{\cos\omega^* t}{e^{\nu\omega t}}B(t) \end{aligned} \qquad (2)$$

where the expression of A(t) and B(t) integrals

$$\begin{aligned} A(t) &= \int_0^t p(\tau)e^{\nu\omega\tau}\cos\omega^*\tau\,\mathrm{d}\tau \\ \mathrm{B}(t) &= \int_0^t p(\tau)e^{\nu\omega\tau}\sin\omega^*\tau\,\mathrm{d}\tau \end{aligned} \qquad (3)$$

can be obtained by means of one of the integration numerical methods. By deriving the Eq. (2), there were obtained the formulae for the velocity and reaction acceleration, having the form of the following formulae:

$$\dot{u}(t) = \frac{\cos\omega^* t}{m\cdot e^{\nu\omega t}}A(t) + \frac{\sin\omega^* t}{m\cdot e^{\nu\omega t}}B(t) - \omega\nu u(t) \qquad (4)$$

$$\ddot{u}(t) = -u(t)\omega^2 - 2\dot{u}(t)\omega\nu \qquad (5)$$

Deducing the response for a structure that has an elastic behavior, with a single degree of freedom, represents the main part of RS-WAVE code, written in the language FORTRAN 90. The formulae of the Eq. (3) were deduced by using the integration rule of Simpsons.

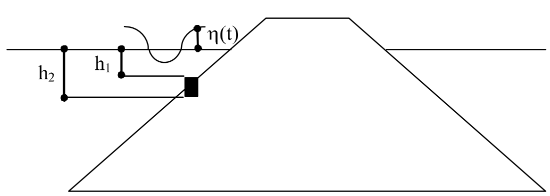
The RS-WAVE code gives the necessary data for to represent the response spectrums, but also those needed in order to represent the response history for a structure of which the own period is known, under the conditions in which the general dynamic actions are directly applied to the oscillating system.

The RS-WAVE code requests the following entry data:

- the oscillograme, described either by the variation of the recorded forces, estimated according to time, or by variation of the wave surface profile;
- the mass of the oscillating system and, if necessary, the own pulsation and the coefficient from the critical amortization.

There could also be produced some oscillogrames of the variation of the wave surface profile by the wave recorders, and other oscillogrames of the forces could be obtained by means of the tension traductors.

If the processed data consist of oscillogrames of the variation of the wave surface profile, the force that is directly applied to the oscillating system is estimated according to the position of the element towards the still water level.



$$F(t) = \frac{p_1(t) + p_2(t)}{2}(h_2 - h_1)$$

$$p_1(t) = \rho g\left[h_1 + \eta(t)\right]$$

$$p_2(t) = \rho g\left[h_2 + \eta(t)\right]$$

Fig. 11 Representation of h1 and h2 heights as required by RS-WAVE code

5 Numerical examples

Figure 12a shows the response spectra of displacement, velocity and acceleration calculated for oscillogrames of the variation for the wave surface profil.

Figure 12b shows the response spectra of displacement, velocity and acceleration calculated for oscillogrames of pressure which could be obtained by means of the tension traductors. The first pressure is 60kPa followed at 150 milisecoonds by second peak pressure of 120kPa, by [5].

6 Results

This paper treats wave loading as a arbitrary general dynamic loading. From the point of view of the structure engineer, the maximal result of these dynamic actions applied directly to the oscillating system could be easily obtained, using dynamic response spectrums.

The wave profile oscillograme becomes the oscillograme of the hydrostatic pressure given by the wave on the structure. By solving numerically the Duhamel integral, the necessary data for a representation of the maximal response under the form of the spectrums can be obtained.

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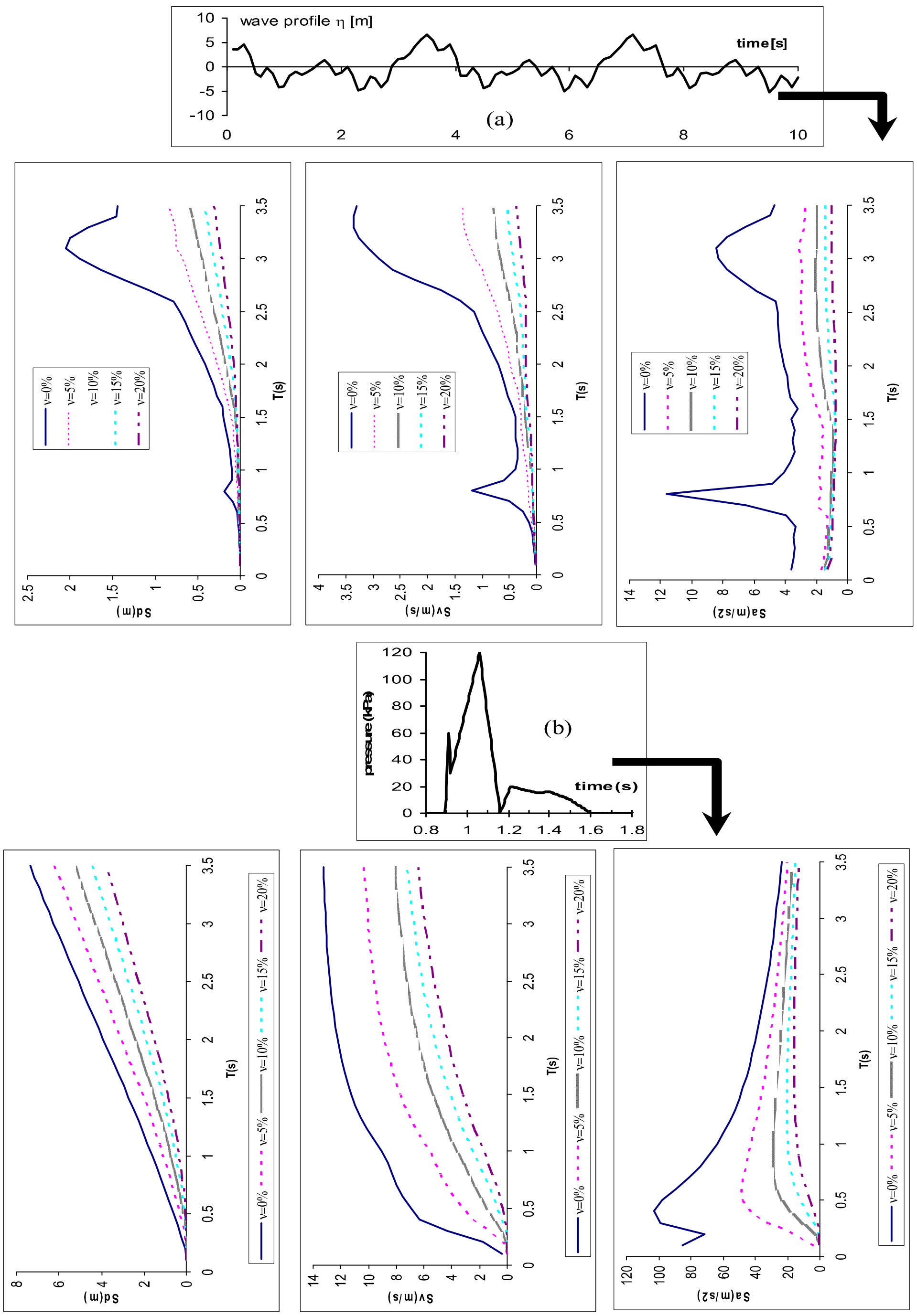

Fig. 12 Response spectra of displacement, velocity and acceleration for: (a) m=1000t , oscillogram for wave surface, h_1=4m, h_2=4.2m; (b) m=1000t, oscillogram for pressure recorded by tension traductor