Explicit Solution for Flow Depth in Open Channels of Trapezoidal Cross-Sectional Area: Classic Problem of Interest

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Abstract: This study presents an explicit solution for the flow depth in open channels of trapezoidal cross-sectional area. Four depths are encountered in most classical open-channel hydraulics problems, namely normal depth, critical depth, alternate depths from the specific energy equation, and sequent depths from the specific force equation. The study proposes new equations derived from dimensional and regression analysis to estimate directly those flow depths. The novelty of this work lies in the proposed regression equations for the prediction of the alternate and sequent flow depths, which can be regarded as a new contribution to the literature of classical open-channel flow because currently only analytical solutions of very complex formulations exist for those depths. The range of used data makes the proposed regression equations applicable to laboratory flumes, irrigation channels, and large rivers as well. The equations are examined over a wide range of flow and geometric conditions, providing satisfactory predictions when compared to the exact solution obtained from the governing hydraulic equations. Maximum and average errors in the flow-depth predictions are under 6 and 2.5%, respectively, which are acceptable in most hydraulic engineering practice. The proposed equations are coupled with fixed-point iterative equations to provide more-accurate predictions of the flow depths when desired. A sensitivity analysis showed that 6% uncertainty in the estimates of hydraulic parameters such as channel longitudinal bed slope and Manning's coefficient provides errors in the flow-depth prediction comparable to the errors encountered in the proposed regression equations. Thus, the accuracy of the proposed regression equations for the prediction of flow depths can be regarded as satisfactory considering the errors that come from other sources of uncertainty. The proposed equations are simple and would be useful in hydraulic engineering practice when quick and accurate estimates are needed for those depths. They can also be used to find the initial values for the flow depth to be utilized in the proposed fixed-point iterative equations or any other alternative numerical scheme if the exact solution is required. **DOI: 10.1061/(ASCE)IR.1943-4774.0001179.**

Introduction

Four flow depths are often encountered in the solution of most classical open-channel hydraulics problems. These are the normal depth, critical depth, alternate depths from the specific energy equation, and sequent depths from the specific force equation. Those flow depths have many applications in hydraulic engineering with the normal and critical depths being used in the design of open channels (Gharabaghi et al. 2001). Alternate depths are used to predict the upstream and downstream depths across a sluice gate, while the sequent depths are used to predict the prejump and postjump depths of a hydraulic jump.

Analytical solutions for those depths exists only for open channels of rectangular cross-sectional area. Other prismatic channels (e.g., circular, trapezoidal, and triangle) have no analytical solution because simplification of the governing hydraulic equations leads to nonlinear equations and fifth-degree polynomials in terms of the flow depth. Therefore, hydraulic engineers have to rely on graphs, regression equations, or numerical methods to find the flow depths in prismatic channels of nonrectangular cross sections (Gharabaghi et al. 1999). Graphical solutions are available for the prediction of the critical and normal flow depths in circular, trapezoidal, and triangular open channels (Silvester 1964; Jeppson 1965). They are included in several classical and recent textbooks in the field of open-channel hydraulics (e.g., Chow 1959; Henderson 1966; French 1985; Chaudhry 2008). Regression equations are also proposed for the prediction of the critical and normal flow depths in open channels of circular, trapezoidal, triangular, and horseshoe cross sections (Straub 1978; Swamee 1993, 1994; Wang 1998; Liu et al. 2010; Vatankhah and Easa 2011). However, few of those equations are included in the standard textbooks of open-channel flow because of the complex nature of their formulations or because they are not sufficiently generalized, but are applicable only to a limited range of flow conditions. Few attempts are made to obtain an explicit solution to the sequent flow depths from the specific force equation and the alternate flow depths from the specific energy equation in circular, triangular, and trapezoidal channels (Straub 1978; Das 2007; He et al. 2009; Vatankhah 2010). However, those methods received less attention and were less attractive to practicing engineers due to the complexity and the high level of mathematics involved in them, which made those methods an academic exercise rather than a practical solution to the problem.

The majority of the standard textbooks in open-channel flow offer numerical and graphical solutions for the prediction of the normal and critical flow depths (e.g., French 1985; Chaudhry 2008). While graphical solutions can be an alternative way to avoid numerical methods, most practicing engineers still prefer to have an equation to obtain the desired flow depth directly. However, a key element for the success of a regression equation, beside precision, is simplicity. Simplicity of a regression equation is often measured

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Note. This manuscript was submitted on September 17, 2016; approved on December 5, 2016; published online on February 16, 2017. Discussion period open until July 16, 2017; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Irrigation and Drainage Engineering***, © ASCE, ISSN 0733-9437.

in terms of the number of coefficients, input parameters, and mathematical operations.

This paper provides a direct solution for the flow depth in open channels of trapezoidal sections proposing new simple regression equations to predict the normal, critical, alternate, and sequent flow depths. The equations are derived from dimensional and regression analysis using a wide range of geometric properties and flow conditions. The range of data used in this study makes the proposed regression equations applicable to laboratory flumes, irrigation channels, and large rivers as well. The proposed equations are coupled with fixed-point iterative equations to provide more-accurate predictions to the flow depth when desired. The predictions of the developed equations are compared to the exact solution obtained from the governing hydraulic equations to identify the accuracy of the proposed equations. In addition, a sensitivity analysis is performed to compare the errors in flow-depth prediction from the developed equations to the errors in flow-depth prediction due to uncertainty in estimates of hydraulic parameters such as channel longitudinal bed slope and Manning's coefficient.

The exercise to find a regression equation for the critical flow depth in trapezoidal channels was easier than the normal flow depth, which explain why several regression equations exist in the literature for the prediction of the critical depth compared to the limited number of regression equations available for the prediction of the normal depth (Swamee 1993, 1994; Wang 1998; Swamee and Rathie 2005; Vatankhah and Easa 2011). Very few methods are available in the literature for the explicit solution of the alternate and sequent flow depths in open channels of trapezoidal sections (Das 2007; He et al. 2009; Vatankhah 2010). Therefore, finding regression equations for the alternate and sequent flow depths was the most difficult task in this study. To the author's best knowledge, there are no attempts to develop regression equations for the prediction of the alternate and sequent flow depths in open channels of trapezoidal sections. Only an analytical solution exists of very complex formulations.

The regression equations of various flow depths are obtained in this study by manipulating the governing hydraulic equations to express the flow parameters as function of the geometric properties of the trapezoidal section. Then, a relationship is developed between the flow depth and flow parameters using multivariate regression analysis.

Trapezoidal Section Geometric Properties

The geometric properties of a trapezoidal section (Fig. 1) can be estimated from the following equations:

$$A = zy^2 + by \tag{1}$$

$$P = b + 2y\sqrt{z^2 + 1} \tag{2}$$

$$T = b + 2yz \tag{3}$$

$$\bar{y} = \frac{y(2b + T)}{3(b + T)} \tag{4}$$

where A = cross-sectional area; b = bed width of the channel; z = side slope of the channel; y = flow depth; P = wetted perimeter; T = top width of the channel at the free surface; and $\bar{y}$ = distance from the centroid of trapezoid area to the water surface.

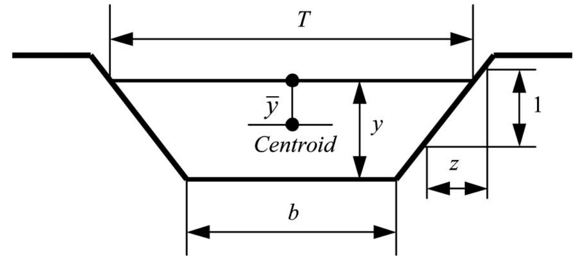

Fig. 1. Dimensions of the trapezoidal section

Dimensional Analysis

Most of the regression equations for the prediction of the flow depth in trapezoidal channels are functions of the dimensionless side-slope z and dimensionless variable zy/b for ranges of $0.25 \leq z \leq 3$ and $0 \leq zy/b \leq 3$ (Swamee 1993, 1994; Vatankhah and Easa 2011). A few regression equations were developed with a larger range of zy/b but their formulations were complex (Wang 1998; Swamee and Rathie 2005). One problem related to the use of the dimensionless variable zy/b is that the upper limit is undefined, which makes most of the regression equations derived using this variable not sufficiently generalized and applicable only to a limited range of flow conditions. A better criterion to define the range of data when deriving a regression equation for the prediction of the flow depth is the dimensionless variable $k = A/yT$. The dimensionless variable k has the following advantages over zy/b: (1) its lower and upper limits are defined, $0.5 < k < 1.0$; and (2) it gives more insight in the trapezoidal section geometric properties. The dimensionless variable k can describe the change in the trapezoidal section shape. For instance, when $k \to 0.5$, the trapezoidal section is converted to a triangle section and when $k \to 1.0$, the trapezoidal section is converted to a rectangular section. Thus, k can explain why the regression equation may provide unsatisfactory predictions for the flow depth near the limits of k, when the trapezoidal section deviates from its original shape to exhibit either a near-triangular or rectangularlike shape.

Regression Equations

The development of a regression equation for the flow depth, in general, requires the rearrangement of the equation governing the flow depth in a way that the hydraulic parameters are on one side of the equation while the geometric properties of the cross-sectional area are on the other side. Then, a relationship can be developed between the flow depth and geometric properties of the cross-sectional area from an accepted range of y, z, and b using multivariate regression analysis or machine-learning methods (e.g., Draper and Smith 1998; Sattar 2014). After that, the geometric properties of the section are replaced by the hydraulic parameters.

A key element that is considered when deriving the regression equations, beside precision, is simplicity. The Akaike information criterion (AIC) is used herein for the selection of the appropriate regression equation by measuring both precision and simplicity. The AIC displays a tradeoff between the regression equation precision and its complexity by seeking an equation that has the best fit to the data but few coefficients (Akaike 1974; Bozdogan 1987). It compromises between the goodness-of-fit of an equation and the number of coefficients incorporated in the equation by imposing a penalty for increasing the number of coefficients. The AIC has an inversely oriented score, where lower values indicate the preferred regression equation that has the fewest number of coefficients but still provides an adequate fit to the data. Mathematically, the AIC is expressed in terms of the variance and bias as

$$\mathrm{AIC} = N\left\{\mathrm{Ln}\left[2\pi\sum_{i=1}^{N}(O_i - P_i)^2 \Big/ N\right] + 1\right\} + 2k \qquad (5)$$

where N = number of data point used in the derivation of equation; O_i and P_i = observed true and predicted values, respectively; and k = number of coefficients used in the regression equation.

Critical Flow Depth

The critical flow depth can be estimated from the Froude number F as

$$\mathsf{F}^2 = \frac{\alpha Q^2 T}{gA^3} = 1 \qquad (6)$$

By separating the hydraulic parameters from the trapezoidal section geometric properties, Eq. (6) can be expressed as

$$\frac{\alpha Q^2}{gb^5} = \frac{A^3}{Tb^5} \qquad (7)$$

where α = energy correction factor; Q = discharge; g = acceleration due to gravity; A = cross-sectional area; b = bed width; and T = top width of the channel.

The left hand side (LHS) of Eq. (7) is defined herein as the hydraulic parameter for the critical flow depth $Q_c = \alpha Q^2/gb^5$. Using multivariate regression analysis, two regression equations are proposed for the critical depth y_c as a function of Q_c and side-slope z. The first equation estimates y_c directly as

$$y_c/b = \ln(2.07z^{-0.15})Q_c^{0.28z^{-0.1}} \qquad (8)$$

The second equation estimates y_c indirectly from the cross-sectional area A_c corresponding to the critical depth as

$$C_c = \frac{zA_c}{b^2} = \frac{z(zy_c^2 + by_c)}{b^2} = 1.4z^{1.16}Q_c^{(0.012\ln z + 0.38)} \qquad (9)$$

Solving for the critical depth y_c in Eq. (9) gives

$$zy_c/b = 0.5\left(\sqrt{4C_c + 1} - 1\right) \qquad (10)$$

Eq. (8) is simpler than Eq. (10), but less accurate. For the range of data given in Table 1, Eq. (8) provides mean and maximum errors of 3.0 and 5.7%, respectively, in the critical depth predictions, while Eq. (10) provides mean and maximum errors of 1.3 and 3.5%, respectively. The proposed regression equations are coupled with the following fixed-point iterative equation to provide more-accurate predictions for the critical flow depth when desired:

$$y_{cI}/b = \frac{Q_c^{1/3}(1 + 2zy_c/b)^{1/3}}{(1 + zy_c/b)} \qquad (11)$$

where y_{cI} = improved value; and y_c = value obtained from either Eq. (8) or (10).

Normal Flow Depth

The normal flow depth can be estimated from Manning's equation as

$$Q = \frac{c}{n}S^{1/2}p^{-2/3}A^{5/3} \qquad (12)$$

By separating the hydraulic parameters from the trapezoidal section geometric properties, Eq. (12) can be expressed as

$$\frac{nQ}{cS^{1/2}b^{8/3}} = \frac{P^{-2/3}A^{5/3}}{b^{8/3}} \qquad (13)$$

where Q = discharge; n = Manning's coefficient; A = cross-sectional area; P = wetted perimeter; S = longitudinal bed slope; b = bed width; and c = conversion factor with a value of 1.0 for SI units and 1.486 for U.S. customary units.

The LHS of Eq. (13) is defined herein as the hydraulic parameter of the normal flow depth $Q_n = nQ/cS^{1/2}b^{8/3}$. Using multivariate regression analysis, two regression equations are proposed also for the normal depth y_n as a function of Q_n and side-slope z. The first equation estimates y_n directly as

$$y_n/b = 0.85z^{-0.28}Q_n^{(0.25z^{-0.28} + 0.28)} \qquad (14)$$

The second equation estimates y_n indirectly from the cross-sectional area A_n corresponding to the normal depth as

$$C_n = \frac{zA_n}{b^2} = \frac{z(zy_n^2 + by_n)}{b^2} = 1.44(z + 0.12)^{1.24}Q_n^{0.74} \qquad (15)$$

Solving for the normal depth y_n in Eq. (15) gives

$$zy_n/b = 0.5\left(\sqrt{4C_n + 1} - 1\right) \qquad (16)$$

Eq. (14) is simpler than Eq. (16), but less accurate. For the range of data given in Table 1, Eq. (14) provides mean and maximum errors of 2.5 and 6.1%, respectively, in the normal depth predictions, while Eq. (16) provides mean and maximum errors of 1.5 and 4.6%, respectively. The proposed regression equations are coupled with the following fixed-point iterative equation to provide more-accurate predictions for the normal flow depth when desired:

$$y_{nI}/b = \frac{Q_n^{0.6}\left(1 + 2\sqrt{z^2 + 1}\,y_n/b\right)^{0.4}}{(1 + zy_n/b)} \qquad (17)$$

where y_{nI} = improved value; and y_n = value obtained from either Eq. (14) or (16).

Alternate Flow Depths

A solution for the alternate flow depths (Fig. 2) can be obtained from the specific energy equation as

Table 1. Range of Data Used in the Development of the Regression Equations for Various Depths

Depth	N	z	b (m)	y (m)	$k = A/yT$	zy/b	F^2
y_c and y_n	72	0.25–10	0.2–200	0.3–30	0.51–0.97	0.038–20	0.038–300
y_1	54	0.25–3.0	0.5–20	0.3–4.0	0.6–0.95	0.06–2.0	3.7–380
y_0	54	0.25–3.0	0.5–20	0.5–503	0.5–0.88	0.16–288	<0.16
y_2	54	0.25–3.0	0.5–20	0.45–44	0.51–0.89	0.13–20	0.01–0.31

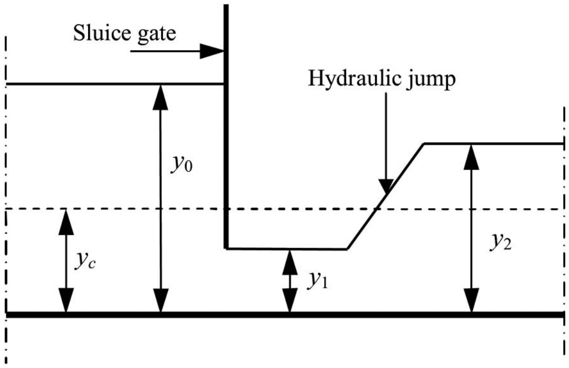

Fig. 2. Definition sketch for the alternate depths across a sluice gate and sequent depths of the hydraulic jump

$$E_s = \frac{\alpha Q^2}{2gA_0^2} + y_0 = \frac{\alpha Q^2}{2gA_1^2} + y_1 \tag{18}$$

where A_0 and A_1 = cross-sectional areas of the alternate depths y_0 and y_1, respectively. It is assumed that y_0 is larger than y_1 (Fig. 2).

In most cases, the term $\alpha Q^2/2gA_0^2$ in Eq. (18) is very small compared to the other terms and can be neglected; thus Eq. (18) can expressed as

$$\frac{y_0}{y_1} = 1 + 0.5\frac{A_1}{T_1 y_1}\frac{\alpha Q^2 T_1}{gA_1^3} \tag{19}$$

Substituting for $A_1/y_1T_1 = k_1$ and $\alpha Q^2 T_1/gA_1^3 = \mathsf{F}_1^2$ gives

$$\frac{y_0}{y_1} = 1 + 0.5k_1\mathsf{F}_1^2 \tag{20}$$

For the ratio $y_0/y_1 > 1$ in the range of $0.5 < k_1 < 1$ and $1 < \mathsf{F}_1^2 \leq 380$, it was found from the regression analysis that a value of 0.93 instead of 1.0 for the numerical constant in Eq. (20) provides better predictions, thus the equation is revised to

$$\frac{y_0}{y_1} = 0.93 + 0.5k_1\mathsf{F}_1^2 \tag{21}$$

Eq. (21) is coupled with the following fixed-point iterative equation to provide a more-accurate prediction for the alternate flow depth y_0 when desired:

$$y_{0I} = E_s - \frac{\alpha Q^2}{2g(zy_0^2 + by_0)^2} \tag{22}$$

where E_s = specific energy as per Eq. (18); and y_{0I} = improved value of y_0.

To obtain an expression for y_1 (Fig. 2), Eq. (18) can be rearranged as

$$\frac{2}{A_0/y_0T_0}\left[\frac{(y_1/y_0)(A_1/A_0)^2 - 1}{(A_1/A_0)^2 - 1} - 1\right] = \frac{\alpha Q^2 T_0}{gA_0^3} \tag{23}$$

The derivation of Eq. (23) is given in Appendix I. From the geometric properties of the trapezoidal section, Eq. (23) can be simplified using the following approximation $(y_1/y_0) \approx (A_1/A_0)^{k_0}$ and substituting for $\alpha Q^2 T_0/gA_0^3 = \mathsf{F}_0^2$ and $A_0/y_0T_0 = k_0$ as

$$\frac{2}{k_0}\left[\frac{(A_1/A_0)^{2+k_0} - 1}{(A_1/A_0)^2 - 1} - 1\right] = \mathsf{F}_0^2 \tag{24}$$

Using multivariate regression analysis, the following regression equation is obtained for the ratio $A_1/A_0 < 1$ in the range of $0.5 < k_0 < 1$ and $\mathsf{F}_0^2 < 1$:

$$C_{a0} = \frac{A_1}{A_0} = \frac{zy_1^2 + by_1}{A_0} = 0.92(k_0\mathsf{F}_0^2)^{(0.13k_0+0.45)} \tag{25}$$

Solving for y_1 in Eq. (25) gives

$$zy_1/b = 0.5\left[\sqrt{4(zA_0/b^2)C_{a0} + 1} - 1\right] \tag{26}$$

And the coupled fixed-point iterative equation is

$$y_{1I} = \left[\frac{\alpha Q^2/(zy_1 + b)^2}{2g(E_s - y_1)}\right]^{0.5} \tag{27}$$

where E_s = specific energy as per Eq. (18); and y_{1I} = improved value of y_1.

For the range of data given in Table 1, Eq. (21) provides mean and maximum errors of 0.8 and 5.7%, respectively, in the alternate flow-depth predictions, while Eq. (26) provides mean and maximum errors of 2.0 and 6.6%, respectively.

Sequent Flow Depths

The sequent flow depths of a hydraulic jump (Fig. 2) can be obtained from the specific force equation as

$$F_s = \frac{\alpha Q^2}{A_1} + g\bar{y}_1 A_1 = \frac{\alpha Q^2}{A_2} + g\bar{y}_2 A_2 \tag{28}$$

where A_1 and A_2 = cross sectional areas at the prejump and postjump depths of the hydraulic jump y_1 and y_2, respectively (Fig. 2); and $\bar{y}_1$ and $\bar{y}_2$ = distances from the centroid to the water surface at cross-sectional areas A_1 and A_2, respectively.

Eq. (28) can be expressed as

$$\frac{1}{A_1/T_1\bar{y}_1}\left[\frac{(\bar{y}_2/\bar{y}_1)(A_2/A_1)^2 - 1}{(A_2/A_1) - 1} - 1\right] = \frac{\alpha Q^2 T_1}{gA_1^3} \tag{29}$$

The derivation of Eq. (29) is given in Appendix I. From the geometric properties of the trapezoidal section, Eq. (29) can be simplified using the following approximations $(\bar{y}_2/\bar{y}_1) \approx (A_2/A_1)^{k_1}$ and $A_1/\bar{y}_1 T_1 \approx k_1 + 1$ and substituting for $\alpha Q^2 T_1/gA_1^3 = \mathsf{F}_1^2$ as

$$\frac{1}{k_1 + 1}\left[\frac{(A_2/A_1)^{2+k_1} - 1}{(A_2/A_1) - 1} - 1\right] = \mathsf{F}_1^2 \tag{30}$$

Using multivariate regression analysis, the following regression equation is obtained for the ratio $A_2/A_1 > 1$ in the range of $0.5 < k_1 < 1$ and $1 < \mathsf{F}_1^2 \leq 380$:

$$C_{s1} = \frac{A_2}{A_1} = \frac{zy_2^2 + by_2}{A_1} = 1.1(\mathsf{F}_1^2)^{(0.9-0.3k_1)} \tag{31}$$

Solving for y_2 in Eq. (31) gives

$$zy_2/b = 0.5\left[\sqrt{4(zA_1/b^2)C_{s1} + 1} - 1\right] \tag{32}$$

Eq. (32) can be coupled with the following fixed-point iterative equation when a more-accurate prediction for the sequent flow depth y_2 is desired:

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$$y_{2I} = \left[\frac{F_s - \alpha Q^2/(zy_2^2 + by_2)}{g(0.33zy_2 + 0.5b)}\right]^{0.5} \tag{33}$$

where F_s = specific force as per Eq. (28); and y_{2I} = improved value of y_2.

In a similar way, Eq. (30) is revised to find the following regression equation for the ratio $A_1/A_2 < 1$ in the range of $0.5 < k_2 < 1$ and $\mathsf{F}_2^2 < 1$:

$$C_{s2} = \frac{A_1}{A_2} = \frac{zy_1^2 + by_1}{A_2} = 1.2(\mathsf{F}_2^2)^{(1.0-0.1k_2)} \tag{34}$$

Solving for y_1 in Eq. (34) gives

$$zy_1/b = 0.5\left[\sqrt{4(zA_2/b^2)C_{s2} + 1} - 1\right] \tag{35}$$

And the coupled fixed-point iterative equation is

$$y_{1I} = \frac{\alpha Q^2/(zy_1 + b)}{F_s - g(0.33zy_1^3 + 0.5by_1^2)} \tag{36}$$

where F_s = specific force as per Eq. (28); and y_{1I} = improved value of y_1.

For the range of data given in Table 1, Eq. (32) provides mean and maximum errors of 2.5 and 4.6%, respectively, in the sequent flow-depth predictions, while Eq. (35) provides mean and maximum errors of 1.5 and 5%, respectively.

The alternate and sequent depths can be written in general form as

$$zy_j/b = 0.5\left[\sqrt{4(zA_i/b^2)C_i + 1} - 1\right] \tag{37}$$

Or in a rational form as

$$y_j/y_i = 0.5(b/zy_i)\left[\sqrt{4(zA_i/b^2)C_i + 1} - 1\right] \tag{38}$$

When applying Eqs. (37) and (38), the subscript notations i and j are interchangeable ($i \neq j$) taken either the value 1 or 0 in case of the alternate depths and the value of 1 or 2 in case of the sequent depths (Fig. 2). In Eqs. (37) and (38), C_i is equal to C_{ai} in case of the alternate depth and is equal C_{si} in case of the sequent depth. The value of C_{ai} is given in Eq. (25), while the value of C_{si} is given in Eqs. (31) and (34).

The proposed regression equations for various flow depths, summarized in Appendix II, can have slightly better predictions if more decimals are considered in their regression coefficients. However, to simplify the equations and make them more attractive to users, only two decimals are considered. Table 2 provides the AIC of various regression equations. AIC is used as a measure of goodness-of-fit of the proposed regression equations when comparing two alternate equations. As can be seen from Table 2, Eq. (9) has a lower AIC value compared to Eqs. (7), and (15) has a lower AIC value compared to Eq. (13), which indicates that Eqs. (9) and (15) provide better predictions than Eqs. (7) and (13), respectively. This is reflected in the errors in the predictions of various equations as indicated in Table 2.

Uncertainty Analysis

The deviation in the predictions of the regression equations from the exact solution obtained from the governing hydraulic equations represents the error or uncertainty in the predictions of the developed equations. Table 2 summarizes the range of errors encountered in the flow-depth prediction obtained from the developed regression equations. For the range of data used in the derivation of the regression equations, the mean and maximum errors encountered in the flow-depth predictions are about 2.5 and 6%, respectively. This range of errors is acceptable in most hydraulic engineering practice considering the additional errors that may be encountered in the flow-depth prediction due to uncertainty in the estimates of the hydraulic parameters, namely α, Q, n, and S. Herein, a first-order analysis of uncertainty (Chow et al. 1988; Mays 2010) is performed to foresee how the errors in the flow-depth prediction due to uncertainty in the hydraulic-parameter estimates are comparable to the errors inherited from the regression equations. The standard error of the flow depth y can be expressed in terms of the hydraulic parameters as

$$\sigma_y^2 = \left(\frac{\partial y}{\partial \alpha}\right)^2 \sigma_\alpha^2 + \left(\frac{\partial y}{\partial Q}\right)^2 \sigma_Q^2 + \left(\frac{\partial y}{\partial n}\right)\sigma_n^2 + \left(\frac{\partial y}{\partial S}\right)\sigma_S^2 \tag{39}$$

where σ_y, σ_α, σ_Q, σ_n, and σ_S = standard errors of the flow depth y, energy correction factor α, discharge Q, Manning's coefficient n, and longitudinal slope S, respectively. The equations describing the standard error of various flow depths are derived in Appendix III.

The standard-error equations (Appendix III) are used to estimate the corresponding relative error in various flow depths by imposing a 2, 4, and 6% variation in the magnitude of various hydraulic parameters, namely α, Q, n, and S. This variation is regarded as the uncertainty or error in the estimates of the hydraulic parameters. The range of errors used in the sensitivity analysis is very conservative. Errors encountered in the hydraulic-parameter estimates are typically in the range of 15–30% (Mays 2010). As indicated in Table 3, little variations in the hydraulic parameters' magnitude provide errors in the flow-depth prediction of the same order of magnitude as the errors encountered in the proposed regression equations. Thus, the accuracy of the proposed regression equations for the prediction of flow depths can be regarded satisfactory considering the errors that come from other sources of uncertainty.

Table 2. Akaike Information Criterion and Errors in Flow Depths Prediction from Various Regression Equations

Depth	Depth equation	AIC	Mean error (%)	Error range (%)
Critical flow depth	y_c [Eq. (7)]	77.67	3.00	0.17–5.74
	y_{cI} [Eqs. (7) and (10)]	—	0.99	0.01–3.00
	y_c [Eq. (9)]	57.55	1.31	0.04–3.50
	y_{cI} [Eqs. (9) and (10)]	—	0.30	0.02–1.27
Normal flow depth	y_n [Eq. (13)]	110.1	2.50	0.02–6.15
	y_{nI} [Eqs. (13) and (16)]	—	0.63	0.00–3.10
	y_n [Eq. (15)]	38.38	1.49	0.09–4.60
	y_{nI} [Eqs. (15) and (16)]	—	0.22	0.01–0.87
Alternate flow depths	y_0 [Eq. (23)]	56.06	0.75	0.03–5.69
	y_{0I} [Eqs. (23) and (24)]	—	0.17	0.00–1.30
	y_1 [Eq. (26)]	20.25	2.24	0.01–4.98
	y_{1I} [Eqs. (26) and (27)]	—	0.39	0.02–1.89
Sequent flow depths	y_2 [Eq. (32)]	25.45	2.47	0.03–4.61
	y_{2I} [Eqs. (32) and (33)]	—	0.71	0.00–1.67
	y_1 [Eq. (35)]	16.14	1.46	0.01–4.97
	y_{1I} [Eqs. (35) and (36)]	—	0.26	0.00–1.00

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Table 3. Errors in Flow Depths Prediction due to Uncertainty in the Hydraulic Parameters Estimates

Depth	Error in the flow depth prediction	Mean error in the hydraulic parameters estimates (%)		
		2	4	6
Critical flow depth	Mean error (%)	1.17	2.34	3.52
	Range of error (%)	0.92–1.47	1.83–2.94	2.75–4.41
Normal flow depth	Mean error (%)	1.51	3.01	4.52
	Range of error (%)	1.15–1.98	2.30–3.96	3.46–5.95
Alternate flow depths	Mean error in flow depth y_0 (%)	2.27	4.53	6.08
	Range of error in flow depth y_0 (%)	1.43–3.04	2.86–6.08	4.29–9.11
	Mean error in flow depth y_1 (%)	1.88	3.77	5.66
	Range of error in flow depth y_1 (%)	1.35–2.49	2.69–4.98	4.04–7.48
Sequent flow depths	Mean error in flow depth y_2 (%)	2.95	3.90	10.99
	Range of error in flow depth y_2 (%)	2.14–5.57	4.28–11.14	6.42–16.72
	Mean error in flow depth y_1 (%)	3.20	6.40	9.60
	Range of error in flow depth y_1 (%)	2.13–4.14	4.27–8.27	6.40–12.41

Application and Verification

Prediction of critical and normal flow depths in open channels has many applications in hydraulic engineering. The critical flow depth is used to identify the physical nature and classify the flow in open channels into supercritical (fast or shooting), subcritical (slow or tranquil), or critical flow. This classification is important when studying wave propagation and routing of kinematic wave (floods of rivers). The normal flow depth is used to predict the flow depth in natural streams and estimate the design depth in irrigation and drainage canals. Both critical and normal depths are used in the calculations for the prediction of water-surface profiles of gradually varied flow and delineation of flood regions.

Solving of the specific energy equation [Eq. (18)] provides two depths known in the literature as the alternate depths y_0 and y_1 (Fig. 2). The alternate depths have several applications in open-channel hydraulics and are often used to predict the flow depths upstream and downstream of sluice gates across the channels (Fig. 2), over broad-crested weirs, and in transition sections along the channel where the change in energy is negligible. Solving the specific force equation [Eq. (28)] provides two depths known in the literature as the sequent depths y_1 and y_2 (Fig. 2). The sequent depths are used to predict the water-surface profile of rapidly varied flow and prejump and postjump flow depths of a hydraulic jump (Fig. 2). They are also used to design spillways and stilling basins of dams.

The predictions of the proposed regression equations were examined against the water depths in a number of concrete lined irrigation canals of various sizes. Table 4 summarizes the geometric and hydraulic properties of the tested canals (Tilp and Scrivner 1964). The predictions of the proposed regression equations were compared to the exact solution for the flow depths in those canals obtained from the governing hydraulic equations. Except for 1 out of the 13 tested canals, the regression equations provided fairly accurate perditions for various flow depths with errors below 5% compared to the exact solution.

The critical and normal depths are estimated from the two proposed regression equations for each depth, namely: the simple form and cross-sectional area equations. As shown in Table 5, the cross-sectional area equations for both the critical and normal depths [Eqs. (9) and (15)] provide better predictions than the simple form equations [Eqs. (7) and (13)]. When compared to the exact solution, the cross-sectional area equations provided errors smaller than 3 and 2% in the critical and normal depths predictions, respectively. Errors in the predictions of the simple form equations were about 6% in both the critical and normal depths.

The predictions of the alternate and sequent flow-depth regression equations are examined using two approaches, defined herein as the forward and backward calculations. With reference to the three flow depths shown in Fig. (2), the forward calculations are carried out assuming that the normal depth given in Table 4 is equal to y_0; then, the alternate depth y_1 is obtained from the specific energy equation [Eq. (18)] and the sequent depth y_2 is obtained from

Table 4. Canals Geometric and Hydraulic Properties

Canal number	Canal name	b (m)	z	S	n	Q (m^3/s)	y_n (m)	y_c (m)
1	Madera-1	2.44	1.25	0.00070	0.0142	23.35	2.17	1.60
2	Madera-2	3.05	1.25	0.00030	0.0142	28.37	2.75	1.64
3	Roza-1	3.66	1.25	0.00039	0.0144	36.88	2.78	1.77
4	Roza-2	4.27	1.25	0.00040	0.0145	62.42	3.42	2.23
5	Charles Hansen	3.66	1.25	0.00131	0.0140	36.88	2.01	1.77
6	Gateway	3.05	1.50	0.00035	0.0141	19.86	2.10	1.31
7	East Low	6.10	1.50	0.00010	0.0144	127.68	5.76	2.80
8	Trail Lake	6.10	1.50	0.00061	0.0151	374.52	6.41	4.95
9	Friant Kern	10.98	1.25	0.00010	0.0143	113.49	4.64	2.04
10	Delta Mendota	14.64	1.50	0.00005	0.0138	130.51	5.06	1.88
11	Long Lake	15.25	1.50	0.00010	0.0146	275.21	6.31	2.91
12	San Luis -1	15.25	2.00	0.00003	0.0140	236.91	7.32	2.58
13	San Luis -2	25.93	2.00	0.00003	0.0139	371.68	7.59	2.57

Table 5. Evaluation of the Critical and Normal Depths Regression Equation Predictions

	Critical depth y_c (m)						Normal depth y_n (m)					
	Simple form			Cross-sectional area form			Simple form			Cross-sectional area form		
Canal number	Exact solution	Prediction Eq. (7)	Error (%)	Exact solution	Prediction Eq. (9)	Error (%)	Exact solution	Prediction Eq. (13)	Error (%)	Exact solution	Prediction Eq. (15)	Error (%)
1	1.60	1.50	6.18	1.60	1.63	1.87	2.17	2.10	2.91	2.17	2.19	0.94
2	1.64	1.54	6.33	1.64	1.68	2.19	2.75	2.67	2.83	2.75	2.77	0.93
3	1.77	1.66	6.21	1.77	1.81	2.30	2.78	2.68	3.58	2.78	2.80	1.02
4	2.23	2.09	6.31	2.23	2.28	2.22	3.42	3.30	3.39	3.42	3.45	1.00
5	1.77	1.66	6.21	1.77	1.81	2.30	2.01	1.93	3.90	2.01	2.03	0.96
6	1.31	1.23	6.14	1.31	1.33	2.10	2.10	2.02	3.86	2.10	2.13	1.35
7	2.80	2.63	6.30	2.80	2.86	2.06	5.76	5.63	2.39	5.76	5.83	1.17
8	4.95	4.69	5.30	4.95	5.00	1.12	6.41	6.30	1.58	6.41	6.47	1.06
9	2.04	2.05	0.23	2.04	2.06	0.68	4.64	4.49	3.20	4.64	4.66	0.57
10	1.88	1.97	4.73	1.88	1.85	1.22	5.06	4.93	2.55	5.06	5.09	0.58
11	2.91	2.90	0.36	2.91	2.93	0.81	6.31	6.10	3.44	6.31	6.38	1.00
12	2.58	2.60	0.53	2.58	2.58	0.14	7.32	7.01	4.25	7.32	7.44	1.59
13	2.57	2.79	8.54	2.57	2.50	2.85	7.59	7.44	2.07	7.59	7.66	0.83

Table 6. Evaluation of the Alternate and Sequent Depths Regression Equations Predictions

	Forward calculations						Backward calculations					
	Alternate depth y_1 (m)			Sequent depth y_2 (m)			Sequent depth y_1 (m)			Alternate depth y_0 (m)		
Canal number	Exact solution	Prediction Eq. (26)	Error (%)	Exact solution	Prediction Eq. (32)	Error (%)	Exact solution	Prediction Eq. (35)	Error (%)	Exact Solution	Prediction Eq. (21)	Error (%)
1	1.23	1.14	7.36	2.02	2.07	2.14	1.13	1.17	3.35	2.45	2.52	3.06
2	1.09	1.06	2.55	2.33	2.34	0.60	0.85	0.85	0.33	4.14	4.12	0.59
3	1.22	1.17	3.77	2.44	2.46	1.04	1.02	1.02	0.46	3.71	3.71	0.06
4	1.57	1.50	4.29	3.03	3.07	1.18	1.33	1.34	0.90	4.42	4.43	0.29
5	1.57	1.37	12.39	1.99	2.07	4.49	1.54	1.68	8.84	2.05	2.37	15.59
6	0.89	0.86	3.25	1.82	1.84	0.84	0.72	0.72	0.11	2.95	2.94	0.31
7	1.64	1.64	0.55	4.35	4.33	0.38	1.02	1.01	1.30	14.66	14.60	0.45
8	3.96	3.61	8.84	6.07	6.22	2.54	3.70	3.85	4.06	7.02	7.28	3.78
9	1.09	1.09	0.19	3.40	3.43	0.94	0.64	0.61	4.03	12.17	12.13	0.32
10	0.90	0.90	0.43	3.35	3.40	1.46	0.43	0.41	3.45	20.89	20.86	0.14
11	1.59	1.60	0.22	4.72	4.74	0.49	0.98	0.95	3.41	15.36	15.30	0.37
12	1.22	1.24	2.10	4.61	4.59	0.39	0.51	0.50	0.53	42.62	42.58	0.08
13	1.16	1.17	0.70	4.74	4.81	1.39	0.48	0.47	2.09	42.02	41.99	0.08

the specific force equation [Eq. (28)]. The exact solutions from Eqs. (18) and (28) are used to examine the predictions of Eq. (26) for the alternate depth y_1 and Eq. (32) for the sequent depth y_2, respectively. Table 6 summarizes the errors of y_1 and y_2 of various canals. The errors in the alternate depth y_1 predictions were lower than 5% for 10 out the tested 13 canals, with a maximum value of 12% in the Charles Hansen canal. The errors in the sequent depth y_2 predictions were below 5%.

With reference to the three flow depths shown in Fig. (2), the backward calculations are carried out in a similar way to the forward calculations, but with the assumption that the normal depth given in Table 4 is equal to y_2. Then, the sequent depth y_1 is obtained from the specific force equation [Eq. (28)] and the alternate depth y_0 is obtained from the specific energy equation [Eq. (18)]. The exact solutions from Eqs. (28) and (18) are used to examine the predictions of Eq. (35) for the sequent depth y_1 and Eq. (21) for the alternate depth y_0, respectively. Table 6 shows that the errors in the sequent depth y_1 and alternate depth y_0 predictions were under 5% for all tested canals except the Charles Hansen canal. The higher errors in the Charles Hansen canal depth prediction, in general, are attributed to the relatively high Froude number (velocity) for the depth y_0, which is not common.

Discussion and Conclusion

Several regression equations are available for explicit solution of the critical depth in trapezoidal sections (e.g., Swamee 1993; Vatankhah and Easa 2011). Most of them are of simple forms similar to the one proposed in this study providing satisfactory predictions with errors of the same order of magnitude of the regression equation proposed herein. Regression equations of higher accuracy (under 1% error), are also proposed for the prediction of the critical depth but they are of complex formulations, which makes them less attractive to users (e.g., Wang 1998). Few regression equations are also available for explicit solution of the normal depth in trapezoidal sections but their formulations are of complex form, while their predictions are of same precision to the equations proposed herein (e.g., Vatankhah and Easa 2011). To the author's best knowledge, regression equations do not exist for direct solution of the alternate and sequent depths in trapezoidal sections. Only an analytical solution exists of very complex formulations (e.g., Das 2007; He et al. 2009; Vatankhah 2010). The method of Das (2007) couples analytical and iterative methods to obtain the alternate and sequent flow depths. The method simplifies the governing equations into quadratic and third-order polynomial equations, then solves the equations iteratively to obtain those flow depths. The solution for either the alternate or sequent depths requires six auxiliary equations of moderate complexity. The methods proposed by He et al. (2009) and Vatankhah (2010) are similar and based on simplification of the equations describing the alternate and sequent depths in trapezoidal sections into polynomial equations of the fourth degree, then find the roots of these polynomials using the classical Ferrari solution. The solution for either the alternate or sequent depths requires eight auxiliary equations of complex nature. The method proposed in this study uses a pair of regression equations, relatively simple compared to the other methods equations, to obtain the alternate flow depths, and another pair of regression equations to obtain the sequent flow depths. Thus, the proposed regression equations for the alternate and sequent depths can be regarded as a novel contribution to the literature on classical open-channel flow and would be useful in hydraulic engineering practice when quick and accurate estimates are needed for those depths. The proposed regression equations can have higher accuracy if more coefficients and decimals are considered in their formulations but this will make them complex and less attractive to users. Alternatively, the proposed equations are coupled with fixed-point iterative equations to provide more-accurate prediction of flow depths when desired. While analytical methods provide exact solutions for flow depth, the complexity of their formulations negates their advantages over regression equations. Nonetheless, they are the only available methods beside the numerical methods for seeking an exact solution. Neither analytical and regression equations require iterations, unlike numerical methods, to obtain the solution.

The choice from the several equations available in the literature is not governed only by the precision of the equation but by its simplicity as well. A third factor is how the users perceive it. For instance, the Manning's equation, which is not dimensionally homogenous, gained more popularity over other rational equations (Rouse and Ince 1963). Therefore, one cannot recommend or direct users to a specific equation because it is a matter of taste, but no question that precision and simplicity will be the most important factors.

In conclusion, this study presents an explicit solution for the flow depth in open channels with a trapezoidal cross-sectional area. The study provides regression equations to estimate directly the normal depth, critical depth, alternate depths from the specific energy equation, and sequent depths from the specific force equation. The range of used data makes the proposed regression equations applicable to laboratory flumes, irrigation channels, and large rivers as well. Maximum and average errors in the flow-depth predictions are less than 6 and 2.5%, respectively, which are acceptable in most hydraulic engineering practice. A sensitivity analysis showed that 6% uncertainty in the estimates of hydraulic parameters such as channel longitudinal bed slope and Manning's coefficient provides errors in flow-depth prediction comparable to the errors encountered in the proposed regression equations. Thus, the accuracy of the proposed regression equations for the prediction of flow depths can be regarded as satisfactory considering the errors that come from other sources of uncertainty.

Appendix I. Derivation of Eqs. (23) and (29)

The specific energy equation can be expressed as

$$E_s = \frac{\alpha Q^2}{2gA_0^2} + y_0 = \frac{\alpha Q^2}{2gA_1^2} + y_1 \tag{40}$$

Rearranging the specific energy Eq. (40) yields

$$y_0\left(1 - \frac{y_1}{y_0}\right) = \frac{\alpha Q^2}{2gA_0^2}\left(\frac{A_0^2}{A_1^2} - 1\right) \tag{41}$$

Multiplying Eq. (41) by $2(T_0/A_0)(A_1/A_0)^2$ gives

$$\left(\frac{2T_0 y_0}{A_0}\right)\left(\frac{A_1^2}{A_0^2} - \frac{y_1}{y_0}\frac{A_1^2}{A_0^2}\right) = \frac{\alpha Q^2 T_0}{gA_0^3}\left(1 - \frac{A_1^2}{A_0^2}\right) \tag{42}$$

Dividing Eq. (42) by $1 - (A_1/A_0)^2$ gives

$$\frac{2T_0 y_0}{A_0}\left[\frac{(A_1/A_0)^2 - (y_1/y_0)(A_1/A_0)^2}{1 - (A_1/A_0)^2}\right] = \frac{\alpha Q^2 T_0}{gA_0^3} \tag{43}$$

Rearranging Eq. (43) gives

$$\frac{2T_0 y_0}{A_0}\left[\frac{(y_1/y_0)(A_1/A_0)^2 - 1 - ((A_1/A_0)^2 - 1)}{(A_1/A_0)^2 - 1}\right] = \frac{\alpha Q^2 T_0}{gA_0^3} \tag{44}$$

Simplifying Eq. (44) gives Eq. (23) in the text

$$\frac{2}{A_0/y_0 T_0}\left[\frac{(y_1/y_0)(A_1/A_0)^2 - 1}{(A_1/A_0)^2 - 1} - 1\right] = \frac{\alpha Q^2 T_0}{gA_0^3} \tag{45}$$

The specific force equation can be expressed as

$$F_s = \frac{\alpha Q^2}{A_1} + g\bar{y}_1 A_1 = \frac{\alpha Q^2}{A_2} + g\bar{y}_2 A_2 \tag{46}$$

Rearranging the specific force Eq. (46) yields

$$\bar{y}_1 A_1 \left(1 - \frac{\bar{y}_2 A_2}{\bar{y}_1 A_1}\right) = \frac{\alpha Q^2}{g A_1}\left(\frac{A_1}{A_2} - 1\right) \tag{47}$$

Multiplying Eq. (47) by $(T_1/A_1^2)(A_2/A_1)$ gives

$$\left(\frac{T_1 \bar{y}_1}{A_1}\right)\left(\frac{A_2}{A_1} - \frac{\bar{y}_2}{\bar{y}_1}\frac{A_2^2}{A_1^2}\right) = \frac{\alpha Q^2 T_1}{g A_1^3}\left(1 - \frac{A_2}{A_1}\right) \tag{48}$$

Dividing Eq. (48) by $1 - (A_2/A_1)$ gives

$$\frac{T_1 \bar{y}_1}{A_1}\left[\frac{(A_2/A_1) - (\bar{y}_2/\bar{y}_1)(A_2/A_1)^2}{1 - (A_2/A_1)}\right] = \frac{\alpha Q^2 T_1}{g A_1^3} \tag{49}$$

Rearranging Eq. (49) gives

$$\frac{T_1 \bar{y}_1}{A_1}\left\{\frac{(\bar{y}_2/\bar{y}_1)(A_2/A_1)^2 - 1 - [(A_2/A_1) - 1]}{(A_2/A_1) - 1}\right\} = \frac{\alpha Q^2 T_1}{g A_1^3} \tag{50}$$

Simplifying Eq. (50) gives Eq. (29) in the text

$$\frac{1}{A_1/T_1\bar{y}_1}\left[\frac{(\bar{y}_2/\bar{y}_1)(A_2/A_1)^2 - 1}{(A_2/A_1) - 1} - 1\right] = \frac{\alpha Q^2 T_1}{g A_1^3} \tag{51}$$

Appendix II. Summary of the Regression Equations for the Prediction of Flow Depth in Open Channels of Trapezoidal Cross-Sectional Area

Parameter	Regression equations of y	Improved values of y
Critical depth (y_c)	Simple form: $y_c/b = \ln(2.07z^{-0.15})Q_c^{0.28z^{-0.1}}$	$\frac{y_{cI}}{b} = \frac{Q_c^{1/3}(1+2zy_c/b)^{1/3}}{(1+zy_c/b)}$
z, b, $Q_c = \alpha Q^2/gb^5$	Cross-sectional area: $zy_c/b = 0.5(\sqrt{4C_c+1} - 1)$ $C_c = 1.4z^{1.16}Q_c^{(0.012\ln z + 0.38)}$	
Normal depth (y_n)	Simple form: $y_n/b = 0.85z^{-0.28}Q_n^{(0.25z^{-0.28}+0.28)}$	$\frac{y_{nI}}{b} = \frac{Q_n^{0.6}(1+2\sqrt{z^2+1}y_n/b)^{0.4}}{(1+zy_n/b)}$
z, b, $Q_n = nQ/cS^{1/2}b^{8/3}$	Cross-sectional area: $zy_n/b = 0.5(\sqrt{4C_n+1} - 1)$ $C_n = 1.44(z+0.12)^{1.24}Q_n^{0.74}$	
Alternate depth (y_j)	$y_j/y_i = 0.93 + 0.5k_i\mathsf{F}_i^2$ for $\mathsf{F}_i^2 > 1$	$y_{jI} = E_s - \frac{\alpha Q^2}{2g(zy_j^2 + by_j)^2}$ for $\mathsf{F}_i^2 > 1$
z, b, A_i, $k_i = A_i/y_iT_i$	$zy_j/b = 0.5[\sqrt{4(zA_i/b^2)C_{ai} + 1} - 1]$	$y_{jI} = \left[\frac{\alpha Q^2/(zy_j + b)^2}{2g(E_s - y_j)}\right]^{0.5}$ for $\mathsf{F}_i^2 < 1$
$\mathsf{F}_i^2 = \alpha Q^2 T_i/gA_i^3$	$C_{ai} = 0.92(k_i\mathsf{F}_i^2)^{(0.13k_i+0.45)}$ for $\mathsf{F}_i^2 < 1$	
Sequent depth (y_j)	$zy_j/b = 0.5[\sqrt{4(zA_i/b^2)C_{si} + 1} - 1]$	$y_{jI} = \left[\frac{F_s - \alpha Q^2/(zy_j^2 + by_j)}{g(0.33zy_j + 0.5b)}\right]^{0.5}$ for $\mathsf{F}_i^2 > 1$
z, b, A_i, $k_i = A_i/y_iT_i$	$C_{si} = 1.1(\mathsf{F}_i^2)^{(0.9-0.3k_i)}$ for $\mathsf{F}_i^2 > 1$	
$\mathsf{F}_i^2 = \alpha Q^2 T_i/gA_i^3$	$C_{si} = 1.2(\mathsf{F}_i^2)^{(1.0-0.1k_i)}$ for $\mathsf{F}_i^2 < 1$	$y_{jI} = \frac{\alpha Q^2/(zy_j + b)}{F_s - g(0.33zy_j^3 + 0.5by_j^2)}$ for $\mathsf{F}_i^2 < 1$

Appendix III. Standard Errors in Various Flow Depths

From Eqs. (6) and (39), the standard error in the critical flow depth y_c is

$$\sigma_{y_c}^2 = (CV_\alpha^2 + 4CV_Q^2)F_{y_c} \tag{52}$$

From Eqs. (12) and (39), the standard error in the normal flow depth y_n is

$$\sigma_{y_n}^2 = (CV_Q^2 + CV_n^2 + 0.25CV_S^2)F_{y_n} \tag{53}$$

From Eqs. (18) and (39), the standard error in the alternate flow depth y_a is

$$\sigma_{y_a}^2 = (0.25CV_\alpha^2 + CV_Q^2)F_{y_a} \tag{54}$$

From Eqs. (28) and (39), the standard error in the sequent flow depth y_s is

$$\sigma_{y_s}^2 = (CV_\alpha^2 + 4CV_Q^2)F_{y_s} \tag{55}$$

where CV = coefficient of variation defined as $CV = \sigma/\mu$, in which μ is the mean

$$F_{y_c} = \left(\frac{3}{A}\frac{dA}{dy} - \frac{1}{T}\frac{dT}{dy}\right)^{-2},$$

$$F_{y_a} = \left\{\left[\frac{1}{A_1}\frac{dA_1}{dy_1} - \frac{T_1}{A_1\mathsf{F}_1^2}\right]\Big/\left[1 - \left(\frac{A_1}{A_2}\right)^2\right]\right\}^{-2},$$

$$F_{y_n} = \left(\frac{5}{3A}\frac{dA}{dy} - \frac{2}{3P}\frac{dP}{dy}\right)^{-2},$$

$$F_{y_s} = \left\{\left[\frac{1}{A_1}\frac{dA_1}{dy_1} - \frac{T_1}{A_1\mathsf{F}_1^2}\left(\frac{\bar{y}_1}{A_1}\frac{dA_1}{dy_1} + \frac{d\bar{y}_1}{dy_1}\right)\right]\Big/\left[1 - \left(\frac{A_1}{A_2}\right)\right]\right\}^{-2}$$

in which $dA/dy = 2zy + b$, $dP/dy = 2\sqrt{z^2+1}$, $dT/dy = 2z$, $\mathsf{F}_1^2 = \alpha Q^2 T_1/gA_1^3$, $\bar{y} = 2zy^2 + 3by/6(zy+b)$, $d\bar{y}/dy = (4zy+3b)(zy+b) - (2z^2y^2 + 3zby)/6(zy+b)^2$.

Notation

The following symbols are used in this paper:

A = cross-sectional area;
A_0 = cross-sectional area of alternate depth y_0;
A_1 = cross-sectional area of alternate depth y_1 or pre-hydraulic-jump depth y_1;
A_2 = cross-sectional area of post-hydraulic-jump depth y_1;
A_c = cross-sectional area corresponding to the critical depth;
A_j = cross-sectional area at section j, where $j = 0$, 1, or 2;
A_n = cross-sectional area corresponding to the normal depth;
b = channel bottom width;
$C_{a0} = A_1/A_0$;
$C_{a1} = A_0/A_1$;
$C_c = zA_c/b^2$;
$C_n = zA_n/b^2$;
$C_{s1} = A_2/A_1$;
$C_{s2} = A_1/A_2$;
CV = coefficient of variation;
c = conversion factor;
E_s = specific energy;
F = Froude number;
F_s = specific force;
$G_c = A^3/Tb^5$;
$G_n = A^{5/3}P^{-2/3}/b^{8/3}$;
g = acceleration due to gravity;
$k = A/yT$;
n = Manning's coefficient;
P = wetted perimeter;
Q = discharge;
$Q_c = \alpha Q^2/gb^5$;
$Q_n = nQ/cS^{1/2}b^{8/3}$;
S = longitudinal slope of the bed;
T = top width of the channel at the free surface;
y = flow depth;
y_0 = alternate depth;
y_{0I} = improved value of the alternate depth y_0;
y_1 = alternate depth or prejump depth of the hydraulic jump;
y_{1I} = improved value of the alternate or sequent depth y_1;
y_2 = postjump depth of the hydraulic jump;
y_{2I} = improved value of the sequent depth y_2;
y_c = critical depth;
y_{cI} = improved value of critical depth y_c;
y_n = normal depth;
y_{nI} = improved value of normal depth y_n;
$\bar{y}$ = distance from the centroid of trapezoid area to the water surface;
$\bar{y}_0$ = distances from the centroid to the water surface at cross-sectional area A_0;
$\bar{y}_1$ = distances from the centroid to the water surface at cross-sectional area A_1;
$\bar{y}_2$ = distances from the centroid to the water surface at cross-sectional area A_2;
z = side slope; and
α = energy correction factor;

Subscripts

0 = Section 0;
1 = Section 1; and
2 = Section 2.

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