Three-Dimensional Model for Stop-Control Intersection Sight Distance

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Abstract: Intersection sight distance (ISD) is the sight distance to be provided at intersections between a minor road and a major road. Current AASHTO policy provides an equation and charts for the required at-grade ISD so that a driver on the minor road can depart (crossing, turning left, or turning right) safely, even though an approaching vehicle on the major road comes into view as the stopped vehicle begins to depart. The AASHTO model is based on two assumptions: (1) both minor and major roads are assumed to be straight without any vertical or horizontal curvature; and (2) the intersection angle is assumed to be 90°. In many practical situations, however, sight distance is required to be checked for an existing or proposed 3D intersection alignment where vertical curves (crest or sag) and horizontal curves overlap. This paper presents a new mathematical model for the analysis of stop-controlled ISD on 3D highway alignments that allows the major road to have vertical and horizontal curves with skewed angle, and the minor road to have a longitudinal grade. Design aids are developed to determine the available ISD for different geometric alignment variables (e.g., radius of horizontal curve, lane width, number of lanes, and vertical curve parameters). Application of the methodology is illustrated using numerical examples.

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Introduction

Intersection sight distance (ISD) is the adequate sight distance to be provided at intersections between a minor road and a major road (TAC 1999; AASHTO 2001). The AASHTO policy provides the required at-grade ISD between a minor road (controlled by a stop sign) and a noncontrolled major road as the sufficient sight distance required for a driver on a minor road for safe departure (crossing, turning left, or turning right), even though an approaching vehicle on the major road comes into view as the stopped vehicle begins to depart. Hence the required sight distance d (Fig. 1) is a function of the time gap t_g required for the stopped vehicle on the minor road to turn left, turn right, or pass through the major road. In most cases, the most critical maneuver is a left turn. However, in some situations it is advised to check the availability of sight distance for crossing maneuvers (AASHTO 2001).

The minimum intersection sight distance in the AASHTO green book (AASHTO 1984) was calculated as a factor in vehicle's speed and driver's perception-reaction time. While AASHTO (1990) adopted the same method to determine ISD with some variations, the new edition of AASHTO (2001) adopted the time gap acceptance method, which is based on previous research work (Harwood et al. 2000). This method assumes that the gap acceptance time does not vary with approach speed on the major road. Extensions to the AASHTO method have been addressed by several authors to consider the effect of different variables on ISD, including vehicle type (Fitzpatrick et al. 1990), driver's eye and vehicle heights (Fitzpatrick et al. 1998), and intersection skew angle (Gattis and Low 1998). Reliability analysis was used to consider the moments (mean and variance) of the probability distribution of each random variable (Easa 2000). Analytical geometry was also used to determine ISD for a horizontally curved roadway with tangential intersection (Gattis 1992).

Sight distance models for uncontrolled intersections that explicitly incorporate design speeds of the intersecting roads and vehicle length have been developed (McGee et al. 1984; Mason et al. 1989). These models have been examined by Easa (1998), who demonstrated that conditions arise when using the design speed that can result in sight distances that are less than desirable. For railroad-highway grade crossings without train-activated warning devices, two sight distance cases for moving and stopped vehicles are described by AASHTO (2001). Although the 85th percentile speed of vehicles is normally used in traffic analysis, the 15th percentile speed should be used for the moving vehicle case (Easa 1993). Comprehensive work on ISD is documented in National Cooperative Highway Research Program (NCHRP) 393 (Harwood et al. 1996). An excellent review of recent geometric design research, including ISD, can be found in NCHRP Synthesis 299 (Fitzpatrick and Wooldridge 2001).

The AASHTO policy has recommended that intersections should not be situated just beyond a short, crest vertical curve or on a sharp horizontal curve, and that intersection legs that operate under stop control preferably should intersect at 90° if possible, but in no case less than 60°. Hence, the two assumptions in the AASHTO model are (1) both minor and major roads are straight

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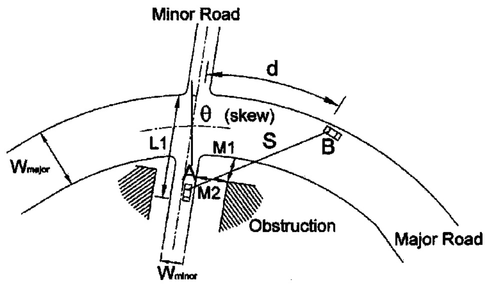

Fig. 1. Intersection sight distance on 3D alignments (vehicle approaching from right)

(tangents) without vertical or horizontal curvature; and (2) the intersection angle is 90°. However, the AASHTO and all previous models are not applicable to 3D intersection alignments where vertical and/or horizontal curves exist. For such alignments, AASHTO has recommended that sight distance be checked graphically.

This paper presents a new analytical model for ISD on 3D alignments where the major road has both horizontal and vertical geometry and the minor road has a grade; the intersection may be skewed. The following sections present the geometry of 3D intersection sight distance and the mathematical model. Design aids and application examples are then presented, followed by some concluding remarks.

Intersection Geometry

One of the key variables in ISD analysis is the required sight distance along the major road, d. This distance is given by (AASHTO 2001)

$$d = 0.278 V_{\text{major}} t_g \tag{1}$$

where d=required sight distance for a vehicle approaching from left or right (m); V_{major}=design speed on major road (km/h); and t_g=time gap required for stopped vehicle on minor road to maneuver (s). The time gap t_g for different types of design vehicles (passenger car P, single unit truck SU, and large semitrailer combination truck WB-50) is provided in the AASHTO model.

To check the required sight distance d, the proposed model assumes the following geometry: (1) the intersection is on a crest vertical curve on the major road; (2) the major road has a horizontal curve combined with the vertical curve; (3) the minor road has a grade, g_m (positive for upgrade and negative for downgrade); (4) the minor road has one lane in each direction; and (5) the intersection has a skew angle, θ.

The sight line S, which is a straight line between the driver's eye on the minor road and the top of an approaching vehicle on the major road at a distance d from the intersection (Fig. 1), is checked against all possible obstructions. If any element representing a sight obstruction intersects the sight line, the sight line is obstructed and the obstruction should be moved to satisfy the required sight distance. Otherwise, it is not obstructed.

The main variables that govern the sight distance are (Fig. 1)

1. Required distance d, which is calculated using Eq. (1);
2. Required sight line S, which is a function of d and road curvature (horizontal and vertical);
3. Distance M_1 between obstruction and edge of major road (perpendicular to tangent); and
4. Distance M_2 between obstruction and edge of minor road.

Table 1. Geometric Data Required as Input to Mathematical Model

Input data	Symbol
Radius of horizontal curve	R
Lane width on major road	w_{major}
Lane width on minor road	w_{minor}
Total number of lanes on major road	n
Median width	u
Total width of minor road	W_{minor}
Driver's eye height	h_d
Height of approaching vehicle on major road	h_o
Length of vertical curve	L
First grade of vertical curve	g_1
Second grade of vertical curve	g_2
Distance between vehicle on minor road and edge of major road	D
Grade of minor road	g_m
Intersection angle (skew)	θ
Distance from the point of vertical curvature to path of minor road vehicle	P

The geometric alignment data required as input to the model are given in Table 1.

Mathematical Model

To simplify the analysis of ISD, analytical geometry is used to determine the required sight distance. The model involves three aspects: establishing coordinates of sight line, checking sight line obstruction, and evaluating existing obstruction or proposed design.

Establishing Coordinates of Sight Line

In our case, the point of origin for the global Cartesian coordinates is located at the road surface under the driver's eye on the minor road with the y-axis being along the centerline of the stopped vehicle on the minor road (Fig. 2). Note that the driver's eye is assumed to be located at the center of the lane. The Cartesian coordinates of Point A (driver's eye) are $(0,0,h_d)$; the Cartesian coordinates of Point B (approaching vehicle) are (x_1, y_1, z_1); and the radius of the horizontal-curve path for a vehicle approaching from left and from right are (Fig. 3)

$$R_n = R - W_{\text{major}}/2 + w_{\text{major}}/2 \quad \text{(vehicle approaching from left)} \tag{2a}$$

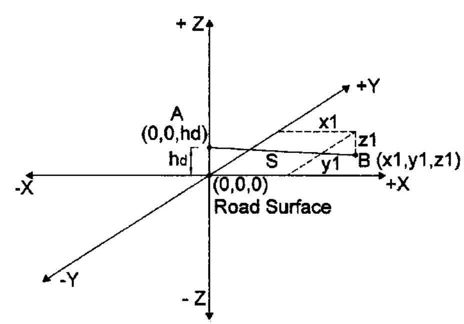

Fig. 2. Coordinates of different points of intersection sight distance on 3D alignments

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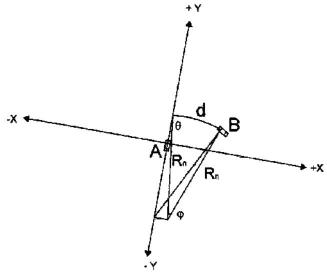

Fig. 3. Intersection sight distance on X–Y view

$$R_n = R + w_{\text{major}}/2 + u/2 \quad \text{(vehicle approaching from right)} \tag{2b}$$

where R_n=radius of horizontal-curve path of approaching vehicle; R=radius of horizontal curve (along centerline of major road); W_{major}=total width of major road; w_{major}=lane width of major road; and u=median width. The total width $W_{\text{major}} = (nw_{\text{major}} + u)$ where n=total number of lanes of the major road. Note that Eqs. (2a) and (2b) assume that the approaching vehicle on the major road is in the nearest lane to the minor road driver (inside lane for a vehicle approaching from right and outside lane for a vehicle approaching from left), as this situation produces the most critical case.

The central angle ϕ (in degrees) for the arc with length d equals $180° d/\pi R_n$. Let θ be measured from the perpendicular line to the tangent to the horizontal curve. For a vehicle approaching from the right, the skew angle θ is considered positive if it is clockwise and negative otherwise (and vice versa for a vehicle approaching from the left). Then

$$x_1 = \frac{R_n \sin\phi}{\cos\theta} + R_n \sin\theta(1 - \sin\phi \tan\theta - \cos\phi) \tag{3a}$$

If there is no skew (θ=0), Eq. (3) becomes

$$x_1 = R_n \sin\phi \tag{3b}$$

Similarly

$$y_1 = L_1 - R_n \cos\theta(1 - \cos\phi - \sin\phi \tan\theta) \tag{4}$$

If there is no skew (θ=0), Eq. (4) will be

$$y_1 = L_1 - R_n(1 - \cos\phi)$$

where

$$L_1 = \frac{W_{\text{major}} - \dfrac{w_{\text{major}}}{2}}{\cos\theta} + D \quad \text{(vehicle approaching from right)}$$

$$L_1 = \frac{w_{\text{major}}}{2\cos\theta} + D \quad \text{(vehicle approaching from left)}$$

Let the minor road grade be g_m in decimals. Then, from Fig. 4

$$z_1 = g_t x_1 + \frac{r x_1^2}{2} + h_o + y_1 g_m \tag{5a}$$

where g_t=slope of vertical curve at minor road vehicle path; r=rate of change of grade of vertical curve; and h_o=height of approaching vehicle on major road. The variables r and g_t are given by $r = (g_2 - g_1)/L$ and $g_t = g_1 + rP$ where g_1 and g_2=first and second grades of vertical curve in decimals; L=length of vertical curve (m); and P=distance from point of vertical curvature (PVC) to path of minor road vehicle (m). Substituting for r in Eq. (5b), then

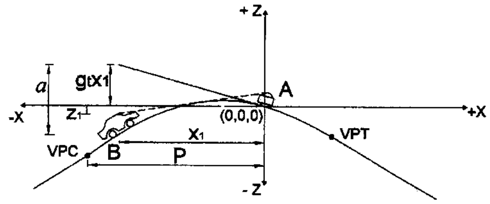

Fig. 4. Intersection sight distance on X–Z view

$$z_1 = g_t x_1 + \frac{(g_2 - g_1) x_1^2}{2L} + h_o + y_1 g_m \tag{5b}$$

Checking Sight Line Obstruction

To ensure that the sight line is not obstructed, two checks are required to ensure that the sight line does not intersect with the road surface and with any potential obstruction.

Major Road Surface Obstruction

To check whether the sight line is obstructed by the major road surface, it is assumed that the driver on the minor road is located on the vertical curve (at the centerline of the major road). This assumption is made to avoid mathematical complexity. Any error that may result from this approximation is expected to be small as the distance between the actual location of the minor road driver and the assumed location is very small compared with the sight distance. In this case, the coordinate z at any point x is given by (Fig. 5)

$$z = \frac{rx^2}{2} + g_t x + y_1 g_m \tag{6}$$

The sight line is a straight line with a start point at the driver's eye $(0,h_d)$ and an end point at the top of the approaching vehicle (x_1, z_1) where z, at any point x, is given by

$$\frac{x}{x_1} = \frac{z - h_d}{z_1 - h_d} \tag{7}$$

Substituting for z_1 and z from Eqs. (5b) and (6) in Eq. (7), then

$$\frac{x}{x_1} = \frac{\dfrac{rx^2}{2} + g_t x + y_1 g_m - h_d}{a + g_t x_1 + h_o + y_1 g_m - h_d} \tag{8}$$

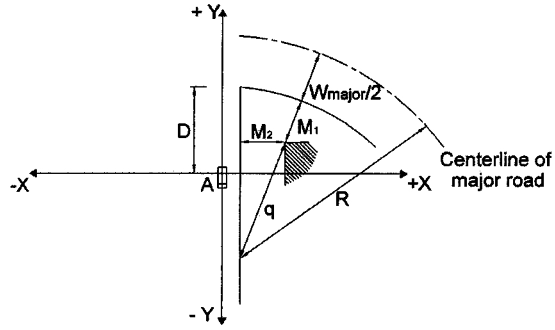

Fig. 5. Coordinates of obstructions

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where $a=rx_1^2/2$. Now Eq. (8) can be written as

$$\frac{rx_1}{2}x^2-(a+h_o-h_d+y_1g_m)x-(h_d-y_1g_m)x_1=0 \qquad (9)$$

The major road surface obstructs the sight line only if Eq. (9) has a solution within the range

$$0\leqslant x\leqslant x_1 \qquad (10)$$

If the sight line is obstructed, the geometric design should be revised either to flatten the curves or to relocate the intersection. Otherwise, we may continue to the second step to check the sight line against potential off-road obstruction. Note that sight line obstruction can be checked more precisely using the finite-element method (FEM) presented in the Appendix. This method, which idealizes the road surface into an element net, uses the actual location of the vehicle on the minor road.

Off-Road Obstruction

The slope of the sight line S in the $x-y$ plan is (Fig. 6)

$$\tan\alpha=y_1/x_1 \qquad (11)$$

For an obstruction at (x_2,y_2)

$$x_2=M_2+w_{\text{minor}}/2 \quad \text{(vehicle approaching from right)} \qquad (12a)$$

$$x_2=M_2+(W_{\text{minor}}-w_{\text{minor}}/2) \quad \text{(vehicle approaching from left)} \qquad (12b)$$

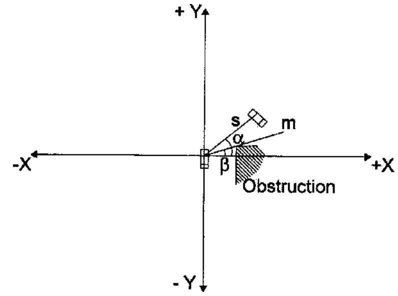

Fig. 6. Slopes of sight line and obstruction line

and

$$y_2=\frac{\sqrt{q^2-M_2^2}+W_{\text{major}}/2+D-R}{\cos\theta}+M_2\sin\theta \qquad (13)$$

where

$$q=R-W_{\text{major}}/2-M_1 \qquad (14)$$

The slope of the obstruction line m in the $x-y$ plan equals $\tan\beta=y_2/x_2$. The sight line S (Fig. 6) is not obstructed if the angle α (for sight line) is greater than or equal to the angle β (for obstruction line m). Therefore, for an unobstructed sight line

$$\beta\leqslant\alpha;\quad \tan\beta\leqslant\tan\alpha;\quad y_2/x_2\leqslant y_1/x_1 \qquad (15)$$

Then Eq. (15) may be written as

$$M_1=R-\frac{W_{\text{major}}}{2}-\sqrt{\left[\frac{y_1}{x_1}\cos\theta\left(M_2+W_{\text{minor}}-\frac{w_{\text{minor}}}{2}\right)+R-D-\frac{W_{\text{major}}}{2}-\cos\theta(M_2\sin\theta)\right]^2+M_2^2} \qquad (16)$$

where x_1, y_1, x_2, and y_2 are given using Eqs. (3), (4), (12b), and (13), respectively. Note that M_1 is the minimum required value that does not obstruct the sight line. Hence for a given highway for any value of M_2, use Eqs. (3) and (4) to calculate x_1 and y_1, respectively, and then use Eq. (16) to calculate M_1. If M_1 is given, calculate M_2 using Eq. (16) by trial and error. The values of M_1 and M_2 should be greater than or equal to zero or other minimum values required by road authority regulations. When y_1 is negative, M_1 and M_2 refer to the corner of obstruction that is farther from the minor road vehicle. Note that the obstruction is located at the corner near the approaching vehicle.

Evaluation of Existing Obstruction

For an existing obstruction with given M_1 and M_2, it is required to determine if the existing obstruction satisfies the required sight distance. The methodology is as follows:

1. Calculate required sight distance d using Eq. (1);
2. Using given M_2, determine minimum required M_1 using Eq. (16);
3. Compare minimum calculated M_1 with existing M_1;
4. If existing $M_1\geqslant$calculated M_1, then sight line is not obstructed; and
5. If existing $M_1<$calculated M_1, then sight line is obstructed and obstruction should be moved.

To save time and effort, this model was coded into computer software in which the user is asked to input all data in an interactive environment; the results can then be shown either on screen or in an attached text file. The results include the following: (1) input data (for verification); (2) required sight distance; (3) road surface obstruction equation; (4) values of x_1, y_1, and z_1; and (5) table for minimum values of M_1 corresponding to different values of M_2. This table may be imported into spreadsheet software (for example, MS Excel) to plot the minimum values for both M_1 and M_2 graphically. The methodology is also applicable to ISD analysis for proposed intersection design.

Design Aids

Using the mathematical model, design aids were established to determine the intersection sight distance for different values of the geometric design variables (for example, radius of horizontal curve, lane width, number of lanes, and vertical curve parameters). The following AASHTO-recommended values were used in developing the design aids (AASHTO 2001):

- Driver's eye height, $h_d=1.08$ m;
- Height of approaching vehicle on major road, $h_o=1.08$ m;
- Minor road vehicle is passenger car;
- Lane width=3.6 m (for both minor and major roads); and
- Distance between vehicle on minor road and edge of major road, $D=5.4$ m.

Since stop-controlled intersections and combined alignments are likely to exist on two-lane rural highways, both major and

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Table 2. Values of x_1 and y_1 for Two-Lane Minor Road Intersecting with Two-Lane Major Road and Lane Width=3.6 m (Major Road Design Speed=40 km/h)[a]

R (m)	R_n (m)	ϕ (deg)	ϕ (rad)	x_1 (m)	y_1 (m)	$\tan\alpha$
100	98.2	48.6	0.84	73.7	−26.2	−0.355
200	198.2	24.1	0.42	81.0	−10.1	−0.125
300	298.2	16.0	0.28	82.4	−4.4	−0.053
400	398.2	12.0	0.21	82.8	−1.5	−0.018
500	498.2	9.6	0.17	83.1	0.3	0.003
600	598.2	7.9	0.14	83.2	1.4	0.017

[a]Required time gap is 7.5 s and required sight distance d is 83.4 m. Values are based on case of vehicle approaching from left.

Table 4. Values of x_1 and y_1 for Two-Lane Minor Road Intersecting with Two-Lane Major Road and Lane Width=3.6 m (Major Road Design Speed=80 km/h)[a]

R (m)	R_n (m)	ϕ (deg)	ϕ (rad)	x_1 (m)	y_1 (m)	$\tan\alpha$
200	198.2	48.2	0.84	147.8	−58.9	−0.399
300	298.2	32.0	0.56	158.2	−38.2	−0.242
400	398.2	24.0	0.42	162.0	−27.2	−0.168
500	498.2	19.2	0.33	163.7	−20.5	−0.125
600	598.2	16.0	0.28	164.6	−15.9	−0.097
700	698.2	13.7	0.24	165.2	−12.6	−0.076

[a]Required time gap is 7.5 s and required sight distance d is 166.8 m. Values are based on case of vehicle approaching from left.

minor roads are assumed to be two-lane highways with a total width of 7.2 m each. No skew or slope of the minor road was assumed.

The values of R_n, x_1, y_1, and $\tan\alpha$ corresponding to different values of R are shown in Tables 2–5 for V_{major} equal to 40, 60, 80, and 100 km/h, respectively. Values of M_1 and M_2 corresponding to different values of R are shown in Figs. 7–10 for the same speeds, respectively. The values are based on the case of a vehicle approaching from the left, which is expected to be the most critical case. The sign for y_1 depends on R_n and d; the less R_n and more d, the more likely for y_1 to be negative.

As the major road has two lanes, according to AASHTO the required time gap is 7.5 s. Note that for $V_{\text{major}}=80$ km/h, the minimum radius is likely to be some value between 195 and 280 m, depending on superelevation, so that a radius of 100 m was not included with this design speed. Also, for $V_{\text{major}}=100$ km/h, the minimum radius is likely to be some value between 330 and 490 m, depending on superelevation, and therefore a radius less than 300 m was not included.

The graphical aids are easy to use as the lines in the graph resemble sight lines. For a proposed intersection or a proposed project (obstruction), the required value of M_1 (or M_2) is plotted on the graph so that the minimum value of M_2 (or M_1) may be obtained, as will be shown in the examples. The design aids are also useful for evaluating an existing obstruction as the values for M_1 and M_2 for an obstruction may be plotted as one point on the graph. If the point lies under the corresponding line, that means M_1 and M_2 are less than the minimum values and the sight line is obstructed. If the point lies above the line, that means M_1 and M_2 are more than the minimum values and the sight line is not obstructed.

A positive slope of a curve in the design graph means that the farther corner of the obstruction controls (that is, y_1 is negative); a negative slope means that the obstruction corner near the minor road vehicle controls. Note that for flat horizontal curves with radii exceeding 300 m, the rate of increase in M_1 is likely to decrease, with the increase in M_2 especially at lower speeds. In other words, the slope of the curves in the design graphs becomes flatter with the increase in M_2 and eventually changes in trend from positive to negative. For M_2 greater than 20 m, the corresponding change in M_1 is very small, especially for flat curves at lower speeds. Also note that M_1 increases as M_2 increases when the farther corner of the obstruction controls, and vice versa when the closer corner controls.

Application Examples

The following examples illustrate how to use both the mathematical model and the design aids for either a proposed or an existing obstruction. Based on AASHTO (2001), the following values were assumed: $w_{\text{major}}=w_{\text{minor}}=3.6$ m; $h_d=1.08$ m; $h_o=1.08$ m; and $D=5.4$ m.

Example 1 (Using Mathematical Model)

A hypothetical example with general geometry is used to illustrate the application of the mathematical model presented. For an intersection where the major road is a two-lane rural highway ($u=0$, $n=2$) and $V_{\text{major}}=60$ km/h, a passenger car requires a time gap $t_g=7.5$ s. The geometric design variables for a proposed intersection are given in Table 6. Note that the skew angle is clockwise. Determining the sight line is required to establish the zone that should be clear of obstruction. The example was solved con-

Table 3. Values of x_1 and y_1 for Two-Lane Minor Road Intersecting with Two-Lane Major Road and Lane Width=3.6 m (Major Road Design Speed=60 km/h)[a]

R (m)	R_n (m)	ϕ (deg)	ϕ (rad)	x_1 (m)	y_1 (m)	$\tan\alpha$
100	98.2	73.0	1.27	93.9	−62.3	−0.663
200	198.2	36.2	0.63	117.0	−31.0	−0.265
300	298.2	24.0	0.42	121.5	−18.7	−0.154
400	398.2	18.0	0.31	123.1	−12.3	−0.100
500	498.2	14.4	0.25	123.8	−8.4	−0.068
600	598.2	12.0	0.21	124.2	−5.8	−0.047

[a]Required time gap is 7.5 s and required sight distance d is 125.1 m. Values are based on case of vehicle approaching from left.

Table 5. Values of x_1 and y_1 for Two-Lane Minor Road Intersecting with Two-Lane Major Road and Lane Width=3.6 m (Major Road Design Speed=100 km/h)[a]

R (m)	R_n (m)	ϕ (deg)	ϕ (rad)	x_1 (m)	y_1 (m)	$\tan\alpha$
300	298.2	40.1	0.70	191.9	−62.8	−0.327
400	398.2	30.0	0.52	199.1	−46.2	−0.232
500	498.2	24.0	0.42	202.5	−35.8	−0.177
600	598.2	20.0	0.35	204.3	−28.8	−0.141
700	698.2	17.1	0.30	205.4	−23.7	−0.115
800	798.2	15.0	0.26	206.1	−19.9	−0.096

[a]Required time gap is 7.5 s and required sight distance d is 208.5 m. Values are based on case of vehicle approaching from left.

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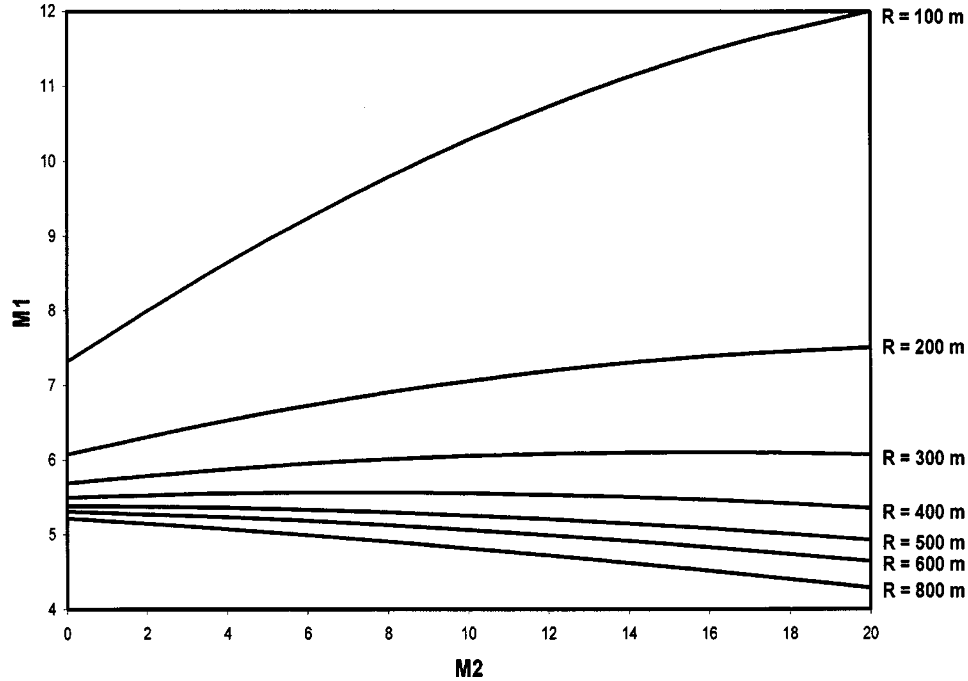

Fig. 7. Design graph for M_1 and M_2 for different R for two-lane minor road intersecting with two-lane major road and lane width = 3.6 m (major road design speed = 40 km/h)

sidering a vehicle approaching from the left, which is the most critical case. The procedure of ISD analysis is as follows:

1. Using Eq. (1), required sight distance, $d = 125.1$ m;
2. Using Eq. (2*a*), radius of horizontal-curve path of approaching vehicle is

$$R_n = 200 - \frac{7.2}{2} + \frac{3.6}{2} = 198.2 \text{ m}$$

3. Central angle ϕ (in degrees) for arc with length d is

$$\phi = \frac{125.1 \times 180}{3.14 \times 198.2} = 36.164°$$

4. Using Eq. (3), $x_1 = 99.47$ m
5. $L_1 = 3.6/2 \cos(-18°) + 5.4 = 7.29$ m
6. Using Eq. (4), $y_1 = -65.22$ m

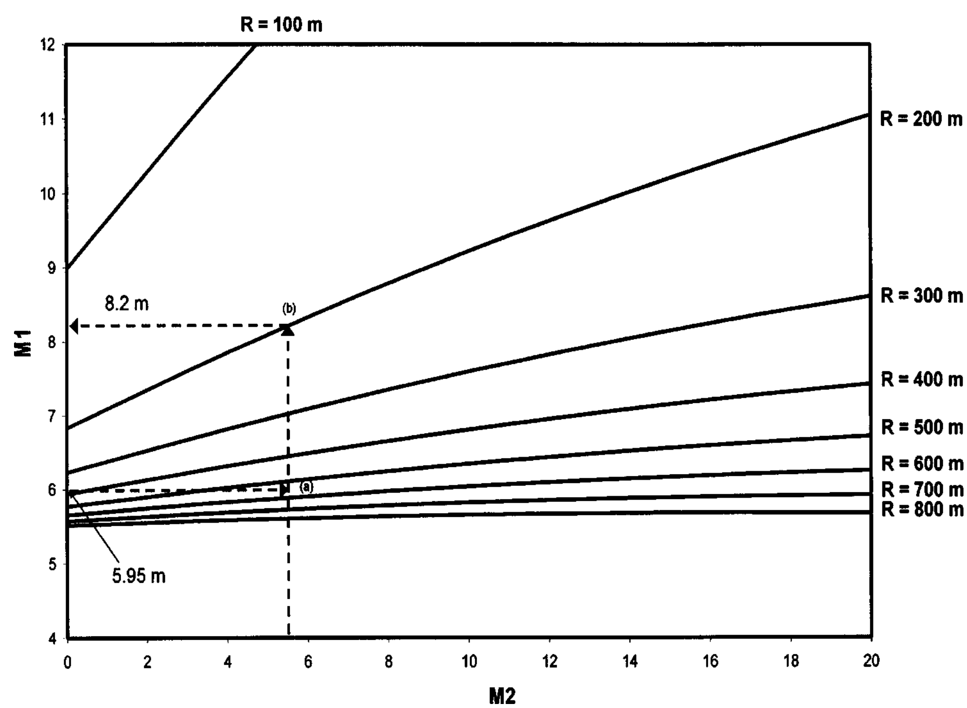

Fig. 8. Design graph for M_1 and M_2 for different R for two-lane minor road intersecting with two-lane major road and lane width = 3.6 m (major road design speed = 60 km/h)

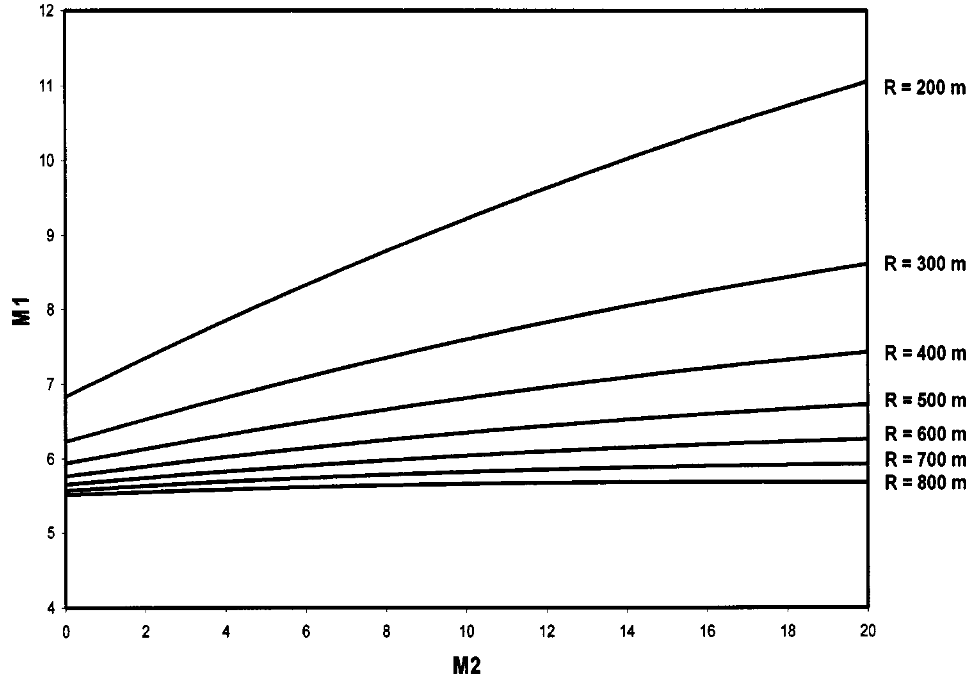

Fig. 9. Design graph for M_1 and M_2 for different R for two-lane minor road intersecting with two-lane major road and lane width=3.6 m (major road design speed=80 km/h)

7. Using Eq. (5*b*), $z_1 = -2.05$ m;
8. Using Eq. (9) to check sight line against road surface, $-0.0039x^2 + 3.13x - 379.9 = 0$, which does not produce any value for x within the range $0 \leq x \leq x_1$. Hence road surface does not obstruct sight line;
9. Using Eq. (11), $\tan\alpha = y_1/x_1 = -0.65$;
10. Using Eq. (15), $y_2/x_2 \leq -0.65$, where $x_2 = M_2 + (7.2 - 1.8) = M_2 + 5.4$ and $y_2 = \sqrt{(196.4 - M_1)^2 - M_2^2} - 191/\cos(-18°) + M_2 \sin(-18°)$.

Then the relationship between minimum M_1 and minimum M_2 may be plotted graphically. M_1 was found to be within the range (8.8–14.3 m) for M_2 in the range (0–20 m). The same

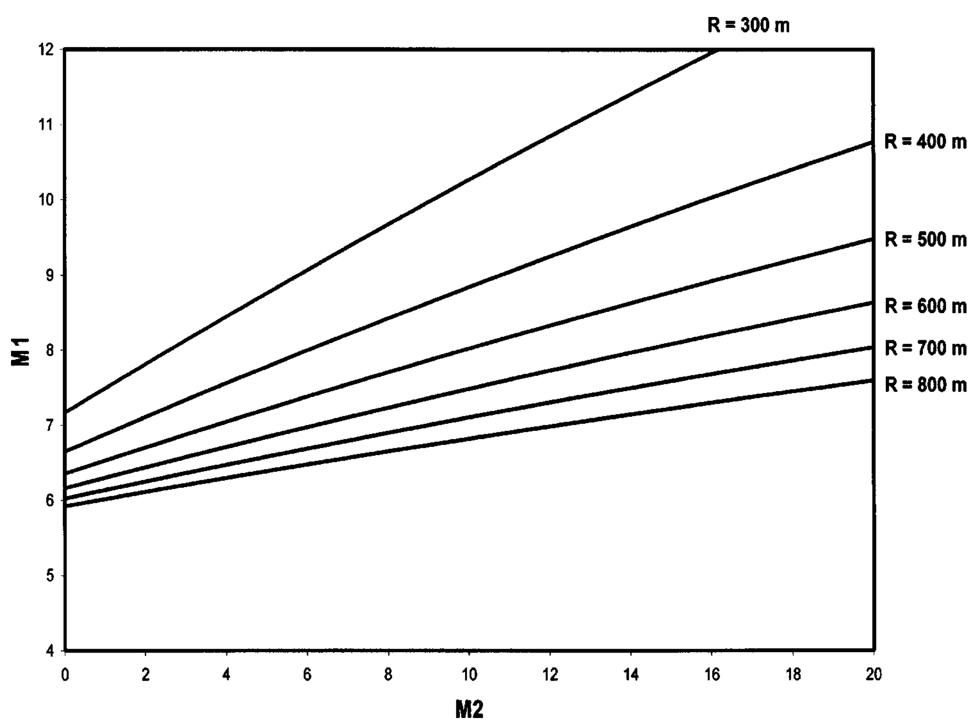

Fig. 10. Design graph for M_1 and M_2 for different R for two-lane minor road intersecting with two-lane major road and lane width=3.6 m (major road design speed=100 km/h)

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Table 6. Geometric Design Variables for Example 1

Variable	Value
R	200 m
w_{major}	3.6 m
w_{minor}	3.6 m
W_{major}	7.2 m
W_{minor}	7.2 m
h_d	1.08 m
h_o	−1.08 m
L	750 m
g_1	2.3%
g_2	−3.6%
D	5.4 m
θ	18° [a]
g_m	4.2%
P	292 m

[a]Clockwise angle.

example was solved considering a vehicle approaching from the right, where M_1 was in the range (5.2–8.3 m) for the same range of M_2 (0–20 m). These results are less than those for a vehicle approaching from the left, which confirms our expectation that the most critical case occurs when the approaching vehicle is coming from the left. The results for a vehicle approaching from the left may be applied for any obstruction, either from the right or the left side, as no significant difference is found (it is also more conservative to consider a vehicle approaching from the left side in all cases). The results of running the computer software were identical to the above manual calculations.

It should be noted that the skew was considered clockwise in the above example. If the skew is counterclockwise, the results for the vehicle approaching from the right may become critical and therefore should be checked separately.

Example 2 (Using Design Aids)

The previous example was solved considering no skew ($\theta=0$) or minor road grade ($g_m=0$). Installing a new road sign with distances $M_1=5.95$ m and $M_2=5.5$ m is also required. In this case, the first step will be to check the sight line against road surface obstruction, and the second will be to evaluate the proposed off-road obstruction (the new road sign) using the design aids. The solution is as follows:

1. From the given data, $r=(g_2-g_1/L)=(-0.036-0.023/750)=-0.000078667$;
2. From Table 3 ($R=200$ m), $x_1=117.0$, $y_1=-31.0$, and $\tan\alpha=-0.265$;
3. Using Eq. (9) to check sight line against road surface, $-0.0046x^2+0.5384x-126.36=0$, which does not produce any value for x within range $0\leqslant x\leqslant x_1$. Hence road surface does not obstruct sight line; and
4. For proposed obstruction with distances $M_1=5.95$ m and $M_2=5.5$ m, using Fig. 8 ($R=200$ m), the point that represents the obstruction is plotted (Point a). Since the point lies under the graph, the sight line is obstructed. To satisfy sight distance requirements, assuming $M_2=5.5$ m, the corresponding minimum distance M_1 should be 8.2 m (corresponding to Point b in the graph).

Concluding Remarks

This paper has presented a new mathematical model for analysis of sight distance at intersections with combined horizontal and vertical alignments. Practical design aids were provided to determine the required coordinates for any obstruction according to different alignment variables. The design aids presented in this paper can be used for sight distance analysis at existing or proposed intersections on horizontal curves combined with crest vertical curves. The model is coded into interactive computer software where the user can input all alignment variables, and then the software outputs the equation of the road surface obstruction along with the available sight distance and different values of M_1 and M_2. Based on this study, the following comments are offered:

1. Intersection sight distance is a key factor in highway safety and should be determined carefully. The mathematical model presented here includes a wide range of geometric features to represent actual 3D intersections as much as possible. These include skewed intersection angles and minor road grades as well as combined alignments. The model was used to establish design aids that can be easily applied to check 3D intersection sight distance on combined alignments involving crest vertical curves.
2. The minor road may intersect with the major road at any point along the combined alignment as long as the approaching vehicle lies on the combined alignment when the driver on the minor road first sees it. For situations in which the approaching vehicle lies on the tangent beyond the combined alignment, the results provided by the presented model generally will be conservative.
3. For crest vertical curves combined with horizontal curves, the finite-element method presented in the Appendix can be used not only to check sight line obstruction by the major road surface, but also to check off-road obstruction and any other type of obstruction. The method is especially relevant when the major road has a median barrier that may obstruct the sight line for a vehicle approaching from the right. In this case, the assumption used to check major road surface obstruction (that is, a minor road vehicle is located at the centerline of the major road) would not be appropriate.
4. The mathematical model is applicable only to the case where the obstruction is located inside the horizontal curve of the major road; further research is required to explore the case where the obstruction is located outside the horizontal curve. In addition, the present study focuses only on intersections with stop control on the minor road (Case B in the AASHTO model). Further research is required to cover other cases of the AASHTO model that are likely to involve combined alignments. Finally, the case of sag vertical curves (where the road surface would not obstruct the sight line, but the sight line may be obstructed by an overpass) should be examined.

Appendix. Finite-Element Method for 3D Intersection Sight Distance

A finite-element model for 3D intersection sight distance that determines precisely whether the sight line is obstructed is developed in this Appendix. In this model, the driver is located on the minor road and the object lies on the major road. The model is similar to the one developed by Hassan et al. (1997) for sight distance analysis along a road with combined alignments, in which both the driver and object are located on the major road.

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For the 3D intersection model, the major road (with length greater than d) and all potential obstructions are idealized into a net of rectangular finite elements. Then the sight line S (corresponding to the distance d) is checked against all elements. If S intersects with any element, then the sight line is obstructed; otherwise, it is not obstructed.

As the major road has a combined alignment, we use the 6-node rectangular element to idealize it. The following notation will be used:

- Assume driver's eye on minor road is at point A (h_d above road surface);
- Assume approaching vehicle on major road is at point B (h_o above road surface); and
- Assume obstruction point C.

The equation of the sight line that starts at point A $(0,0,h_d)$ and ends at point B (x_1,y_1,z_1) is

$$\frac{x-0}{x_1-0}=\frac{y-0}{y_1-0}=\frac{z-h_d}{z_1-h_d} \tag{17}$$

which represents two linear equations:

$$x-\frac{x_1}{y_1}y=0 \tag{18}$$

$$x-\frac{x_1}{(z_1-h_d)}z+\frac{h_d x_1}{(z_1-h_d)}=0 \tag{19}$$

Then the local coordinates η and ξ for the point of intersection (between the sight line and an element) are obtained using the following equations:

$$\begin{aligned}(-c_{10}c_6+c_{12}c_4)\eta^4+(-c_{10}c_5-c_8c_6+c_{11}c_4+c_{12}c_2)\eta^3\\ +(-c_{10}c_3-c_7c_6-c_8c_5+c_{11}c_2+c_{12}c_1+c_9c_4)\eta^2\\ +(-c_7c_5-c_8c_3+c_{11}c_1+c_9c_2)\eta+(-c_7c_3+c_9c_1)=0\end{aligned} \tag{20}$$

$$\xi=\frac{c_1+c_2\eta+c_4\eta^2}{c_3+c_5\eta+c_6\eta^2} \tag{21}$$

where c_1 through c_{12} are as given by Hassan et al. (1997), noting that $x_{s1}=y_{s1}=0$; $z_{s1}=h_d$; $x_{s2}=x_1$; $y_{s2}=y_1$; $z_{s2}=z_1$; $S_{xy}=x_1/y_1$; and $S_{xz}=x_1/(z_1-h_d)$.

No sight obstruction exists if the solution of Eqs. (20) and (21) produces one of the following cases: (1) no solution; (2) division by zero; (3) imaginary roots only (two identical real roots, which represent the point of tangency); and (4) one real root, where one point of intersection exists (the point of intersection must be within the limits of the element, which happens only if $-1\leqslant\xi\leqslant+1$ and $-1\leqslant\eta\leqslant+1$). If the solution produces two different real roots, this means two points of intersection exist, and we have to check which one (or none) of them is within the range $-1\leqslant\xi\leqslant+1$ and $-1\leqslant\eta\leqslant+1$. If the solution produces more than two real roots, this case can happen only with distorted elements and is therefore not applicable for highways. This model can be coded into computer software where the user inputs the geometric and operational variables.

Notation

The following symbols are used in this paper:

D = distance between vehicle on minor road and edge of major road;
d = required sight distance for vehicle approaching from left or right;
g_m = grade of minor road (positive for upgrade and negative for downgrade);
g_t = slope of vertical curve at minor road vehicle path;
g_1,g_2 = first and second grades of vertical curve;
h_d = driver's eye height on minor road;
h_o = height of approaching vehicle on major road;
L = length of vertical curve;
M_1 = distance between obstruction and edge of major road (perpendicular to tangent);
M_2 = distance between obstruction and edge of minor road;
n = total number of lanes on major road;
P = distance from the point of vertical curvature to path of minor road vehicle;
R = horizontal curve radius;
R_n = radius of horizontal-curve path for approaching vehicle;
r = rate of change of grade of vertical curve;
t_g = time gap required for stopped vehicle on minor road to maneuver;
u = median width of major road;
V_{major} = design speed on major road;
W_{major} = total width of major road;
W_{minor} = total width of minor road;
w_{major} = lane width on major road;
w_{minor} = lane width on minor road;
x_1,y_1,z_1 = Cartesian coordinates of approaching vehicle;
η,ξ = local coordinates of point of intersection between sight line and obstruction;
θ = skew angle of intersection; and
ϕ = central angle for arc with length d.

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