Global Finance Journal 58 (2023) 100897

Contents lists available at ScienceDirect

Global Finance Journal

journal homepage: www.elsevier.com/locate/gfj

Pricing of European currency options considering the dynamic information costs

Wael Dammak [a,b,*], Salah Ben Hamad [b,c], Christian de Peretti [a], Hichem Eleuch [d,e,f]

[a] University of Lyon, University Claude Bernard Lyon 1, Institute of Financial and Insurance Sciences, LSAF-EA2429, F-69007 Lyon, France
[b] Faculty of Economics and Management of Sfax, LREG-LR18ES27, Sfax, Tunisia
[c] University of Tunis El Manar, Tunisia
[d] Department of Applied Physics and Astronomy, University of Sharjah, Sharjah 27272, United Arab Emirates
[e] Department of Applied Sciences and Mathematics, College of Arts and Sciences, Abu Dhabi University, Abu Dhabi 59911, United Arab Emirates
[f] Institute for Quantum Science and Engineering, Texas A&M University, College Station, TX 77843, USA

ARTICLE INFO

JEL classification:
C61
F47
G13
G14

Keywords:
Currency options
Garman and Kohlhagen's model
Dynamic information costs
Derivatives
Market imperfections
Beer–Lambert law

ABSTRACT

Dynamic costs arising from the variable impact of information on asset pricing present a challenge for accurate European currency option pricing. The Garman and Kohlhagen model, though influential in the literature, does not adequately account for these costs. This study extends the model by integrating an intensity function into the interest rates to measure dynamic information costs. Inspired by the Beer–Lambert law, the function is applied to a decade-long dataset of daily futures continuous calls on the EUR/USD pair from September 21, 2012, to September 23, 2022. The augmented model reduces pricing errors and manages implied volatility better than the 1983 model, consistent across different categories of maturity and moneyness. Our findings emphasize the need to consider dynamic information costs in asset pricing, demonstrating that their inclusion can significantly enhance the accuracy and reliability of currency option pricing.

1. Introduction

Using options for hedging or speculative purposes has gained popularity in recent decades. Consequently, businesses have adopted options to mitigate interest rate risk and minimize price fluctuations in real commodities and securities. Furthermore, options are employed in foreign exchange as a derivative instrument based on a fixed amount of foreign currency. Firms with substantial international operations and exposure to currency exchange rates often utilize foreign exchange options, commonly called "forex" options. Forex options have gained popularity as both speculative tools and components of investment portfolios; however, the widely acclaimed Black–Scholes model Black and Scholes (1973), which won the Nobel Prize, is unsuitable for pricing forex options. The Black–Scholes model is primarily designed for non-dividend paying stocks following geometric Brownian motion, and it considers only the domestic risk-free interest rate in its calculations. Currency exchange pricing requires a more comprehensive approach incorporating both domestic and foreign risk-free rates. To address this, Garman and Kohlhagen proposed a forex option pricing model in

* Corresponding author at: University of Lyon, University Claude Bernard Lyon 1, Institute of Financial and Insurance Sciences, LSAF-EA2429, F-69007 Lyon, France.

E-mail addresses: Wael.dammak55@gmail.com (W. Dammak), benhamad_salah@yahoo.fr (S.B. Hamad), christian.de-peretti@univ-lyon1.fr (C. de Peretti), hichemeleuch@yahoo.fr (H. Eleuch).

https://doi.org/10.1016/j.gfj.2023.100897
Received 19 February 2023; Received in revised form 26 September 2023; Accepted 27 September 2023
Available online 30 September 2023
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1983, which accounts for interest rate parity and the equalization of forward premiums with interest rate differentials. This model is grounded on several key hypotheses. First, it assumes that exchange rate changes evolve gradually and do not exhibit abrupt jumps (H1). Second, the model assumes continuous exchange in the market (H2). Additionally, it assumes constant interest rates in both the domestic and international markets (H3). Moreover, the model assumes an ideal foreign exchange market where transaction and information costs are absent (H4). Lastly, it posits that option prices are determined by a single stochastic variable, denoted as S. Consequently, the exchange rate price follows a geometric Brownian motion or the Gauss–Wiener process (H5).

The pioneering Garman and Kohlhagen model (G–K model) and its subsequent extensions exhibit several limitations closely tied to the validity of its underlying assumptions. These assumptions include the hypothesis of constant interest rates, constant volatility, absence of transaction and information costs, and the assumption of market sentiment. In contrast, the growth of the options market can be attributed to various factors, primarily driven by episodes of excessive volatility and financial crises during the era of internationalization. These circumstances created a demand for more flexible risk coverage beyond traditional techniques; thus, significant theoretical research on currency option pricing can be traced back to the pioneering work of Garman and Kohlhagen (1983). Subsequently, the research conducted by Black, Scholes, and Merton served as a source of inspiration for both this study and the work of Grabbe (1983). Over time, these models underwent further development and improvement, with various researchers making notable advancements (Blenman & Ayadi, 1997; Carr & Wu, 2007; Heston, 1993; Hilliard, Madura, & Tucker, 1991; Hoque, Chan, & Manzur, 2008; Liu et al., 2022; Liu, Chen, & Ralescu, 2015; Liu, Lin, & Hung, 2016; Lv, Xiao, Fan, & Ren, 2016; Melino & Turnbull, 1990; Sarwar & Krehbiel, 2000; Shokrollahi, Kılıçman, & Magdziarz, 2016; Swishchuk, Tertychnyi, & Elliott, 2014; Van Haastrecht & Pelsser, 2011; Wang & Ning, 2017; Xiao, Zhang, Zhang, & Wang, 2010). Each model has contributed to the refinement of currency option evaluation.

Still, empirical studies based on surveys of the G–K model have revealed the presence of biases in the derived theoretical prices. The hypotheses of Garman and Kohlhagen's model Garman and Kohlhagen (1983) do not hold in the financial market, prompting the need to develop more suitable and realistic new valuation models. Nonetheless, pricing financial instruments, including options, relies on an accurate assessment of various factors, with information being critical. In the context of European currency options, the influence of pricing information is particularly significant; however, the impact of information on option pricing is not static but varies over time, leading to dynamic information costs. These dynamic costs reflect the changing nature of information availability, market conditions, and investor behavior. Existing literature has recognized the importance of information in asset pricing, but limited research has explicitly focused on its time-dependent nature and the associated dynamic information costs in the context of option pricing; therefore, there is a need to examine how these dynamic costs affect the performance of currency option pricing models. This situation raises the following questions for which we endeavor to provide a preliminary response:

i. Would Garman–Kohlhagen's modified model, considering the dynamic information costs, reduce the evaluation mistakes caused by the standard G–K model?
ii. How does introducing these imperfections affect the economic agent's behavior?
iii. Are the parameters of the option assessment perfectly observed or measured?

This study addresses this research gap by proposing extending the Garman and Kohlhagen (G–K) model for pricing European currency options. The extension incorporates an intensity function based on interest rates to measure dynamic information costs. This approach draws inspiration from the Beer–Lambert law, an empirical relationship between light absorption and the properties of the medium it passes through. By analogy to the Beer−Lambert law, we introduce the dynamic information costs into the interest rates in the G–K model.

This paper employs a specific methodology to address the questions above. Our analysis focuses on the pricing of options, considering their dependence on maturity and moneyness. To evaluate the performance of the pricing models, we utilize the average mean squared errors (MSE) in option prices and implied volatility. We conduct a comprehensive empirical analysis by categorizing the data based on maturity and moneyness. Our empirical methodology utilizes daily data from future continuous calls on the EUR/USD pair from September 21, 2012, to September 23, 2022. We compare the pricing errors and implied volatility obtained from the proposed model, which incorporates dynamic information costs, with those derived from the original G–K model. This approach allows us to assess the effectiveness and accuracy of the extended model in currency option pricing. By employing the average MSE in option prices and implied volatility across different maturity and moneyness categories, we gain valuable insights into the performance of the pricing models under various conditions. This method comprehensively assesses the models' ability to accurately estimate option prices and implied volatility in different market scenarios. The significance of this research lies in its contribution to a more comprehensive understanding of the relationship between information costs and currency option pricing. By considering the time-dependent nature of information and incorporating dynamic costs, we aim to enhance the accuracy and reliability of currency option pricing models. This enhanced model approach has practical implications for investors and market participants, enabling more informed decision-making in currency options trading.

The structure of this paper is as follows. Section 2 provides a descriptive overview of existing literature on real anomalies in currency options pricing, highlighting the limitations of the G–K model. Section 3 introduces an extended version of the G–K model that incorporates dynamic information costs, and it presents a novel derivation of the standard currency option formula, which has not been published. Section 4 of this study presents a comprehensive overview of the adopted methodology, providing a detailed explanation of the data utilized, the variables considered, and the empirical models employed for the analysis. In Section 5, the empirical results are presented and discussed. Finally, the paper concludes in Section 6, where the main findings are summarized, the study's limitations are discussed, and suggestions are provided for future research directions.

2. Literature review

Numerous theoretical and empirical research has demonstrated the significance of imperfections in option valuation. The literature in this field distinguishes between classic anomalies and real anomalies. Classic anomalies are models that scrutinize factors associated with the underlying asset's price movement. Conversely, real anomalies encompass models that focus on assumptions related to the market.

2.1. Classic anomalies: models for assessment under stochastic volatility and interest

As employed by Black and Scholes (1973) for asset pricing, the geometric Brownian motion has faced criticism for multiple reasons. One significant criticism stems from the assumption that the instantaneous profitability of the underlying asset follows a Gaussian distribution with a constant variance. In light of this limitation, researchers have sought to generalize the geometric Brownian motion by incorporating stochastic volatility. Black (1976) argues that if volatility changes over time, an assessment formula based on constant volatility is inaccurate. Moreover, the Black–Scholes model is unsuitable for valuing all options simultaneously in a given market due to the observed skew effect commonly encountered in practice. Various researchers have thoroughly investigated the issue of stochastic volatility (Aït-Sahalia & Kimmel, 2007; He & Lin, 2021; He & Lin, 2023; Heston, 1993; Hull & White, 1988; Hurn, Lindsay, & McClelland, 2015; Johnson & Shanno, 1987; Kirkby & Nguyen, 2020; Lyu, Ma, & Sun, 2022; Schöbel & Zhu, 1999; Wang, Wang, & Zhou, 2017; Wiggins, 1987). Empirical research commonly addresses the stochastic volatility problem through the solutions proposed by Hull and White (1988) and Heston (1993). Hull and White (1988) propose the assumption of constant volatility for the underlying asset price and suggest using a geometric Brownian motion with uncorrelated increments for the variance process. In contrast, Heston (1993) presents an alternative approach by advocating for the direct modeling of volatility instead of the variance. In this framework, the variance is governed by a process that exhibits mean-reversion, often called the "squared root" process.

Furthermore, some consider the presence of a stochastic interest rate in option assessment models. This area of research focuses on extending the Black and Scholes (B&S) model by questioning the assumption of a constant interest rate. Assuming that the interest rate is deterministic can be overly restrictive since anticipated interest rate fluctuations constitute a significant source of market uncertainty for bondholders. Merton (1973) relaxed the restrictive assumption of interest rate stability by generalizing the B&S formula for risk-free interest rates that vary according to a Gaussian Wiener process. Following the works of Merton (1973) and Vasicek (1977), subsequent models have been developed (Amin & Jarrow, 1991, 1992; Brennan & Schwartz, 1980; Grabbe, 1983; He & Zhu, 2018; Ho & Lee, 1986; Jaimungal & Wang, 2006; Liang & Xu, 2020; Liu et al., 2016; Lyu et al., 2022; Pearson & Sun, 1994; Van Haastrecht & Pelsser, 2011; Zhang & Wang, 2013).

2.2. Real anomalies

The assumption of market efficiency plays a significant role in the framework proposed by Black and Scholes (1973) and is also an essential assumption in Garman and Kohlhagen's model Garman and Kohlhagen (1983). This assumption implies that the absence of imperfections in financial markets is assumed and that economic agents are perfectly rational.

The market microstructure theory reveals the presence of information costs, which play a crucial role in forming international portfolios. Consequently, these costs should be considered in pricing financial asset models. Thus, the underlying asset's price reflects the availability or arrival of new information, as Fama (1991) highlighted. Indeed, several factors contribute to the imperfections observed in the market, including information asymmetry (Allen & Gorton, 1993; Benkraiem, Bensaad, & Lakhal, 2022; De Long, Shleifer, Summers, & Waldmann, 1990; Kadan, Michaely, & Moulton, 2018; Kashefi Pour, 2017; Leland & Pyle, 1977; Pan & Misra, 2022; Zghal, Ben Hamad, Eleuch, & Nobanee, 2020), transaction and information costs (Ardalan, 1999; Govindaraj, Li, & Zhao, 2020; Grossman & Stiglitz, 1980; Gu, Liang, & Zhang, 2012; Hansen Henten & Maria Windekilde, 2016, Ou-Yang and Wu, 2017; Jensen, 1978; Li & Fang, 2022), and restrictions on short selling (Bohl, Reher, & Wilfling, 2016; Duffie, Gârleanu, & Pedersen, 2002; Duong, Kalev, & Tian, 2023; Feng & Chan, 2016; He, Ma, & Wei, 2022; Scheinkman & Xiong, 2003). Incorporating these imperfections is crucial as it prevents arbitrage operations from eliminating evaluation errors. Additionally, behavioral finance plays a role in investors' decision-making, contributing to market inefficiency and challenging the assumptions of investor rationality. Research has shown that evaluation errors can arise when rational investors interact with irrational ones, such as the presence of noise traders (Gao & Ladley, 2022; Ramiah, Xu, & Moosa, 2015; Ryu & Yang, 2020) and over-confident investors (Abreu & Brunnermeier, 2003; Du & Budescu, 2018).

The extant literature also shows that several studies on option pricing models have overlooked the impact of information costs, highlighting the need to examine various types of information costs and investor motivations. These costs can be categorized into search and information acquisition costs, where Grossman and Stiglitz (1980) highlight the equilibrium price implications of acquiring costly information. Easley and O'Hara (1987) emphasize the additional costs of learning and experience in assessing information quality. Bellalah and Jacquillat (1995) introduce a new cost of information, considering its indispensable role in collection and analysis. Gu et al. (2012) and Hansen Henten and Maria Windekilde (2016) demonstrate that information costs can lead to evaluation errors. As studied by Marin and Rahi (2000), information transmission costs correlate with the number of informed investors, while Duffie and Rahi (1995) highlight the importance of private information transmission in asset allocation decisions. Gul, Kim, and Qiu (2010) and Kashefi Pour (2017) find a positive correlation between transaction volume and information. Furthermore, adverse selection or asymmetric information costs, explored by Grossman (1976), Harrison & Kreps (1978), and Hellwig (1980), assume heterogeneous information among investors, with some possessing specific knowledge about future asset performance. Kovalenkov and

Vives (2014) extend this to endogenous information acquisition, while Easley, O'Hara, and Srinivas (1998), Cremers, Halling, and Weinbaum (2015), and Goncalves-Pinto, Grundy, Hameed, van der Heijden, and Zhu (2020) show that informed investors in the options market can cause deviations in the call-put relationship. Gapeev and Li (2022) highlight the incorporation of asymmetric information into option pricing models and introduce dynamic exercise strategies based on stochastic boundaries. Market sentiment, as examined by Mahani and Poteshman (2008) and Bauer, Cosemans, and Eichholtz (2009), influences option pricing without a specific theoretical model. Moreover, empirical findings reveal systematic effects of sentiment metrics from underlying and options markets on option pricing (Boutouria, Ben Hamad, & Medhioub, 2021; Daniel, Hirshleifer, & Teoh, 2002; Maghyereh, Abdoh, & Awartani, 2022; Nagarajan & Malipeddi, 2009; Yang, Gao, & Yang, 2016; Zghal et al., 2020).

Therefore, the extension of option pricing models is motivated by the lack of transparency and liquidity observed in certain markets, resulting in the search for costly information. This extension introduces the dynamic nature of information costs, highlighting that the cost depends on the speed of information acquisition. Consequently, investors who receive information earlier can reduce their risk exposure; however, conventional option pricing methods neglect the impact of information costs. Building upon this notion, previous studies by Bellalah (2006), Minehan and Simons (1995), Bellalah and Jacquillat (1995), and Merton (1987) incorporated the costs associated with statistical information. Additionally, Ben Hamad and Eleuch (2008a, 2008b) addressed the dynamic imperfections related to information costs. These studies provide valuable insights into the biases present in option pricing models.

3. The Garman and Kohlhagen model considering the dynamic information costs

Developing a new model in the presence of dynamic information costs highlights the significance of introducing the theoretical model of Garman and Kohlhagen (1983). This model suggests that the sale of a call option for a foreign currency can be effectively hedged by purchasing a specific quantity of foreign government bonds, considering the interest rate on these bonds (r_f). The effectiveness of this hedging strategy is based on the assumptions mentioned earlier. Consequently, under these assumptions, the price of a European currency call can be determined using the following formula:

$$C_t = S_t e^{-r_f\tau} N(d_1) - X e^{-r_d\tau} N(d_2) \tag{1}$$

Then, the price of a European currency *put* is given by this formula:

$$P_t = S_t e^{-r_f\tau} N(-d_1) + X e^{-r_d\tau} N(-d_2) \tag{2}$$

In this context, the variables used in the currency option pricing framework are defined as follows. C_t Is the value of a European currency call paying an interest rate r_f, P_t is the value of a European currency put, and S_t is the price of the underlying asset. X is the strike price of the call, r_f is the interest rate of the foreign currency, and r_d is the interest rate of the domestic currency. Finally, τ is the time calculated in years or a fraction of a year, and $N()$ is the cumulative distribution function of a normal distribution: $N(0,1)$; d_1 and d_2 are calculated as follows:

$$d_1 = \frac{ln\left(\frac{S}{X}\right) + \left(r_d - r_f\right)\tau + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}} \tag{3}$$

and,

$$d_2 = d_1 - \sigma\sqrt{\tau} \tag{4}$$

3.1. Study of the new function of dynamic information costs by the analogy of Beer–Lambert law

The extension of the classical model in this study builds upon most of its assumptions; however, it introduces a new hypothesis that considers the presence of dynamic information costs. Accordingly, we propose to expand Garman and Kohlhagen's model by incorporating these dynamic imperfections, utilizing the risk-neutral framework established by Black and Scholes (1973). Therefore, we propose a function inspired by the Beer–Lambert law, also known as the Beer–Lambert–Bouguer law. This empirical relationship describes light absorption as it passes through different media. The law was initially discovered by Pierre Bouguer in 1729 and published in his book "An Optical Essay on the Gradation of Light" (Claude Jombert, Paris, 1729). Later, in 1760, Johann Heinrich Lambert reformulated the law, further developed by August Beer in 1852. This reformulation establishes that the intensity of light propagating through an optical medium is given by the following:

$$\mathbf{I_t(l) = I_0 e^{-\alpha l}} \tag{5}$$

Therefore, the absorbed light through the medium can be expressed by the following:

$$I_{ab} = I_0 - I_t = I_0\left(1 - e^{-\alpha l}\right) \tag{6}$$

where α is the light absorption coefficient, and l is the optical length in the medium.

Drawing on the principles of this law, we propose introducing a function in our extended model that captures the dynamic

Table 1
The evolution of the values acquired by the time advantage function.

l	w(l) for α = 1	w (l)for α = 0.85	w(l)for α = 0.65	w(l)for α = 0.5	w(l)for α = 0.25
0	0	0	0	0	0
1	0.632120559	0.572585068	0.477954223	0.39346934	0.221199217
2	0.864664717	0.817316476	0.727468207	0.632120559	0.39346934
3	0.950212932	0.921918334	0.857725928	0.77686984	0.527633447
4	0.981684361	0.96662673	0.925726422	0.864664717	0.632120559
5	0.993262053	0.985735766	0.961225792	0.917915001	0.713495203
6	0.997521248	0.993903253	0.979758089	0.950212932	0.77686984
7	0.999088118	0.997394159	0.989432796	0.969802617	0.826226057
8	0.999664537	0.998886225	0.994483436	0.981684361	0.864664717
9	0.99987659	0.999523956	0.997120101	0.988891003	0.894600775
10	0.9999546	0.999796532	0.998496561	0.993262053	0.917915001
11	0.999983298	0.999913035	0.999215136	0.995913229	0.936072139
12	0.999993856	0.99996283	0.999590265	0.997521248	0.950212932
13	0.99999774	0.999984113	0.9997861	0.998496561	0.961225792
14	0.999999168	0.99999321	0.999888334	0.999088118	0.969802617
15	0.999999694	0.999997098	0.999941705	0.999446916	0.976482254
16	0.999999887	0.99999876	0.999969568	0.999664537	0.981684361
17	0.999999959	0.99999947	0.999984113	0.999796532	0.985735766
18	0.999999985	0.999999773	0.999991706	0.99987659	0.988891003
19	0.999999994	0.999999903	0.99999567	0.999925148	0.991348305
20	0.999999998	0.999999959	0.99999774	0.9999546	0.993262053
21	0.999999999	0.999999982	0.99999882	0.999972464	0.994752482

Note: The table below illustrates the values obtained by the time advantage function for the temporal advantage "*l*" ranging from 0 to 21, with varying coefficients of the temporal advantage "*α*".

characteristics of information costs. This function is referred to as the temporal advantage function of information, denoted as $w(l)$, and can be defined as follows:

$$\mathrm{W}:]-\infty, +\infty[\rightarrow]-\infty, +\infty]$$

$$l \rightarrow 1 - e^{\alpha l} \tag{7}$$

where "*l*" represents the temporal advantage, defined as the difference between the time of information acquisition and the maturity of a currency option. The quality of the acquired information influences the value of this temporal advantage. To quantify its impact on the currency option value, we introduce a coefficient "*α*" that ranges between zero and one. Specifically, when the information is deemed less significant, the coefficient approaches zero, while it tends toward one for highly relevant information.

The derivative of $w(l)$ concerning l is given by the following formula:

$$w^{'}(l) = \alpha e^{-\alpha l} > 0 \tag{8}$$

Then, $w(l)$ is strictly increasing on $\mathbb{R}$,
and,

$$\begin{cases} \lim\limits_{l \to -\infty} w(l) = -\infty \\ \lim\limits_{l \to +\infty} w(l) = 1 \end{cases} \tag{9}$$

The temporal advantage function *w(l)* is enhanced by increasing the temporal advantage l or reducing the coefficient α.

Table 1 includes values for the temporal advantage "*l*" going from 0 to 21 and for different coefficients of the temporal advantage "*α*" ($\alpha = 1$; $\alpha = 0.85$;$\alpha = 0.65$,$\alpha = 0.5$ and $\alpha = 0.25$).

Fig. 1 shows the evolution of the temporal advantage function, indicating that the temporal advantage function increases as "l" increases and tends to 1, whatever the value of the temporal advantage coefficient "α" is. Nevertheless, the speed of evolution of w (l) varies depending on the value of "α"; In fact, w (l) tends more quickly toward 1 when α increases. We also see that the value of w(l) for a temporal advantage l increases when α tends toward 1. For example, for the same temporal advantage of 5, the value of the temporal advantage function is 0.993262053 for α = 1, 0.629633437 for α = 0.5, and 0.467212935 for α = 0.25. The effect of obtaining information five days before the maturity date varies depending on the degree of relevance of this information as measured by α. This means that the obtained information's relevance modifies the temporal advantage's impact on the option value.

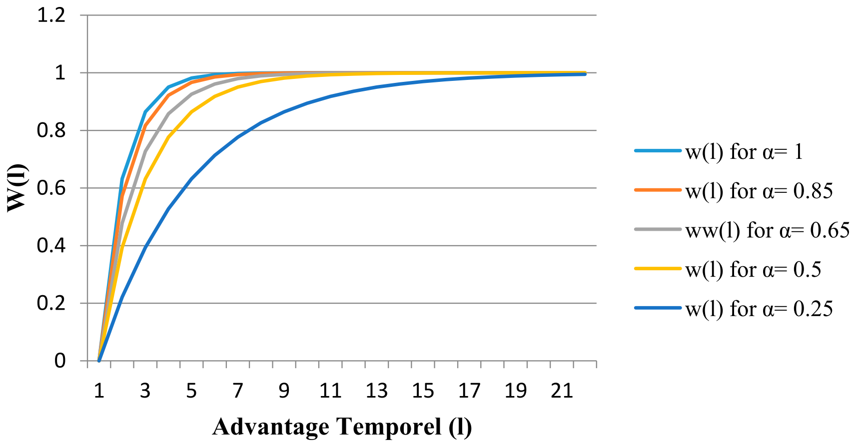

Fig. 1. The evolution of the temporal advantage function w(l).
Note: This figure shows the evolution of the temporal advantage function w(l). The horizontal (vertical) axis presents the advantage temporal (l) and the temporal advantage function w(l).

3.2. The actual discount rate in the presence of dynamic information costs

Consequently, it is necessary to include dynamic information costs in the Garman and Kohlhagen formula by incorporating them alongside the risk-free domestic and foreign interest rates. The effect of information costs on currency option prices can be likened to applying an additional discount rate. *"When investors move from a risk-neutral world to a risk-averse world, two things happen: the expected growth rate in the stock price changes and the discount rate that must be used for any pay-off from the derivative changes." (John* Hull, 2008*).*

In this context, Thaler (1981), Merton (1987), Bellalah and Jacquillat (1995), Ben Hamad and Eleuch (2008a), and Zghal et al. (2020) assert that the discount rate and expected growth rate of the underlying asset are presumed to be equal to the risk-free rates plus the costs associated with these anomalies. By deviating from the market efficiency hypothesis, including dynamic information costs in the model is akin to applying an additional discount rate that accounts for the dynamic information costs. It can be written as follows:

$$r_1 = r + \text{dynamic Information Costs} \tag{7}$$

$$r_1 = r + \lambda w(l) \tag{8}$$

where, r_f is the effective interest rate, r represents the nominal interest rate, l is the temporal advantage, $w(l) = 1 - e^{-\alpha l}$ represents the temporal advantage function, and λ designates the information costs. Furthermore, several information-related costs arise: search, acquisition, transmission, and adverse selection. These costs are represented in the new model by "λ." If λ is equal to zero, then there are no information costs; therefore, we are in an efficient market in which we find the initial model of Garman and Kohlhagen (1983).

In our case, we define two functions of dynamic information costs, denoted as $w(l)$ and $w(l_1)$, which are related to the domestic and foreign markets, respectively. These functions capture the varying nature of information costs over time, considering each market's specific dynamics and characteristics. By incorporating these functions into our model, we aim to reflect the impact of dynamic information costs on currency option pricing.

In this scenario, the effective domestic interest rate r_{1d} is presented as follow:

$$r_{1d} = r_d + \lambda\, w(l) \tag{9}$$

where, r_d is the real risk-free domestic interest rate, and λ is the information costs amplitude associated with domestic information. The function $w(l)$ corresponds to the advantage function related to domestic information and is equal to $1 - e^{-\alpha l}$. Here, α is the advantage coefficient related to domestic information, and l is the temporal advantage associated with domestic information.

Therefore, the effective foreign interest rate r_{1f} is presented as follows:

$$r_{1f} = r_f + \lambda_1\, w(l_1) \tag{10}$$

where, r_f is the real risk-free foreign interest rate and $\lambda_1$is the information cost amplitude associated with foreign information. The function $w(l_1)$ corresponds to the advantage function related to foreign information and is equal to $1 - e^{-\alpha_1 l_1}$. Here, α_1 is the advantage coefficient related to foreign information and l_f is the temporal advantage associated with foreign information.

3.3. Derivation of the model in the presence of dynamic information costs

The extension of the new model relies on most of the assumptions of the classical model. We introduce a new assumption based on dynamic information costs in the currency options pricing.

According to the Black–Scholes's model,[1] it is possible to create a short position composed of the sale of $\frac{1}{\frac{\partial C(S,T)}{\partial S}}$ for an option against a long position on the shares.

Following the same approach of Black and Scholes (1973) and replacing $Se^{-\delta\tau}$ of the model of Black, Scholes, and Merton by $Se^{-\tau r_f}$ of the model of Garman, we can deduce that if the price of the active support changes by a small amount ΔS, the option will change by $\frac{\partial C(S,T)}{\partial S}\Delta S$. This cover can be maintained without interruption so that the return of the short position becomes completely independent of the change of the value of the active support, i.e., the return to the covered position becomes sure.

As a result, the change in the long position of the currency is approximately offset by the change in $\frac{1}{\frac{\partial C(S,T)}{\partial S}}$ options.

The value of PF is composed of the purchase of a currency and the sale of $\frac{1}{\frac{\partial C(S,T)}{\partial S}}$ options, is as follows::

$$V = S - \frac{1}{\frac{\partial C(S,T)}{\partial S}} C(S,T) \tag{11}$$

During one-time intervals, the change of this position is given by the following:

$$\Delta V - r_f\, S - \frac{1}{\frac{\partial C(S,T)}{\partial S}} r_f\, C(S,T) \tag{12}$$

where:

$$r_f\, C(S,T) = C\left(S + r_f S, T + r_f\, T\right) - C(S,T) \tag{13}$$

Geometric Brownian motion governs the currency spot price, i.e., the differential representation of the spot price movements is as follows:

$$ds = \mu\, S\, dt + \sigma\, S\, dZ \tag{14}$$

where dS is the evolution of the spot currency price. The μ between t and dt is the mathematical expectation of equity return, and dZ is the Gauss–Wiener standard process, $N\,(0, dt)$, $V\,(dz) = \varnothing dt$ Finally, σ is the standard deviation of instantaneous equity return, which is supposed to be a known constant.

Using the differential stochastic calculation and Itô's lemma, we get:

$$\Delta C = \frac{\partial C}{\partial t}\Delta S + \frac{\partial C}{\partial t} dt + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} dt \tag{15}$$

The return to the covered position is sure and must be equal to $\left(r_{1d} - r_{1f}\right)$, where r_{1d} is the effective risk-free domestic interest rate and r_{1f} is the effective risk-free foreign interest rate.

A specific condition is required to deduce the partial derivative equation: the lack of arbitrage opportunity from the continuous change in portfolio composition (it remains risk-free).

We get:

$$\left(-\frac{\partial C}{\partial t} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}\right) dt = \left(r_{1d} - r_{1f}\right)\left(-C + \frac{\partial C}{\partial S} S\right) dt \tag{16}$$

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \left(r_{1d} - r_{1f}\right) S \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} - \left(r_{1d} - r_{1f}\right) C = 0, \tag{17}$$

which is the partial differential equation in the presence of dynamic information costs. The problem is solving this partial differential equation (PDE) (see Appendix 1: the resolution of the PDE); thus, to determine the value of the currency call option, we can proceed as follows. First, we assume that the underlying asset's return equals the effective risk-free rate, accounting for dynamic information costs. Next, we calculate the expected pay-off of the derivative asset at the option's maturity. This expectation is then discounted using

[1] The derivation of Black–Scholes is founded on arbitration, which constitutes a covered position formed by a long one (or purchase) on the actions and a short position (or sale) on the options, and vice versa. Then, the composition of the PF of arbitration depends on the type of the option:

- Case of option of purchase "call": the position detained on the action must be the inverse to that of the call [purchase (sale) of the call + sale (purchase) of the action].
- Case of sale "option": the same position detained on the action and the option of sale [purchase (sale) of the call+ purchase (sale) of the action].

the effective risk-free domestic interest rate (r_{1d}) and the effective, risk-free foreign interest rate (r_{1d}), considering the dynamic information costs. As a result, the modified pricing formula for European currency call options in the presence of dynamic information costs can be expressed as follows:

$$C(S,t) = Se^{-r_{1f}T}N\left(d_1^{'}\right) - Xe^{-r_{1d}T}N\left(d_2^{'}\right) \tag{18}$$

with,

$$d_1^{'} = \frac{ln\left(\frac{S}{X}\right) + \left(\frac{\sigma^2}{2} + \left(r_{1d} - r_{1f}\right)\right)T}{\sigma\sqrt{T}} \tag{19}$$

and,

$$d_2^{'} = d_1^{'} - \sigma\sqrt{T} \tag{20}$$

Then, the parity relationship in the presence of dynamic information is as follows:

$$C_t - P_t = Se^{-r_{1f}T} + Xe^{-r_{1d}T} \tag{21}$$

where C is the price of the call, P is the price of the put, and S is the price of the underlying asset.X is the strike price, T is the time to maturity, r_{1f} is the effective foreign (riskless) interest rate, and r_{1d} is the effective domestic (riskless) interest rate.

Therefore, we can write:

$$S_t e^{-r_{1f}T}N\left(d_1^{'}\right) - X_t e^{-r_{1d}T}N\left(d_2^{'}\right) - P_t = Se^{-r_{1f}T} - X_t e^{-r_{1d}T} \tag{22}$$

As a result, we can deduct the value of the European currency put in the presence of dynamic information costs as follows:

$$P_t = -S_t e^{-r_{1f}T}\left[1 - N\left(d_1^{'}\right)\right] + Xe^{-r_{1d}T}\left[1 - N\left(d_2^{'}\right)\right] \tag{23}$$

We know that $N(d) + N(-d) = 1$.

The price of the European currency put is then:

$$P_t = S_t e^{-r_{1f}T}N\left(-d_1^{'}\right) + Se^{-r_{1d}T}N\left(-d_2^{'}\right) \tag{24}$$

4. Methodological issues

This section empirically tests the modified Garman and Kohlhagen model in dynamic imperfections. Our objective is to assess the accuracy of the modified model in approximating currency option prices compared to the theoretical Garman–Kohlhagen model Garman and Kohlhagen (1983); therefore, we formulated a set of research questions that guide our empirical analysis, aiming to evaluate different aspects of the modified model's performance and its ability to capture market dynamics. Through our empirical approach, we apply an inductive technique to verify the presence and impact of dynamic information costs in financial markets. By analyzing real-world data and market dynamics, we seek to provide empirical evidence supporting dynamic information costs and their influence on currency option pricing. This approach bridges the gap between theoretical insights and empirical observations, enhancing our understanding of financial market dynamics.

Our approach will be structured as follows:

- We calculate the price of the currency call option using both models.
- We evaluate the models' performance through analysis of the valuation gaps.
- We examine the impact of introducing dynamic imperfections on market participants' behavior in the financial market.

4.1. Data sources and variables

Our sample comprises 2.612 observations of currency call options on the EUR/USD pair traded on the Russian Trading System. Specifically, we focus on the prices of European-type foreign currency options obtained from the DATASTREAM database. The study period spans September 9, 2012, to September 9, 2022, and the options included in our analysis are at the money and out-of-the-money options. We utilize daily data to conduct our empirical analysis and examine the performance of the modified model in capturing the dynamics of these currency options.

We use currency option evaluation formulas with and without information costs and their corresponding variables to implement our analysis. The variables include the underlying price (S), exercise price (X), time remaining until maturity (T), risk-free domestic interest rate (r_d), risk-free foreign interest rate (r_f), and volatility (σ). These variables are identified and specified as follows:

- The underlying price (EUR/USD exchange rate), represented by a continuous series with the data type (OU), reflects the closing price of the underlying asset in the options market, providing daily activity information.
- The option strike price, represented by the data type (OS), indicates the option's strike price at the money.
- The implied volatility with a constant maturity of 3 months, represented by the data type (O3), displays the ATM implied volatility with a fixed time to maturity of 90 days.
- The market price, represented by the data type (OM), shows the price of the option series at the money.
- The number of days remaining to maturity is a known contract characteristic, denoted as T, calculated as the number of days until maturity divided by 360.
- The domestic interest rate is derived from the "LIBOR," with a constant time to maturity of 90 days.
- The foreign interest rate is derived from the "EURIBOR," with a constant time to maturity of 90 days.

Hull (2008) posits that these rates serve as accurate estimators of the risk-free rate.

4.2. Study of the model performance

This study primarily aims to assess the performance of the extended model in capturing market prices compared to the standard Garman–Kohlhagen model Garman and Kohlhagen (1983). Our methodology encompasses multiple steps: first, we simulate the dynamic information cost parameters; next, we calculate option prices using the modified Garman–Kohlhagen and standard G-K model. We then analyze the model's performance by examining evaluation errors, including errors reported by maturity, skewness, and overall errors, and comparing theoretical prices with market prices. To compare the models, we define the following variables that need to be calculated. C_{GK} is the price of a European currency call option calculated using the G–K model Garman and Kohlhagen (1983) formula. Furthermore, C_{GKDIC} is the price of a European currency call option calculated using the model in the presence of the dynamic information costs, and C_M is the observed price of a European currency call option on the market.

To simplify the calculation of the currency call option using the formula derived from the theoretical model of Garman–Kohlhagen, we have created a Visual Basic (VBA) procedure called *Call_GK* (refer to Appendix 2). This procedure allows us to use the function in Excel just like any other calculation function. Additionally, considering the dynamic information costs, we need to simulate the values of λ, λ_1, $w(l)$ and $w\ (l_1)$ to calculate the value of the currency call option using the new model. We have utilized a simulation method in Excel using VBA. Initially, we assign initial values to λ, λ_1, α, α_1, l, and l_1, and then calculate the effective interest rates "r_{1d}" and "r_{1f}." Next, we iteratively adjust these parameters to minimize the average difference. This statistical procedure aims to reduce the difference toward zero by modifying the values of the information cost parameters. To achieve this, we utilize the "goal seek" feature available in the tools menu of Excel. This method helps us estimate the implicit values of λ, λ_1, α, α_1, l, and $l_1$that minimize the average difference. Lastly, to simplify the calculation of the currency call option using the formula derived from the new model, we have developed a Visual Basic procedure named *Call_GKIC* (refer to Appendix 3).

Then, to compare the models, we consider the following variables. GK represents the price of a European currency call option calculated using the Garman–Kohlhagen (G–K) theoretical model formula. GKIC represents the price of a European currency call option calculated using the modified model with information costs, and VM represents the observed market prices of the currency options. Subsequently, we define the following variables:

- $EGK = GK - VM,$ which represents the deviation of the price obtained by the Garman–Kohlhagen (G–K) model Garman and Kohlhagen (1983) compared to the market prices of the currency options.
- $EGKIC\ = GKIC - \ VM$, which represents the deviation of the price obtained by the model considering dynamic information costs compared to the market prices of the currency options.

Then, if we observe a positive (negative) difference, it indicates that the option is overvalued (undervalued), suggesting that the theoretical model of Garman and Kohlhagen (1983) either overestimates or underestimates the market price. These differences are computed for each call on every sample day, allowing us to determine the average error for each call. Moreover, since option prices are influenced by factors such as maturity and moneyness, we calculate the average squared deviation by considering maturity and skewness, in addition to overall errors, across the entire sample of currency calls. This approach enables us to derive the average squared error evaluation *EGKM* and *EGKICM*, respectively:

$$EGKM\ \left(EGK^2\right) \tag{25}$$

$$EGKICM\ \left(EGKIC^2\right) \tag{26}$$

To demonstrate that our new model provides a more accurate approximation of currency option values compared to the basic G–K model, we aim to test the null hypothesis as follows:

$$H0 : EGKICM - EGKM < 0$$

We must establish our expectations regarding this type of test, considering how the question is posed moving forward:

In the case of rejecting the null hypothesis (H0), several reasons can be considered:

- The model incorporating dynamic information costs is either equivalent or less effective than the Garman and Kohlhagen (1983) model.
- The estimation of the variables *EGKM* and *EGKICM* in the presence of dynamic information costs and Garman–Kohlhagen's model may have resulted in distortions.
- The data quality may have affected the results, including issues such as the synchronization of the call option value with the underlying asset, exchange rate, and simultaneous measurement of the interest rate.

If the null hypothesis (H0) is not rejected, both models are adequately estimated, which suggests that the new model provides a better average approximation of currency options' market value than the Garman and Kohlhagen (1983) model.

To further explore the comparison between the model considering dynamic information costs and the G–K model, we utilize the relative deviation of the average errors (*RDAE*), which is calculated as follows:

$$RDAE = \frac{EGKICM - EGKM}{EGKICM} \tag{27}$$

where *EGKM* represents the average errors of Garman and Kohlhagen's model Garman and Kohlhagen (1983). Additionally, *EGKICM* represents the average errors of our modified model considering dynamic information costs, and *RDAE* denotes the relative deviation of the average errors of the model considering dynamic information costs compared to the Garman–Kohlhagen's theoretical model Garman and Kohlhagen (1983).

4.3. Analysis of implied volatility and its implications for economic agent behavior in financial markets

Implied volatility refers to the volatility implied by an option's market price. It plays a crucial role as it reflects the market's collective perception of the expected future volatility of the underlying asset (Poterba & Summers, 1986). The supply and demand dynamics in the currency options market influence this measure of volatility; thus, by introducing dynamic information costs into the assessment of currency options, we can examine how it affects the behavior of economic agents. This comparative study analyzes the impact of these dynamic information costs on implied volatility and compares it with the implied volatility derived from both the modified and basic models.

We follow a systematic procedure to calculate implied volatility. First, we input the relevant parameters into the currency option assessment model, including the underlying price, domestic and foreign interest rates, dynamic information costs, strike price, and time to maturity. The goal is to align the calculated value with the observed market value, with the only unknown parameter being the volatility. We employ the Newton algorithm to estimate this implied volatility, which can be implemented through a VBA script named IVGKICM (refer to Appendix 4). By iteratively adjusting the volatility parameter, we can find the value that brings the calculated model price in line with the market price. This process is repeated daily in our sample, enabling us to derive implied volatility for the entire dataset.

To further analyze the impacts of introducing these imperfections on the behavior of economic agents in financial markets, we examine the relative deviation of the average implied volatility, referred to as "DAIVM," in the model incorporating dynamic information costs compared to the theoretical Garman and Kohlhagen (1983) model. This relative deviation allows us to assess the extent to which dynamic information costs affect the implied volatility and, consequently, the behavior of economic agents in the currency options market. It is defined as follows:

$$\mathrm{DAIVM} = \frac{\mathrm{IVGKICM} - \mathrm{IVGKM}}{\mathrm{VIVGKICM}} \tag{28}$$

where IVGKM represents the average implied volatility extracted from the Garman and Kohlhagen (1983) model, while IVGKICM represents the average implied volatility extracted from the model considering dynamic information costs. Comparing these two averages allows us to analyze the impact of introducing dynamic information costs on the implied volatility.

Table 2
Mean squared errors by maturity and moneyness.

Moneyness	Total observations/ category			At the money			Out of the money		
Maturity (in days)	N° of obs	EGKICM - EGKM	RDAE in %	N° of obs	EGKICM–EGKM	RDAE in %	N° of obs	EGKICM - EGKM	RDAE in %
1–14	12	6.031E-06	18.189%	3	6.037E-06	19.338%	9	6.029E-06	17.835%
15–30	356	−0.0036	−41.105%	161	−0.005	−39.393%	195	−0.002	−44.594%
31–60	747	−0.013	−44.492%	419	−0.015	−45.484%	328	−0.009	−42.514%
61–90	1021	−0.045	−47.05%	437	−0.033	−46.846%	584	−0.055	−47.151%
91–112	475	−0.078	−46.384%	263	−0.091	−45.548%	212	−0.061	−48.018%
Total	2612	−0.036	−45.816%	1283	−0.036	−45.816%	1329	−0.036	−47.035%

Note: The table below illustrates the Mean Squared Errors by maturity and moneyness. N° of obs is the number of studied observations. EGKICM – EGKM represents the difference between the average errors of our modified model considering dynamic information costs and the average errors of Garman and Kohlhagen's model Garman and Kohlhagen (1983). RDAE denotes the relative deviation of the average errors of both models.

5. Empirical results and discussion

5.1. Evaluating performance of the extended model and analyzing valuation discrepancies

We first assign initial values to the parameters λ, λ_1, α, α_1, l, and l_1. Then, we employ a simulation method in Excel to optimize the average difference between EGKICM and EGKM. We utilize the "goal seek" feature located in the tools menu. This approach allowed us to refine the estimations of the new parameters. After performing the simulation, we obtained the following values: $\lambda = 0.8$, $\lambda_1 = 0.01$, $\alpha = 0.8$, $\alpha_1 = 0.7$, $l = 1$, and $l_1 = 4$. Using these adjusted parameter values, we calculate the prices of currency options using both the modified and basic models. The resulting MSE are then computed for each combination of maturity and moneyness, and these MSE values are presented in Table 2.

An in-depth review of Table 2 reveals how our new model, incorporating dynamic information costs, performs relative to the traditional Garman–Kohlhagen model. The primary goal was to prove that our model yields a more accurate valuation of currency options. To support this, we examined the average errors (EGKICM – EGKM) and the relative deviation of average errors (RDAE) over various maturity and moneyness categories.

The EGKICM – EGKM values indicate negative differences in all categories, signifying that our advanced GK model produces fewer average errors than the original model, indicating a better approximation of the real currency option values. Fig. 2 graphically demonstrates this finding. Evaluating the RDAE, we found negative figures throughout all categories, corroborating that our novel model provides enhanced accuracy regarding percentage error than the original. These negative figures suggest that our model incorporating dynamic information costs tends to undervalue option values, implying a closer approximation to the true currency option values. This outcome is beneficial for market participants and investors, as the increased precision of our model aids better risk evaluation and decision-making for currency options trading strategies.

Moving on to a more detailed analysis, we observe that the RDAE values generally increase as the options' term lengthens, suggesting that the models' estimates and market prices deviate more for longer-term options. This result indicates the more significant impact of dynamic information costs on long-term options.

Examining the maturity categories, for 1–14 days, with a small sample size of 12, we have an RDAE of 18.189% for "at the money" and 17.835% for "out of the money" options, showing our model undervalues option prices. For the 15–30 days' maturity category, we find negative RDAE values across all moneyness categories, ranging from −39.393% to −44.594%, indicating a consistent underestimation of option values. This pattern continues for the 31–60 days, 61–90 days, and 91–112 days' maturity categories, with negative RDAE values of up to −48.018%, signifying a continued underestimation of option values.

In terms of moneyness, the RDAE values vary. For instance, "at the money" options show relatively low RDAE values, suggesting close approximations to market prices; however, "out of the money" options reveal higher RDAE values, indicating greater disparities between the models' estimates.

Looking at "at the money" options, we see negative RDAE values across all maturities, increasing from −41.105% for 15–30 days' maturity to −47.05% for 61–90 days' maturity, indicating price underestimations. Similarly, for "out of the money" options, negative RDAE values continue across all maturities, with values ranging from −39.393% to −48.018%. This finding means our model consistently undervalues these options compared to market observations. Overall, the variability in RDAE values is influenced by the moneyness category, with "at the money" options showing closer alignment with market prices than "out of the money" options.

Our data indicates that our extended model, which incorporates dynamic information costs, outperforms the basic model across all option categories and maturities. The consistent underestimation of prices, as evidenced by the negative RDAE values, underscores the model's capacity to effectively integrate dynamic information costs in pricing currency options. As such, it offers a more precise approximation of the real-world values of currency options compared to the basic model. These findings underscore the benefits for investors and market participants, as the extended model can provide a more accurate valuation of currency options. The results reiterate the importance for market participants to acknowledge the influence of information costs when pricing options across diverse

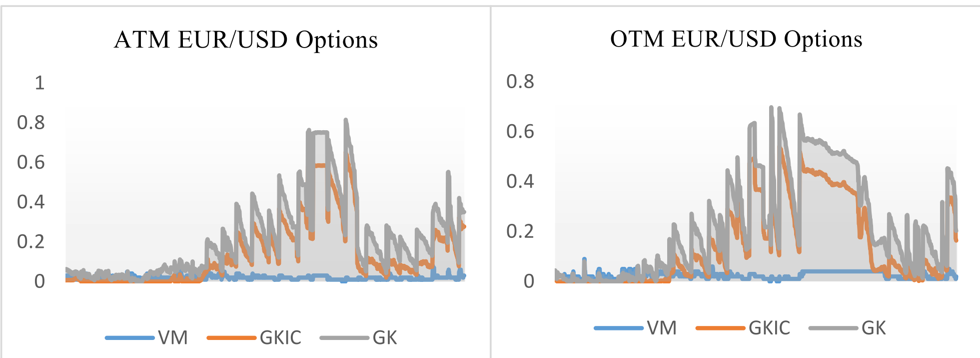

Fig. 2. Comparison of currency call option prices: base model vs. extended model by moneyness.
Note: This figure displays charts comparing call options calculated using the Garman–Kohlhagen model (GK in gray), the extended model (GKIC in orange), and the observed market call prices (VM in blue) categorized by moneyness.

currency pairs. This analysis is in harmony with existing financial literature that posits the role of informed investors in the options market can cause a deviation from the call-put parity relationship driven by privately held information (Alexander, Deng, Feng, & Wan, 2023; Anand, Hua, & Puckett, 2023; Cremers & Weinbaum, 2010; Easley et al., 1998; Gapeev & Li, 2022; Goncalves-Pinto et al., 2020; Wu, Liu, Yuan, & Huang, 2022; Zhou, 2022). The amplified precision offered by our extended model can facilitate superior risk assessments and inform decision-making in currency options trading strategies.

These results bear considerable implications for professionals operating in the currency options market, offering a model that enables more accurate trading and risk management decisions. The data suggests that incorporating dynamic information costs can substantially impact longer-term and out-of-the-money options, which could reflect the higher levels of uncertainty and potential for larger price swings inherent in these categories. Thus, from a practical standpoint, market professionals need to consider the maturity and moneyness of options when making trading evaluations. Utilizing our extended model could aid in garnering more accurate estimations of market prices by integrating dynamic information costs into the calculation.

5.2. Analyzing implied volatility

Table 3 presents the IVGKICM – IVGKM values for the “at the money” and “out of the money” categories across various maturities. These values are uniformly positive across all maturities, illustrating that the average implied volatility values are consistently higher in the revised model compared to the traditional model, a conclusion that is further corroborated by Fig. 3. This discovery lends empirical credence to our initial supposition that integrating information costs in the valuation process of European currency call options results in escalated implied volatility. The integration of dynamic information costs signifies that the market anticipates a more pronounced future volatility of the underlying price compared to a market scenario devoid of these costs. This finding is consistent with Stein (1989) model, which asserts that the participation of informed agents can cause a surge in volatility due to their potential destabilizing impact. Similarly, Easley and O’Hara (1987) observed that the involvement of uninformed investors can engender surplus volatility. In situations of information asymmetry, economic agents tend to foresee a higher volatility in markets where information costs are present compared to those with perfectly symmetric information. Consequently, the pricing of currency options in the presence of information costs intimates the existence of informational asymmetry, culminating in elevated implied volatility values.

Our assessment of implied volatility values by maturity highlights distinct patterns that elucidate the effect of the modified Garman and Kohlhagen’s model compared to the traditional one. A closer look at the different maturity categories yields the following insights. In the 1–14 days’ maturity category, the “at the money” and “out of the money” observations all yield positive IVGKICM – IVGKM values, indicating an increase in implied volatility in the modified model. The DAIVM percentages span from 5.962% to 19.338%, signifying a notable surge in implied volatility relative to the traditional model. We observe the same trend in the 15–30 days’ maturity category, with both “at the money” and “out of the money” options displaying positive IVGKICM – IVGKM values, thus pointing to heightened implied volatility in the modified model. The accompanying DAIVM percentages fall between 45.606% and 58.289%, representing a substantial jump in implied volatility relative to the traditional model. A similar pattern persists in the 31–60 days’ maturity category, with both “at the money” and “out of the money” options registering positive IVGKICM – IVGKM values, suggesting increased implied volatility in the modified model. The DAIVM percentages, ranging from 56.921% to 65.058%, underscore a marked increase in implied volatility relative to the traditional model. The 61–90 days and 91–112 days’ maturity categories also consistently yield positive IVGKICM – IVGKM values for both “At the Money” and “Out of the Money” options, insinuating a higher implied volatility in the modified model. The DAIVM percentages vary from 61.646% to 69.092% for the 91–112 days’ category and from 65.805% to 69.092% for the 61–90 days’ category. These percentages demonstrate a sizable increase in implied volatility relative to the traditional model for these extended maturities.

Our examination across different maturity categories reveals a consistent trend of the modified Garman and Kohlhagen’s model exhibiting higher implied volatility than the traditional model. This finding bolsters the theory that integrating information costs in the valuation of currency options leads to a rise in the anticipated volatility of the underlying price. Our results align with the understanding that the involvement of informed investors and the existence of information asymmetry can drive volatility levels higher in

Table 3
Deviation of the average of implied volatility by maturity and moneyness.

Moneyness	Total observations/category			At the money			Out of the money		
Maturity (in days)	N° of obs	IVGKICM - IVGKM	DAIVM in %	N° of obs	IVGKICM - IVGKM	DAIVM in %	N° of obs	IVGKICM - IVGKM	DAIVM in %
1–14	12	0.006	8.295%	3	0.014	12.038%	9	0.004	5.962%
15–30	356	0.109	52.378%	161	0.143	58.289%	195	0.081	45.606%
31–60	747	0.146	61.987%	419	0.170	65.058%	328	0.115	56.921%
61–90	1021	0.197	66.982%	437	0.170	69.092%	584	0.218	65.805%
91–112	475	0.191	64.891%	263	0.180	61.646%	212	0.205	68.848%
Total	2612	0.169	63.643%	1283	0.169	64.881%	1329	0.168	62.483%

Notes: The table below illustrates the deviation of the average of implied volatility by maturity and moneyness. N° of obs is the number of studied observations. IVGKICM – IVGKM represents the difference between the average implied volatility extracted from the model considering dynamic information costs and the average implied volatility extracted from the Garman and Kohlhagen (1983) model. DAIVM denotes the relative deviation of the average implied volatility.

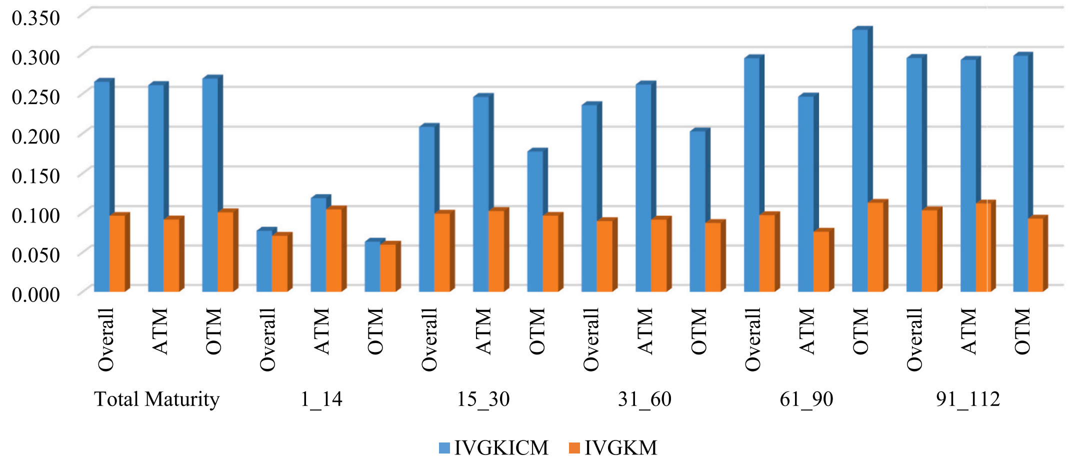

Fig. 3. Comparison of implied volatility: base model vs. extended model by maturity and moneyness.
Notes: The histograms presented in this figure depict the volatility for two different models; the Garman–Kohlhagen model with dynamic information costs (IVGKICM) is displayed in blue, and the new model with dynamic information costs (IVGKICM) is displayed in orange. The data is presented based on moneyness (ATM = at the money, and OTM = out of the money) and maturity (1–14 days, 15–30 days, 31–60 days, 61–90 days, and 91–112 days).

financial markets (Attanasio, 1990; Dávila & Parlatore, 2023; Vlastakis & Markellos, 2012) and specifically in the currency options market (Baek, 2022; Jin, Livnat, & Zhang, 2012; Zhou, 2022). These detailed insights underscore the importance of accounting for the impact of information costs on implied volatility when pricing and trading currency options across different maturities. Market participants should recognize the escalated volatility linked to information asymmetry and fine-tune their risk management strategies accordingly.

When analyzing the implied volatility values by moneyness, we encounter unique patterns illuminating the impact of the dynamic information costs-infused Garman and Kohlhagen model concerning the traditional model.

In the realm of "at the money" options, we consistently register positive IVGKICM – IVGKM values across all maturity categories, signifying a higher level of implied volatility in the modified model. Within the 1–14 days' maturity segment, DAIVM percentages fluctuate between 5.962% and 8.295%. Progressing toward the 15–30 days, 31–60 days, 61–90 days, and 91–112 days' maturity segments, DAIVM percentages span from 45.606% to 66.982%. This notable uptick in implied volatility underscores the considerable impact of integrating information costs into the valuation model for "at the money" options. Mirroring this pattern, "out of the money" options also demonstrate consistently positive IVGKICM – IVGKM values across all maturity categories, indicative of heightened implied volatility in the modified model. Within the 1–14 days' maturity bracket, DAIVM percentages vary from 12.038% to 19.338%. Shifting toward the 15–30 days, 31–60 days, 61–90 days, and 91–112 days' maturity brackets, DAIVM percentages extend from 58.289% to 69.092%. These figures underline the pronounced increase in implied volatility when contrasted with the traditional model for "out of the money" options. In summary, the moneyness-centered examination reveals that both "at the money" and "out of the money" options display higher implied volatility within the modified Garman and Kohlhagen model than the classical one. This insinuates that incorporating information costs escalates the perceived volatility for both types of options. These meticulous findings accentuate the significance of considering the influence of information costs on implied volatility when valuing and trading currency options across varying levels of moneyness. It is crucial for market participants to acknowledge the inflated volatility associated with the inclusion of information costs and to subsequently calibrate their risk management strategies. The elevated implied volatility within the modified model implies a heightened perception of risk that should be considered when valuing and transacting options across diverse moneyness. When pricing and trading options across a range of moneyness categories, we notice a larger increase in implied volatility for "at the money" options versus "out of the money" options when dynamic information costs are factored in. This phenomenon can be attributed to several contributing elements. First, "at the money" options possess strike prices closely aligned with the underlying asset's current price. Consequently, even minor shifts in the asset's price can significantly influence the option's value. In the presence of dynamic information costs, an augmented sense of volatility within the marketplace can instigate larger fluctuations in the option's value, yielding a higher implied volatility for "at the money" options. Second, dynamic information costs infer the presence of informed traders who hold private, market-influencing information. Due to their superior comprehension of market trends and anticipated price shifts, these traders can inject additional volatility into the market. As a result, the implied volatility of "at the money" options may be more profoundly impacted than "out of the money" options. Additionally, "at the money" options are generally seen as more susceptible to market conditions changes and tend to carry a higher risk level than "out of the money" options. When dynamic information costs are woven into the pricing model, the market's perception of risk may be elevated, leading to higher implied volatility for "at the money" options. Lastly, "at the money" options draw higher trading volumes and more market engagement than "out of the money" options. In situations where dynamic information costs are present, increased trading activity and the involvement of informed traders can culminate in higher implied volatility for "at the money" options.

The subsequent examination of implied volatility values indicates that the modified Garman and Kohlhagen's model, which accounts for information costs, consistently generates higher average implied volatility compared to its classical counterpart. This revelation underscores the criticality of acknowledging information costs in the pricing of currency options and amplifies the ramifications of information asymmetry within the market.

From a policy standpoint, these outcomes hold substantial implications. They underscore the urgency of factoring in information costs and their impact on implied volatility during the pricing and trading of currency options. Market players must understand the escalated volatility resulting from information asymmetry and calibrate their risk management strategies accordingly. By acknowledging the influence of information costs on implied volatility, market participants can better navigate the complex landscape of the currency options market and make more informed decisions. These findings fortify the significance of integrating dynamic information costs into option pricing models and illuminate the intricacies that information asymmetry introduces. Moreover, by identifying and accounting for these elements, market participants can deepen their comprehension of implied volatility and enhance their risk management practices within the currency options market.

6. Conclusion and perspectives

This study thoroughly examined the pricing of European currency options, uniquely incorporating dynamic information costs into the traditional Garman and Kohlhagen model. This research utilized an intensity function inspired by the Beer–Lambert law, which was integrated into domestic and foreign interest rates to effectively capture the variable nature of information costs. We analyzed daily data from EUR/USD futures continuous calls from September 21, 2012, to September 23, 2022. This analysis provided a wide-lens perspective on the fluctuations in currency option pricing over a considerable period. This empirical examination, focused on understanding the currency options market in greater depth, adds a valuable dimension to the prevailing discourse on option pricing. The research findings have proven that the revised model incorporating dynamic information costs surpasses the classical model in pricing accuracy. This superior accuracy, highlighted in longer-term and out-of-the-money options, is a testament to the profound impact of dynamic information costs. By calibrating the new parameters of information costs, we have successfully minimized the discrepancies between our model's predictions and the actual market values of currency options, augmenting the precision of our estimations. The maturity and moneyness-based analysis conducted post-simulation reveals a significant error reduction for our model that integrates dynamic information costs. This enhancement reinforces the model's validity and practicability for accurately valuing currency options. Furthermore, the analysis further showed that the extended model persistently displays a higher average implied volatility than the classical model. This finding supports the hypothesis that integrating information costs into option pricing correlates with a surge in implied volatility. These results are congruent with existing literature, emphasizing the role of informed investors and the destabilizing effect of information asymmetry in the options market.

From a practical standpoint, these findings are pivotal for the currency options market. They improve the accuracy of option pricing and refine risk assessment. Market participants must take note of the elevated volatility due to information asymmetry and adjust their risk management strategies.

Although this research underscores the necessity of considering dynamic information costs in currency option pricing and offers valuable insights for stakeholders, it has some limitations. The study does not compare the extended model with other potential extensions like stochastic volatility models or those incorporating stochastic interest rates. Moreover, it relies on daily data, which might not fully capture intraday market dynamics and potential changes in implied volatility. Future research should overcome these limitations by delving into comparisons with other model extensions and utilizing more granular, high-frequency data to deepen the understanding of the effects of dynamic information costs on option pricing. While this study primarily evaluates the extended Garman and Kohlhagen model vis-à-vis the baseline model, the inclusion of other model extensions and intraday data in future research could further illuminate the dynamics of option pricing in the presence of dynamic information costs.

CRediT authorship contribution statement

Wael Dammak: Conceptualization, Formal analysis, Investigation, Methodology, Writing – review & editing, Data curation, Visualization, Writing – original draft, Validation. **Salah Ben Hamad:** Supervision, Methodology, Visualization, Formal analysis, Methodology. **Christian de Peretti:** Supervision, Formal analysis, Investigation, Validation. **Hichem Eleuch:** Investigation, Methodology, Validation.

Declaration of Competing Interest

None.

Data availability

Data will be made available on request.

Appendices

Appendix A: The resolution of the PDE

The value of the purchase option must fill the condition to the boundary mark, expressing the value of the call at the date of maturity:

$$C(S,t^*) = Max[0; S(t^*) - X] \tag{A.1}$$

The search for the solution requires a change of the variables that lead to the equation of heat. We start by writing the price of the option in the following form:

$$C(S,t) = f(t)y(u_1,u_2) \tag{A.2}$$

where, $f(t)$ and $y(u_1,u_2)$ are unknown functions.

Therefore, we calculate the derivative of the option price regarding time and compared it to the price of the underlying asset:

$$\frac{\partial C}{\partial t} = \frac{\partial f}{\partial t}y + f\frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial t} + f\frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial t} \tag{A.3}$$

$$\frac{\partial C}{\partial S} = f\left(\frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial S} + \frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial S}\right) \tag{A.4}$$

$$\frac{\partial^2 C}{\partial S^2} = f\left(\frac{\partial^2 y}{\partial {u_1}^2}\left(\frac{\partial u_1}{\partial S}\right)^2 + \frac{\partial y}{\partial u_1}\frac{\partial^2 u_1}{\partial S^2}\right) + f\left(\frac{\partial^2 y}{\partial {u_2}^2}\left(\frac{\partial u_2}{\partial S}\right)^2 + \frac{\partial y}{\partial u_2}\frac{\partial^2 u_2}{\partial S^2}\right) + 2f\left(\frac{\partial^2 y}{\partial u_1 \partial u_2}\frac{\partial u_2}{\partial S}\frac{\partial u_1}{\partial S}\right) \tag{A.5}$$

Then, we replace these derivatives in Eq. (20). We get:

$$\frac{1}{2}\sigma^2 S^2 f\left(\frac{\partial^2 y}{\partial {u_1}^2}\left(\frac{\partial u_1}{\partial S}\right)^2 + \frac{\partial y}{\partial u_1}\frac{\partial^2 u_1}{\partial S^2}\right) + \frac{1}{2}\sigma^2 S^2 f\left(\frac{\partial^2 y}{\partial {u_2}^2}\left(\frac{\partial u_2}{\partial S}\right)^2 + \frac{\partial y}{\partial u_2}\frac{\partial^2 u_2}{\partial S^2}\right) + \sigma^2 S^2 f\left(\frac{\partial^2 y}{\partial u_1 \partial u_2}\frac{\partial u_2}{\partial S}\frac{\partial u_1}{\partial S}\right)$$
$$+ (r_{1d} - r_{1f})S f\left(\frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial S} + \frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial S}\right) + \frac{\partial f}{\partial t}y$$

$$+f\frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial t} + f\frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial t} - (r_{1d} - r_{1f})fy = 0 \tag{A.6}$$

Now, we can solve Eq. (26) by using the « Head Transfer » equation for the function y:

$$\frac{\partial y}{\partial u_2} = \frac{\partial^2 y}{\partial {u_1}^2} \tag{A.7}$$

Let us search $f(t), u_1(S,t), u_2(S,t)$; thus:

$$-(r_{1d})fy + \frac{\partial f}{\partial y} = 0 \rightarrow (r_{1d})f = \frac{\partial f}{\partial y} \rightarrow f(t) = e^{r_{1d}T} \tag{A.8}$$

We designate by the following: $a^2 = \frac{1}{2}\sigma^2 S^2 \left(\frac{\partial u_1}{\partial S}\right)^2$, simplify by $f(t)$, and rewrite the equation to identify the different terms.

So, $\frac{1}{2}\sigma^2 S^2\left(\frac{\partial^2 y}{\partial {u_1}^2}\left(\frac{\partial u_1}{\partial S}\right)^2\right) + \frac{1}{2}\sigma^2 S^2\left(\frac{\partial y}{\partial u_2}\frac{\partial^2 u_2}{\partial S^2}\right) + (r_{1d} - r_{1f})S\left(\frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial S}\right) + \frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial t}$ is canceled when: $-a^2 = \frac{1}{2}\sigma^2 S^2\left(\frac{\partial^2 u_2}{\partial S^2}\right) + (r_{1d} - r_{1f})S\left(\frac{\partial u_2}{\partial S}\right) + \frac{\partial f}{\partial t}y + \frac{\partial u_2}{\partial t}$

Assuming that

$$\frac{\partial u_2}{\partial S} = \frac{\partial^2 u_2}{\partial S^2} = 0 \tag{A.9}$$

We find that $\frac{\partial u_2}{\partial t} = -a^2$, then,

$$u_2 = -a^2 T \tag{A.10}$$

In these conditions, the EDP is written as follows:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 u_1}{\partial S^2} + (r_{1d} - r_{1f})S\frac{\partial u_1}{\partial S} + \frac{\partial u_1}{\partial t} = 0 \tag{A.11}$$

Using the expression of a^2, we get:

$$\frac{\partial u_1}{\partial S} = \sqrt{2}\,\frac{a}{\sigma}\frac{1}{S} \tag{A.12}$$

Then,

$$u_1 = \sqrt{2}\ \frac{a}{\sigma} ln\left(\frac{S}{X}\right) + b(t) \tag{A.13}$$

Replacing (33) in (31), we get:

$$\frac{1}{2}\sigma^2 S^2\left(-\frac{1}{S^2}\sqrt{2}\ \frac{a}{\sigma} + \frac{\partial b}{\partial t}\right) + \left(r_{1d} - r_{1f}\right)S\sqrt{2}\ \frac{a}{\sigma}\frac{1}{S} = 0 \tag{A.14}$$

But,

$$-\frac{1}{2}\sigma\sqrt{2} + \frac{\partial b}{\partial t} + \left(r_{1d} - r_{1f}\right)\frac{a}{\sigma}\sqrt{2} = 0 \tag{A.15}$$

Then,

$$\frac{\partial b}{\partial t} = \frac{\sigma\sqrt{2}}{\sigma}\left(\frac{\sigma^2}{2} - \left(r_{1d} - r_{1f}\right)\right) \tag{A.16}$$

$$=> b(t) = \frac{\sigma\sqrt{2}}{\sigma}\left(\frac{\sigma^2}{2} - \left(\left(r_{1d} - r_{1f}\right)\right)\right)T \tag{A.17}$$

$$=> u_1 = \frac{\sigma\sqrt{2}}{\sigma}\left[ln\frac{S}{X} + \left(\frac{\sigma^2}{2} - \left(r_{1d} - r_{1f}\right)\right)T\right] \tag{A.18}$$

Because $C(S,t^*) = f(t)y(\ u_1,u_2)$ and $\frac{\partial y}{\partial u_2} = \frac{\partial^2 y}{\partial {u_1}^2}$, the result at maturity is written as follows:

$$C(S,t^*) = [y(u_1(S,t^*),u_2(S,t^*))] = y\left[\frac{a\sqrt{2}}{\sigma} ln\frac{S}{X}, 0\right] \tag{A.19}$$

The solution of the Head Transfer equation is of the following form:

$$y(\ u_1,u_2) = \frac{1}{\sqrt{2\pi u_2}}\int_{-\infty}^{+\infty} u_0(\ \in\ )e^{\frac{-\epsilon^2}{2u_2}} d\in \tag{A.20}$$

The value of the call at the date of maturity can be expressed as follows:

$$C(S,t^*) = y(k,0) = \begin{cases} X\left[e^{\frac{-\epsilon^2}{a\sqrt{2}}} - 1\right], \text{if } k \geq 0 \\ 0, \text{otherwise} \end{cases} \tag{A.21}$$

where, $k = \frac{a\sqrt{2}}{\sigma} ln\left(\frac{S}{X}\right)$

Appendix B: Visual Basic program for computing European currency call option by using the standard G–K model

```
Function Call_GK(S, K, r_d, r_f, sigma, T)
    d1 = ((Application.Ln(S / (K))) / ((sigma) * Sqr(T)) + (rd - rf) * T / ((sigma) * Sqr(T)) + (0.5 *
    sigma) * Sqr(T))
    d2 = d1 - sigma * Sqr(T)
    nd1 = Application.NormSDist(d1)
    nd2 = Application.NormSDist(d2)
    Call_GK = (S * Exp(−r_f * T) * nd1) - (K * Exp(−r_d * T) * nd2)
    End Function
```

Appendix C: Visual Basic program for computing European currency call option by using the new G–K model

```
Function Call_GKIC(S, K, r_1d, r_1f, sigma, T)
    d1 = ((Application.Ln(S / (K))) / ((sigma) * Sqr(T)) + (r_1d - r_1f) * T / ((sigma) * Sqr(T)) + (0.5 *
    sigma) * Sqr(T))
    d2 = d1 - sigma * Sqr(T)
    nd1 = Application.NormSDist(d1)
```

(*continued on next page*)

(*continued*)

```
nd2 = Application.NormSDist(d2)
Call_GKIC = (S * Exp(−r1f * T) * nd1) - (K * Exp(−r1d * T) * nd2)
End Function
```

Appendix D: Visual Basic program for computing implied volatility

```
Function IVGKICM (Market_Call As Double, K As Double, S As Double, r1d As Double, r1f As Double, T
    As Double)
    sigma = 0.2
    dv = 0.0001 + 1
    While Abs(dv) > 0.0001
    d1 = ((Application.Ln(S / (K))) / ((sigma) * Sqr(T)) + (r1d - r1f) * T / ((sigma) * Sqr(T)) + (0.5 *
    sigma) * Sqr(T))
    d2 = d1 - sigma * Sqr(T)
    nd1 = Application.NormSDist(d1)
    nd2 = Application.NormSDist(d2)
    erreurprice = (S * Exp(−r1f * T) * nd1) - (K * Exp(−r1d * T) * nd2)–Market_Call
    Vega = S * Sqr(T / 3.1415926 / 2) * Exp(−0.5 * d1 ^ 2)
    dv = erreurprice / Vega
    sigma = sigma - dv
    Wend
    IVGKICM = sigma
    End Function
```

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