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Impact of inspection errors on the performance measures of a general repeat inspection plan

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Multi-characteristic critical components exist in many systems. Such components may be a part of an aircraft, space shuttle or a gas ignition system. An inspection plan for such components has been proposed in quality control that deals with several types of classification errors made by the inspector. In this paper, performance measures for this plan are defined and the statistical and economic impact of the several types of inspection errors on these measures is investigated. The impact of the errors is studied by conducting sensitivity analysis on the errors utilizing computer software which implements an algorithm that determines the optimal parameters of the model of the plan. The behaviour of the performance measures upon variation in the levels of errors is investigated. The results indicate that these errors have a considerable effect on the performance measures of the inspection plan.

Keywords: Inspection errors; General repeat inspection plan; Performance measures

1. Introduction

A component is critical if it causes a disaster or a very high cost upon failure. Such components may be a part of an aircraft, a space shuttle or a complex gas ignition system. For critical components, a common practice in industry is to institute multiple inspections (Chandra and Schall 1998). The reason for multiple inspections is simply that inspection is never perfect. There is always the possibility of misclassification. All misclassifications have costs such as the cost of misclassifying a good component as rework. In the case of critical components the cost of false acceptance is much higher than the cost of false rejection, because falsely accepted components may result in system failure, which may involve human losses. Therefore, it is perceived and shown that repeat inspections are likely to reduce the costs of the errors and increase the cost of inspection.

Considering the fact that an inspector, in general, can make three types of classifications of a product, an inspection plan and model have been proposed in quality

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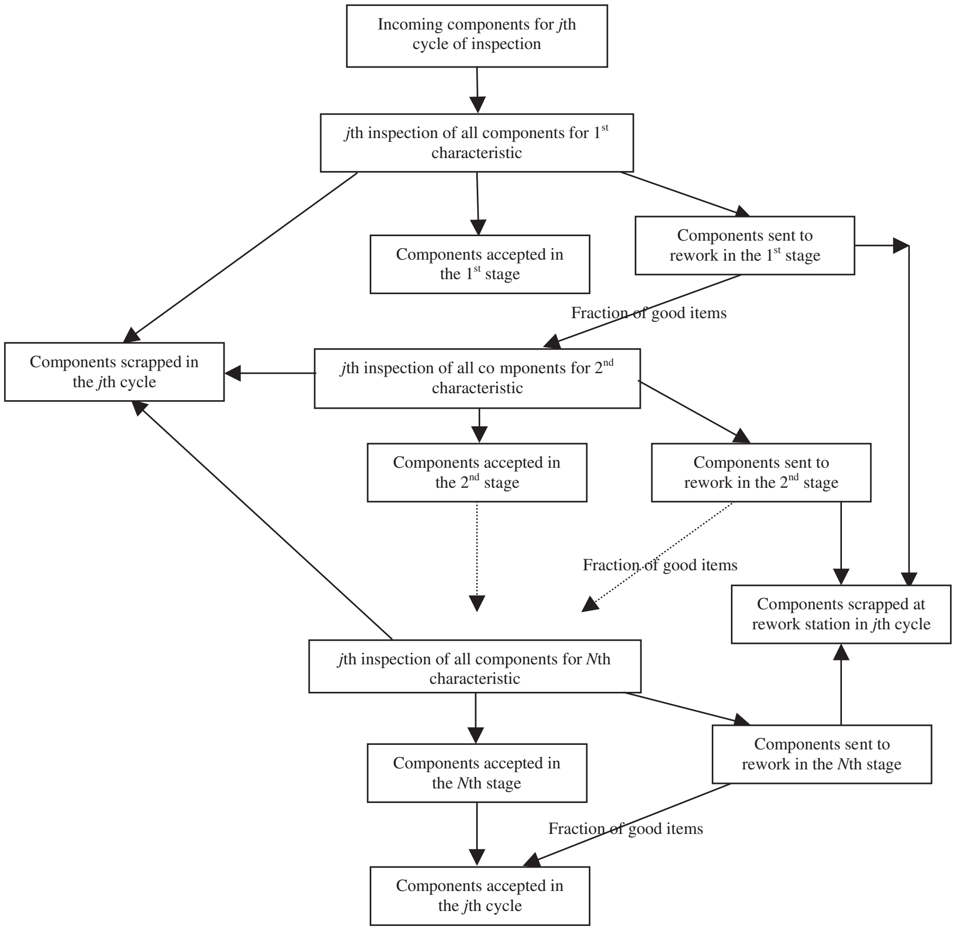

Figure 1. Inspection plan for the *j*th cycle, $j = 1, 2, \ldots, n$.

control. This model assumes several characteristics in a product with known incoming quality and suggests six types of inspection errors. The inspection plan for such components is described as follows: an inspector inspects one particular characteristic for each component entering the inspection process, and classifies it as meeting specifications, scrap or rework. All the accepted components and those that are found to be meeting specifications at the rework station go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics have been inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and this process is repeated a total of n times before the components are finally accepted. Here, n is the optimal number of inspections necessary to minimize the total cost per accepted component. Finally, the accepted components will be those that are accepted in the nth cycle, and the total scrapped components will be the sum of those scrapped in the first, second, ..., nth cycle. Figure 1 depicts the inspection. The model that depicts the above plan is used to determine the optimal number of repeat inspections.

Inspection errors have a significant effect on the performance measures of an inspection plan. These errors are taken to be fixed in the literature. However, these errors are not fixed, they change with many factors such as time allowed for inspection, work environment, inspector fatigue, and other work- and inspection-related factors. A need exists to study the impact of errors on the performance of the above repeat inspection plan and to examine their practical implications.

The purpose of this paper is to study the statistical and economic impact of several types of classification errors on the performance measures of a general inspection plan. The performance measures used in the study are average outgoing quality (AOQ) and average total inspection (ATI). The impact of the errors is investigated by conducting sensitivity analysis using the model in section 3. In this study, the errors were varied between 0.01 and 0.15 (Duffuaa 1996).

2. Literature review

Ayoub *et al.* (1970) defined the mean inspection error as the average number of defectives classified as good items by the inspector. They gave a formula for Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) from a single sampling plan under inspection error. Collins *et al.* (1973) considered the effects of inspection error on the probability of acceptance, average outgoing quality and average total inspection. They examined these measures under both replacement and non-replacement assumptions. Bennet *et al.* (1974) investigated the effect of error on a single sampling plan with known incoming quality. Sylla and Drury (1995) proposed a model that uses a form of SDT to predict inspector performance in order to improve system performance. They presented the concept of lability to characterize the inspector's ability to respond to the costs, penalties and probabilities involved in the inspection decision.

Raouf *et al.* (1983) were the first to develop a model for determining the optimal number of repeat inspections for multi-characteristic components to minimize the total expected cost per accepted component due to Type I error, Type II error and the cost of inspection. Tang and Schneider (1987) investigated the economic and statistical effects of inspection error on the complete inspection plan. They developed two models with consideration of inspection error under different rework schemes and then made a comparison with a model without consideration of inspection error. Lee (1988) provided a simplified version of the model given by Raouf *et al.* (1983) to evaluate the costs involved in the multiple-cycle inspection schemes for multi-characteristic components. He extended the results for the case where the probabilities of defectives are random. Chandra and Schall (1998) studied the effect of replicate measurements on average outgoing quality and the average total inspection. They obtained the optimum number of replications based on the total cost of inspection. Duffuaa and Raouf (1989) developed three mathematical optimization models for multi-characteristic repeat inspection. Duffuaa and Raouf (1990) established an optimal rule for sequencing characteristics for inspection in the plan proposed by Raouf *et al.* (1983). Duffuaa and Nadeem (1994) extended the model proposed by Raouf *et al.* (1983) to cases where defective rates are statistically dependent. They proposed an algorithm to determine the optimal number of repeat inspections and sequenced the characteristics for inspection in order to minimize the total expected cost. Duffuaa and Al-Najjar (1995) proposed a new inspection plan for critical multi-characteristic components with a variable number of inspections for different characteristics. They proposed an algorithm to determine the optimal number of repeat inspections and sequenced the characteristics for inspection in order to minimize the total expected cost. Duffuaa (1996) investigated the statistical and economic impact of inspector errors on the performance measures, i.e. ATI, AOQ and ETC, of a complete inspection plan. He concluded that Type I and Type II errors have a significant effect on the performance measures of repeat inspection plans. Chen and Labbrecht (1997) proposed a model to optimize the sequence and frequency of inspections of multi-characteristic components. They used marginal analysis and gave an efficient algorithm to find the optimal plan. Hong *et al.* (1998) developed economic screening procedures when the rejected items are reworked. Screening procedures based on the performance variable of interest and a correlated variable are considered. They considered the cost incurred by imperfect quality, reprocessing cost and inspection cost. Duffuaa and Khan (2002) provided a more realistic formulation to the models for inspecting multi-characteristic components. They incorporated the misclassifications of scrap and rework into the total cost of inspection. Khan and Duffuaa (2002) studied the effect of inspection errors on the performance measures of different inspection plans. Most of the models in the literature are based on the plan given by Raouf *et al.* (1983) and Duffuaa and Al-Najjar (1995).

3. Assumptions of the model

In order to develop the model in the next section the following assumptions have been made:

1. Estimates for the cost of inspection, the cost of false rejection and the cost of false acceptance are available.
2. The probabilities of scrap and rework rates are assumed to be known.
3. Characteristics' defective rates are assumed to be statistically independent.
4. The probabilities with which the inspector makes a misclassification are assumed to be known.
5. The inspection process at the rework station is assumed to be error-free.

Assumptions 1, 2 and 4 are not limiting factors for developing the model. However, they must be at hand in order to apply the model to any situation. Estimates for the parameters assumed known in 1, 2 and 4 can be obtained via the statistical design of experiments. Also, production process capabilities can be used to obtain good estimates of the quantities assumed known in 2. The reliability of the estimates of these parameters may impact the results of the model and affect the decision to be made in practical applications. Two approaches can be used to mitigate the effect of inaccurate estimates of the parameters of the model. The first approach is to develop estimates with high confidence and the second is to conduct a sensitivity analysis on these parameters to investigate their impact on the model results.

Assumption 3 is a limiting factor in the case where the characteristics' defective rates are statistically dependent. Independence must be checked prior to applying the model. However, the model can easily be extended to the dependency situation. In many situations, independence is a good starting point for the dependency case.

Assumption 5 is limiting for using the model when manual inspection is used, due to the errors resulting from the inspectors and instrument drift. However, technology has advanced and in an automated inspection environment, errors can be negligible. In many situations this error-free inspection may be costly.

4. Model description

In order to investigate the impact of errors on the inspection plan, it is necessary to describe the model that depicts the plan. The model has been developed for components having several characteristics, such as an aircraft, a space shuttle or a complex ignition system. Each characteristic can be in one of the following three classes: good (meeting specifications), rework or scrap. The incoming quality of the components, i.e. the probabilities of being good, scrap or rework, is assumed to be known. A component is classified as good only if all the characteristics meet the quality specifications. The probabilities with which the inspector makes a misclassification are assumed to be known. Three different types of costs are considered: (i) the cost of false rejection of good components (i.e. classifying these components as scrap or rework); (ii) the cost of false acceptance of components that are either rework or scrap; and, finally, (iii) the cost of inspection. The inspection process at the rework station is assumed to be error-free and therefore it costs more to inspect there. The estimates for these costs are assumed to be available.

The model notation is given in table 1.

4.1 *Basic relationships*

The probability of the ith characteristic being good, rework or scrap varies from cycle to cycle. The relationship between ${}^{j}P_{i}$ and P_i is given as

$$ {}^{j}P_{i\mathrm{g}} = \frac{P_{i\mathrm{g}}(1-E_{i\mathrm{gs}})^{j-1}}{P_{i\mathrm{g}}(1-E_{i\mathrm{gs}})^{j-1} + P_{i\mathrm{r}}E_{i\mathrm{rg}}^{j-1} + P_{i\mathrm{s}}E_{i\mathrm{sg}}^{j-1}}, \tag{1} $$

$$ {}^{j}P_{i\mathrm{r}} = \frac{P_{i\mathrm{r}}E_{i\mathrm{rg}}^{j-1}}{P_{i\mathrm{g}}(1-E_{i\mathrm{gs}})^{j-1} + P_{i\mathrm{r}}E_{i\mathrm{rg}}^{j-1} + P_{i\mathrm{s}}E_{i\mathrm{sg}}^{j-1}}, \tag{2} $$

$$ {}^{j}P_{i\mathrm{s}} = \frac{P_{i\mathrm{s}}E_{i\mathrm{sg}}^{j-1}}{P_{i\mathrm{g}}(1-E_{i\mathrm{gs}})^{j-1} + P_{i\mathrm{r}}E_{i\mathrm{rg}}^{j-1} + P_{i\mathrm{s}}E_{i\mathrm{sg}}^{j-1}}. \tag{3} $$

The probability of a component being good, rework or scrap changes in each cycle and is given by

$$ {}^{j}PG = \prod_{i=1}^{N}\left[\frac{P_{i\mathrm{g}}(1-E_{i\mathrm{gs}})^{j-1}}{P_{i\mathrm{g}}(1-E_{i\mathrm{gs}})^{j-1} + P_{i\mathrm{r}}E_{i\mathrm{rg}}^{j-1} + P_{i\mathrm{s}}E_{i\mathrm{sg}}^{j-1}}\right], \tag{4} $$

$$ {}^{j}PR = \sum_{i=1}^{N} P_{i\mathrm{r}} - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} {}^{j}P_{i\mathrm{r}}{}^{j}P_{j\mathrm{r}} + \sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^{N} {}^{j}P_{i\mathrm{r}}{}^{j}P_{j\mathrm{r}}{}^{j}P_{k\mathrm{r}} - \cdots \pm {}^{j}P_{1\mathrm{r}}{}^{j}P_{2\mathrm{r}}\ldots{}^{j}P_{N\mathrm{r}}, \tag{5} $$

Table 1. Model notation.

M_j	Number of components entering the *j*th cycle of inspection
N	Number of characteristics in each component to be inspected
${}^jP_{is}$	Probability of the *i*th characteristic in the sequence of inspection being scrap while entering the *j*th cycle of inspection
${}^jP_{ir}$	Probability of the *i*th characteristic in the sequence of inspection being rework while entering the *j*th cycle of inspection
${}^jP_{ig}$	Probability of the *i*th characteristic in the sequence of inspection being good while entering the *j*th cycle of inspection
${}^jPS$	Probability of a component being scrap while entering the *j*th cycle of inspection
${}^jPR$	Probability of a component being rework while entering the *j*th cycle of inspection
${}^jPG$	Probability of a component being good while entering the *j*th cycle of inspection
E_{igr}	Probability of classifying the *i*th good characteristic in the sequence of inspection as rework
E_{igs}	Probability of classifying the *i*th good characteristic in the sequence of inspection as scrap
E_{irg}	Probability of classifying the *i*th rework characteristic in the sequence of inspection as good
E_{irs}	Probability of classifying the *i*th rework characteristic in the sequence of inspection as scrap
E_{isg}	Probability of classifying the *i*th scrap characteristic in the sequence of inspection as good
E_{isr}	Probability of classifying the *i*th scrap characteristic in the sequence of inspection as rework
$M_{i,j}$	Number of components entering the *i*th stage of inspection in the *j*th cycle
$PG_{i,j}$	Probability of a component being good in the *i*th stage of the *j*th cycle
F_{iab}	Number of components of type a misclassified as type b in the *i*th stage
$FR_{i,j}$	Number of components falsely sent to rework in the *i*th stage of the *j*th cycle
$FS_{i,j}$	Number of components falsely sent to scrap in the *i*th stage of the *j*th cycle
$FA_{i,j}$	Number of falsely accepted components in the *i*th stage of the *j*th cycle
$CA_{i,j}$	Number of correctly accepted components in the ith stage of the *j*th cycle
$R_{i,j}$	Rate of rejection of components due to the *i*th characteristic in the sequence of inspection in the *j*th cycle
C_{fgs}	Cost of misclassifying a good component to be scrap
C_{frs}	Cost of misclassifying a rework component to be scrap
$A(j)$	Number of accepted components in the *j*th cycle
$CFR(j)$	Cost of false rejection in the *j*th cycle
$CFA(j)$	Cost of false acceptance in the *j*th cycle
$CI(j)$	Cost of inspection in the *j*th cycle
$TCFR$	Total cost of false rejection
$TCFA$	Total cost of false acceptance
TCI	Total cost of inspection
TA	Total number of accepted components
$E(tc)\|j$	Expected total cost per accepted component after *j* cycles of inspection
$E(\cdot)$	Expected value of the argument inside the parentheses

and

$$ {}^jPS = 1 - {}^jPG - {}^jPR. \tag{6} $$

When there is no inspection, the expected total cost per accepted unit will simply be the cost due to the false acceptance of all defective components, i.e.

$$ E(tc)|_{j=0} = C_a(1 - PG), \tag{7} $$

where C_a is the cost of false acceptance per accepted component and PG is given by equation (4).

The expected total cost per accepted component, after n cycles of inspection, can be written as

$$E(tc)|_{j=n} = [TCFR + TCFA + TCI]/TA, \tag{8}$$

where $TCFR$, $TCFA$, TCI and TA are defined in table 1.

4.2 *Cost minimization model*

The objective of this model is to determine the optimal inspection plan for multi-characteristic components, where an inspector has to make several classifications. The model minimizes the total cost per accepted component resulting from the several types of misclassification errors made by the inspector, and the cost of inspection. Given the basic relationships in the previous section, a mathematical expression for the expected cost subject to the relationships governing this situation is derived.

In order to derive the expected cost of inspection after n cycles of inspection, an analysis of cycle 1 of the inspection is necessary. All the components entering cycle 1 go to the first inspector, who inspects the first characteristic of each component in order to classify it as good, rework or scrap. This is the first stage of inspection.

4.2.1 Stage 1 of cycle 1. The number of components entering into the first stage is

$$M_{1,1} = M_1. \tag{9}$$

A component may have the following probabilities:

$$PG_{1,1} = PG, \tag{10}$$

$$PR_{1,1} = {}^1PR = \sum_{i=1}^{N} P_{ir} - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} P_{ir}P_{jr} + \sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^{N} P_{ir}P_{jr}P_{kr} - \cdots \pm P_{1r}P_{2r}\ldots P_{Nr}, \tag{11}$$

$$PS_{1,1} = 1 - PG_{1,1} - PR_{1,1}. \tag{12}$$

E(the number of components falsely sent to rework) is

$$FR_{1,1} = M_{1,1}(PG_{1,1}E_{1gr} + PS_{1,1}E_{1sr}). \tag{13}$$

E(the number of components falsely sent to scrap) is

$$FS_{1,1} = M_{1,1}(PG_{1,1}E_{1gs} + PR_{1,1}E_{1rs}). \tag{14}$$

E(the number of falsely accepted components) is

$$FA_{1,1} = M_{1,1}[P_{1r}E_{1rg} + P_{1s}E_{1sg} + (1 - PG - P_{1r} - P_{1s})(1 - E_{1gr} - E_{1gs})]. \tag{15}$$

E(the number of correctly accepted components) is

$$CA_{1,1} = M_{1,1}PG_{1,1}(1 - E_{1gr} - E_{1gs}). \tag{16}$$

All accepted components in this stage go to the second inspector, who inspects the second characteristic of each component. The following can be expected in the second stage of inspection.

4.2.2 Stage 2 of cycle 1

$$\begin{aligned} M_{2,1} &= FA_{1,1} + CA_{1,1} + FGR_{1,1} \\ &= M_{1,1}[P_{1\mathrm{r}}E_{1\mathrm{rg}} + P_{1\mathrm{s}}E_{1\mathrm{sg}} + (1 - P_{1\mathrm{r}} - P_{1\mathrm{s}})(1 - E_{1\mathrm{gr}} - E_{1\mathrm{gs}}) + PGE_{1\mathrm{gr}}], \end{aligned} \tag{17}$$

$$PG_{2,1} = (1 -{}^2P_{1\mathrm{r}} -{}^2P_{1\mathrm{s}}) \prod_{i=2}^{N} (1 - P_{i\mathrm{r}} - P_i), \tag{18}$$

$$PR_{2,1} = {}^2P_{1\mathrm{r}} + \sum_{i=2}^{N} P_{i\mathrm{r}} - \sum_{i=2}^{N} {}^2P_{1\mathrm{r}}P_{i\mathrm{r}} + \sum_{i=2}^{N-1}\sum_{j=i+1}^{N} {}^2P_{i\mathrm{r}}P_{j\mathrm{r}} - \cdots \pm {}^2P_{1\mathrm{r}}P_{2\mathrm{r}} \ldots P_{N\mathrm{r}},$$

$$PS_{2,1} = 1 - PG_{2,1} - PR_{2,1}, \tag{19}$$

$$FR_{2,1} = M_{2,1}(PG_{2,1}E_{2\mathrm{gr}} + PS_{2,1}E_{2\mathrm{sr}}), \tag{20}$$

$$FS_{2,1} = M_{2,1}(PG_{2,1}E_{2\mathrm{gs}} + PR_{2,1}E_{2\mathrm{rs}}), \tag{21}$$

$$\begin{aligned} FA_{2,1} &= M_{2,1}[P_{2\mathrm{r}}E_{2\mathrm{rg}} + P_{2\mathrm{s}}E_{2\mathrm{sg}} + (1 - PG_{2,1} - P_{2\mathrm{r}} - P_{2\mathrm{s}} +{}^2P_{1\mathrm{r}}P_{2\mathrm{r}} +{}^2P_{1\mathrm{s}}P_{2\mathrm{s}}) \\ &\quad \times (1 - E_{2\mathrm{gr}} - E_{2\mathrm{gs}})], \end{aligned} \tag{22}$$

$$CA_{2,1} = M_{2,1}PG_{2,1}(1 - E_{2\mathrm{gr}} - E_{2\mathrm{gs}}). \tag{23}$$

Similarly, we can generalize these expressions for the Nth stage.

4.2.3 Stage N

$$\begin{aligned} M_{N,1} = M_{N-1,1}\Bigg[& P_{N-1\mathrm{r}}E_{N-1\mathrm{r}} + P_{N-1\mathrm{s}}E_{N-1\mathrm{sg}} \\ & + \Bigg(1 - P_{N-1\mathrm{r}} - P_{N-1\mathrm{s}} + \sum_{k=1}^{N-2} P_{k\mathrm{r}}P_{N-1,\mathrm{r}} - \sum_{k=1}^{N-3}\sum_{l=k+1}^{N-2} P_{k\mathrm{r}}P_{l\mathrm{r}}P_{N-1,\mathrm{r}} \\ & + \cdots \pm P_{1\mathrm{r}}P_{2\mathrm{r}} \ldots P_{N-1,\mathrm{r}} + \sum_{k=1}^{N-2} P_{k\mathrm{s}}P_{N-1\mathrm{s}} + \sum_{k=1}^{N=3}\sum_{l=k+1}^{N=2} P_{k\mathrm{s}}P_{l\mathrm{s}}P_{N-1\mathrm{s}} \\ & + \cdots \pm P_{1\mathrm{s}}P_{2\mathrm{s}} \ldots P_{N-1\mathrm{s}}\Bigg)(1 - E_{N-1\mathrm{gr}} - E_{N-1\mathrm{gs}}) + PG_{N-1,1}E_{N-1\mathrm{gr}}\Bigg], \end{aligned} \tag{24}$$

$$PG_{N,1} = (1 - P_{N\mathrm{r}} - P_{N\mathrm{s}}) \prod_{i=1}^{N-1} (1 -{}^2P_{i\mathrm{r}} -{}^2P_{i\mathrm{s}}), \tag{25}$$

$$\begin{aligned} PR_{N,1} &= \sum_{i=1}^{N-1} {}^2P_{i\mathrm{r}} + P_{N\mathrm{r}} - \sum_{i=1}^{N-1} {}^2P_{i\mathrm{r}}P_{N\mathrm{r}} + \sum_{i=2}^{N-1} {}^2P_{1\mathrm{r}}{}^2P_{i\mathrm{r}}P_{i\mathrm{r}} \\ &\quad - \cdots \pm {}^2P_{1\mathrm{r}}{}^2P_{2\mathrm{r}} \ldots {}^2P_{N-1\mathrm{r}}P_{N\mathrm{r}}, \end{aligned} \tag{26}$$

$$PS_{N,1} = 1 - PG_{N,1} - PR_{N,1}, \tag{27}$$

$$FR_{N,1} = M_{N,1}(PG_{N,1}E_{N\mathrm{gr}} + M_{N,1}PS_{N,1}E_{N\mathrm{sr}}), \tag{28}$$

$$FS_{N,1} = M_{N,1}(PG_{N,1}E_{N\mathrm{gs}} + PR_{N,1}E_{N\mathrm{rs}}), \tag{29}$$

$$FA_{N,1} = M_{N,1}\Bigg[P_{N\mathrm{r}}E_{N\mathrm{rg}} + P_{N\mathrm{s}}E_{N\mathrm{sg}} + \Bigg(1 - PG_{N,1} - P_{N\mathrm{r}} - P_{N\mathrm{s}} + \sum_{k=1}^{N-1} P_{k\mathrm{r}}P_{N\mathrm{r}} - \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} P_{k\mathrm{r}}P_{l\mathrm{r}}P_{N\mathrm{r}} + \cdots \pm P_{1\mathrm{r}}P_{2\mathrm{r}}\ldots P_{N-1,r} + \sum_{k=1}^{N-1} P_{k\mathrm{s}}P_{N-1\mathrm{s}} + \sum_{k=1}^{N-2}\sum_{l=k+1}^{N-1} P_{k\mathrm{s}}P_{l\mathrm{s}}P_{N\mathrm{s}} + \cdots \pm P_{1\mathrm{s}}P_{2\mathrm{s}}\ldots P_{N-1\mathrm{s}}\Bigg)(1 - E_{N\mathrm{gr}} - E_{N\mathrm{gs}})\Bigg], \quad (30)$$

$$CA_{N,1} = M_{N,1}PG_{N,1}(1 - E_{N\mathrm{gr}} - E_{N\mathrm{gs}}). \quad (31)$$

This completes one cycle of inspection and the results of this cycle are described by the following equations.

The number of accepted components after completing the first cycle is

$$A(1) = FA_{N,1} + CA_{N,1} + FR_{N,1\mathrm{g}}. \quad (32)$$

The cost of false rejection is

$$CFR(1) = C_{\mathrm{fgs}}\sum_{i=1}^{N} F_{i\mathrm{gs}} + C_{\mathrm{frs}}\sum_{i=1}^{N} F_{i\mathrm{rs}}. \quad (33)$$

The cost of false acceptance is

$$CFA(1) = C_a FA_{N,1}. \quad (34)$$

The cost of inspection is

$$CI_1(1) = \sum_{i=1}^{N} C_{i1}M_{i1}, \quad (35)$$

$$CI_2(1) = \sum_{i=1}^{N} C_{i2}\big[M_{i,1}PG_{i,1}E_{i\mathrm{gr}} + M_{i,1}PS_{i,1}E_{i\mathrm{sr}} + M_{i,1}PR_{i,1}(1 - E_{i\mathrm{rg}} - E_{i\mathrm{rs}})\big] \quad (36)$$

where C_{i1} is the cost of inspecting a component at the ith stage, and C_{i2} is the cost of inspecting a component at the ith rework station.

We assume that very sensitive equipment (which is error-free) is installed there for inspection. So,

$$C_{i2} > C_{i1}.$$

Therefore, the total cost of inspection for cycle 1 is

$$CI(1) = CI_1(1) + CI_2(1). \quad (37)$$

E(the total cost per accepted component after one cycle of inspection) is

$$E(tc)|_{j=1} = [CFR(1) + CFA(1) + CI(1)]/[A(1)], \quad (38)$$

where $CFR(1)$, $CFA(1)$, $CI(1)$ and $A(1)$ are given by equations (42), (43), (46) and (41), respectively.

Before proceeding to the next cycle, it is worth noting that it was reported by Raouf *et al.* (1983) that the cost of false acceptance and the cost of false rejection are independent of the way the characteristics are sequenced for inspection. However, the cost of inspection $CI(n)$ does depend on the way the characteristics are sequenced for inspection. As $CI(n)$ is a function of the cost of inspection of the characteristics and the rejection rate, we would have inspected the characteristics with the highest rejection rate first and those with the lowest rejection rate last if the cost of inspection of all the characteristics were the same. And if the rejection rates for all the characteristics were the same, we would have inspected the characteristic with the lowest cost of inspection first and that with the highest cost of inspection last to minimize the cost of inspection. These two observations suggest that the characteristic with the minimum ratio of the cost of inspection to rejection rate should be inspected first. Based on this, Duffuaa and Raouf (1990) gave a rule that proved to be optimal for minimizing the cost of inspection within each inspection cycle. To minimize the cost of inspection, this rule should be applied in each cycle. The rule states: at inspection cycle j, compute the ratio $C_i/R_{i,j}$ for all i. The formula for $R_{i,j}$ is given by equation (36). Then, for each component, first inspect the characteristic with the smallest ratio and, last, inspect that with the largest ratio. This rule ensures that $CI(j)$ is minimized within cycle j.

From the analysis of cycle 1, it can easily be seen that, after this cycle, we can proceed in a similar manner to compute the cost of false rejection, the cost of false acceptance and the cost of inspection in the following cycles. Hence, by symmetry, the results of the nth cycle can be obtained:

$$A(n) = FA_{N,n} + CA_{N,n} + FR_{N,ng}, \tag{39}$$

$$CFR(n) = C_{\text{fgs}} \sum_{i=1}^{N} F_{i\text{gs}} + C_{\text{frs}} \sum_{i=1}^{N} F_{i\text{rs}}, \tag{40}$$

$$CFA(n) = C_a FA_{N,n}, \tag{41}$$

$$CI(n) = CI_1(n) + CI_2(n). \tag{42}$$

The ratio to determine the optimal ordering of characteristics in the nth cycle is $C_i/R_{i,n}$, $i = 1, \ldots, N$ where

$$C_i = C_{i1} + P_{i\text{r}} C_{i2}, \tag{43}$$

$$R_{i,n} = {}^{n}P_{i\text{s}}(1 - E_{i\text{sg}} - E_{i\text{sr}}) + (1 - {}^{n}P_{i\text{s}})(E_{i\text{gs}} + E_{i\text{rs}}). \tag{44}$$

After n cycles of inspection we must determine the expected total cost of inspection per accepted component. It is determined from the total cost of false rejection, $TCFR$, the total cost of false acceptance, $TCFA$, and the total cost of inspection:

$$TCFR = \sum_{j=1}^{n} [CFR(j)], \tag{45}$$

$$TCFA = CFA(n), \tag{46}$$

$$TCI = \sum_{j=1}^{n} [CI1(j) + CI2(j)]. \tag{47}$$

Total accepted components:

$$TA = A(n). \tag{48}$$

The above set of equations provides the basic relationships for the model; the purpose is to find the value of n that minimizes the expected total cost per accepted component. Table 2 gives an algorithm that can be used to find a local optimal n.

Table 2. Algorithm used to find a local optimal n.

Step 1	Compute PG and $E(tc)|_{j=0}$ using equations (4) and (7), respectively
Step 2	Set $j=j+1$. Compute ${}^{j}P_{ir}$ and ${}^{j}P_{is}$ using equations (2) and (3). Sequence the characteristics according to the ratio $C_i/R_{i,j}$ using equations (43) and (44)
Step 3	Compute ${}^{j}PG$, ${}^{j}PR$, ${}^{j}PS$, $PG_{i,j}$, $PR_{i,j}$, $PS_{i,j}$, $M_{i,j}$ and M_j using equations (4), (5), (6), (25), (26), (27), (24) and (32), respectively
Step 4	Compute $A(j)$, $CFR(j)$, $CFA(j)$, and $CI(j)$ using equations (39), (40), (41) and (42), respectively
Step 5	Compute $TCFR$, $TCFA$, TCI and TA using equations (45), (46), (47) and (48), respectively
Step 6	Compute $E(tc)|_j$ using equation (8)
Step 7	If $E(tc)|_j < E(tc)|_{j=1}$, go to step 2. Otherwise, stop ($n=j-1$)

5. Impact of inspection errors on performance measures

The above model depicts the plan that is employed in this section to investigate the impact of errors Esg (i.e. misclassifying a scrap component to be good) and Egs (i.e. misclassifying a good component to be scrap) on ATI, AOQ and ETC. A batch of 100 components, each with three characteristics, was inspected with known incoming quality, i.e. the probability that the component would be required to be reworked was 0.10, 0.05 and 0.05, and requiring scrap was 0.10, 0.05 and 0.15, respectively. The cost of inspection for the three characteristics is taken to be 100 at the inspection station and 5000 at the rework station. The cost of false acceptance is 100 000. The cost of misclassification of a good component to scrap is 10 000 and that of a rework component to scrap is 5000. The two errors, i.e. misclassifying a good component to be scrap or a scrap component to be good, were taken as 0.01, 0.03, 0.05, 0.10 and 0.15 to observe the effect on the expected total cost per accepted component (ETC), the average total inspection (ATI) and the average outgoing quality (AOQ). This range of errors was obtained from Bennett *et al*. (1974) and Duffuaa (1996) and the values of the costs were obtained from Duffuaa (1996). Fixing the other misclassification errors at the above five levels, the model was run 25 times each using the above five values of Esg and Egs. The model determines the optimal number of repeat inspections and simulates the inspection process for each pair of errors. The results of the runs for the five misclassification error levels are shown in tables A1–A5 of Appendix A. Each of these tables shows the pairs of errors and optimal values for ETC and AOQ, which are (1–PG), n and $A(n)$ for that pair of errors. Also, the tables show the total cost of false rejection ($TCFR$), the total cost of false acceptance ($TCFA$) and the total cost of inspection (TCI). All of these costs are taken per accepted component. Tables A1–A5 and figures 2–11 can now be examined to determine the impact of the errors on the two performance measures of the inspection plan, viz. AOQ and ATI.

5.1 *Impact of inspection errors on average outgoing quality (AOQ)*

The average quality of the lot leaving the inspection process is defined as

$$\text{AOQ} = \frac{\text{Number of defective components after inspection}}{\text{Total number of accepted components}}.$$

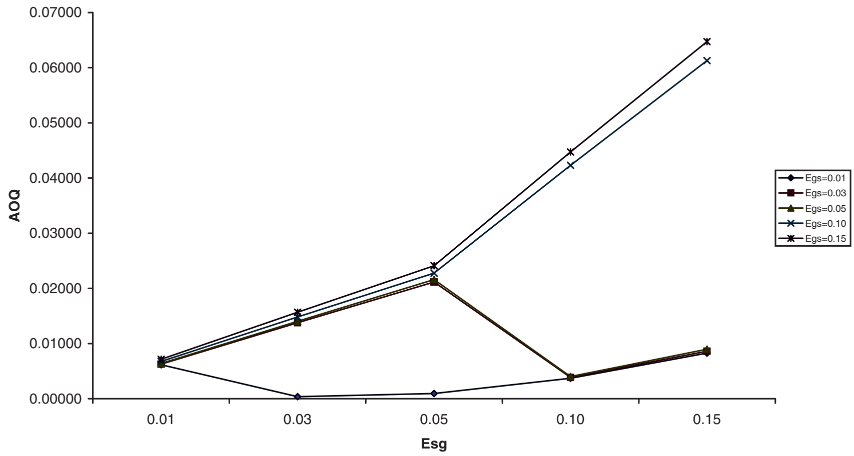

Figure 2. Effect of error Esg on average outgoing quality (AOQ). Egr=Erg=Ers=Esr=0.01.

Figures 2–6 show five curves each. Each curve shows the relationship between *E*sg and AOQ for a fixed level of *E*gs. The other misclassification errors are set at 0.01, 0.03, 0.05, 0.10 and 0.15, respectively, in the five figures. There is an unusual decrease in AOQ in each figure when *E*sg goes from one level to another for some fixed values of *E*gs. This can be explained by looking at tables A1–A5 in Appendix A: AOQ becomes so large that the model, in order to minimize AOQ, is forced to take one more cycle of inspection.

Figure 2 shows that AOQ increases as *E*gs increases. For *E*gs levels of 0.10–0.15 the increase in AOQ is similar to the case when *E*sg varies from 0.01 to 0.15. However, at an *E*gs level of 0.03 and 0.05, AOQ decreases by 81% when *E*sg goes from 0.05 to 0.10.

Figure 3 shows that AOQ increases as *E*gs increases. For *E*gs levels of 0.10–0.15 the increase in AOQ is similar to when *E*sg varies from 0.01 to 0.15. However, at an *E*gs level of 0.03 and 0.05, AOQ decreases by 84% when *E*sg goes from 0.05 to 0.10.

Figure 4 shows that AOQ increases as *E*gs increases. The decrease in AOQ at an *E*gs level of 0.05, when *E*sg goes from 0.10 to 0.15, is 80.7%, and the decrease in AOQ at an *E*gs level of 0.03, when *E*sg goes from 0.05 to 0.10, is 85.6%.

Figure 5 shows that AOQ increases as *E*gs increases. The decrease in AOQ at an *E*gs level of 0.05, when *E*sg goes from 0.10 to 0.15, is 80.9%, and the decrease in AOQ at an *E*gs level of 0.03, when *E*sg goes from 0.05 to 0.10, is 84.8%.

Similarly, figure 6 shows that AOQ increases as *E*gs increases. However, at an *E*gs level of 0.01 or 0.03, AOQ decreases by 82% when *E*sg goes from 0.05 to 0.10.

In general, it can be concluded that the AOQ level increases with increasing *E*gs and the extent of this increase in AOQ for increasing *E*sg decreases as the level of the other misclassification errors increases. This dictates cautious planning of the inspection process.

5.2 *Impact of inspection errors on average total inspection (ATI)*

The average total inspection is defined as the total number of inspections conducted in the optimal inspection plan. The inspection followed here is such that inspection is

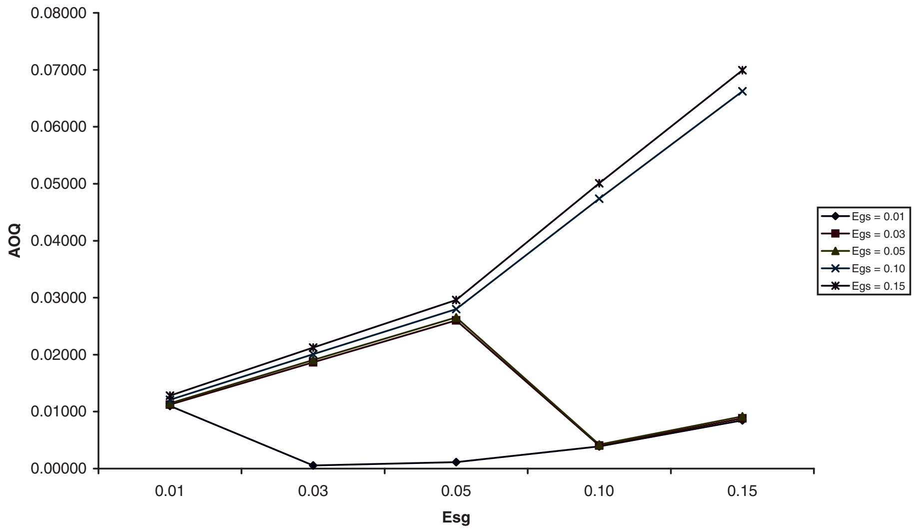

Figure 3. Effect of error Esg on average outgoing quality (AOQ). Egr=Erg=Ers=Esr=0.03.

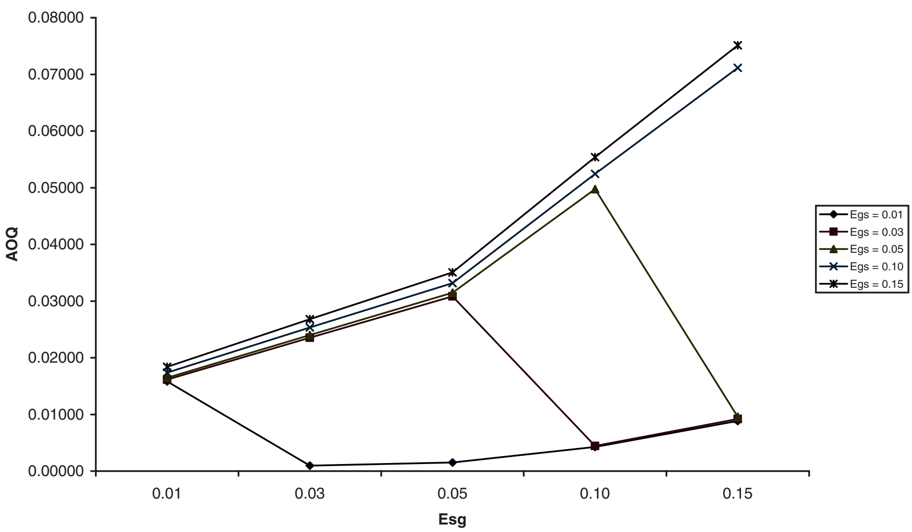

Figure 4. Effect of error Esg on average outgoing quality (AOQ). Egr=Erg=Ers=Esr=0.05.

carried out at characteristic inspection stations and at rework stations. Thus, for a batch of M components ATI is computed as

$$\mathrm{ATI} = \sum_{j=i}^{n}\left(\sum_{i=1}^{N}\left[M_{i,j} + M_j{}^{j}PG\left[\prod_{k=0}^{i-1}(1-E_{k\mathrm{gs}})\right]E_{i\mathrm{gr}} + M_{i,j}PS_{i,j}E_{i\mathrm{sr}} + M_{i,j}PR_{i,j}(1-E_{i\mathrm{rg}}-E_{i\mathrm{rs}})\right]\right).$$

ATI can be interpreted as the inspection load in the plan.

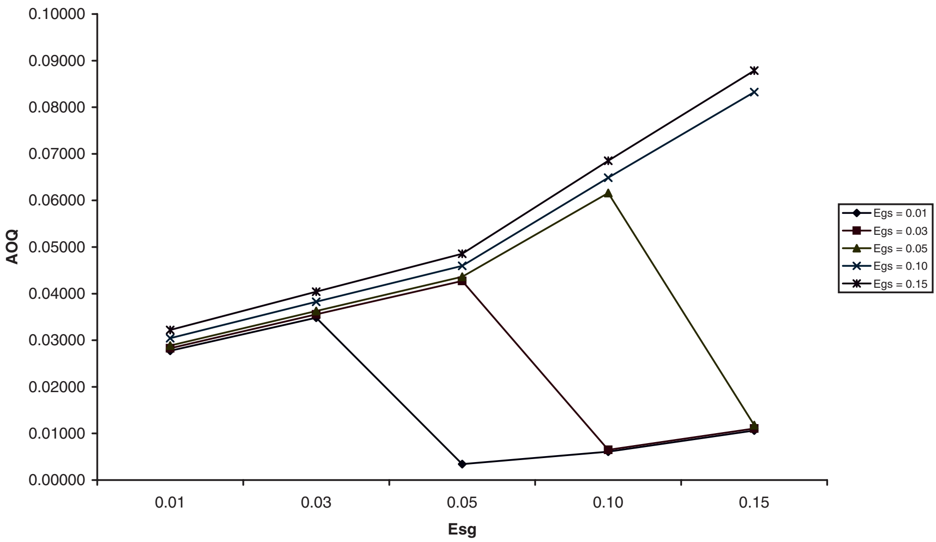

Figure 5. Effect of error Esg on average outgoing quality (AOQ). Egr=Erg=Ers=Esr=0.10.

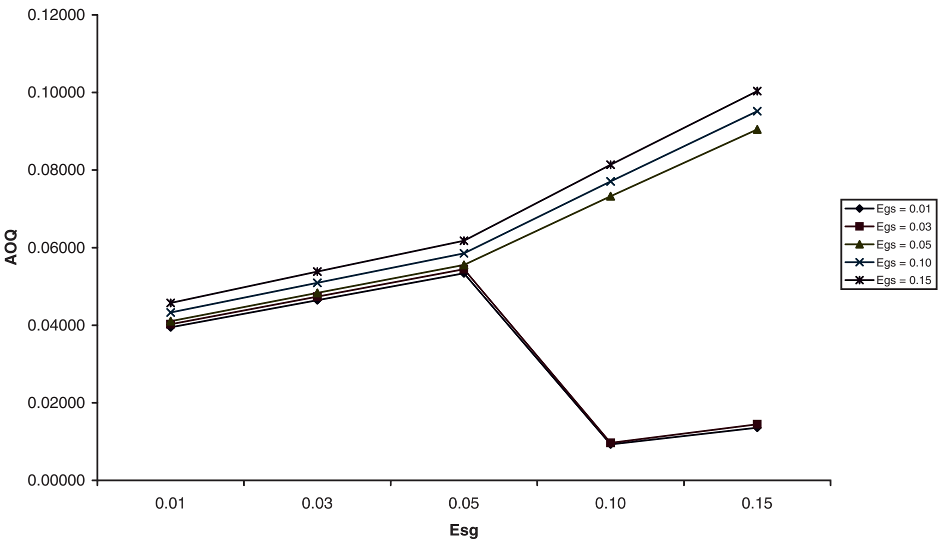

Figure 6. Effect of error Esg on average outgoing quality (AOQ). *E*gr=*E*rg=*E*rs=*E*sr=0.15.

Figures 7–11 show five curves each. Each curve shows the relationship between *E*sg and ATI for a fixed level of *E*gs. The other misclassification errors are set at levels of 0.01, 0.03, 0.05, 0.10 and 0.15, respectively, in the five figures. There is an unusual increase in ATI in each figure when *E*sg goes from one level to another for some fixed values of *E*gs. This can be explained by looking at tables A1–A5 in Appendix A: AOQ becomes so large that the model, in order to minimize AOQ, is forced to take one more cycle of inspection.

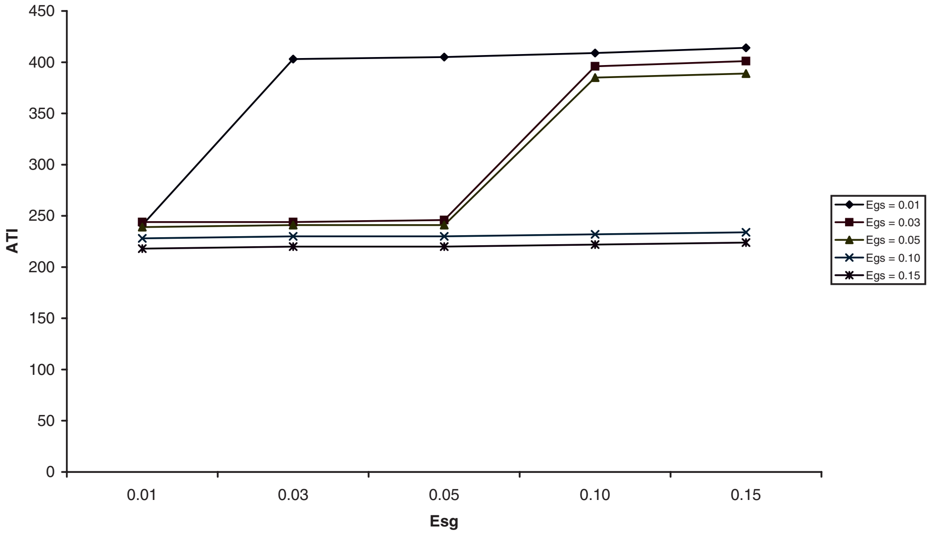

Figure 7. Effect of error *E*sg on average total inspection (ATI). *E*gr=*E*rg=*E*rs=*E*sr=0.01.

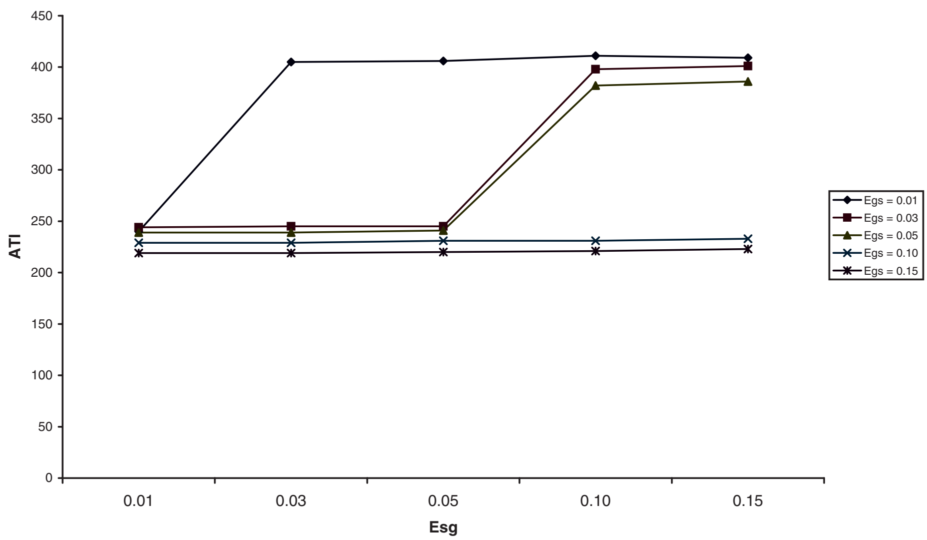

Figure 8. Effect of error *E*sg on average total inspection (ATI). *E*gr=*E*rg=*E*rs=*E*sr=0.03.

Figure 7 shows that the inspection load decreases with an increase in *E*gs. On the other hand, the inspection load increases in a piecewise linear fashion as *E*sg increases. At an *E*gs level of 0.01, ATI increases by 67.2% on changing *E*sg from 0.01 to 0.03. However, ATI increases by 61% at an *E*gs level of 0.03 for *E*sg varying from 0.05 to 0.10.

Figure 8 shows that the inspection load decreases with an increase in *E*gs. On the other hand, the inspection load increases in a piecewise linear fashion as *E*sg increases. At an *E*gs level of 0.01, ATI increases by 68.75% on changing *E*sg from

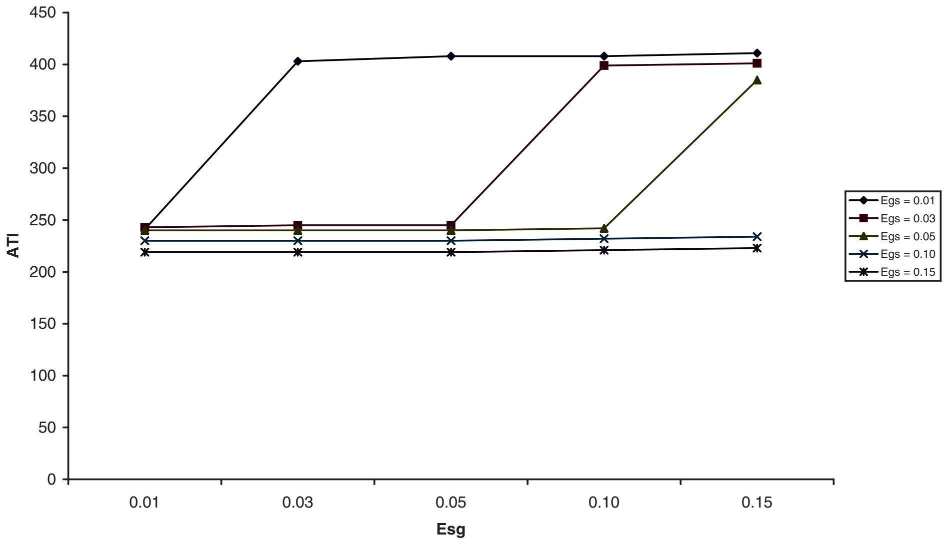

Figure 9. Effect of error *E*sg on average total inspection (ATI). *E*gr=*E*rg=*E*rs=*E*sr=0.05.

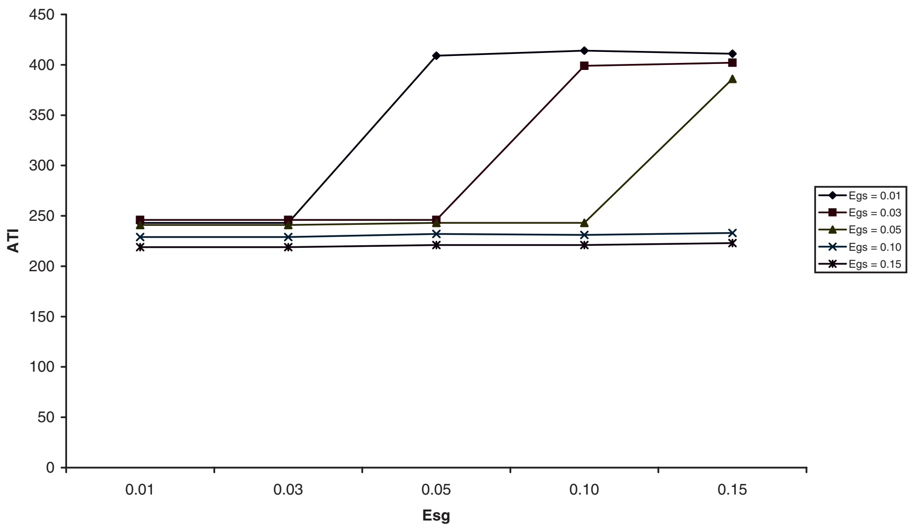

Figure 10. Effect of error *E*sg on average total inspection (ATI). *E*gr=*E*rg=*E*rs=*E*sr=0.10.

0.01 to 0.03. However, ATI increases by 62.4% at an *E*gs level of 0.03 for *E*sg varying from 0.05 to 0.10.

Figure 9 shows that the inspection load decreases with an increase in *E*gs. On the other hand, the inspection load increases in a piecewise linear fashion as *E*sg increases. At an *E*gs level of 0.01, ATI increases by 67.2% on changing *E*sg from 0.03 to 0.05. However, ATI increases by 62.8% at an *E*gs level of 0.03 for *E*sg varying from 0.05 to 0.10.

Figure 10 shows that the inspection load decreases with an increase in *E*gs. On the other hand, the inspection load increases in a piecewise linear fashion as *E*sg

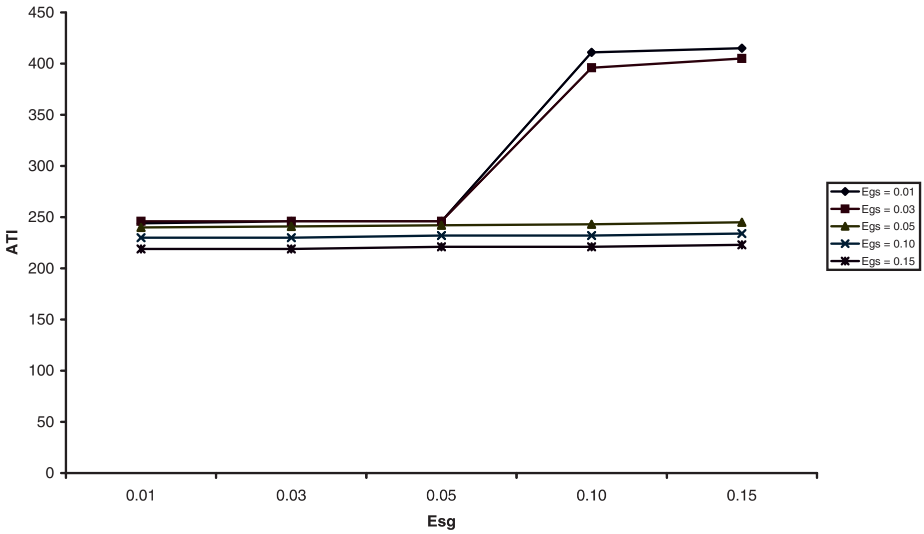

Figure 11. Effect of error *E*sg on average total inspection (ATI). *E*gr=*E*rg=*E*rs=*E*sr=0.15.

increases. At an *E*gs level of 0.01, ATI increases by 68.3% on changing *E*sg from 0.03–0.05. However, ATI increases by 62.2% at an *E*gs level of 0.03 for *E*sg varying from 0.05 to 0.10.

Similarly, figure 11 shows that the inspection load decreases with an increase in *E*gs, whereas it increases on varying *E*sg from 0.01–0.15. At an *E*gs level of 0.01, ATI increases by 67% on changing *E*sg from 0.05–0.10.

In general, it can be concluded from the above description that the inspection load decreases with *E*gs and increases with *E*sg.

The impact of larger values of *E*sg on the inspection model at a fixed level of the other misclassifications can be summarized as

1. An increased inspection load.
2. A higher expected total cost of inspection.
3. A higher value of the average outgoing quality.

The above three results are in line with intuition, since, by increasing *E*sg, more scrap compounds will be classified as good. This will result in a need for more inspections to improve the quality and hence increase the inspection load and the average outgoing quality.

On the other hand, the impact of larger values of *E*gs on the inspection model at a fixed level of the other misclassifications can be summarized as

1. A reduced inspection load.
2. A higher expected total cost of inspection.
3. A higher value of the average outgoing quality.

The above results make sense, since a larger value of *E*gs tends to classify good components as scrap. These components leave the inspection process early and therefore reduce the inspection load and increase average outgoing quality.

Similarly, we can summarize the impact of the other misclassifications at a certain level of both *E*sg and *E*gs as

1. A higher total cost of inspection.
2. A higher value of average outgoing quality.

The practical implications of the above misclassification errors are in the planning and design of such inspection plans. Higher values of the inspection load lead to careful planning of the time needed for inspection, and a cautious design of the inspection station. Higher cost of inspection dictates the need to invest more in the means of controlling errors such as training operators, maintaining and controlling the inspection equipment, etc. Also, higher values of AOQ indicate that we should probably change the criterion minimized in the model in cases where AOQ is a critical measure.

6. Conclusion

The purpose of this paper is to quantify the effect of inspection errors, mainly *E*sg and *E*gs, on the performance measures of a complete repeat inspection plan. This is accomplished by conducting a sensitivity analysis on the errors and then observing the changes in ETC, ATI and AOQ. As a result of this analysis, the following can be concluded for the inspection model described:

(a) The errors *E*sg and *E*gs both increase ETC, but *E*gs increases ETC at a faster rate, especially at values ≥ 0.05.
(b) AOQ is higher for higher values of both *E*sg and *E*gs.
(c) ATI decreases as *E*gs increases. This is in line with intuition, since an increase in the probability of rejecting good components will reduce the number of components to be inspected in subsequent inspection cycles. On the other hand, as *E*sg increases, ATI increases. Therefore, larger values of *E*sg increase the inspection load in the plan.

It can be concluded that *E*gs and *E*sg have a significant effect on the performance measures of the repeat inspection plan studied in this paper.

7. Applications and extensions

In this section, we present some practical applications and possible extensions of the work presented in this paper.

7.1 *Practical applications*

Multi-characteristic components are found in many industrial systems. These systems include gas ignition systems (Raouf *et al.* 1983), aircraft avionics systems, the space shuttle and nuclear reactors. Details of some of these are presented below in order to demonstrate the use of the model.

Printed circuit boards in aircraft avionics systems have many connections, welding points and wires that need to be inspected for quality assurance. Failure of any of these connections or wires could be catastrophic. These boards are thus treated as critical components.

Gas ignition systems in the heavy mechanical industry and ships have many valves, switches and circuits that need to be inspected for quality assurance. Failure of any of these switches or circuits could result in a heavy loss or fatality.

Vacuum pumps on conveyors in the food handling industry have many seals and coatings that need to be inspected for quality assurance. Failure of any of these characteristics could result in large-scale casualties.

7.2 *Extensions*

Some suggestions regarding possible future research directions are as follows:

1. The presented inspection plan can be extended under varying inspection errors. In this extension, the error can be made a function of the incoming quality.
2. The approach of Duffuaa and Al-Najjar (1995) can be used to extend the presented model by relaxing the assumption of equality of the number of inspections and by allowing a variable number of inspections for different characteristics.
3. A sensitivity analysis can also be carried out, similar to that in point 2 above.
4. The presented inspection plan can be generalized by incorporating material handling costs.
5. A more detailed study of the errors could be conducted.

Acknowledgement

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Appendix

Table A1. Effect of inspection errors on the inspection plan

$Egr = Erg = Ers = Esr = 0.01$.

No.	Egs	Esg	$1-PG$	$A(n)$	ATI	$TCFA$	$TCFR$	TCI	ETC	n
1	0.01	0.01	0.00615	55	241	611.56	340.35	3609.21	4561.11	1
2	0.01	0.03	0.00036	52	403	36.25	668.27	4309.26	5013.78	2
3	0.01	0.05	0.00094	52	405	96.25	669.52	4319.98	5085.75	2
4	0.01	0.10	0.00370	52	409	376.84	670.38	4350.06	5397.28	2
5	0.01	0.15	0.00827	53	414	841.98	660.11	4300.38	5802.47	2
6	0.03	0.01	0.00627	52	244	670.94	994.76	4132.01	5797.70	1
7	0.03	0.03	0.01376	52	244	1480.96	992.00	4131.97	6604.93	1
8	0.03	0.05	0.02116	53	246	2276.81	979.33	4082.21	7338.35	1

(*continued*)

Table A1. Continued.

No.	*E*gs	*E*sg	1−*PG*	*A*(*n*)	ATI	*TCFA*	*TCFR*	*TCI*	ETC	*n*
9	0.03	0.10	0.00385	45	396	394.00	2121.53	5334.88	7850.41	2
10	0.03	0.15	0.00861	46	401	879.73	2084.61	5261.49	8225.83	2
11	0.05	0.01	0.00640	48	239	694.18	1730.01	4400.90	6825.08	1
12	0.05	0.03	0.01404	49	241	1524.35	1705.54	4341.58	7571.48	1
13	0.05	0.05	0.02160	49	241	2353.33	1700.71	4341.55	8395.58	1
14	0.05	0.10	0.00402	41	385	413.23	3725.52	5773.40	9912.16	2
15	0.05	0.15	0.00897	41	389	923.42	3731.14	5817.46	10 472.02	2
16	0.10	0.01	0.00676	41	228	723.47	3798.54	4960.10	9482.10	1
17	0.10	0.03	0.01482	42	230	1585.88	3733.46	4877.60	10 196.93	1
18	0.10	0.05	0.02278	42	230	2447.90	3722.56	4877.55	11 048.01	1
19	0.10	0.10	0.04231	43	232	4551.26	3644.53	4798.83	12 994.61	1
20	0.10	0.15	0.06129	44	234	6598.48	3569.79	4723.68	14 891.95	1
21	0.15	0.01	0.00715	34	218	780.19	6505.10	5768.34	13 053.63	1
22	0.15	0.03	0.01568	35	220	1704.95	6365.26	5646.26	13 716.47	1
23	0.15	0.05	0.02410	35	220	2631.24	6346.19	5646.21	14 623.64	1
24	0.15	0.10	0.04472	36	222	4876.90	6186.32	5530.83	16 594.05	1
25	0.15	0.15	0.06473	37	224	7049.83	6034.58	5421.69	18 506.11	1

Table A2. Effect of inspection errors on the inspection plan $Egr = Erg = Ers = Esr = 0.03$.

No.	*E*gs	*E*sg	1−*PG*	*A*(*n*)	ATI	*TCFA*	*TCFR*	*TCI*	ETC	*n*
1	0.01	0.01	0.01100	54	240	1073.43	405.42	3934.56	5413.41	1
2	0.01	0.03	0.00055	52	405	55.10	734.00	4961.52	5750.61	2
3	0.01	0.05	0.00114	51	406	113.99	747.83	5068.82	5930.65	2
4	0.01	0.10	0.00389	52	411	389.27	736.63	5013.62	6139.52	2
5	0.01	0.15	0.00846	51	409	846.32	740.79	5100.79	6687.91	2
6	0.03	0.01	0.01122	51	244	1188.61	1080.85	4481.70	6751.16	1
7	0.03	0.03	0.01866	52	245	1980.02	1062.19	4409.26	7451.48	1
8	0.03	0.05	0.02601	52	245	2772.23	1059.46	4409.17	8240.86	1
9	0.03	0.10	0.00405	45	398	406.84	2202.17	6041.13	8650.14	2
10	0.03	0.15	0.00881	45	401	884.10	2199.18	6083.12	9166.39	2
11	0.05	0.01	0.01146	48	239	1204.97	1795.40	4679.25	7679.62	1
12	0.05	0.03	0.01905	48	239	2015.91	1790.55	4679.15	8485.61	1
13	0.05	0.05	0.02655	49	241	2808.57	1765.36	4616.98	9190.90	1
14	0.05	0.10	0.00422	40	382	427.25	3849.20	6624.25	10 900.71	2
15	0.05	0.15	0.00918	40	386	928.34	3856.30	6678.42	11 463.06	2
16	0.10	0.01	0.01209	41	229	1274.85	3879.79	5285.15	10 439.79	1
17	0.10	0.03	0.02009	41	229	2132.33	3868.74	5285.03	11 286.10	1
18	0.10	0.05	0.02800	42	231	2964.77	3802.47	5198.10	11 965.34	1
19	0.10	0.10	0.04739	42	231	5059.78	3775.70	5197.81	14 033.28	1
20	0.10	0.15	0.06624	43	233	7078.81	3697.17	5114.75	15 890.73	1
21	0.15	0.01	0.01280	34	219	1373.69	6597.07	6140.39	14 111.15	1
22	0.15	0.03	0.02125	35	219	2231.44	6389.73	5964.80	14 585.98	1
23	0.15	0.05	0.02961	35	220	3122.80	6402.14	5990.89	15 515.83	1
24	0.15	0.10	0.05008	36	221	5280.63	6211.99	5844.19	17 336.80	1
25	0.15	0.15	0.06994	37	223	7370.54	6061.00	5730.22	19 161.76	1

Table A3. Effect of inspection errors on the inspection plan $Egr = Erg = Ers = Esr = 0.05$.

No.	*Egs*	*Esg*	1−*PG*	*A*(*n*)	ATI	*TCFA*	*TCFR*	*TCI*	ETC	*n*
1	0.01	0.01	0.01582	54	241	1517.24	468.41	4213.07	6198.72	1
2	0.01	0.03	0.00096	50	403	92.55	826.83	5787.12	6706.50	2
3	0.01	0.05	0.00152	51	408	150.23	818.23	5726.75	6695.20	2
4	0.01	0.10	0.00427	51	408	420.28	812.01	5725.70	6958.00	2
5	0.01	0.15	0.00884	50	411	885.74	827.84	5878.65	7592.23	2
6	0.03	0.01	0.01614	50	243	1698.83	1165.86	4819.86	7684.55	1
7	0.03	0.03	0.02353	51	245	2477.86	1150.34	4760.03	8388.23	1
8	0.03	0.05	0.03083	51	245	3260.26	1147.58	4759.89	9167.73	1
9	0.03	0.10	0.00445	45	399	438.92	2280.33	6720.41	9439.65	2
10	0.03	0.15	0.00920	45	401	906.77	2272.55	6746.02	9925.33	2
11	0.05	0.01	0.01648	48	240	1713.98	1870.14	4968.08	8552.19	1
12	0.05	0.03	0.02401	48	240	2511.06	1865.30	4967.92	9344.29	1
13	0.05	0.05	0.03147	48	240	3303.69	1860.50	4967.77	10 131.97	1
14	0.05	0.10	0.04976	49	242	5240.58	1827.33	4902.27	11 970.18	1
15	0.05	0.15	0.00959	39	385	979.39	4007.09	7598.48	12 584.95	2
16	0.10	0.01	0.01738	41	230	1812.12	3962.12	5606.94	11 381.17	1
17	0.10	0.03	0.02533	41	230	2654.12	3951.09	5606.75	12 211.96	1
18	0.10	0.05	0.03318	41	230	3491.18	3940.16	5606.57	13 037.91	1
19	0.10	0.10	0.05243	42	232	5525.39	3856.74	5514.97	14 897.10	1
20	0.10	0.15	0.07114	43	234	7505.05	3776.94	5427.63	16 709.62	1
21	0.15	0.01	0.01839	34	219	1912.84	6654.57	6485.05	15 052.46	1
22	0.15	0.03	0.02679	34	219	2800.81	6635.42	6484.82	15 921.05	1
23	0.15	0.05	0.03508	34	219	3683.25	6616.45	6484.60	16 784.30	1
24	0.15	0.10	0.05539	35	221	5812.08	6447.05	6349.65	18 608.78	1
25	0.15	0.15	0.07511	36	223	7872.77	6286.51	6222.19	20 381.47	1

Table A4. Effect of inspection errors on the inspection plan $Egr = Erg = Ers = Esr = 0.10$.

No.	*Egs*	*Esg*	1−*PG*	*A*(*n*)	ATI	*TCFA*	*TCFR*	*TCI*	ETC	*n*
1	0.01	0.01	0.02773	54	243	2554.19	627.74	4874.30	8056.23	1
2	0.01	0.03	0.03485	54	243	3223.95	626.72	4874.03	8724.69	1
3	0.01	0.05	0.00341	50	409	315.60	1017.23	7398.51	8731.33	2
4	0.01	0.10	0.00607	50	414	583.21	1022.44	7475.42	9081.08	2
5	0.01	0.15	0.01062	48	411	1018.96	1051.79	7745.17	9815.92	2
6	0.03	0.01	0.02829	51	246	2850.67	1326.29	5402.98	9579.94	1
7	0.03	0.03	0.03555	51	246	3592.89	1323.54	5402.75	10 319.18	1
8	0.03	0.05	0.04274	51	246	4331.15	1320.82	5402.52	11 054.48	1
9	0.03	0.10	0.00647	43	399	620.43	2585.88	8736.89	11 943.20	2
10	0.03	0.15	0.01106	43	402	1079.18	2584.73	8799.97	12 463.87	2
11	0.05	0.01	0.02887	47	241	2946.55	2086.14	5753.48	10 786.16	1
12	0.05	0.03	0.03628	47	241	3713.26	2081.28	5753.22	11 547.77	1
13	0.05	0.05	0.04361	48	243	4453.26	2051.44	5677.19	12 181.88	1
14	0.05	0.10	0.06160	49	243	6202.95	1998.02	5560.73	13 761.69	1
15	0.05	0.15	0.01177	38	386	1136.16	4318.59	9635.46	15 090.21	2
16	0.10	0.01	0.03044	40	229	3060.16	4215.02	6479.41	13 754.59	1

(*continued*)

Table A4. Continued.

No.	*Egs*	*Esg*	1−*PG*	*A*(*n*)	ATI	*TCFA*	*TCFR*	*TCI*	ETC	*n*
17	0.10	0.03	0.03824	40	229	3855.15	4204.07	6479.11	14 538.33	1
18	0.10	0.05	0.04596	41	232	4614.69	4130.40	6372.46	15 117.55	1
19	0.10	0.10	0.06487	41	231	6558.26	4103.88	6371.73	17 033.86	1
20	0.10	0.15	0.08326	42	233	8420.74	4018.51	6269.74	18 708.99	1
21	0.15	0.01	0.03219	33	219	3291.19	7029.19	7542.86	17 863.25	1
22	0.15	0.03	0.04043	33	219	4144.66	7009.94	7542.49	18 697.09	1
23	0.15	0.05	0.04857	34	221	4945.04	6854.64	7382.70	19 182.38	1
24	0.15	0.10	0.06850	34	221	7023.58	6808.25	7381.79	21 213.63	1
25	0.15	0.15	0.08786	35	223	8988.75	6635.50	7230.62	22 854.87	1

Table A5. Effect of inspection errors on the inspection plan

$E_{gr} = E_{rg} = E_{rs} = E_{sr} = 0.15.$

No.	*Egs*	*Esg*	1−*PG*	*A*(*n*)	ATI	*TCFA*	*TCFR*	*TCI*	ETC	*n*
1	0.01	0.01	0.03944	53	244	3496.65	804.28	5594.25	9895.18	1
2	0.01	0.03	0.04645	54	246	4121.08	794.43	5533.39	10 448.91	1
3	0.01	0.05	0.05338	54	246	4748.39	793.39	5533.07	11 074.85	1
4	0.01	0.10	0.00928	47	411	835.64	1295.86	9461.98	11 593.48	2
5	0.01	0.15	0.01358	48	415	1246.24	1271.92	9342.19	11 860.35	2
6	0.03	0.01	0.04023	49	246	3996.97	1556.98	6237.79	11 791.75	1
7	0.03	0.03	0.04737	50	246	4692.46	1529.51	6136.15	12 358.11	1
8	0.03	0.05	0.05444	50	246	5402.00	1526.75	6135.88	13 064.63	1
9	0.03	0.10	0.00966	41	396	901.89	2928.71	10 762.34	14 592.94	2
10	0.03	0.15	0.01447	43	405	1335.66	2828.80	10 445.79	14 610.26	2
11	0.05	0.01	0.04105	46	240	4046.69	2304.17	6515.01	12 865.86	1
12	0.05	0.03	0.04834	47	241	4747.69	2260.55	6401.00	13 409.23	1
13	0.05	0.05	0.05555	47	242	5464.96	2265.76	6427.77	14 158.48	1
14	0.05	0.10	0.07323	48	243	7207.09	2216.81	6317.52	15 741.43	1
15	0.05	0.15	0.09046	49	245	8911.33	2179.30	6237.72	17 328.35	1
16	0.10	0.01	0.04327	39	230	4272.26	4505.44	7379.54	16 157.23	1
17	0.10	0.03	0.05093	39	230	5035.46	4494.43	7379.18	16 909.06	1
18	0.10	0.05	0.05851	40	232	5754.18	4413.49	7255.49	17 423.16	1
19	0.10	0.10	0.07709	40	232	7620.32	4386.83	7254.62	19 261.77	1
20	0.10	0.15	0.09516	41	234	9398.32	4294.57	7136.47	20 829.36	1
21	0.15	0.01	0.04573	33	219	4460.54	7206.35	8360.51	20 027.40	1
22	0.15	0.03	0.05381	33	219	5255.17	7187.55	8360.08	20 802.80	1
23	0.15	0.05	0.06180	34	221	5989.59	7029.24	8185.83	21 204.66	1
24	0.15	0.10	0.08136	34	221	7926.67	6983.91	8184.78	23 095.36	1
25	0.15	0.15	0.10036	35	223	9750.23	6808.89	8019.89	24 579.01	1

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