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Quantum coherence and nonlocality of two qubits in the presence of a common dephasing environment

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ABSTRACT

We study the coherence and quantum correlations for a system consisting of two interacting qubits under the effect of a common environment, where the system of qubits has the dephasing coupling with its surrounding. We study the dynamical behavior of the quantumness measures when the influence of the qubits' interaction on the dephasing is analyzed. We display the effect of the qubit–qubit interaction and qubits–environment interaction on the measures considering the case of symmetry and asymmetry interaction between the qubits and environment. Finally, we explicitly illustrate the relation between the coherence and quantum correlations and show that quantum coherence and correlations can be preserved during the dynamics.

Introduction

Many phenomena have been discovered during the past several decades as resources to perform various tasks of quantum optics and information, both theoretically and experimentally (QIAP). Quantum correlations like nonlocal correlation and discord, in general, require the coherent superposition of quantum states [1–8]. Many quantum effects in nano-thermodynamics [9,10], highly sensitive and high-resolution measurements [11–15], quantum biology [16–18], etc. are based on quantum coherence. In light of the basic importance of quantum coherence, it has only lately been stressed that a precise theory of coherence with a few essential limitations is required to guarantee that quantum coherence could be a physical resource [19]. Consequently, some quantifiers have been suggested that confirm these limits, such as those that rely on the norm and relative entropy [19]. On the other side, quantum coherence may be measured using the convex-roof construction and nonlocal correlation [20,21]. In addition, a quantum coherence operational theory has been proposed [22].

Nonlocality poses a challenge to our perception of nature and intuition as a fundamental feature. The focal point is the "EPR paradox" [23] which was first proposed by Einstein, Podolsky, and Rosen. They claimed that the concept of "spooky action at a distance" may be explained by quantum physics. Steering, as defined by E. Schrodinger, is the ability of local measurements to influence a remote system without having access to it [24]. The Bell inequality was then used by Bell to show how this "spooky activity" might result in a correlation that defies all local classical explanations [25]. It is demonstrated in [26] that EPR steering is thought of one of two factors that defines the degree of Bell nonlocality. Heisenberg [27] initially proposed the uncertainty principle, which was further extended by Roberston [28] and Deutsch [29]. The prediction of some measurements must have an intrinsic limit, according to the theory of quantum mechanics. In other words, there are certain observables that cannot be predicted simultaneously with certainty because they are incompatible. This degree of uncertainty is viewed as describing how much quantum information can be extracted from the system.

With the advent of experimental methods to detect and manipulate systems, quantum coherence has recently attracted a lot of interest. It plays a significant role in quantum mechanics. Realistic quantum systems are always compelled to interact with their surroundings, which causes the phenomenon of decoherence as the system evolves. The Markov approximation has often been used in the dynamics of open quantum systems. Studies considering open quantum systems without the Markov approximation have attracted a lot of interest recently in a wide range of domains, from comprehending the fundamental concepts of quantum mechanics to using cutting-edge experimental methods for coherent control in quantum technology, such as measurement of the non-Markovian feature [30]. It has been shown, both experimentally and theoretically, that the presence of non-Markovian dynamics leads to partial loss of coherence and causes the open quantum system to have a lengthy correlation time with its environment. Recent research

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on the decoherence of multi-qubit systems moving across correlated noisy channels has shown two unique phenomena that might be used to maintain quantum coherence [31]. In light of the aforementioned techniques, it would also be interesting to present other techniques to maintain and improve coherence and correlation.

Motivated by the mentioned considerations, the purpose of the present manuscript is to examine the coherence and quantum correlations for a system consisting of two interacting qubits under the influence of a common environment, where the system of qubits has the dephasing coupling with the environment. We show the time variation of the quantumness measures when the effect of the qubit–qubit (q–q) interaction on the dephasing is analyzed. We display the effect of the q–q interaction and qubits–environment interaction on the measures considering the case of symmetry and asymmetry interaction between the qubits and environment. Finally, we explicitly illustrate the relation between the coherence and quantum correlations during their evolution and show that quantum coherence and correlations can be created during the time evolution.

In Section "The physical model", we present the physical model and the quantum formalism that explains the dependence on different parameters of the model. Sections "Coherence and nonlocality" and "Coherence versus quantum correlations" describe the quantum quantifiers and illustrate the obtained results, respectively. In the last section, we summarize the work.

The physical model

We consider two interacting qubits in the presence of a common dephasing environment. The Hamiltonian system is

$$H_T = H_{12} + H_R + H_I \tag{1}$$

The first part of the Hamiltonian represents the Hamiltonian of the two interacting qubits given by

$$H_{12} = \hbar\omega S_1^z + \hbar\omega S_2^z + \hbar\lambda\left(S_1^- S_2^+ + S_1^+ S_2^-\right), \tag{2}$$

where S_i^z is the spin half operator acting on its eigenkets as $S_i^z|0\rangle_i = 1/2|0\rangle_i$ and $S_i^z|1\rangle_i = 1/2|1\rangle_i$ and that $S_i^{\pm} = S_i^x \pm S_i^y$. The term $\hbar\lambda\left(S_1^- S_2^+ + S_1^+ S_2^-\right)$ describes the XY-coupling of the two qubits. $\pm\hbar\omega$ (with eigenkets $|\phi_0\rangle = |00\rangle$ and $|\phi_1\rangle = |11\rangle$) and $\pm\hbar\lambda$ (with eigenkets $|\psi_\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2})$ are the eigenvalues of the Hamiltonian H_{12}. The term H_R in Eq. (1) represents the environment Hamiltonian and the last term is the interaction Hamiltonian of the qubits–environment system expressed as

$$H_I = \hbar R S, \tag{3}$$

with

$$S = \gamma_1 S_1^z + \gamma_2 S_2^z,$$

where γ_i denotes the coupling strength between the qubits and environment, and R represents some Hermitian operator of the environment.

The Liouville–von Neumann equation that governs the dynamics of density matrix of the total system is

$$\dot{\varrho}(t) = \frac{1}{i\hbar}\left[H_T, \varrho(t)\right]. \tag{4}$$

The master equation for the reduced matrix operator can be constructed utilizing the projection operator approach up to the second order in accordance with system–reservoir interaction. The quantum master equation of the two qubits in the Schrödinger picture is [32,33]

$$\dot{\varrho}_{12}(t) = \frac{1}{i\hbar}\left[H_{12}, \varrho_{12}(t)\right] + L_{12}^{(0)}(t)\varrho_{12}(t) + L_{12}^{(1)}(t)\varrho_{12}(t) \quad \text{with} \quad \varrho_{12}(t) = Tr_R\varrho(t). \tag{5}$$

and that

$$L_{12}^{(0)}\varrho_{12}(t) = \Phi(t)[S\varrho_{12}(t), S] + \Phi^*(t)[S, \varrho_{12}(t)S] \tag{6}$$

$$\begin{aligned} L_{12}^{(1)}\varrho_{12}(t) = &-\Psi_-(t)[S|\psi_-\rangle\langle\psi_-|\varrho_{12}(t), S] - \psi_-^*(t)[S, \varrho_{12}(t)|\psi_-\rangle\langle\psi_-|S] \\ &-\Psi_+(t)[S|\psi_+\rangle\langle\psi_+|\varrho_{12}(t), S] - \psi_+^*(t)[S, \varrho_{12}(t)|\psi_+\rangle\langle\psi_+|S]. \end{aligned} \tag{7}$$

The function $\Phi(t)$ and $\Psi_\pm(t)$ are defined by [32,33]

$$\Phi(t) = \int_0^t dt'\langle R(t')R(0)\rangle_R \tag{8}$$

$$\Psi_\pm(t) = \int_0^t dt'\langle R(t')R(0)\rangle_R\left(1 - e^{\pm 2i\lambda t'}\right). \tag{9}$$

The damping operator $L_{12}^{(0)}$ ($L_{12}^{(1)}$) does not include (depend on) the q–q interaction effect. In several cases, the effect of interaction on the relaxation process can be ignored. In such an approximated treatment, the operator $L_{12}^{(1)}$ does not appear [32]. The operator $L_{12}^{(1)}$ has a negligible effect on the qubits dephasing for very small asymmetry interaction and it becomes considerable when the interaction is not small. By using the solution of the master equation, we study the coherence and quantum correlations for a system consisting of two interacting qubits under the influence of a common environment, considering the case of symmetry and asymmetry interaction between the qubits and environment. For this purpose, the two-qubit initial state considered in this manuscript is the Bell-like state:

$$|\Psi\rangle = \alpha|01\rangle + \sqrt{1-\alpha^2}|10\rangle, \qquad \alpha^2 \le 1. \tag{10}$$

In the next sections, we provide the quantumness measures and results for the measures of coherence, quantum correlations and Bell nonlocality in the two interacting qubits in the presence of dephasing effect.

Coherence and nonlocality

The features of the coherence for system states are depending on the diagonal elements of its density operator. The l_1 norm of coherence is used to detect the amount of quantum coherence considering the concept of the absolute value. The measure of coherence using the l_1 norm is given by [19]

$$C_l = \min_{\tau\in\eta}\|\varrho - \tau\|_{l_1} = \sum_{i\neq j}\left|\varrho_{ij}\right|. \tag{11}$$

Here η defines a set of incoherent states. The concept of distance is used to express the coherence measurement in terms of relative entropy as [19]

$$C_r = S\left(\varrho \parallel \varrho_{\text{diag}}\right) = S\left(\varrho_{\text{diag}}\right) - S\left(\varrho\right), \tag{12}$$

Here C_r represents the distance between the concerned state and the closest incoherent state. The function $(\varrho) = -Tr\left(\varrho\log_2\varrho\right)$ designs the von Neumann entropy and ϱ_{diag} denotes the quantum incoherent state. It has been shown that the coherence measures, C_{l_1} and C_r, obey the property of monotonicity, and for pure states C_{l_1} defines the upper bound of C_r.

Let us now consider Bell Clauser–Horne–Shimony–Holt (BCHSH) inequality to examine the Bell nonlocality. By considering the criterion of Horodecki, the nonlocality is detected through $B_l = 2\sqrt{\max_{i<j}\left(x_i + x_j\right)}$ with $i, j = 1, 2, 3$ and x_i are related to the density elements by

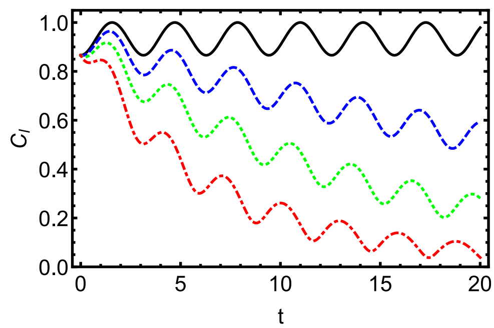
$$\begin{aligned} x_1 &= 4\left(|\varrho_{14}| + |\varrho_{23}|\right)^2 \;\; x_2 = 4\left(|\varrho_{14}| - |\varrho_{23}|\right)^2; \\ x_3 &= \left(\varrho_{11} - \varrho_{22} - \varrho_{33} + \varrho_{44}\right)^2. \end{aligned} \tag{13}$$

We have always x_1 is larger than x_2, and therefore the BCHSH inequality maximum violation is given by [34,35]

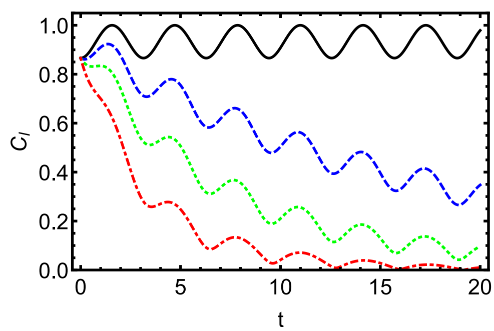
$$B_l = 2\max\{B_{l_1}, B_{l_2}\}, \;\; B_{l_1} = \sqrt{x_1 + x_2}, \;\; B_{l_2} = \sqrt{x_1 + x_3}. \tag{14}$$

The Bell nonlocality defined in Eq. (13) is only considered for the case of two-qubit X states.

To display the influence of the main parameters of the model on the dynamical comportment of the quantum coherence in the system

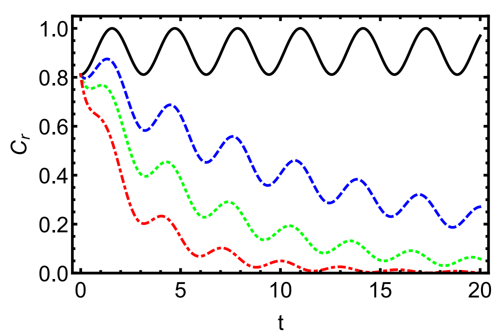


(a)

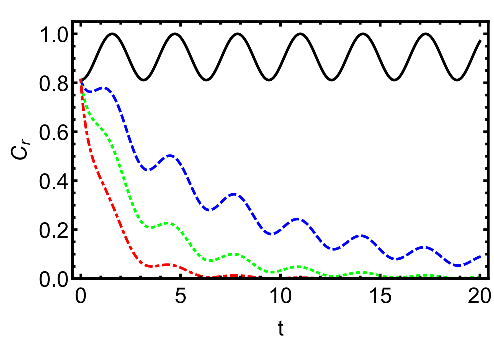


(b)

Fig. 1. The temporal evolution of the coherence measure C_l with respect to the different values of the model parameters. The Panels (a) and (b) represent the dynamics of C_l with and without the effect of the q–q interaction, respectively, for various values of the parameter $\Delta\gamma$ for two-qubit initial state. The solid line is considered for $\Delta\gamma = 0$, dashed line is for the case of $\Delta\gamma = 0.5$, dotted line corresponds to the case of $\Delta\gamma = 0.8$ and dot-dashed line is for $\Delta\gamma = 1.2$. In general, we can note that the figures reflect some considerable dynamic features of the measure of coherence with an oscillatory behavior, exhibiting decay and revival phenomena of coherence.



(a)



(b)

Fig. 2. The temporal evolution of the coherence measure C_r with respect to the different values of the model parameters. The Panels (a) and (b) represent the dynamics of C_r with and without the effect of the q–q interaction, respectively, for various values of the parameter $\Delta\gamma$ for two-qubit initial state. The solid line is considered for $\Delta\gamma = 0$, dashed line is for the case of $\Delta\gamma = 0.5$, dotted line corresponds to the case of $\Delta\gamma = 0.8$ and dot-dashed line is for $\Delta\gamma = 1.2$. In general we can note that the figures reflect some considerable dynamic features of the measure of coherence with an oscillatory behavior, exhibiting decay and revival phenomena of coherence during the time evolution.

of two qubits, we have depicted in Figs. 1 and 2 the time variation of the functions C_l and C_r in the absence and presence of q–q interaction for symmetry and asymmetry interaction between the qubits and environment according to the initial state of the qubits. Figs. 1(a)–2(a) and 1(b)–2(b) show the dynamics of coherence with and without the effect of the q–q interaction, respectively, for different values of $\Delta\gamma$. The solid line is considered for $\Delta\gamma = 0$, dashed line is for the case of $\Delta\gamma = 0.5$, dotted line illustrates the case of $\Delta\gamma = 0.8$ and dot-dashed line is for $\Delta\gamma = 1.2$. In general, the figures reflect some considerable dynamic features of the measures of coherence with an oscillatory behavior, exhibiting decay and revival phenomena of coherence. In the $\Delta\gamma \rightarrow 0$ limit, we can observe that the functions C_l and C_r display a periodic oscillatory behavior and take their maximal value during the dynamics. In this context, the q–q interaction does not affect the amount of two-qubit coherence in the case of symmetry interaction. On the other hand, the dynamics of C_l and C_r is largely affected in the presence of asymmetry interaction between the qubits and environment $\Delta\gamma \neq 0$. From the figures, we can observe damped oscillations of coherence and that the peculiar selection of the parameters γ_1 and γ_2 can significantly preserve the amount of coherence from the decoherence effect. Indeed, the smaller the difference $\delta\gamma$ is, the larger amount of coherence is during the dynamics. Moreover, the presence of the q–q interaction leads to retard the loss of the functions C_l and C_r during the time evolution. The q–q interaction effect reduces the dephasing effect in the case of asymmetry interaction. This indicate that the quantum coherence can be enhanced through the energy exchange between the qubits. From the other side, we can see that the presence of the q–q interaction makes short the periodic time of the oscillatory behavior of the functions C_l and C_r. The results illustrate how the two factors participating in the control and preservation of coherence. These results point to a useful method for suppressing the decoherence effect, which is crucial for a number of quantum information and optical processes.

Based on Eq. (14), we can examine the Bell nonlocality in the two qubits in the absence and presence of q–q interaction for symmetry and asymmetry interactions between the qubits and environment. The time variation of this class of correlation for different values of $\Delta\gamma$ is shown in Fig. 3. In general, we observe that the Bell nonlocality behaves like quantum coherence, and the two factors operate similarly to nonlocality. On the other hand, we find that only in finite time intervals for symmetry interaction, the state of two qubits can violate the inequality for which the Bell nonlocality's is satisfied. For asymmetry interaction condition, the two-qubit state cannot violate the CHSH inequality with the presence of quantum coherence.

Coherence versus quantum correlations

In this section we make a comparison between the quantum dynamics of coherence and quantum correlation considering the impact of the two factors.

In order to detect qubits' entanglement, we used the entanglement of formation as a measure of entanglement. It is written as a function of the concurrence as [36],

$$E_F = -\Gamma \log_2 \Gamma - (1-\Gamma)\log_2(1-\Gamma), \tag{15}$$

where $\Gamma = 1/2 - 1/2\sqrt{1-C^2}$ with $C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ and that λ_i being the eigenvalues giving in the nonincreasing order of the density matrix $\varrho_{12}\tilde{\varrho}_{12}$ with $\tilde{\varrho}_{12} = (\sigma_y \otimes \sigma_y)\,\varrho^*_{12}\,(\sigma_y \otimes \sigma_y)$ where ϱ^*_{12} denotes the conjugate of ϱ_{12} and σ_y is the y-component of the Pauli matrix.

By using the concept of local quantum uncertainty (LQU), the total quantum correlation is defined by [37]

$$U_{1D} = \min_{K_1^\Gamma} I\left(\varrho, K_1^\Gamma\right). \tag{16}$$

Γ represents the spectrum of K_1^Γ and I designs the quantum Skew information. For the system of two qubits, the LQU can be formulated as

$$U_{1D}\left(\varrho_{12}\right) = 1 - \lambda_{\max}\{W_{12}\}, \tag{17}$$

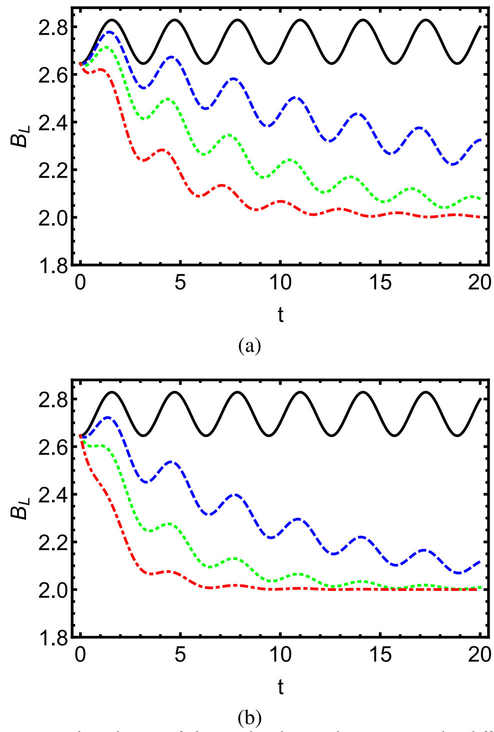


Fig. 3. The temporal evolution of the nonlocality with respect to the different values of the model parameters. The Panels (a) and (b) represent the dynamics of B with and without the effect of the q–q interaction, respectively, for various values of the parameter $\Delta\gamma$ for two-qubit initial state. The solid line is considered for $\Delta\gamma = 0$, dashed line is for the case of $\Delta\gamma = 0.5$, dotted line corresponds to the case of $\Delta\gamma = 0.8$ and dot-dashed line is for $\Delta\gamma = 1.2$.

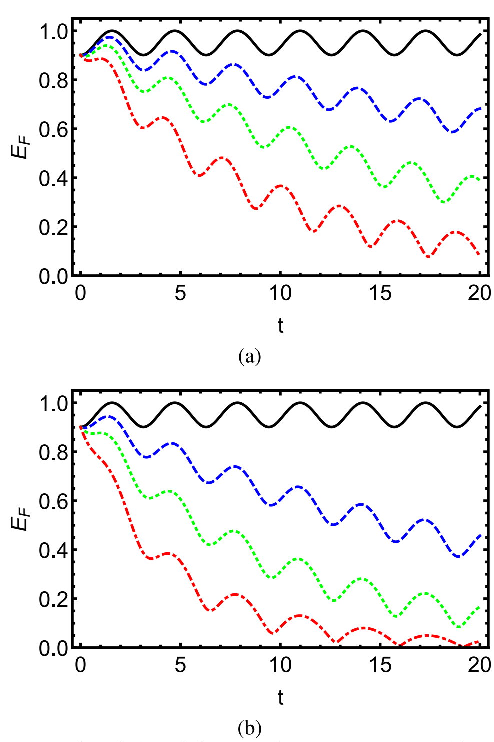


Fig. 4. The temporal evolution of the entanglement measure E_F with respect to the different values of the model parameters. The Panels (a) and (b) represent the dynamics of E_F with and without the effect of the q–q interaction, respectively, for various values of the parameter $\Delta\gamma$ for two-qubit initial state. The solid line is considered for $\Delta\gamma = 0$, dashed line is for the case of $\Delta\gamma = 0.5$, dotted line corresponds to the case of $\Delta\gamma = 0.8$ and dot-dashed line is for $\Delta\gamma = 1.2$. Generally, the figures reflect some considerable dynamic features of the entanglement with an oscillatory behavior, exhibiting decay and revival phenomena of coherence during the time evolution.

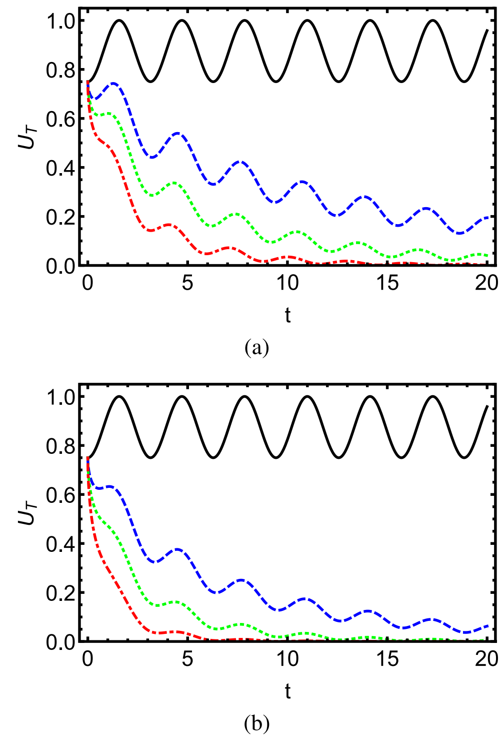


Fig. 5. The temporal evolution of the total correlation measure U_1 with respect to the different values of the model parameters. The Panels (a) and (b) represent the dynamics of U_1 with and without the effect of the q–q interaction, respectively, for various values of the parameter $\Delta\gamma$ for two-qubit initial state. The solid line is considered for $\Delta\gamma = 0$, dashed line is for the case of $\Delta\gamma = 0.5$, dotted line corresponds to the case of $\Delta\gamma = 0.8$ and dot-dashed line is for $\Delta\gamma = 1.2$. Generally, the figures reflect some considerable dynamic features of the total correlation with an oscillatory behavior, exhibiting decay and revival phenomena of coherence.

where $\lambda_{\max}$ is the largest eigenvalue of the matrix W_{12} with elements

$$\left(W_{12}\right)_{ij} = \mathrm{Tr}\{\sqrt{\varrho_{12}}\left(\sigma_{i1} \otimes I\right)\sqrt{\varrho_{12}}\left(\sigma_{j1} \otimes I\right)\}, \tag{18}$$

where, $i, j = \{x, y, z\}$ numbers the components of the Pauli matrix.

Figs. 4 and 5 display the temporal evolution of the entanglement and total quantum correlation in the presence and absence of q–q interaction for symmetry and asymmetry interaction with environment. In the $\Delta\gamma \to 0$ limit, both the entanglement and total quantum correlation dynamics for the two qubits present the same behavior as the quantum coherence. When the $\Delta\gamma$ gets farther from zero, the total correlation and coherence keep the same behavior while entanglement and coherence behave quite differently. From the obtained results, we can conclude that the presence of q–q interaction enhances the exchange of the quantum correlations between the two qubits and then enhance the coherence during the temporal evolution. This demonstrates the value and importance of coherence by extracting information from quantum systems and applying it to various areas of quantum optics and information.

Conclusion

We have investigated the coherence and quantum correlations for a system consisting of two interacting qubits under the influence of a common environment, where the system of qubits subjected to the dephasing coupling with its environment. We have displayed the dynamical behavior of the quantumness measures when the effect of the qubit–qubit interaction on the dephasing is considered or ignored. We have analyzed the effect of the q–q interaction and qubits–environment interaction on the quantumness measures considering the case of symmetry and asymmetry interaction between the qubits and environment.

In the case of symmetric interaction, we have found that the coherence exhibits a periodic dynamics that does not depend on the q–q interaction. On the other hand, we have demonstrated that q–q interaction significantly affects the decay of the coherence for asymmetric interaction between the qubits and environment. We have noticed that the decay of coherence can be suppressed when the parameter $\Delta\gamma$ gets closer to zero. Finally, we have explicitly displayed the connection between the coherence and quantum correlations. This current work in terms of phenomenology could be helpful for explaining experimental findings on the coherence and quantum nonlocality and giving further guidance for future investigation on this subject. Here, we have introduced the Markovian environment, then it is very significant to examine and analyze the effect of the interaction between qubits and the properties of non-Markovian environments on the coherence and correlations dynamics.

CRediT authorship contribution statement

K. Berrada: Writing – original draft, Writing – review & editing. **H. Eleuch:** Writing – review & editing.

Declaration of competing interest

The authors claim no conflict of interest statement.

Data availability

No data was used for the research described in the article.

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