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Integration of a Hybrid Method into an Efficient Reliability-Based Design Optimization Technique for Large Structures

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Abstract. The computational burden imposed by the repetitive and lengthy nature of the structural analyses of large structures has lead researchers to continuously devise efficient methods for conducting their reliability analysis in addition to powerful methods for solving the design optimization problem. The presence of a considerably large number of random variables in the reliability analyses of large structure is another issue that tends to slow down the design optimization problem. In this paper, a hybrid method that was recently presented by the first author will be integrated into a structural reliability-based design optimization technique for large structures. In this technique, modified concepts of the weighted average simulation method are used to determine the most probable point of failure in a computationally efficient manner. The most probable point of failure is then transferred into the standard normal space, where the reliability index is calculated in closed-form. The firefly algorithm will be used for solving the optimization problem. It will be shown that the integrated approach significantly reduces the computational resources required for solving the design problem. The technique will be tested on a truss bridge. The results will shed light on the efficiency of the proposed technique in solving reliability-based design optimization problems.

1. Introduction

To determine the reliability of a large structure, a considerably large number of variables typically need to be introduced to the problem. When conducting a simulation-based reliability analysis, the structure needs to be analysed multiple times for each sample point to determine whether it fails under the applied loads, given the random values generated at that specific sample. Accordingly, using a simulation-based reliability analysis in a Reliability-Based Design Optimization (RBDO) can be computationally prohibitive because RBDO problems lend themselves to be solved with iterative optimization methods. Thus, the many times the structural analysis is conducted to find the reliability of the structure is repeated numerous during the optimization problem solution. Modern powerful optimization techniques are nature-inspired, where huge numbers of samples are commonly employed in the search process. This exacerbates the RBDO problem even further if nature-inspired optimization methods, such as the firefly algorithm [1,2], are used.

Despite, the obstacles laid out above, researchers have strived over the past years to develop simulation methods that could find the reliability in a reduced amount of computations. The Monte Carlo Simulation (MCS) [3] is one of the oldest known simulation-based techniques, but has been proven to



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be inefficient if the evaluation of the performance function is computationally expensive or if the probability of failure is very small [3-5]. Ziha [6] developed a stratified sampling technique while Olsson *et al.* [7] presented the Latin Hypercube Sampling. Other methods include the Importance Sampling [8,9], the Response Surface Methods [10,11], the Directional Sampling [12-14], the Line Sampling [15,16], the Subset Simulation [17,18] and the Local Domain Monte Carlo simulation [19]. One of the most recent and more powerful simulation methods is the Weighted Average Simulation Method (WASM) [20-23], which has been proven to be capable of determining the reliability with a reasonably small number of generated samples.

The WASM provides the probability of failure and the Most Probable Point (MPP). The MPP is a point where the highest likelihood among all points in the failure region exists [24,25]. It is, therefore, the point that has the highest probability density on the limit state function. One of the advantages of using the WASM in finding the MPP is that there is no need for an explicit limit state function. Okasha [26] proposed a hybrid technique for computing the reliability of large structures, where the MPP is determined first using a modified version of the WASM. The WASM is modified in order to find the MPP such that the performance function is evaluated for a small part of the generated samples. After the MPP is determined, Okasha [26] transferred it into the standard normal space where the reliability index is calculated in closed-form.

In this paper, the hybrid method proposed in Okasha [26] will be integrated into a structural RBDO technique for large structures. The firefly algorithm will be used for solving the optimization problem. It will be shown that the integrated approach significantly reduces the computational resources required for solving the design problem. The technique will be tested on a truss bridge. The results will show the efficiency of the proposed technique in solving reliability-based design optimization problems.

2. Determination of the MPP by WASM

The WASM is a method that can be used to determine the reliability and the MPP. It can find the MPP in the absence of an explicit limit state function, which is one of the strengths of WASM. The MPP can be found by the WASM following these steps:

- 1- Determine proper intervals for each random variable in the problem. It is suggested in Rashki *et al.* [20] that an MCS can be used to determine the upper and lower points for the interval of each random variable.
- 2- Generate samples for all random samples in a random variable space. The uniform distribution can be used for generating these random samples [20].
- 3- Determine the weight index for each sample as the product of the probability density functions (PDFs) of the variables as follows [20]

$$W(i) = \prod_{j=1}^n f_j(i) \quad (1)$$

where $W(i)$ is the weight index of the i^{th} sample, f_j is the PDF of the j^{th} variable, and n is the number of random variables.

- 4- Establish the index function, $I(i)$, for the i^{th} sample by evaluating the performance function. The $I(i)$ is determined as follows [20]

$$I(i) = \begin{cases} 1 & \text{for } g_i < 0 \\ 0 & \text{for } g_i \geq 0 \end{cases} \quad (2)$$

- 5- The MPP is the point with the highest failure potential. Accordingly, the MPP in the WASM is the point in the failure region with the highest weight index [20]. The MPP can thus be determined as follows

$$\text{MPP} = \max_{i=1}^N \{I(i) \cdot W(i)\} \quad (3)$$

where N is the number of samples.

3. Structural Reliability

The reliability index is the minimum distance between the origin and the failure surface in standard normal space [27]. This distance is demonstrated in Figure 1, which shows a hypothetical space for random variables X_1 and X_2 in addition to the limit state function $g(X_1, X_2) = 0$. The random variables and performance function are transformed to the standard normal space $U_1 - U_2$ in order to determine the reliability index [27] as shown in Figure 1.

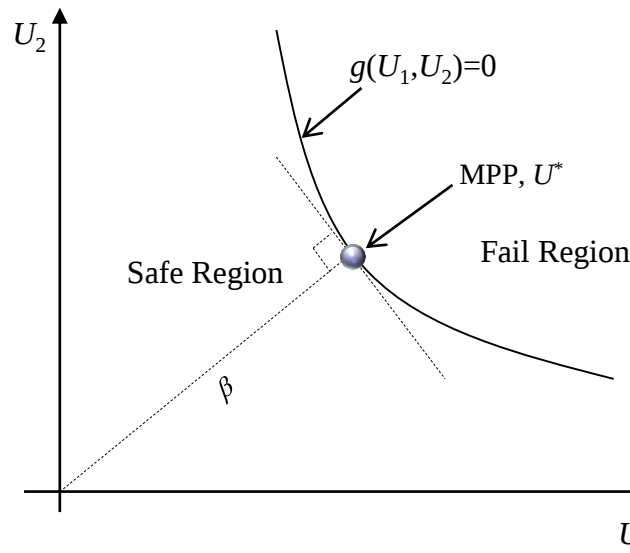


Figure 1. Limit state function $g = 0$, and random variables X_1 and X_2 in standard normal space $U_1 - U_2$.

One can transform n normally distributed random variables $\mathbf{X} = [X_1, X_2, \dots, X_n]$ into a set of standard variables $\mathbf{U} = [U_1, U_2, \dots, U_n]$ with zero means and unit covariance matrix as follows [28]

$$\mathbf{U} = \mathbf{\Gamma D}(\mathbf{X} - \mathbf{M}) \quad (4)$$

where $\mathbf{D} = \text{diag}[\sigma_i]$ is the diagonal matrix for the standard deviations, σ_i is the standard deviation of variable X_i , $\mathbf{M} = [m_1, m_2, \dots, m_n]^T$ is the mean vector where m_i is the mean of variable X_i , and $\mathbf{\Gamma} = \mathbf{L}^{-1}$, where $\mathbf{L}$ is the lower-triangle matrix obtained from Cholesky decomposition [3] of the correlation matrix $\mathbf{R}$ which is equal to [28]

$$\mathbf{R} = \mathbf{D}^{-1} \mathbf{C} \mathbf{D}^{-1} \quad (5)$$

where $\mathbf{C} = [\rho_{ij}\sigma_i\sigma_j]$ is the covariance matrix, ρ_{ij} is the correlation coefficient of variables X_i and X_j . The performance function can be transformed into the standard space as follows [28]

$$g(\mathbf{U}) = g(\mathbf{M} + \mathbf{DLU}) \quad (6)$$

Figure 1 shows that the MPP is the closest point on the limit state function to the origin in the standard normal space. The MPP coordinates are $\mathbf{U}^* = [U_1^*, U_2^*, \dots, U_n^*]$. The reliability index is the Euclidean distance from this point to the origin. Accordingly, the reliability index, β , can be calculated as follows

$$\beta = \|\mathbf{U}^*\| = \sqrt{\mathbf{U}^* \cdot \mathbf{U}^*} \quad (7)$$

The reliability index associated with probability of failure, P_f , are related by

$$\beta = \Phi^{-1}(1 - P_f) \quad (8)$$

where Φ^{-1} is the inverse of the standard normal distribution function.

4. The Hybrid Approach

The premise of the hybrid approach proposed in [26] is that modified concepts from WASM are used to find the MPP, where it is transformed to the standard normal space to find the reliability index. It was illustrated in [26] that the multiplication of small numbers in Equation (1) may cause numerical problems that can be avoided by calculating the natural logarithm of the weight index for each sample as follows

$$\ln(W(i)) = \sum_{j=1}^n \ln(f_j(i)) \quad (9)$$

Hence, the samples are sorted according to the values of the natural logarithm of their weight indices in a descending order. Starting from the sample with highest $\ln(W(i))$, and going through the samples one by one in the order of decreasing $\ln(W(i))$, the index function, $I(i)$, is calculated according to Equation 2. The process stops once the sample that provides a value of $I(i) = 1.0$ is reached. This sample is the MPP. The steps of the method can be listed as follows [26]:

- 1- The random variables and their properties are identified.
- 2- Proper intervals for each random variable in the problem are determined.
- 3- Samples for all random samples are generated in a random variable space. The uniform distribution can be used for generating these random samples.
- 4- The natural logarithm of the weight index is determined for each sample.
- 5- The samples are sorted in a descending order of $\ln(W(i))$. Let $q_1, q_2, \dots, q_n$ be the indexes of the sorted samples in order.
- 6- Let $t = 1$.
- 7- The performance function is evaluated for the sample having the index q_t .
- 8- If $I(q_t) = 1.0$, the q_t^{th} sample is the MPP, i.e., $\mathbf{X}^* = S(q_t)$. Otherwise, $t = t + 1$ and repeat from Step 7.
- 9- Transform $\mathbf{X}^*$ into the standard normal space to obtain $\mathbf{U}^*$.
- 10- $\beta = \|\mathbf{U}^*\|$

The applicability of this algorithm in an RBDO of a large structure is investigated in this paper. The firefly algorithm [1,2] will be used to find the solution of the optimization problem.

Table 1. Member nodal connectivity and grouping for the 10 groups case.

Group Number	Member ID	Node <i>i</i>	Node <i>j</i>	Group Number	Member ID	Node <i>i</i>	Node <i>j</i>
1	1	1	2	6	41	5	22
	2	2	3		43	6	23
	3	3	4		45	7	24
	4	4	5		47	8	25
	13	13	14		49	9	26
	14	14	15		51	27	10
	15	15	16		53	28	11
	16	16	17		55	29	12
2	5	5	6	7	57	30	13
	6	6	7		34	18	2
	7	7	8		36	19	3
	8	8	9		62	15	33
	9	9	10		64	16	34
	10	10	11		66	1	19
	11	11	12		67	2	20
	12	12	13		80	16	32
3	17	18	19	8	81	17	33
	18	19	20		38	20	4
	19	20	21		40	21	5
	20	21	22		58	13	31
	29	30	31		60	14	32
	30	31	32		68	3	21
	31	32	33		69	4	22
	32	33	34		78	14	30
4	21	22	23	9	79	15	31
	22	23	24		42	22	6
	23	24	25		44	23	7
	24	25	26		54	11	29
	25	26	27		56	12	30
	26	27	28		70	5	23
	27	28	29		71	6	24
	28	29	30		76	12	28
5	33	1	18	10	77	13	29
	35	2	19		46	24	8
	37	3	20		48	25	9
	39	4	21		50	9	27
	59	31	14		52	10	28
	61	32	15		72	7	25
	63	33	16		73	8	26
	65	34	17		74	10	26
					75	11	27

5. A 81 Bars Truss

A 81-bar truss bridge is considered in this paper to investigate the applicability of the hybrid algorithm in an RBDO of a large structure. The bridge is shown in Figure 2 and it is adopted from [29]. The members are numbered as shown in Table 1. As also shown in Table 1, the elements are grouped into 10 groups, where the elements in each group are identified in the table. Accordingly, the RBDO problem involves 10 design variables where each represents the cross-sectional area assigned to the elements in one group. These design variables are constrained by a minimum value of 0.5 in² and a maximum of 5.0 in². The problem is also solved where only 3 groups are considered: chord members, vertical members and diagonal members. It should be noted that these three groups correspond to the ones in Table 1 as follows: Groups 1 to 4 are chord members, 5 and 6 are vertical members, and 7 to 10 are diagonal members.

The modulus of elasticity of the truss members are treated as normally distributed random variables with coefficient of variation 0.1 and mean, 29000 ksi, respectively. The load P is treated as a normally distributed random variable with coefficient of variation 0.1 and mean of 40 kips. Accordingly, 82 random variables are treated in this problem.

The mode of failure considered is caused by excessive displacement of node 26. The performance functions associated with this mode of failure is written as follows

$$g = U_{max} - U_{26} \quad (10)$$

where $U_{max} = 0.5$ ft is the maximum allowed displacement, and U_{26} is the absolute value of the displacement of node 26 determined by conducting a structural analysis of the truss bridge.

The problem was solved by MATLAB [30] codes written by the authors, where 1000 samples of WASM, 50 generations of 50 fireflies are used. The objective of the optimization problem was to minimize the volume of the truss bridge while keeping the values of the design variables within the previously specified limits and satisfying a structural reliability corresponding to a probability of failure of at least 10^{-6} . The problem is solved once for the 10 group case and once for the 3 group case.

Figure 3 shows the progress of improvement in the objective function over the generations of the firefly operations. Table 2 shows the optimum values of the design variables in both solved cases. The optimum volume in the 3 groups case was found as 58691 in³ and for the 10 groups was 56668 in³. As expected, the higher freedom provided in the 10 group solution allowed for a design with lower volume.

In terms of the numerical performance of the approach, the average number of times the structural analysis of the bridge had to be conducted each time the reliability of a firefly is computed was about 600 times. Compared to any simulation based RBDO, this is a huge reduction in the number of times the structural analysis is required to be performed in an RBDO problem. It is clear from these results that the hybrid technique provides an efficient platform for conducting RBDO problems of large structures with a significantly reduced computational cost.

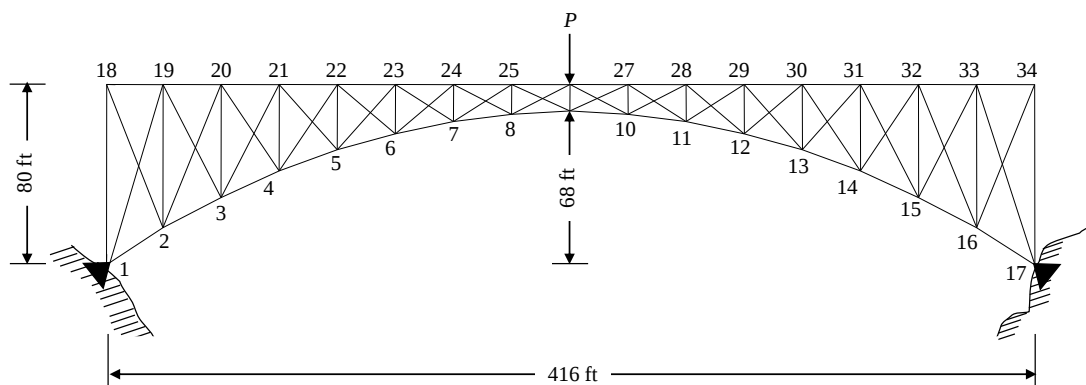


Figure 2. 81 Bar truss bridge

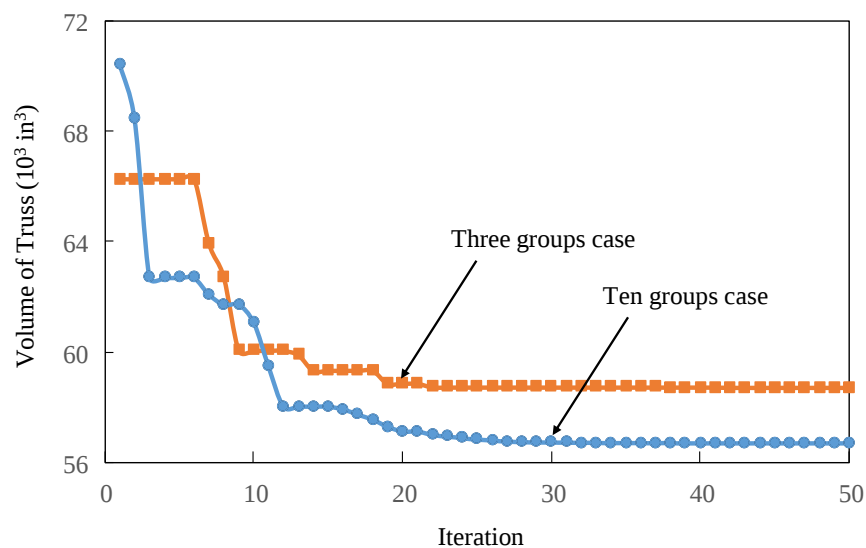


Figure 3. Variation of the total volume of optimum solution with progress in iterations for the 81 bar truss bridge.

Table 2. Optimum solution of the 81 bar truss bridge.

Group Number		Optimum Cross Sectional Areas of Member in the Group (in ²)	
3 Groups Case	10 Groups Case	3 Groups Case	10 Groups Case
1 Chord Members	1	3.6377	2.8256
	2		3.5944
	3		0.5917
	4		2.3803
2 Vertical Members	5	0.5586	0.5000
	6		0.5076
	7		0.7147
3 Diagonal Members	8	0.9751	0.8955
	9		2.3654
	10		4.1904

6. Conclusions

In this paper, the hybrid method proposed by the first author in a previous publication is integrated into a structural RBDO technique for large structures. The firefly algorithm was used for solving the optimization problem. In the hybrid technique, modified concepts of the WASM were used to determine the MPP. The WASM concepts are modified in order to handle the problem of large random variables present in large structures and also in order to efficiently find the MPP. The samples generated by the WASM were arranged according to the natural logarithm of the weight index in a descending order. Once the MPP is determined, it is transferred into the standard normal space. There, the reliability index is calculated in closed-form.

The technique was tested on a truss bridge example. It was observed that the proposed technique required a significantly small number of structural analyses. The integrated approach clearly reduces the computational resources required for solving the design problem significantly. The results showed the efficiency of the proposed technique in solving RBDO problems of large structures.

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