Global exponential stabilization of impulsive neural networks with unbounded continuously distributed delays

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Global exponential stabilization of impulsive neural networks with unbounded continuously distributed delays

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Abstract

This paper investigates the stabilization problem of neural networks with unbounded continuously distributed delays via impulsive control. By establishing an impulsive infinite delay differential inequality from the impulsive control point of view, some sufficient conditions ensuring the existence and global exponential stabilization of the unique equilibrium point are derived. Our result shows that impulses can be used to globally and exponentially stabilize the neural networks with unbounded continuously distributed delays if they are not originally stable or exponentially stable. Finally, two examples with their simulations are given to show the effectiveness of the proposed method.

Key words: Neural networks; Equilibrium point; Global exponential stabilization; Impulsive control; Unbounded continuously distributed delays; Impulsive delay differential inequality.

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1. Introduction

Impulsive effects on the stability and other dynamical behaviors of neural networks has been extensively investigated in the literature because of theoretical interest and applications, see [1–4]. In fact, stimuli from the body or the external environment is received by receptors, the electrical impulses will be conveyed to the neural net and impulsive effects arise naturally[5, 6]. Among the existing investigations, one problem is very interesting, namely the problem that a given model in neural networks is originally stable, and it can remain stable under some impulsive perturbations, which usually is called robustness of stability of neural networks. Recently, the robustness of stability of neural networks has been investigated by many authors via different approaches, see [7–18]. For example, Gopalsamy [7] studied the existence and asymptotic stability of a unique equilibrium for Hopfield neural network by using a fixed point theorem and some analysis techniques. In [10], Stamova and Ilarionov investigated the global exponential stability of cellular neural networks with time-varying delays by using piecewise continuous Lyapunov functions and the Razumikhin technique combined with Young's inequality. As is well known, neural networks usually have a spatial nature due to the presence of parallel pathways with a variety of axon sizes and lengths. So it is desirable

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to model them by introducing unbounded distributed delays. Mixed-type time delays with both time-varying and unbounded distributed delays are the most general case and more difficult to deal with. In [12], Xia investigated the dynamics of impulsive Cohen-Grossberg-type BAM neural networks with continuously distributed delays via fixed point theory, Lyapunov-Kravsovskii functional, homeomorphism theory and the matrix spectral theory. In particular, Mohamad et al. [15] considered a model of neural networks with continuously distributed delays and large impulses as follows:

$$\begin{cases} \dot{x}_i(t) &= -a_i x_i(t) + \displaystyle\sum_{j=1}^{n} b_{ij} f_j\left(\int_0^{\infty} K_{ij}(s) x_j(t-s)ds \right) + c_i, \ t \neq t_k, \ t \geq 0, \\ \Delta x_i(t_k) &= I_k(x_i(t_k^-)), \ i = 1, 2, \cdots, n, \ k \in \mathbb{Z}_+. \end{cases} \tag{1}$$

Some sufficient conditions for the existence and global exponential stability of a unique equilibrium point were obtained by using Lyapunov functions and Halanay inequalities techniques, which can be applied to neural networks with persistent impulsive perturbations.

However, most of the results (e.g. [7–18]) focus on the robustness of stability of impulsive neural networks and do not tackle the problem on impulsive stabilization of neural networks. In fact, even if the corresponding model is originally unstable, the model can be stabilized by utilizing impulsive control. Recently, there have been some initial studies on the impulsive stabilization of neural networks with time delays [19–22]. For example, Yang and Xu [19] initially considered the impulsive control on stability of Cohen-Grossberg neural networks with time-varying delays via inequality techniques. Wang and Liu [20] studied the exponential stability of impulsive cellular neural networks with time delays via Lyapunov functionals and stability theory. However, one may note that the development methods in [19–22] are only valid for neural networks with bounded delays and there is little work done on stabilization problem of neural networks with unbounded delays. Hence, it is an very interesting and challenging work to develop and explore the method for stabilization problem of neural networks with unbounded delays. Recently, from the impulsive perturbation point of view, we established an impulsive infinite delay differential inequality and studied the dynamics of impulsive Cohen-Grossberg-type BAM neural networks with unbounded continuously distributed delays [23]. Inspired by the work, our objective in the paper is to study the stabilization problem of neural networks models (1) by establishing a new impulsive infinite delay differential inequality from the impulsive control point of view. The differential inequality we obtained, which is different from that in [23], can be applied to study impulsive stabilization of systems.

The rest of this paper is organized as follows: In Section 2, we introduce the model of neural networks. In Section 3, some sufficient conditions ensuring the existence and global exponential stabilization of the unique equilibrium point are presented. Two examples with their simulations are given to show the effectiveness of the proposed method in Section 4. Finally, the paper is concluded in Section 5.

2. Preliminaries

Notations. Let $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}_+$ the set of nonnegative numbers, $\mathbb{Z}_+$ the set of positive integers, $\mathbb{R}^n$ the n-dimensional real spaces with the norm $\|x\| = \sum_{i=1}^n |x_i|$ for any $x \in \mathbb{R}^n$. Let $[x]^+ = (|x_1|, |x_2|, \cdots, |x_n|)^T$ for any $x \in \mathbb{R}^n$. Let $\mathscr{A}$ be a matrix. The notation $\mathscr{A} > 0 (\mathscr{A} < 0)$ means that $\mathscr{A}$ is symmetric and positive (negative) definite. Moreover, $\mathscr{A}^T$ and

$\mathscr{A}^{-1}$ denote the transpose and the inverse of $\mathscr{A}$, respectively. I denotes the identity matrix with appropriate dimensions. For any interval $J \subseteq \mathbb{R}$, set $V \subseteq \mathbb{R}^k (1 \le k \le n), C(J, V) = \{\phi : J \to V$ is continuous $\}$, $PC(J, V) = \{\varphi : J \to V$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+), \varphi(t^-)$ exist and $\varphi(t^+) = \varphi(t)\}$ and $PCB(J, V) = \{\varphi \in PC(J, V) : \varphi$ is bounded $\}$. Let $\mathbb{C} \doteq PCB((-\infty, 0], \mathbb{R}^n)$ and for $\phi \in \mathbb{C}$, $\|\phi\|_\infty = \sup_{-\infty \le \theta \le 0} \|\phi(\theta)\|$. $\mathbb{K} = \{h \in C(\mathbb{R}_+, \mathbb{R}_+) |\, h(0) = 0$ and $h(s) > 0$ for $s > 0$ and h is strictly increasing in $s\}$. The impulse times $t_k, k \in \mathbb{Z}_+$, satisfy $0 \le t_0 < t_1 < \ldots < t_k \to +\infty$ as $k \to +\infty$. In addition, let $\Lambda = \{1, 2, \cdots, n\}$.

Consider network model (1) endowed with initial value described by

$$\begin{cases} \dot{u}_i(t) = -a_i u_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(\int_0^\infty K_{ij}(s) u_j(t-s) ds \right) + c_i, \ t \neq t_k, \ t \ge t_0, \\ \Delta u_i(t_k) = I_{ik}(u_i(t_k^-)), \quad k \in \mathbb{Z}_+, \ i \in \Lambda, \\ u_i(s) = \phi_i(s), \quad -\infty \le s \le t_0, \end{cases} \tag{2}$$

where $(\phi_1, \phi_2, \cdots, \phi_n) \in \mathbb{C}$,

$$\Delta u_i(t_k) = u_i(t_k) - u_i(t_k^-), \quad u_i(t_k^-) = \lim_{t \to t_k^-} u_i(t), \ i \in \Lambda,$$

$n \ge 2$ corresponds to the number of units in a neural network, u_i corresponds to the state of the ith neuron at time t, $a_i > 0$ denotes the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network and external inputs. Now b_{ij} denotes the connection weight of the unit j on the unit i, c_i denotes the input of the unit i, I_{ik} shows the impulsive perturbation of the ith neuron at time t_k and f_j denotes the activation function which is Lipschitz continuous, i.e., there exist positive constants L_j, $j \in \Lambda$, such that

$$|f_j(u) - f_j(v)| \le L_j |u - v|, \ \forall\, u, v \in \mathbb{R}, \ \ j \in \Lambda.$$

$K_{ij} : \mathbb{R}_+ \to \mathbb{R}_+, i, j \in \Lambda$, denotes the delay kernel function which is piecewise continuous, satisfying $K_{ij}(s) \le \mathscr{K}(s)$ for any $i, j \in \Lambda, \ s \in \mathbb{R}_+$ and

$$\int_0^\infty K_{ij}(s) ds \doteq \kappa_{ij}, \quad \int_0^\infty \mathscr{K}(s) e^{\lambda_0 s} ds < \infty,$$

where $\mathscr{K}(s) : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous and $\kappa_{ij} > 0, \lambda_0 > 0$ are two real constants.

Remark 2.1. In [15], the existence-uniqueness and global exponential stability of the equilibrium point of networks model (2) has been extensively investigated from the impulsive perturbation point of view. In this paper, we further investigate the dynamics of networks model (2) but from the impulsive control point of view.

Definition 2.1. A real matrix $D = (d_{ij})_{n \times n}$ is said to be an M-matrix if $d_{ii} > 0$, $d_{ij} \le 0, i \neq j, i, j \in \Lambda$, and the successive principle minors of A are positive.

Before presenting the main result, we establish the following Lemma which gives an estimate of the rate of decay of solutions to an impulsive infinite delay differential inequality from the impulsive control point of view.

Lemma 2.1. *Let $\alpha \in \mathbb{R}, \beta > 0, M > 1$ and $\theta_k \geq 0, k \in \mathbb{Z}_+$ denote some constants, and function $f \in PC(\mathbb{R}, \mathbb{R}_+)$ satisfies the following scalar impulsive infinite delay differential inequality*

$$\begin{cases} D^+ f(t) \leq \alpha f(t) + \beta \int_0^\infty \mathcal{K}(s) f(t-s) ds, & t \neq t_k, t \geq t_0, \\ f(t_k) \leq (1+\theta_k) M^{-1} f(t_k^-), & k \in \mathbb{Z}_+, \end{cases} \tag{3}$$

where D^+ denotes the upper right-hand Dini derivative, $\mathcal{K} \in PC(\mathbb{R}_+, \mathbb{R}_+)$, $\int_0^\infty \mathcal{K}(s) e^{\mu_0 s} ds < \infty$ for some positive constant $\mu_0 > 0$. Suppose that there exist constant $\tau > 0$ and function $g \in \mathbb{K}$ such that $t_k - t_{k-1} \leq \tau$, $M^{-1}s \leq g(s) < s$ and

$$\alpha + \beta M \int_0^\infty \mathcal{K}(s) ds < \frac{1}{\tau} \inf_{s>0} \ln \frac{s}{g(s)}. \tag{4}$$

Then

$$f(t) \leq M \overline{f}(t_0) e^{-\lambda(t-t_0)} \prod_{t_0 < t_k \leq t} (1+\theta_k), \ t \geq t_0,$$

where $\overline{f}(t_0) = \sup_{-\infty \leq s \leq t_0} f(s)$, and $\lambda \in (0, \mu_0]$ satisfies

$$\lambda < \frac{1}{\tau} \inf_{s>0} \ln \frac{s}{g(s)} - \alpha - \beta M \int_0^\infty \mathcal{K}(s) e^{\lambda s} ds. \tag{5}$$

Proof. First, assumption (4) implies that there exists a constant $\lambda \in (0, \mu_0]$ small sufficiently such that the inequality (5) holds.

Define

$$\mathscr{F}(t) = \begin{cases} f(t)\, e^{\lambda(t-t_0)}, & t \geq t_0, \\ f(t), & -\infty \leq t \leq t_0. \end{cases}$$

It is obvious that $\mathscr{F}(t) \geq f(t), t \in \mathbb{R}$ and $\mathscr{F}(t) \leq \overline{f}(t_0), t \in (-\infty, t_0]$.
We claim that

$$\mathscr{F}(t) \leq M \overline{f}(t_0) \prod_{k=0}^{n-1} (1+\theta_k), \ t \in [t_{n-1}, t_n), n \in \mathbb{Z}_+.$$

where $\theta_0 = 0$. Taking $n = 1$, we first claim that $\mathscr{F}(t) \leq M\overline{f}(t_0), t \in [t_0, t_1)$. Suppose on the contrary. Then in view of $\mathscr{F}(t) \leq \overline{f}(t_0), t \in (-\infty, t_0]$, we know that there exists a $t^\star \in (t_0, t_1)$ such that

$$\mathscr{F}(t^\star) = M\overline{f}(t_0), \quad \mathscr{F}(t) \leq M\overline{f}(t_0), t \in (-\infty, t^\star]. \tag{6}$$

Considering $M^{-1}s \leq g(s) < s$, we get $\mathscr{F}(t^\star) = M\overline{f}(t_0) > g(M\overline{f}(t_0))$, $\mathscr{F}(t_0) \leq \overline{f}(t_0) \leq g(M\overline{f}(t_0))$, which implies that there exists a $t^{\star\star} < t^\star$ such that $\mathscr{F}(t^{\star\star}) = g(M\overline{f}(t_0))$ and $\mathscr{F}(t) > g(M\overline{f}(t_0))$, $t \in (t^{\star\star}, t^\star]$. This together with (6) and $g \in \mathbb{K}$ yield $g^{-1}(\mathscr{F}(t)) \geq M\overline{f}(t_0) > \mathscr{F}(t -$

$s), \ s \in [0, \infty), \ t \in [t^{\star\star}, t^{\star}]$. Then calculating the upper right derivative $\mathscr{F}$ along the solutions of (3) on $[t^{\star\star}, t^{\star}]$, we get

$$\begin{aligned} D^{+}\mathscr{F}(t) &\leq \left[\alpha f(t) + \beta \int_0^{\infty} \mathcal{K}(s) f(t-s) ds \right] e^{\lambda(t-t_0)} + \lambda f(t) e^{\lambda(t-t_0)} \\ &= (\lambda+\alpha) f(t) e^{\lambda(t-t_0)} + \beta \int_0^{\infty} \mathcal{K}(s) e^{\lambda s} f(t-s) e^{\lambda(t-s-t_0)} ds \\ &\leq (\lambda+\alpha)\mathscr{F}(t) + \beta \int_0^{\infty} \mathcal{K}(s) e^{\lambda s} \mathscr{F}(t-s) ds \end{aligned}$$

$$\begin{aligned} &\leq (\lambda+\alpha)\mathscr{F}(t) + \beta \int_0^{\infty} \mathcal{K}(s) e^{\lambda s} g^{-1}(\mathscr{F}(t)) ds \\ &\leq \mathscr{F}(t) \left[\lambda + \alpha + \beta \int_0^{\infty} \mathcal{K}(s) e^{\lambda s} ds \frac{g^{-1}(\mathscr{F}(t))}{\mathscr{F}(t)} \right] \\ &\leq \mathscr{F}(t) \left[\lambda + \alpha + \beta M \int_0^{\infty} \mathcal{K}(s) e^{\lambda s} ds \right], \ t \in [t^{\star\star}, t^{\star}], \end{aligned}$$

in view of $M^{-1}s \leq g(s) < s$. Integrating the above inequality from $t^{\star\star}$ to $t^{\star}$, we get

$$\int_{t^{\star\star}}^{t^{\star}} \frac{d\mathscr{F}(t)}{\mathscr{F}(t)} \leq \tau \left[\lambda + \alpha + \beta M \int_0^{\infty} \mathcal{K}(s) e^{\lambda s} ds \right]. \tag{7}$$

On the other hand, by (4) we note

$$\int_{t^{\star\star}}^{t^{\star}} \frac{d\mathscr{F}(t)}{\mathscr{F}(t)} = \int_{\mathscr{F}(t^{\star\star})}^{\mathscr{F}(t^{\star})} \frac{ds}{s} = \int_{g(M\overline{f}(t_0))}^{M\overline{f}(t_0)} \frac{ds}{s} \geq \inf_{s>0} \ln \frac{s}{g(s)} > \tau \left[\lambda + \alpha + \beta M \int_0^{\infty} \mathcal{K}(s) e^{\lambda s} ds \right], \tag{8}$$

which is a contradiction with (7) and thus $\mathscr{F}(t) \leq M\overline{f}(t_0), \ t \in [t_0, t_1)$.
Now we assume that

$$\mathscr{F}(t) \leq M\overline{f}(t_0) \prod_{k=0}^{N-1} (1+\theta_k) \doteq \Gamma(N-1), \ t \in [t_0, t_N), \ \text{for some} \ N \in \mathbb{Z}_+.$$

Note that $\theta_k \geq 0, k \in \mathbb{Z}_+$, implies

$$\mathscr{F}(t) \leq \Gamma(N-1), \ t \in (-\infty, t_N). \tag{9}$$

Thus $\mathscr{F}(t_N) = f(t_N)e^{\lambda(t_N - t_0)} \leq (1+\theta_N)M^{-1} f(t_N^{-})e^{\lambda(t_N - t_0)} \leq (1+\theta_N)M^{-1}\Gamma(N-1) \leq M^{-1}\Gamma(N)$. We shall show that $\mathscr{F}(t) \leq \Gamma(N), t \in [t_N, t_{N+1})$. Suppose on the contrary. Then there exists a $t^{*} > t_N$ such that $\mathscr{F}(t^{*}) = \Gamma(N)$ and $\mathscr{F}(t) \leq \Gamma(N), t \in (-\infty, t^{*}]$, in view of (9). Using the condition $M^{-1}s \leq g(s) < s$ again, we get $\mathscr{F}(t^{*}) = \Gamma(N) > g(\Gamma(N)), \ \mathscr{F}(t_N) \leq M^{-1}\Gamma(N) \leq g(\Gamma(N))$, which implies that there exists a $t^{**} < t^{*}$ such that $\mathscr{F}(t^{**}) = g(\Gamma(N))$ and $\mathscr{F}(t) > g(\Gamma(N)), \ t \in (t^{**}, t^{*}]$. This, together with the fact that $\mathscr{F}(t) \leq \Gamma(N), t \in (-\infty, t^{*}]$, yields $g^{-1}(\mathscr{F}(t)) \geq \Gamma(N) > \mathscr{F}(t - s), \ s \in [0, \infty), \ t \in [t^{**}, t^{*}]$. Then applying exactly the same argument as in (7) and (8) yields a contradiction and thus $\mathscr{F}(t) \leq \Gamma(N), \ t \in [t_N, t_{N+1})$. By the method of induction, we conclude

$$\mathscr{F}(t) \leq M\overline{f}(t_0) \prod_{k=0}^{n} (1+\theta_k), \ t \in [t_n, t_{n+1}), \ n \in \mathbb{Z}_+,$$

which implies

$$f(t) \le M\overline{f}(t_0)e^{-\lambda(t-t_0)} \prod_{t_0<t_k\le t} (1+\theta_k),\ t \ge t_0,$$

where λ satisfies (5). The proof of Lemma 2.1 is therefore complete. ■

Corollary 2.1. *Let $\alpha \in \mathbb{R}, \beta > 0$ and $M > 1$ denote some constants, and function $f \in PC(\mathbb{R}, \mathbb{R}_+)$ satisfies the following scalar impulsive infinite delay differential inequality*

$$\begin{cases} D^+ f(t) \le \alpha f(t) + \beta \displaystyle\int_0^\infty \mathcal{K}(s) f(t-s) ds, & t \ne t_k, t \ge t_0, \\ f(t_k) \le M^{-1} f(t_k^-), & k \in \mathbb{Z}_+, \end{cases}$$

where $\mathcal{K} \in PC(\mathbb{R}_+, \mathbb{R}_+)$, $\int_0^\infty \mathcal{K}(s)e^{\mu_0 s} ds < \infty$ for some positive constant $\mu_0 > 0$. Suppose that there exists a constant $\tau > 0$ such that $t_k - t_{k-1} \le \tau$ and

$$\alpha + \beta M \int_0^\infty \mathcal{K}(s) ds < \frac{\ln M}{\tau}.$$

Then

$$f(t) \le M\overline{f}(t_0)e^{-\lambda(t-t_0)},\ t \ge t_0,$$

where $\overline{f}(t_0) = \sup_{-\infty\le s\le t_0} f(s)$, and $\lambda \in (0, \mu_0]$ satisfies

$$\lambda < \frac{\ln M}{\tau} - \alpha - \beta M \int_0^\infty \mathcal{K}(s)e^{\lambda s} ds.$$

Proof. Let $g(s) = M^{-1}s$ and $\theta_k \equiv 0, k \in \mathbb{Z}_+$ and by Lemma 2.1, we can easily derive the above result. ■

Remark 2.2. Recently, we have derived an exponential estimate of the rate of decay of solutions to inequality (3) with $\alpha < 0$, see [23] for details. The result is given from the impulsive perturbation point of view, that is, the estimate still holds when there is no impulsive effects (i.e., $f(t_k) = f(t_k^-)$). Note in this paper, an exponential estimate for inequality (3) with $\alpha \in \mathbb{R}$ is established from the impulsive control point of view, that is, the solutions of (3) without impulsive effects may be divergent, but it admits an effective exponential estimate via proper impulsive control. Such result can be applied to the stabilization problems of unstable systems.

3. Main results

In this section, based on Lemma 2.1, some conditions ensuring the existence and global exponential stabilization of the unique equilibrium point of system (2) are obtained. First, we present some conditions for the existence and uniqueness of the equilibrium point, which are different from those in [15].

Theorem 3.1. *System (2) has at least one equilibrium point if $I - A^{-1}B^\star L$ is an M-matrix, where A =diag$(a_1, a_2, \cdots, a_n)$, $B^\star = (|b_{ij}|\kappa_{ij})_{n\times n}$ and L =diag$(L_1, L_2, \cdots, L_n)$.*

Proof. Suppose that $u^\star = (u_1^\star, u_2^\star, \cdots, u_n^\star) \in \mathbb{R}^n$ is an equilibrium point of system (2). Then it satisfies

$$-a_i u_i^\star + \sum_{j=1}^{n} b_{ij} f_j\left(\int_0^\infty K_{ij}(s) u_j^\star ds \right) + c_i = 0,\ i \in \Lambda,$$

i.e.,

$$a_i u_i^\star - \sum_{j=1}^{n} b_{ij} f_j\left(\kappa_{ij} u_j^\star\right) - c_i = 0,\ i \in \Lambda.$$

Set $h(x) = (h_1(x), h_2(x), \cdots, h_n(x))^T, x = (x_1, x_2, \cdots, x_n)^T \in \mathbb{R}^n$, where

$$h_i(x) = x_i - \sum_{j=1}^{n} \frac{b_{ij}}{a_i} f_j\left(\kappa_{ij} x_j\right) - \frac{c_i}{a_i}, i \in \Lambda.$$

Obviously, system (2) has an equilibrium point $u^\star$ if and only if $h(u^\star) = 0$. Thus we now only need to show that equation $h(x) = 0$ has at least one solution. Consider a homotopic mapping

$$H(x, \lambda) = \lambda h(x) + (1 - \lambda)x, \lambda \in [0, 1].$$

It follows that

$$\begin{aligned}
[H(x,\lambda)]^+ &= [\lambda h(x) + (1-\lambda)x]^+ \\
&\geq [x]^+ - \lambda\Big(\sum_{j=1}^{n} \frac{|b_{1j}|}{a_1} |f_j(\kappa_{1j}x_j)|, \sum_{j=1}^{n} \frac{|b_{2j}|}{a_2} |f_j(\kappa_{2j}x_j)|, \cdots, \sum_{j=1}^{n} \frac{|b_{nj}|}{a_n} |f_j(\kappa_{nj}x_j)|\Big)^T - \lambda c \\
&\geq [x]^+ - \lambda\Big(\sum_{j=1}^{n} \frac{|b_{1j}|}{a_1} \kappa_{1j} L_j |x_j|, \sum_{j=1}^{n} \frac{|b_{2j}|}{a_2} \kappa_{2j} L_j |x_j|, \cdots, \sum_{j=1}^{n} \frac{|b_{nj}|}{a_n} \kappa_{nj} L_j |x_j|\Big)^T \\
&\quad -\lambda\Big(\sum_{j=1}^{n} \frac{|b_{1j}|}{a_1} |f_j(0)|, \sum_{j=1}^{n} \frac{|b_{2j}|}{a_2} |f_j(0)|, \cdots, \sum_{j=1}^{n} \frac{|b_{nj}|}{a_n} |f_j(0)|\Big)^T - \lambda c \\
&= [x]^+ - \lambda A^{-1} B^\star L [x]^+ - \lambda A^{-1} B [f(0)]^+ - \lambda c \\
&= (1-\lambda)[x]^+ + \lambda\Big\{(I - A^{-1}B^\star L)[x]^+ - A^{-1}B[f(0)]^+ - c\Big\},
\end{aligned} \tag{10}$$

where

$$B = (|b_{ij}|)_{n\times n},\ c = \Big(\frac{|c_1|}{a_1}, \frac{|c_2|}{a_2}, \cdots, \frac{|c_n|}{a_n}\Big)^T.$$

Since $I - A^{-1}B^\star L$ is an M-matrix, there exists a $T \in R_+^n$ such that $(I - A^{-1}B^\star L)T > 0$. Note that $(I - A^{-1}B^\star L)^{-1} \geq 0$. Then one may define a non-empty set

$$\Pi = \Big\{x \in R^n :\ [x]^+ \leq T + (I - A^{-1}B^\star L)^{-1}\Big(A^{-1}B[f(0)]^+ + c\Big)\Big\}.$$

For any $x \in \partial\Pi$, it follows from (10) that

$$[H(x, \lambda)]^+ \geq (1 - \lambda)[x]^+ + \lambda(I - A^{-1}B^\star L)T > 0, \lambda \in [0, 1],$$

which implies that $[H(x, \lambda) \neq 0$, for any $x \in \partial\Pi$ and $\lambda \in [0, 1]$, where $\partial\Pi$ denotes the boundary of Π. Thus $H(x, \lambda)$ is a nonsingular continuous deformation between $h(x)$ and x on $\partial\Pi$. By employing topological degree invariance theory [24], we get

$$\deg(h(x), \Pi, 0) = \deg(H(x, 1), \Pi, 0) = \deg(H(x, 0), \Pi, 0) = 1.$$

Thus equation $h(x) = 0$ has at least one solution on Π, which implies that system (2) has at least one equilibrium point, completing the proof. ■

Theorem 3.2. *System* (2) *has at least one equilibrium point if f_j, $j \in \Lambda$ are bounded on $\mathbb{R}$.*

Proof. Suppose that $|f_j| \leq M_j$, $j \in \Lambda$. Consider a mapping $\Psi(v) = (\Psi_1(v), \Psi_2(v), \cdots, \Psi_n(v))^T$, where $v = (v_1, v_2, \cdots, v_n)^T \in \triangle$, and

$$\Psi_i(v) = \sum_{j=1}^{n} \frac{b_{ij}}{a_i} f_j\Big(\kappa_{ij} v_j\Big) + \frac{c_i}{a_i},\ i \in \Lambda, \qquad \triangle = \Big\{x \in \mathbb{R}^n : \|x\| \leq \rho = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{|b_{ij}|}{a_i} M_j + \sum_{i=1}^{n} \frac{|c_i|}{a_i}\Big\}.$$

It is easy to check that $\triangle$ is an invariant set of Ψ. Moreover, $\Psi : \triangle \to \triangle$ is continuous. By employing the well-known Brouwer's fixed point theorem, there exists a $v^* \in \triangle$ such that $\Psi(v^*) = v^*$, which implies that system (2) has at least one equilibrium point. The proof is complete. ■

Theorem 3.3. *System* (2) *has at least one equilibrium point if there exists vector* $(\delta_1, \delta_2, \cdots, \delta_n)^T \in \mathbb{R}^n$ *such that*

$$\sum_{j=1}^{n} b_{ij} f_j\Big(\kappa_{ij} \frac{c_j + \delta_j}{a_j}\Big) = \delta_i,\ i \in \Lambda.$$

Proof. Note that $u^\star = (u_1^\star, u_2^\star, \cdots, u_n^\star)$ is an equilibrium point of system (2), which satisfies

$$-a_i u_i^\star + \sum_{j=1}^{n} b_{ij} f_j\Big(\kappa_{ij} u_j^\star\Big) + c_i = 0,\ i \in \Lambda.$$

Let $u_i = \frac{c_i + \delta_i}{a_i}$, $i \in \Lambda$, then we can derive the above result, completing the proof. ■

Theorem 3.4. *Under the conditions in Theorem* 3.1, 3.2 *or* 3.3, *system* (2) *has a unique equilibrium point if there exists a $i \in \Lambda$ such that $|b_{ij}|\kappa_{ij} L_j < a_i$ for all $j \in \Lambda$.*

Proof. By Theorem 3.1, 3.2 or 3.3, system (2) has at least one equilibrium point. For the sake of uniqueness, suppose that $u^\star = (u_1^\star, u_2^\star, \cdots, u_n^\star) \in \mathbb{R}^n$ and $v^\star = (v_1^\star, v_2^\star, \cdots, v_n^\star) \in \mathbb{R}^n$ are the different equilibrium points of system (2). Then it can be deduced that

$$\sum_{i=1}^{n} |u_i^\star - v_i^\star| = \sum_{i=1}^{n} \Big| \sum_{j=1}^{n} \frac{b_{ij}}{a_i} f_j\Big(\kappa_{ij} u_j^\star\Big) - \sum_{j=1}^{n} \frac{b_{ij}}{a_i} f_j\Big(\kappa_{ij} v_j^\star\Big)\Big| \leq \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{|b_{ij}|\kappa_{ij} L_j}{a_i} |u_j^\star - v_j^\star|$$

Obviously, it follows from our assumption that there is a contradiction in the above inequality. Thus the uniqueness is proved and the proof is complete. ■

Assume that $u^\star = (u_1^\star, u_2^\star, \cdots, u_n^\star)^T$ is an equilibrium point of system (2). As usual, the impulsive operator is viewed as a perturbation of the equilibrium point $u^\star$ defined by

$$u_i(t_k) - u_i(t_k^-) = I_{ik}(u_i(t_k^-)) = -\gamma_{ik}(u_i(t_k^-) - u_i^\star),\ i \in \Lambda,\ k \in \mathbb{Z}_+, \tag{11}$$

where γ_{ik} denote some real numbers. Let $z_i = u_i - u_i^\star$, then system (2) becomes

$$\begin{cases} \dot{z}_i(t) &= -\alpha_i z_i(t) + \sum_{j=1}^n b_{ij} g_j\Big(\int_0^\infty K_{ij}(s) z_j(t-s)ds\Big),\ t \neq t_k,\ t \geq 0, \\ z_i(t_k) &= (1-\gamma_{ik}) z_i(t_k^-),\ k \in \mathbb{Z}_+,\ i \in \Lambda, \\ \varphi_i(s) &= \phi_i(s) - u^\star, \quad -\infty \leq s \leq 0, \end{cases} \tag{12}$$

where $(\varphi_1, \varphi_2, \cdots, \varphi_n) \in \mathbb{C}$ and

$$g_j\Big(\int_0^\infty K_{ij}(s) z_j(t-s)ds\Big) = f_j\Big(\int_0^\infty K_{ij}(s) z_j(t-s)ds + u_j^\star \kappa_{ij}\Big) - f_j\big(u_j^\star \kappa_{ij}\big).$$

Obviously, the impulsive stabilization of the equilibrium point $u^\star$ of (2) can now be transformed to the impulsive stabilization of the trivial solution $z = 0$ of (12). Set $\Upsilon_k = (\gamma_{1k}, \gamma_{2k}, \cdots, \gamma_{nk})^T$ and denote by $(t_k, \Upsilon_k)_{k\in\mathbb{Z}_+}$ the impulsive controller that describes the fact that system (2) will encounter the impulsive effects in terms of (11) at time t_k. Next we shall present some conditions to design the impulsive controller $(t_k, \Upsilon_k)_{k\in\mathbb{Z}_+}$ such that the unstable (non-exponential stable) system (2) becomes globally exponentially stable.

Theorem 3.5. *Under the conditions in one of Theorems* 3.1-3.4, *assume that there exist* $M > 1, \tau > 0$, $\Upsilon_k = (\gamma_{1k}, \gamma_{2k}, \cdots, \gamma_{nk})^T \in \mathbb{R}^n, k \in \mathbb{Z}_+$, *and function* $g \in \mathbb{K}$ *such that* $t_k - t_{k-1} \leq \tau$, $M^{-1}s \leq g(s) < s, s \in \mathbb{R}_+$,

$$-\min a_i + \sum_{i=1}^n \Big(\max_{j\in\Lambda} |b_{ij}| L_j\Big) M \int_0^\infty \mathscr{K}(s)ds < \frac{1}{\tau}\inf_{s>0} \ln\frac{s}{g(s)},$$

and

$$\prod_{k=1}^\infty \max\{1, \Omega_k\} < \infty,\quad \Omega_k = M \max_{i\in\Lambda} |1-\gamma_{ik}|.$$

Then system (2) *has a unique equilibrium point which can be globally exponentially stabilized via the impulsive controller* $(t_k, \Upsilon_k)_{k\in\mathbb{Z}_+}$. *Also, the approximate exponential convergent rate* $\lambda \in (0, \lambda_0]$ *is derived, which satisfies*

$$\lambda < \frac{1}{\tau}\inf_{s>0}\ln\frac{s}{g(s)} + \min a_i - \sum_{i=1}^n \Big(\max_{j\in\Lambda}|b_{ij}|L_j\Big) M \int_0^\infty \mathscr{K}(s) e^{\lambda s} ds. \tag{13}$$

Proof. It follows from Theorems 3.1-3.4 that the equilibrium point of the system (2) exists. For any $\varphi = (\varphi_1, \varphi_2, \cdots, \varphi_n)^T \in \mathbb{C}$, let $z(t) = (z_1, z_2, \cdots, z_n)^T$ be a solution of system (12) through (t_0, φ). Calculating the upper right derivative $D^+|z_i(t)|$ along the solutions of system (12), we get

$$\begin{aligned} D^+|z_i(t)| &= \left[-\alpha_i z_i(t) + \sum_{j=1}^{n} b_{ij} g_j \left(\int_0^\infty K_{ij}(s) z_j(t-s) ds \right) \right] sgn(z_i(t)) \\ &\leq -a_i|z_i(t)| + \sum_{j=1}^{n} b_{ij} L_j \int_0^\infty K_{ij}(s)|z_j(t-s)|ds,\ t \neq t_k, t \geq t_0, k \in \mathbb{Z}_+. \end{aligned} \tag{14}$$

Choose a Lyapunov function

$$V(t) = ||z(t)|| = \sum_{i=1}^{n} |z_i(t)|,\ t \in \mathbb{R},$$

and define

$$\overline{V}(t) = \sup_{-\infty \leq s \leq t} V(s) = \sup_{-\infty \leq s \leq t} \left(\sum_{i=1}^{n} |z_i(s)| \right), t \geq t_0.$$

Then by (14), we obtain the time derivative of V along the solutions of (12), for $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$,

$$\begin{aligned} D^+V(t) = \sum_{i=1}^{n} D^+|z_i(t)| &\leq -\sum_{i=1}^{n} a_i|z_i(t)| + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}L_j \int_0^\infty K_{ij}(s)|z_j(t-s)|ds \\ &\leq -\min_{i\in\Lambda} a_i \sum_{i=1}^{n} |z_i(t)| + \sum_{i=1}^{n} \left(\max_{j\in\Lambda} b_{ij}L_j \right) \int_0^\infty \mathscr{K}(s) \sum_{j=1}^{n} |z_j(t-s)|ds \\ &= -\min_{i\in\Lambda} a_i V(t) + \sum_{i=1}^{n} \left(\max_{j\in\Lambda} b_{ij}L_j \right) \int_0^\infty \mathscr{K}(s) V(t-s)ds. \end{aligned} \tag{15}$$

On the other hand,

$$V(t_k) = \sum_{i=1}^{n} |z_i(t_k)| = \sum_{i=1}^{n} |1-\gamma_{ik}||z_i(t_k^-)| \leq \Omega_k M^{-1} V(t_k^-) \leq \max\{1, \Omega_k\} M^{-1} V(t_k^-). \tag{16}$$

It then follows from (15), (16) and Lemma 2.1 that

$$V(t) \leq M\overline{V}(0)e^{-\lambda(t-t_0)} \prod_{k=1}^{\infty} \max\{1, \Omega_k\},\ t \geq t_0,$$

i.e.,

$$\sum_{i=1}^{n} |z_i(t)| \leq \mathbb{M}||\phi||_\infty e^{-\lambda(t-t_0)},\ t \geq t_0,$$

where $\mathbb{M} = \prod_{k=1}^{\infty} \max\{1, \Omega_k\} M$ and $\lambda \in (0, \lambda_0]$ satisfies (13). The proof is complete. ■

Corollary 3.1. *Under the conditions in Theorem* 3.5, *assume that there exist* $M > 1$ *and* $\tau > 0$ *such that* $t_k - t_{k-1} \leq \tau$, $|1-\gamma_{ik}| \leq M^{-1}$ *and*

$$-\min a_i + \sum_{i=1}^{n} \left(\max_{j\in\Lambda} |b_{ij}| L_j \right) M \int_0^\infty \mathscr{K}(s)ds < \frac{\ln M}{\tau}.$$

Then system (2) *has a unique equilibrium point which can be globally exponentially stabilized via the impulsive controller* $(t_k, \Upsilon_k)_{k\in\mathbb{Z}_+}$. *Also, the approximate exponential convergent rate* $\lambda \in (0, \lambda_0]$ *is derived, which satisfies*

$$\lambda < \frac{\ln M}{\tau} + \min a_i - \sum_{i=1}^{n} \Big(\max_{j\in\Lambda} |b_{ij}|L_j \Big) M \int_0^\infty \mathscr{K}(s) e^{\lambda s} ds.$$

Proof. Note that $\Omega_k = |1 - \gamma_{ik}| M \le 1$ and let $g(s) = M^{-1}s$, then we can derive the above result. ■

Remark 3.1. By Theorems 3.1-3.3, it should be noted that the equilibrium point of system (2) exist but it is not always unique. Theorem 3.5 provides the conditions for global exponential stabilization of the equilibrium point of system (2), which implies the uniqueness of the equilibrium point. Hence, Theorem 3.5 can be applied to not only the global stabilization problem but also the uniqueness problem of the equilibrium point.

4. Numerical examples

Example 4.1. Consider an impulsive neural networks model

$$\begin{pmatrix} \dot u_1(t) \\ \dot u_2(t) \end{pmatrix} = \begin{pmatrix} -0.1 & 0 \\ 0 & -0.1 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} + \begin{pmatrix} 0.5 & 0.1 \\ -0.1 & 0.5 \end{pmatrix} \begin{pmatrix} \left| \int_0^\infty e^{-s} u_1(t-s) ds - 1 \right| \\ \left| \int_0^\infty e^{-s} u_2(t-s) ds + 2 \right| \end{pmatrix} + c, \tag{17}$$

subject to

$$\begin{pmatrix} \Delta u_1(t_k) \\ \Delta u_2(t_k) \end{pmatrix} = \begin{pmatrix} -\gamma_{1k} & 0 \\ 0 & -\gamma_{2k} \end{pmatrix} \begin{pmatrix} u_1(t_k^-) - 1 \\ u_2(t_k^-) + 2 \end{pmatrix}, \ k \in \mathbb{Z}_+, \tag{18}$$

where $\gamma_{1k} = \gamma_{2k} = \frac{1}{3} - \frac{2}{3k^2}$, $t_k = 0.25k$, $k \in \mathbb{Z}_+$, $K_{ij}(s) = e^{-s}, i, j = 1, 2, c = (0.1, -0.2)^T$. Then $f_1(u) = |u - 1|$, $f_2(u) = |u + 2|$, $L_1 = L_2 = 1$, $\mathscr{K}(s) = K_{ij}(s) = e^{-s}, \tau = 0.25, \lambda_0 < 1$. Choose $\delta_i = 0, i \in \Lambda$. By Theorem 3.3, it is easy to check that system (17) has an equilibrium point $(1, -2)^T$. Numerical result shows that the equilibrium point $(1, -2)^T$ of system (17) without impulsive effects is unstable (see Fig.1.(a)).

Now we claim that system (17) has a unique equilibrium point $(1, -2)^T$ which can be globally exponentially stabilized by impulses (18). In fact, one may choose $M = 1.5 > 1$ such that

$$-\min a_i + \sum_{i=1}^{n} \Big(\max_{j\in\Lambda} |b_{ij}|L_j \Big) M \int_0^\infty \mathscr{K}(s) ds = 1.4 < 1.6219 \approx \frac{\ln M}{\tau},$$

$$\prod_{k=1}^{\infty} \max\{1, \Omega_k\} = \prod_{k=1}^{\infty} 1 + \frac{1}{k^2} < \infty.$$

Then it is easy to check that all conditions of Theorem 3.5 are satisfied. Therefore, the unique equilibrium point $(1, -2)^T$ of system (17) can be globally exponentially stabilized by impulses

(18), which is shown in Fig.1.(b). Also, the approximate exponential convergent rate λ is derived as follows: $\lambda \in (0,1)$ satisfies

$$\lambda < 1.7219 - 1.5\int_0^{\infty} e^{-(1-\lambda)s}ds.$$

Remark 4.1. One can observe that condition (3.2) in [15] is not satisfied for model (17), which means that the theorems in [15] cannot be used to ascertain the existence and stability of the equilibrium point of model (17).

Example 4.2. Consider another impulsive neural networks model

$$\begin{pmatrix} \dot{u}_1(t) \\ \dot{u}_2(t) \end{pmatrix} = \begin{pmatrix} -0.3 & 0 \\ 0 & -0.2 \end{pmatrix}\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} + \begin{pmatrix} -1 & 0.3 \\ 0.1 & 1 \end{pmatrix}\begin{bmatrix} \tanh\left(\int_0^{\infty} 0.65e^{-s}u_1(t-s)ds\right) \\ \tanh\left(\int_0^{\infty} 0.65e^{-s}u_2(t-s)ds\right) \end{bmatrix}, \tag{19}$$

subject to

$$\begin{pmatrix} \Delta u_1(t_k) \\ \Delta u_2(t_k) \end{pmatrix} = \begin{pmatrix} -\frac{1}{6} & 0 \\ 0 & -\frac{1}{5} \end{pmatrix}\begin{pmatrix} u_1(t_k^-) \\ u_2(t_k^-) \end{pmatrix}, \; k \in \mathbb{Z}_+, \tag{20}$$

where $t_k = 0.12k$, $k \in \mathbb{Z}_+$, $K_{ij}(s) = 0.65e^{-s}, i, j = 1, 2$. Then $f_1(u) = f_2(u) = \tanh(u)$, $L_1 = L_2 = 1$, $\mathscr{K} = 0.65e^{-s}, \tau = 0.12, \lambda_0 < 1$. By Theorem 3.2, it is obvious that system (19) has an equilibrium point $(0,0)^T$. Numerical result shows that the equilibrium point $(0,0)^T$ is unstable (see Fig.2.(a)).

Now we claim that system (19) has a unique equilibrium point $(0,0)^T$ which can be globally exponentially stabilized by impulses (20). In fact, one may choose $M = 1.2 > 1$ such that

$$-\min a_i + \sum_{i=1}^{n}\Big(\max_{j\in\Lambda} |b_{ij}|L_j\Big)M\int_0^{\infty}\mathscr{K}(s)ds \approx 1.36 < 1.5193 \approx \frac{\ln M}{\tau}.$$

Note that $|1-\gamma_{ik}|M \leq 1$ and by Corollary 3.1, the unique equilibrium point $(0,0)^T$ of system (19) can be exponentially stabilized by impulses (20), which is shown in Fig.2.(b). Also, the approximate exponential convergent rate λ is derived as follows: $\lambda \in (0,1)$ satisfies

$$\lambda < 1.7193 - 1.56\int_0^{\infty} e^{-(1-\lambda)s}ds.$$

5. Conclusion

From the impulsive control point of view, we have studied the stabilization problem of neural networks with unbounded continuously distributed delays by establishing a new impulsive infinite delay differential inequality. Some conditions for the existence, uniqueness and global exponential stabilization of the equilibrium point have been presented. The criteria can be applied to exponentially stabilize the neural networks with unbounded delays if they are not originally stable or exponentially stable. To the best of our knowledge, so far few authors have considered the dynamics of neural networks with unbounded delays via impulsive control. Two numerical examples have illustrated the effectiveness of the proposed method.

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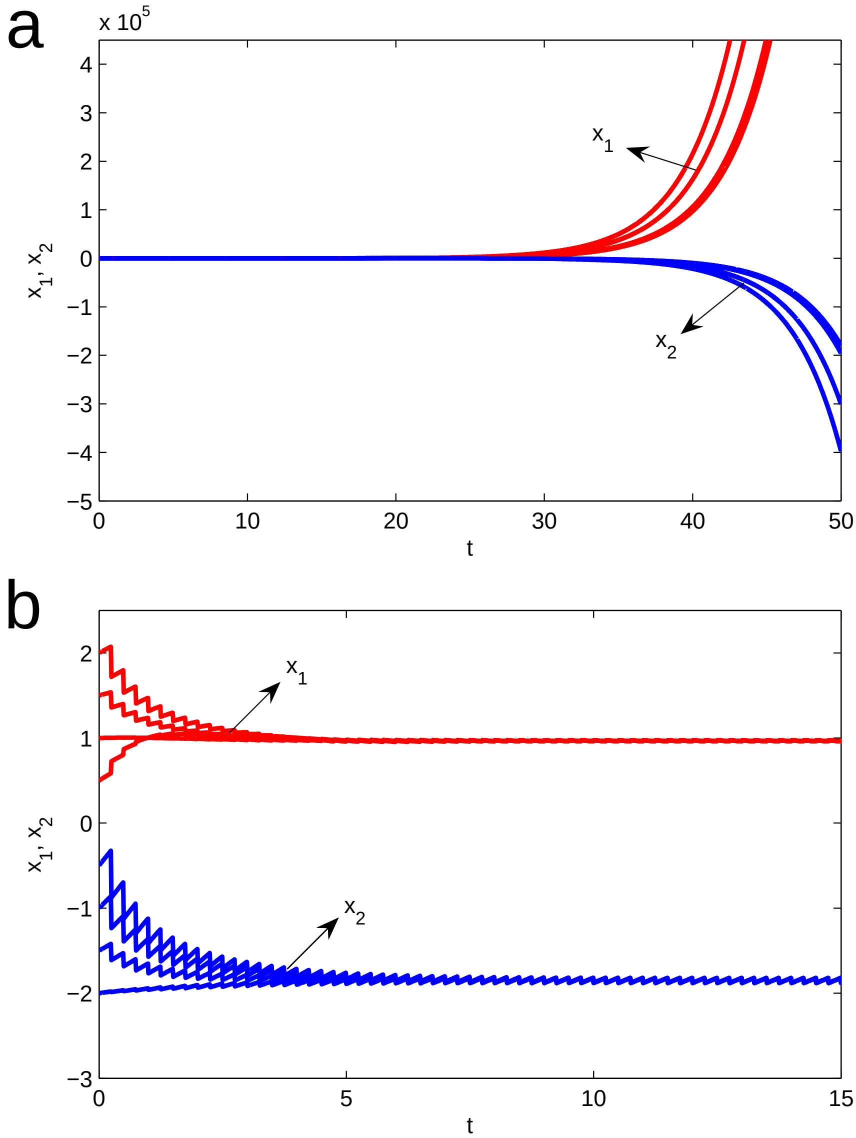

Fig.1. (a) State trajectories of system (17) without impulsive effects.
(b) State trajectories of impulsive system (17)-(18).

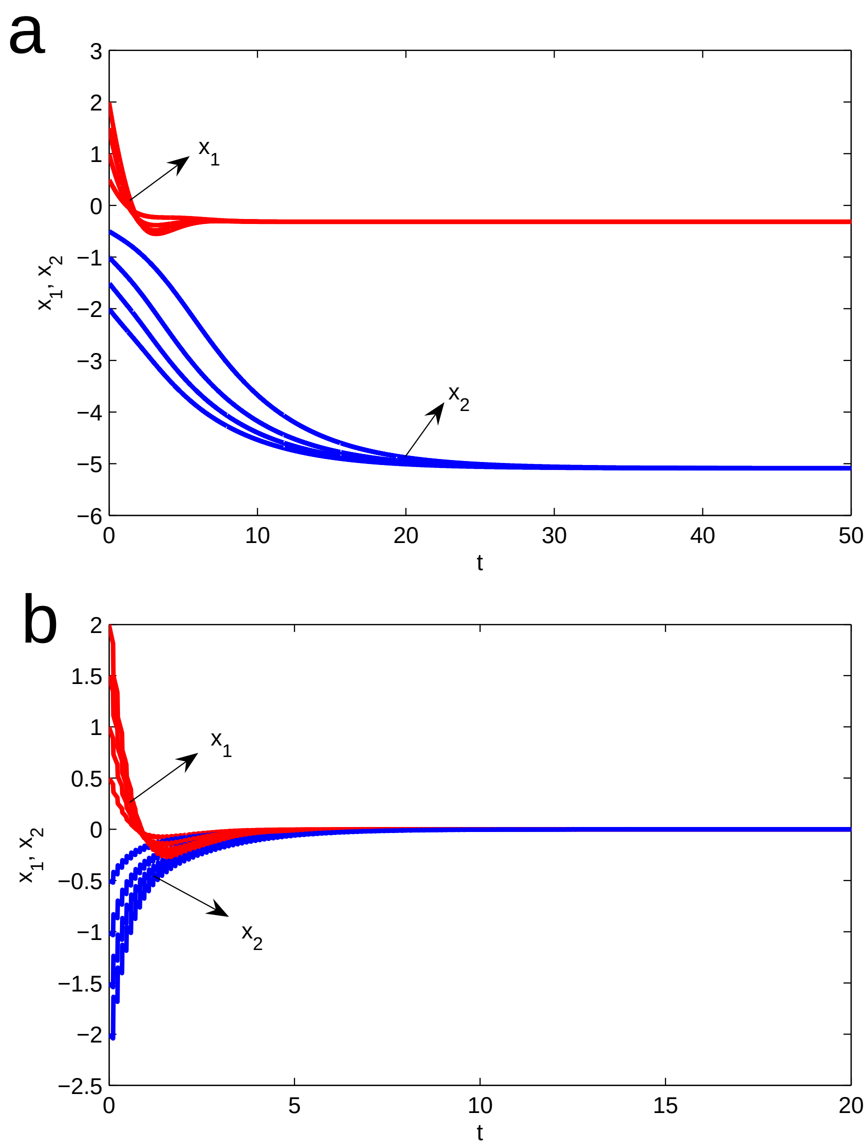

Fig.2. (a) State trajectories of system (19) without impulsive effects. (b) State trajectories of impulsive system (19)-(20).