Expert Systems With Applications 290 (2025) 128433

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Finite-time bipartite synchronization for coupled neural networks: An aperiodically intermittent and impulsive control approach

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ARTICLE INFO

Keywords:
Coupled neural networks
Aperiodically intermittent control
Impulsive control
Finite-time bipartite synchronization.

ABSTRACT

This paper delves into the issue of finite-time bipartite synchronization of neural networks (NNs) with signed graphs. A novel hybrid control approach, incorporating aperiodically intermittent control (AIC) and impulsive control (IC), is presented. This strategy significantly reduces the control costs and the duration of continuous input control in the networks. Firstly, the sufficient conditions and settling time for the system to reach finite-time synchronization under AIC are presented using a Lyapunov function. Moreover, the integration of IC into the coupled NNs is explored. On the one hand, the synchronization speed of coupled NNs is accelerated by assuming that the IC occurs within the controlled segments of the AIC. On the other hand, when the IC occurs in the uncontrolled segments of the AIC, it can diminish the controlled periods of the AIC. Subsequently, based on the two hybrid controllers designed above, the synchronization criteria for the coupled NNs are established and the corresponding settling time is estimated. Finally, examples are provided to corroborate the efficacy of the designed AIC and IC.

1. Introduction

The real-world applications of neural networks (NNs) in areas such as secure communication, signal processing and various other fields have garnered significant attention (Arik, 2000; He et al., 2020). Unlike traditional single network, coupled NNs involve multiple interconnected NNs, such as smart grids, social networks and transportation networks (Guimera & Amaral, 2005; Huang, Zhang, & Zhou, 2017; Sepehr, Gomis-Bellmunt, & Pouresmaeil, 2022). Up to now, research on coupled NNs has predominantly concentrated on two primary areas: the internal characteristics of the networks, including the coupling configuration among nodes (Ravasz & Barabasi, 2003; Strogatz, 2001), and the network's cluster behavior, encompassing network synchronization and control (Li, Cui, Cao, Qiu, & Sun, 2024; Zhang, Lu, Jiang, & Zhong, 2024a).

In general, the coupled NNs are characterized by a particular topology that reflects the interplay of information among the nodes of the networks. However, the majority of current research tends to focus on unsigned graphs, where the nodes only exhibit cooperative relationships (Lakshmanan et al., 2018; Lu, Ho, & Cao, 2010). In reality, networks frequently incorporate both cooperative and competitive interactions, as seen in ecosystems and social networks. For instance, the two-peak alliance structure is commonly observed in many adversarial systems, including a country's two-party political landscape and sports competitions. As documented in Wasserman and Faust (1994), in such scenarios, the network naturally splits into two sharply contrasting factions. Furthermore, with respect to network synchronization, in networks modeled as signed graphs, the nodes are unable to achieve full synchronization because of the coexistence of both positive and negative edges. Instead, the bipartite synchronization can be achieved. In 2013, Altafini introduced the definition of bipartite consensus (BC) for signed graphs, which incorporate both cooperative and competitive interactions (Altafini, 2013). Since then, some scholars have explored the issue of bipartite synchronization for NNs (Li & Zheng, 2020; Shen, Wang, Duan, Cao, & Wang, 2023; Yaghmaie, Su, Lewis, & Olaru, 2017). For instance, two concepts pertaining to the bipartite synchronization of network nodes were first defined. After that, by introducing a specific matrix M, the criteria for the bipartite synchronization of networks exhibiting cooperative and competitive relationships were deduced (Li & Zheng, 2020).

It is important to note that the bipartite synchronization of NNs considered above is asymptotic, which cannot guarantee the fast convergence of the networks. However, in numerous real-world networks,

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https://doi.org/10.1016/j.eswa.2025.128433
Received 8 April 2025; Received in revised form 20 May 2025; Accepted 31 May 2025
Available online 9 June 2025

particularly in engineering systems (Galicki, 2015), the realization of bipartite synchronization is required within a finite time frame, i.e., finite-time bipartite synchronization (FTBS). Moreover, the objective of research into network synchronization is to engineer a suitable control strategy that facilitates the attainment of the desired goal state within the networks. Consequently, several scholars have delved into the investigation of the FTBS problems of coupled NNs under various control strategies (Chang, Park, & Yang, 2023; Meng, Jia, & Du, 2016; Zhang, Yang, Yang, & Alsaedi, 2021). For example, in reference (Meng et al., 2016), the authors achieved finite-time BC of systems by devising a control strategy incorporating a sign function. To mitigate the undesirable chattering phenomenon stemming from the sign function, a negative feedback controller utilizing a saturation function was subsequently proposed and the criteria for FTBS in memristive NNs were also established in Chang et al. (2023). However, these control strategies necessitate continuous control and continuous communication among the network nodes, which are very costly to implement. Furthermore, some scholars have also investigated the potential of discontinuous control (Liu, Cao, & Xie, 2019; Mao, Liu, Cao, & Hu, 2022; Zhang, Zhu, Wen, & Huang, 2024b). For instance, the finite-time BC of multiple signed networks was achieved through implementation of aperiodically intermittent control (AIC) and the node-Lyapunov method (Zhang et al., 2024b). Although the AIC designed in Zhang et al. (2024b) can significantly reduce control time, it still required the use of sign functions and strict negative feedback control components within the control intervals, which impose stringent conditions. Hence, the primary challenge we address in this paper is to explore how to remove these constraints and design more appropriate AIC.

In recent years, another form of discontinuous control, known as impulsive control (IC), has garnered significant attention. Different from traditional control methods, IC causes sudden changes in the network's state at specific discrete moments (Liang, Gu, Li, & Tao, 2025; Yang, Li, & Song, 2022). This approach eliminates the need for continuous information exchange between network nodes during intervals between these impulsive instants. The structural simplicity of IC and its effectiveness in significantly reducing network information transmission have made it a popular choice in both engineering and ecological systems (Gopalsamy, 1992; Jiang, Lu, & Liu, 2020). Recently, researchers have explored the potential of combining the benefits of AIC and IC to enhance the stability and consensus of complex systems (Wang, Wang, Liu, Wu, & Sun, 2024; Yang, Li, Lu, & Cheng, 2020; You, Wu, & Li, 2025). For instance, a novel hybrid control strategy, which integrates AIC and IC, has been presented to address the finite-time boundedness issue of nonlinear systems in You et al. (2025). However, this study only considered IC within the uncontrollable intervals of intermittent control, limiting the ability of systems to reach finite-time boundedness, which may be insufficient for practical applications. Further, the finite-time BC issue of multi-agent systems under intermittent communication and IC has been investigated in Wang et al. (2024). However, this approach necessitates strong negative feedback control and restricts IC to occur only within the intervals of intermittent control. Therefore, how to implement FTBS under conditions where the intensity of the intermittent control and the timing of the IC are reduced is an urgent problem to be addressed. In essence, the key challenge we confront is to design the appropriate hybrid controllers that combine AIC and IC. Moreover, the criteria for FTBS of coupled NNs under such hybrid control, together with an estimate of the settling time (ST), are other issues we have to consider.

Based on the above discussion, the FTBS of coupled NNs under a new hybrid control is the main research problem in this paper. The main contributions are as follows.

1. In this paper, we design a novel hybrid control strategy that integrates AIC and IC. The fusion of the two forms of discontinuous control significantly reduces control cost and accelerates the convergence pace of the coupled NNs towards synchronization.
2. This study separately addresses two scenarios of impulsive control within the AIC framework: one where IC acts within the controlled segments and another where it operates in the uncontrollable segments. In the first scenario, the IC enhances the synchronization process of the networks, that is, IC can synchronize networks that are unstable under AIC. In the second case, the inclusion of the IC compresses the control periods of the AIC, thereby substantially reducing the duration of continuous information exchange.
3. This paper explores signed graphs, where cooperative and competitive relationships among network nodes coexist, reflecting a more realistic scenario. The criteria for FTBS of coupled NNs with signed graphs are elucidated under the above two hybrid control strategies, accompanied by an estimation of the associated ST.

Notations: Let $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{R}^n$, $\mathbb{R}^{n\times n}$, $\mathbb{N}$ and $\mathbb{N}_+$ be the sets of real numbers, positive real numbers, n dimensional vectors, $n\times n$ dimensional matrices, natural numbers and positive integers, respectively. $\mathrm{sign}(\cdot)$ is the sign function. $|\cdot|$ represents the absolute value. $\Lambda(1,N)\triangleq\{1,\cdots,N\}$. $K=\mathrm{diag}\{k_1,\cdots,k_N\}\triangleq\mathrm{diag}(k_l)_{l\in\Lambda(1,N)}\in\mathbb{R}^{N\times N}$ is a diagonal matrix. $\ulcorner M\urcorner$ denotes the value of M rounded up.

2. Preliminaries

2.1. Signed graph theory

$\mathcal{G}^s=\{\mathcal{V},\mathcal{E},A^s\}$ is a signed graph with node set $\mathcal{V}$, edge set $\mathcal{E}$ and adjacency matrix $A^s=(a^s_{pq})_{N\times N}\in\mathbb{R}^{N\times N}$. In signed graph $\mathcal{G}^s$, $a^s_{pp}=0$, $a^s_{pq}=0$ means that $(p,q)\notin\mathcal{E}$, otherwise, $a^s_{pq}\neq 0$, $p\neq q\in\mathcal{V}$. Furthermore, if the relationship between nodes p and q is cooperative, then $a^s_{pq}>0$; if it is competitive, then $a^s_{pq}<0$. The corresponding signed Laplacian matrix is $L^s=(l^s_{pq})_{N\times N}=\mathrm{diag}(\sum_{q=1}^N|a^s_{lq}|)_{l\in\Lambda(1,N)}-A^s$. It should be noted that the row sum of matrix L^s is not 0 since a^s_{pq} may be less than 0. For the corresponding unsigned graph $\mathcal{G}^u$, the adjacency matrix is $A^u=(a^u_{pq})_{N\times N}=(|a^s_{pq}|)_{N\times N}$, and the Laplacian matrix is $L^u=(l^u_{pq})_{N\times N}=\mathrm{diag}(\sum_{q=1}^N|a^s_{lq}|)_{l\in\Lambda(1,N)}-A^u$.

Definition 1. (Altafini, 2013) If the set $\mathcal{V}$ of the graph $\mathcal{G}^s$ can be divided into $\{\mathcal{V}_1,\mathcal{V}_2\}$ such that $\mathcal{V}_1\cap\mathcal{V}_2=\emptyset$ and $\mathcal{V}_1\cup\mathcal{V}_2=\mathcal{V}$. Further, it holds that $a^s_{pq}>0$, $p,q\in\mathcal{V}_{d'}$ $(d'\in\{1,2\})$, and $a^s_{pq}<0$, $p\in\mathcal{V}_{d'}$, $q\in\mathcal{V}_{d''}$ $(d'\neq d''$, $d',d''\in\{1,2\})$. Then, the graph $\mathcal{G}^s$ is structurally balanced.

2.2. Model description

Consider the coupled NNs with signed graph $\mathcal{G}^s$ as follows: for $p\in\Lambda(1,N)$,

$$\dot{x}_p(t)=-Hx_p(t)+Kf(x_p(t))+c\sum_{q\in\mathcal{E}_p}|a^s_{pq}|(\mathrm{sign}(a^s_{pq})x_q(t)-x_p(t))+u_p(t),\quad t\geq 0,\tag{1}$$

where $H=\mathrm{diag}(h_l)_{l\in\Lambda(1,n)}$ and $K\in\mathbb{R}^{n\times n}$ are proper real matrices. $x_p(t)\in\mathbb{R}^n$ is the p-th neuron's state. $f(\cdot)$ is an odd activation function. $c>0$, $\mathcal{E}_p=\{q|(p,q)\in\mathcal{E}\}$ and $u_p(t)$ is control input.

In cooperative-competition model, the cooperation relationship between nodes p and q can be shown as $x_q(t)-x_p(t)$, and the competition relationship can be represented as $x_q(t)+x_p(t)$ (Li & Zheng, 2020). Then, combining the above theory of signed graphs can have

$$\begin{aligned}\sum_{q\in\mathcal{E}_p}&|a^s_{pq}|(\mathrm{sign}(a^s_{pq})x_q(t)-x_p(t))\\&=\sum_{q\in\mathcal{E}_p}a^{s+}_{pq}(x_q(t)-x_p(t))+\sum_{q\in\mathcal{E}_p}a^{s-}_{pq}(x_q(t)+x_p(t))\\&=-\sum_{q\in\mathcal{E}_p}l^s_{pq}x_q(t)-l^s_{pp}x_p(t)=-\sum_{q=1}^N l^s_{pq}x_q(t),\end{aligned}\tag{2}$$

where a_{pq}^{s+} means $a_{pq}^{s} > 0$, and a_{pq}^{s-} represents $a_{pq}^{s} < 0$. Therefore, the coupled NNs (Eq. 1) can be rewritten as

$$\dot{x}_p(t) = -Hx_p(t) + Kf(x_p(t)) - c\sum_{q=1}^{N} l_{pq}^{s} x_q(t) + u_p(t), \quad t \geq 0. \tag{3}$$

Moreover, the leader of the NNs (Eq. 1) is expressed as

$$\dot{s}(t) = -Hs(t) + Kf(s(t)), \quad t \geq 0. \tag{4}$$

Based on the above consideration of the NNs model, some the necessary assumptions and preparatory knowledge are presented below.

Assumption 1. For any $\mathfrak{x}, \mathfrak{z} \in \mathbb{R}^n$, there is $R \in \mathbb{R}^{n\times n}$ such that $(f(\mathfrak{x}) - f(\mathfrak{z}))^T(f(\mathfrak{x}) - f(\mathfrak{z})) \leq (\mathfrak{x} - \mathfrak{z})^T R^T R(\mathfrak{x} - \mathfrak{z})$.

Remark 1. The nonlinear function f within the NNs (Eq. 1) considered in this paper satisfies the Lipschitz condition, as stated in Assumption 1. This assumption is both widely adopted and highly reasonable in relevant research contexts. First, it guarantees the existence and uniqueness of the system solution $x(t)$. Second, it ensures that the rate of change of f is bounded by a fixed constant within any interval, which promotes function smoothness. Moreover, in many natural systems, including the Lorenz and Rössler systems, there exist nonlinear terms that adhere to the global Lipschitz condition.

Assumption 2. The graph $\mathcal{G}^s$ is connected and structurally balanced.

Lemma 1 (Altafini 2013). *If $\mathcal{G}^s$ is structurally balanced, then, there is a matrix $Z = diag(z_l)_{l\in\Lambda(1,N)}$ such that $ZL^sZ = L^u$, where $z_p = 1$ if $p \in \mathcal{V}_1$ and $z_p = -1$ if $p \in \mathcal{V}_2$.*

Definition 2 (Meng, Jia, and Du 2015). The coupled NNs (Eqs. 1 and 4) can reach the FTBS, if there exist set $\mathfrak{D} \subseteq \mathbb{R}^n$ and constant $\mathcal{T} > 0$ such that for any $x_p(0) - z_p s(0) \in \mathfrak{D}$, $\lim_{t\to\mathcal{T}} ||x_p(t) - z_p s(t)|| = 0$ and $||x_p(t) - z_p s(t)|| = 0$, $t \geq \mathcal{T}$, $p \in \Lambda(1, N)$.

Lemma 2 (Khalil 2022). *Suppose $\varpi_1, \cdots, \varpi_n \geq 0$, and $0 < \gamma \leq 1$, then, one gets*

$$\sum_{i=1}^{n} \varpi_i^{\gamma} \geq \left(\sum_{i=1}^{n} \varpi_i\right)^{\gamma}. \tag{5}$$

Lemma 3. *If δ_1, $\delta_2 > 0$, $0 < \gamma < 1$, and the continuous positive definite function $V(x(t)) \triangleq V(t)$ satisfy*

$$\begin{cases} \dot{V}(t) \leq -\delta_1 V^{\gamma}(t) + \delta_2 V(t),\ t \in [t_k, t_k + d_k), k \in \mathbb{N}, \\ \dot{V}(t) \leq \delta_2 V(t),\ t \in [t_k + d_k, t_{k+1}), \end{cases}$$

then, we get

$$e^{-\delta_2(1-\gamma)t}\left(\check{V}(t) - \frac{\delta_1}{\delta_2}\right) \leq \check{V}(0) - \frac{\delta_1}{\delta_2}, \quad 0 \leq t \leq T_0, \tag{6}$$

where $t_0 = 0$, $V^{1-\gamma}(0) \triangleq \check{V}(0) < \frac{\delta_1}{\delta_2}$, $V^{1-\gamma}(t) \triangleq \check{V}(t)$, $T_0 = \frac{\ln(1-\frac{\delta_2\check{V}(0)}{\delta_1})}{\delta_2(\gamma-1)}$, and for $t \geq T_0$, $V(t) = 0$.

Proof. Let $\theta(t) = e^{-\delta_2(1-\gamma)t}\big(\check{V}(t) - \frac{\delta_1}{\delta_2}\big)$, $t \geq 0$, and $Y = \check{V}(0) - \frac{\delta_1}{\delta_2}$ is a constant. Then, we will prove that (Eq. 6) holds. Firstly, when $t = 0$, (Eq. 6) is obviously true.

Next, for $t \in (0, d_0)$, we get

$$Q_{01}(t) = \theta(t) - \xi Y + \frac{\delta_1}{\delta_2}e^{-\delta_2(1-\gamma)t} < 0, \tag{7}$$

where $0 < \xi < 1$. Then, suppose that (Eq. 7) is false, that is, there exists $\hat{t}_0 \in (0, d_0)$ such that $Q_{01}(\hat{t}_0) = 0$, $\dot{Q}_{01}(\hat{t}_0) \geq 0$, and $Q_{01}(t) < 0$, $t \in (0, \hat{t}_0)$. Moreover, it holds that

$$\begin{aligned} \dot{Q}_{01}(\hat{t}_0) &= \dot{\theta}(\hat{t}_0) - \delta_1(1-\gamma)e^{-\delta_2(1-\gamma)\hat{t}_0} \\ &= -\delta_2(1-\gamma)e^{-\delta_2(1-\gamma)\hat{t}_0}\check{V}(\hat{t}_0) + \delta_1(1-\gamma)e^{-\delta_2(1-\gamma)\hat{t}_0} \\ &\quad - \delta_1(1-\gamma)e^{-\delta_2(1-\gamma)\hat{t}_0} + e^{-\delta_2(1-\gamma)\hat{t}_0}(1-\gamma)V^{-\gamma}(\hat{t}_0)\dot{V}(\hat{t}_0) \\ &\leq -\delta_1(1-\gamma)e^{-\delta_2(1-\gamma)\hat{t}_0} < 0, \end{aligned}$$

which is a contradiction. Hence, (Eq. 7) holds. Further, one gets $\theta(t) < \xi Y - \frac{\delta_1}{\delta_2}e^{-\delta_2(1-\gamma)t} < \xi Y$.

For $t \in [d_0, t_1)$, we claim that

$$Q_{02}(t) = \theta(t) - \xi Y + \frac{\delta_1}{\delta_2}e^{-\delta_2(1-\gamma)(t-d_0)} < 0. \tag{8}$$

Then, suppose that (Eq. 8) is false, that is, there is $\check{t}_0 \in [d_0, t_1)$ satisfying that $Q_{02}(\check{t}_0) = 0$, $\dot{Q}_{02}(\check{t}_0) \geq 0$, and $Q_{02}(t) < 0$, $t \in [d_0, \check{t}_0)$. Moreover, it holds that

$$\begin{aligned} \dot{Q}_{02}(\check{t}_0) &= \dot{\theta}(\check{t}_0) - \delta_1(1-\gamma)e^{-\delta_2(1-\gamma)(\check{t}_0-d_0)} \\ &\leq \delta_1(1-\gamma)e^{-\delta_2(1-\gamma)\check{t}_0}(1 - e^{\delta_2(1-\gamma)d_0}) < 0, \end{aligned}$$

which is a contradiction. Hence, (Eq. 8) holds. Further, we have $\theta(t) < \xi Y - \frac{\delta_1}{\delta_2}e^{-\delta_2(1-\gamma)(t-d_0)} < \xi Y$.

By mathematical induction, for any $t \in [t_k, t_k + d_k)$, $k \in \mathbb{N}$,

$$Q_{k1}(t) = \theta(t) - \xi Y + \frac{\delta_1}{\delta_2}e^{-\delta_2(1-\gamma)(t-t_k)} < 0,$$

and for $t \in [t_k + d_k, t_{k+1})$,

$$Q_{k2}(t) = \theta(t) - \xi Y + \frac{\delta_1}{\delta_2}e^{-\delta_2(1-\gamma)(t-t_k-d_k)} < 0.$$

Therefore, let $\xi \to 1^-$, it holds that $\theta(t) \leq Y$, that is, (Eq. 6) holds. In addition, according to (Eq. 6), let

$$\check{V}(t) \leq e^{\delta_2(1-\gamma)t}(\check{V}(0) - \frac{\delta_1}{\delta_2}) + \frac{\delta_1}{\delta_2} = 0.$$

Then, it holds that $T_0 = \frac{\ln(1-\frac{\delta_2\check{V}(0)}{\delta_1})}{\delta_2(\gamma-1)}$. □

2.3. Hybrid controller design

Firstly, let $e_p(t) = x_p(t) - z_p s(t)$. Then by simple calculation, the error network is expressed as follows:

$$\dot{e}_p(t) = -He_p(t) + K\tilde{f}(e_p(t)) - c\sum_{q=1}^{N} l_{pq}^{s} e_q(t) + u_p(t), \quad t \geq 0, \tag{9}$$

where $\tilde{f}(e_p(t)) = f(x_p(t)) - f(z_p s(t))$.

In this paper, the hybrid controller $u_p(t)$ has two design forms, that is, intermittent control and IC occur simultaneously or not. The specific mathematical expressions are as follows.

(1) AIC and IC occur in the same time periods: $u_p(t) = \bar{u}_p(t) + \hat{u}_p(t)$, where

$$\bar{u}_p(t) = \begin{cases} -\varsigma e_p^{\frac{v_1}{v_2}}(t), & t \in [t_k, t_k + d_k), \\ 0, & t \in [t_k + d_k, t_{k+1}), k \in \mathbb{N}, \end{cases} \tag{10}$$

and

$$\hat{u}_p(t) = \sum_{i=1}^{+\infty}(\nu - 1)e_p(s_i^{k-})\delta(t - s_i^k), \quad k \in \mathbb{N}, s_i^k \in [t_k, t_k + d_k). \tag{11}$$

$\varsigma > 0$ is a tunable constant, $0 < v_1 < v_2$ are odd numbers, $\{t_k\}_{k\in\mathbb{N}}$ is aperiodically intermittent time sequence satisfies $0 = t_0 < t_1 < t_2 < \cdots$. Aperiodically intermittent width sequence $\{d_k\}_{k\in\mathbb{N}}$ satisfies $t_{k+1} - t_k > d_k > 0$. $\nu > 0$ is the impulsive intensity. Impulsive sequence $\{s_i^k\}_{i,k\in\mathbb{N}}$ satisfies $t_k = s_0^k < s_1^k < \cdots < s_{\mathfrak{N}(t_k,t_k+d_k)}^k < t_k + d_k$. And $\mathfrak{N}(t_k, t_k + d_k)$ denotes the number of impulsive instants on $[t_k, t_k + d_k)$.

(2) AIC and IC occur in different time periods: $u_p(t) = \bar{u}_p(t) + \check{u}_p(t)$, where

$$\check{u}_p(t) = \sum_{j=1}^{+\infty}(\nu - 1)e_p(\theta_j^{k-})\delta(t - \theta_j^k), \ k \in \mathbb{N}, \theta_j^k \in [t_k + d_k, t_{k+1}). \tag{12}$$

In addition, impulsive sequence $\{\theta_j^k\}_{j,k\in\mathbb{N}}$ satisfies $t_k + d_k = \theta_0^k < \theta_1^k < \cdots < \theta_{\mathfrak{N}(t_k+d_k,t_{k+1})}^k < t_{k+1}$. And $\mathfrak{N}(t_k + d_k, t_{k+1})$ represented the number of impulsive instants on $[t_k + d_k, t_{k+1})$.

Then, based on the above controller design, we can get the corresponding error NNs as follows. When AIC and IC occur in the same time periods, we have

$$\begin{cases} \dot{e}_p(t) = -He_p(t) + K\tilde{f}(e_p(t)) - c\sum_{q=1}^N l_{pq}^s e_q(t) - \varsigma e_p^{\frac{v_1}{v_2}}(t), \\ \qquad t \in [t_k, t_k + d_k), t \neq s_i^k, \\ e_p(t) = ve_p(t^-), \quad t = s_i^k, k \in \mathbb{N}, i \in \mathbb{N}^+, \\ \dot{e}_p(t) = -He_p(t) + K\tilde{f}(e_p(t)) - c\sum_{q=1}^N l_{pq}^s e_q(t), \quad t \in [t_k + d_k, t_{k+1}). \end{cases} \tag{13}$$

When AIC and IC occur in different time periods, one has

$$\begin{cases} \dot{e}_p(t) = -He_p(t) + K\tilde{f}(e_p(t)) - c\sum_{q=1}^N l_{pq}^s e_q(t) - \varsigma e_p^{\frac{v_1}{v_2}}(t), \\ \qquad t \in [t_k, t_k + d_k), k \in \mathbb{N}, \\ \dot{e}_p(t) = -He_p(t) + K\tilde{f}(e_p(t)) - c\sum_{q=1}^N l_{pq}^s e_q(t), t \in [t_k + d_k, t_{k+1}), t \neq \theta_j^k, \\ e_p(t) = ve_p(t^-), \quad t = \theta_j^k, j \in \mathbb{N}^+. \end{cases} \tag{14}$$

Remark 2. In most of the previous works on finite-time synchronization problems, scholars often used continuous controls (Du, He, & Cheng, 2014; Shen & Xia, 2008), such as finite-time control term $(-\varsigma e_p^{v_1/v_2}(t), t \geq 0)$. Later, some researchers began to consider discontinuous controls, such as IC and intermittent control. However, the intermittent control here is always assumed to be periodic (Jiang, Li, Tu, & Feng, 2019; Mei, Lu, Hu, & Fan, 2022), which seems unreasonable. Therefore, the problem of finite-time synchronization under AIC is studied in Lemma 3. By constructing auxiliary functions and mathematical induction, we obtain the criterion and ST of finite-time synchronization for the systems under AIC.

Remark 3. Two new hybrid controllers are designed in this paper. The first type of AIC occurs simultaneously with IC. Compared with a single AIC, the addition of IC may stabilize a previously unstable system (see Example 1). The second is to inject IC in the intervals without AIC, that is, the intervals $[t_k + d_k, t_{k+1}), k \in \mathbb{N}$. Then, we can significantly reduce the time of continuous control (see Example 2), that is, the intervals $t \in [t_k, t_k + d_k), k \in \mathbb{N}$. In general, the two hybrid controllers designed above can effectively avoid the shortcomings of single intermittent control or IC.

3. Main results

Two types of hybrid control will be discussed in this section. For the first circumstance, we assume that the IC occurs in the controlled intervals of the AIC. In the second case, the IC occurs in the uncontrolled intervals of the AIC. Theorems 1 and 2 respectively give the sufficient conditions for FTBS of coupled NNs in the above two cases. In addition, in Corollaries 1 and 2 we give the criteria of FTBS of coupled NNs under periodically intermittent control and IC respectively.

3.1. AIC and IC occur in the same time periods

Based on the design considerations outlined in the second part for the first form of hybrid controller and the associated error NNs (Eq. 13), we are able to derive the sufficient conditions for achieving FTBS of the coupled NNs, as detailed in the subsequent theorem. Before giving the main theorem of this section, we will first introduce some notations for convenience. Denote

$$\tilde{V}(0) = 1 - \frac{\delta_2 \check{V}(0)}{\delta_1} > 0,$$

$$\omega_k = \mu^{1-\gamma}(e^{\delta_2(\gamma-1)(t_{k-1}+d_{k-1})} - e^{\delta_2(\gamma-1)t_k}) > 0,$$

$$W_{1k} = \frac{(1-\mu^{1-\gamma})e^{\delta_2(\gamma-1)s_1^k} - \omega_k}{\tilde{V}(0)\prod_{l=0}^{k-1} M_l^{\mathfrak{N}(t_l, t_l+d_l)}},$$

$$W_k = \frac{(1-\mu^{1-\gamma})e^{\delta_2(\gamma-1)s^k_{\mathfrak{N}(t_k,t_k+d_k)}}}{\tilde{V}(0)\prod_{l=0}^{k-1} M_l^{\mathfrak{N}(t_l, t_l+d_l)}},$$

where the constants $\delta_1 = 2\varsigma c_1 c_2^{-\frac{v_1+v_2}{2v_2}}$, $c_1 = \lambda_{\min}(P)$, $c_2 = \lambda_{\max}(P)$, $\gamma = \frac{v_1+v_2}{2v_2}$, $\mu = v^2$, $\check{V}(0) = V^{1-\gamma}(0)$, $\delta_2 > 0$, $M_l > 1$ and matrix $P > 0$ are later given in Theorem 1.

Theorem 1. *Under Assumptions 1–2, the coupled NNs (Eqs. 1 and 4) can achieve FTBS, if there exist matrix $P > 0$ and constant $\delta_2 > 0$ such that*

$$\begin{pmatrix} I_N \otimes \chi - 2c(L^s \otimes P) & I_N \otimes PK \\ * & -I_N \otimes I_n \end{pmatrix} < 0, \tag{15}$$

where $\chi = -2PH + R^T R - \delta_2 P$. Moreover, constants $M_k > 1$, $k \in \mathbb{N}$ satisfy

$$M_k - \mu^{1-\gamma} \leq W_{1k}, \ t \in [s_1^k, s_2^k), \tag{16}$$

and

$$M_k^{\mathfrak{N}(t_k,t_k+d_k)-1}(M_k - \mu^{1-\gamma}) \leq W_k, \ t \in [s^k_{\mathfrak{N}(t_k,t_k+d_k)}, t_k + d_k), \tag{17}$$

where $\omega_0 = 0$, $\prod_{l=0}^{-1} M_l^{\mathfrak{N}(t_l,t_l+d_l)} = 1$ and $k \leq k_0$, $k_0 \in \mathbb{N}$ is a constant.

In addition, the ST is

$$T_1 \leq \frac{1}{\delta_2(\gamma-1)}[\ln \prod_{l=0}^{k_0-1} M_l^{\mathfrak{N}(t_l,t_l+d_l)} + \ln \tilde{V}(0)]. \tag{18}$$

Proof. Let $V(t) = \sum_{p=1}^N e_p^T(t)Pe_p(t) = e^T(t)(I_N \otimes P)e(t)$. Then, for $t \in [t_k, t_k + d_k)$, $t \neq s_i^k$, that is, the intervals $[t_k, s_1^k)$, $[s_1^k, s_2^k)$, $\cdots$, $[s^k_{\mathfrak{N}(t_k,t_k+d_k)}, t_k + d_k)$, it follows from Assumption 2 and (Eq. 13) that

$$\begin{aligned} \dot{V}(t) &= 2\sum_{p=1}^N e_p^T(t)P\left[-He_p(t) + K\tilde{f}(e_p(t)) - c\sum_{q=1}^N l_{pq}^s e_q(t) - \varsigma e_p^{\frac{v_1}{v_2}}(t)\right] \\ &\leq \sum_{p=1}^N e_p^T(t)(-2PH + PKK^TP + R^TR)e_p(t) \\ &\quad - 2e^T(t)(cL^s \otimes P)e(t) - 2\varsigma\sum_{p=1}^N e_p^T(t)Pe_p^{\frac{v_1}{v_2}}(t), \end{aligned} \tag{19}$$

where $L^s = (l_{pq}^s)_{N\times N} = \mathrm{diag}(\sum_{q=1}^N |a_{lq}^s|)_{l\in\Lambda(1,N)} - A^s$ is the signed Laplacian matrix. According to (Eq. 15), we have

$$I_N \otimes (\chi + PKK^TP) - 2c(L^s \otimes P) < 0, \tag{20}$$

where $\chi = -2PH + R^TR - \delta_2 P$. Combined with Lemma 2, one has

$$\begin{aligned} 2\varsigma\sum_{p=1}^N e_p^T(t)Pe_p^{\frac{v_1}{v_2}}(t) &\geq 2\varsigma\lambda_{\min}(P)\sum_{p=1}^N (e_p^T(t)e_p(t))^{\frac{v_1+v_2}{2v_2}} \\ &\geq 2\varsigma c_1 c_2^{-\frac{v_1+v_2}{2v_2}}(e^T(t)(I_N \otimes P)e(t))^{\frac{v_1+v_2}{2v_2}}, \end{aligned} \tag{21}$$

where $c_1 = \lambda_{\min}(P)$ and $c_2 = \lambda_{\max}(P)$. Then, substituting (Eqs. 20 and 21) into (Eq. 19) implies

$$\begin{aligned} \dot{V}(t) &\leq e^T(t)[I_N \otimes (-2PH + PKK^TP + R^TR) \\ &\quad - 2c(L^s \otimes P)]e(t) - 2\varsigma c_1 c_2^{-\frac{v_1+v_2}{2v_2}} V^{\frac{v_1+v_2}{2v_2}}(t) \\ &\leq -\delta_1 V^\gamma(t) + \delta_2 V(t), \end{aligned} \tag{22}$$

where $\gamma = \frac{v_1+v_2}{2v_2}$ and $\delta_1 = 2\varsigma c_1 c_2^{-\frac{v_1+v_2}{2v_2}}$. When $t = s_i^k$, $i \in \mathbb{N}_+$, $k \in \mathbb{N}$, it holds that

$$V(s_i^k) = \sum_{p=1}^N e_p^T(s_i^k)Pe_p(s_i^k) = v^2 V(s_i^{k-}) \triangleq \mu V(s_i^{k-}).$$

Then, (Eqs. 19 and 20) imply

$$\dot{V}(t) \leq \delta_2 V(t), t \in [t_k + d_k, t_{k+1}). \tag{23}$$

Further, for $t \in [0, d_0)$, we have $0 < s_1^0 < \cdots < s^0_{\mathfrak{N}(0,d_0)} < d_0$. Specifically, for $t \in [0, s_1^0)$,

$$\check{V}(t) \le e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.$$

For $t \in [s_1^0, s_2^0)$,

$$\begin{aligned}\check{V}(t) &\le e^{\delta_2(1-\gamma)(t-s_1^0)}\left(\check{V}(s_1^0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}\\ &\le e^{\delta_2(1-\gamma)(t-s_1^0)}\left[\mu^{1-\gamma}\left(e^{\delta_2(1-\gamma)s_1^0}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}\right) - \frac{\delta_1}{\delta_2}\right] + \frac{\delta_1}{\delta_2}\\ &= \mu^{1-\gamma}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}(\mu^{1-\gamma}-1)e^{\delta_2(1-\gamma)(t-s_1^0)} + \frac{\delta_1}{\delta_2}\\ &\le M_0 e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.\end{aligned}$$

By parity of reasoning, for $t \in [s_i^0, s_{i+1}^0)$, $i \in \Lambda(1, \mathfrak{N}(0, d_0) - 1)$, one gets

$$\check{V}(t) \le M_0^i e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2},$$

and

$$\check{V}(s^{0-}_{\mathfrak{N}(0,d_0)}) \le M_0^{\mathfrak{N}(0,d_0)-1} e^{\delta_2(1-\gamma)s^0_{\mathfrak{N}(0,d_0)}}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.$$

Further, for $t \in [s^0_{\mathfrak{N}(0,d_0)}, d_0)$, one has

$$\begin{aligned}\check{V}(t) &\le e^{\delta_2(1-\gamma)(t-s^0_{\mathfrak{N}(0,d_0)})}\left[\check{V}(s^0_{\mathfrak{N}(0,d_0)}) - \frac{\delta_1}{\delta_2}\right] + \frac{\delta_1}{\delta_2}\\ &\le \mu^{1-\gamma}M_0^{\mathfrak{N}(0,d_0)-1}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right)\\ &\quad + (\mu^{1-\gamma}-1)\frac{\delta_1}{\delta_2}e^{\delta_2(1-\gamma)(t-s^0_{\mathfrak{N}(0,d_0)})} + \frac{\delta_1}{\delta_2}\\ &\le M_0^{\mathfrak{N}(0,d_0)}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.\end{aligned}$$

Therefore, for $t \in [0, d_0)$, $\check{V}(t) \le M_0^{\mathfrak{N}(0,t)}e^{\delta_2(1-\gamma)t}(\check{V}(0) - \frac{\delta_1}{\delta_2}) + \frac{\delta_1}{\delta_2}$. Then, for $t \in [d_0, t_1)$, (Eq. 23) represents that $V(t) \le e^{\delta_2(t-d_0)}V(d_0)$. That is,

$$\check{V}(t) \le e^{\delta_2(1-\gamma)(t-d_0)}\check{V}(d_0) \le M_0^{\mathfrak{N}(0,d_0)}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + e^{\delta_2(1-\gamma)(t-d_0)}\frac{\delta_1}{\delta_2}.$$

Similarly, for $t \in [t_1, t_1 + d_1)$, we have $t_1 < s_1^1 < \cdots < s^1_{\mathfrak{N}(t_1,t_1+d_1)} < t_1 + d_1$. Specifically, for $t \in [t_1, s_1^1)$, one has

$$\check{V}(t) \le e^{\delta_2(1-\gamma)(t-t_1)}\left(\check{V}(t_1) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2},$$

and

$$\begin{aligned}\check{V}(s_1^{1-}) \le{}& M_0^{\mathfrak{N}(0,d_0)}e^{\delta_2(1-\gamma)s_1^1}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} + e^{\delta_2(1-\gamma)(s_1^1-d_0)}\frac{\delta_1}{\delta_2}\\ &- e^{\delta_2(1-\gamma)(s_1^1-t_1)}\frac{\delta_1}{\delta_2}.\end{aligned}$$

For $t \in [s_1^1, s_2^1)$, one gets

$$\begin{aligned}\check{V}(t) &\le e^{\delta_2(1-\gamma)(t-s_1^1)}\left(\check{V}(s_1^1) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}\\ &\le M_0^{\mathfrak{N}(0,d_0)}M_1 e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.\end{aligned}$$

By parity of reasoning, for $t \in [s_i^1, s_{i+1}^1)$, $i \in \Lambda(1, \mathfrak{N}(t_1, t_1 + d_1) - 1)$, we have

$$\check{V}(t) \le M_1^i M_0^{\mathfrak{N}(0,d_0)}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.$$

Further, for $t \in [s^1_{\mathfrak{N}(t_1,t_1+d_1)}, t_1 + d_1)$, one gets

$$\begin{aligned}\check{V}(t) &\le e^{\delta_2(1-\gamma)(t-s^1_{\mathfrak{N}(t_1,t_1+d_1)})}\left[\check{V}(s^1_{\mathfrak{N}(t_1,t_1+d_1)}) - \frac{\delta_1}{\delta_2}\right] + \frac{\delta_1}{\delta_2}\\ &\le M_1^{\mathfrak{N}(t_1,t_1+d_1)}M_0^{\mathfrak{N}(0,d_0)}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.\end{aligned}$$

Therefore, for $t \in [t_1, t_1 + d_1)$, it holds that $\check{V}(t) \le M_1^{\mathfrak{N}(t_1,t)}M_0^{\mathfrak{N}(0,d_0)}e^{\delta_2(1-\gamma)t}(\check{V}(0) - \frac{\delta_1}{\delta_2}) + \frac{\delta_1}{\delta_2}$. Then, for $t \in [t_1 + d_1, t_2)$, (Eq. 23) implies that $V(t) \le e^{\delta_2(t-t_1-d_1)}V(t_1 + d_1)$. That is,

$$\begin{aligned}\check{V}(t) &\le e^{\delta_2(1-\gamma)(t-t_1-d_1)}\check{V}(t_1 + d_1)\\ &\le M_1^{\mathfrak{N}(t_1,t_1+d_1)}M_0^{\mathfrak{N}(0,d_0)}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + e^{\delta_2(1-\gamma)(t-t_1-d_1)}\frac{\delta_1}{\delta_2}.\end{aligned}$$

By mathematical induction, for $t \in [t_k, t_k + d_k)$, and $t_k < s_1^k < \cdots < s^k_{\mathfrak{N}(t_k,t_k+d_k)} < t_k + d_k$,

$$\check{V}(t) \le M_k^{\mathfrak{N}(t_k,t)}\prod_{l=0}^{k-1}M_l^{\mathfrak{N}(t_l,t_l+d_l)}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}. \tag{24}$$

Suppose that there is constant $k_0 \in \mathbb{N}$ such that $V(t_{k_0}) > 0$ and $V(t_{k_0} + d_{k_0}) \le 0$. If $k_0 = 0$, then, the network (Eq. 1) does not need intermittent control, and can achieve FTBS under appropriate IC. If $k_0 > 0$, there is $T_1 \in (t_{k_0}, t_{k_0} + d_{k_0})$, which is the ST. According to (Eq. 24), let

$$\check{V}(t) \le M_{k_0}^{\mathfrak{N}(t_k,t)}\prod_{l=0}^{k_0-1}M_l^{\mathfrak{N}(t_l,t_l+d_l)}e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} = 0.$$

Then, the ST is $T_1 \le \frac{1}{\delta_2(\gamma-1)}[\ln\prod_{l=0}^{k_0-1}M_l^{\mathfrak{N}(t_l,t_l+d_l)} + \ln\tilde{V}(0)]$, which means that $V(t) \to 0$ when $t \to T_1$. Therefore, the coupled NNs (Eqs. 1 and 4) can achieve FTBS under hybrid controllers (Eqs. 10 and 11). □

Remark 4. Different from Lemma 3, which only considers the finite-time synchronization of network under AIC, we also add IC to improve the network synchronization performance. The criteria for FTBS of coupled NNs (Eq. 1) are derived by designing suitable AIC (Eq. 10) and IC (Eq. 11) in Theorem 1. Moreover, the ST T_1 is significantly reduced compared to T_0 of Lemma 3.

Further, if both intermittent control and IC occur periodically, the result of the following Corollary 1 can be derived by Theorem 1. We will first give the following notations for convenience. Denote

$$\hat{\omega}_k = \mu^{1-\gamma}e^{\delta_2(\gamma-1)k(d+\eta)}(e^{\delta_2(1-\gamma)\eta} - 1) > 0,$$

$$\hat{W}_{1k} = \frac{(1-\mu^{1-\gamma})e^{\delta_2(\gamma-1)((k+h)d+k\eta)} - \hat{\omega}_k}{\tilde{V}(0)\prod_{l=0}^{k-1}\hat{M}_l^{\bar{N}}},$$

$$\hat{W}_k = \frac{(1-\mu^{1-\gamma})e^{\delta_2(\gamma-1)((k+\bar{N}h)d+k\eta)}}{\tilde{V}(0)\prod_{l=0}^{k-1}\hat{M}_l^{\bar{N}}},$$

where the constants $\delta_1 = 2\varsigma c_1 c_2^{-\frac{v_1+v_2}{2v_2}}$, $c_1 = \lambda_{\min}(P)$, $c_2 = \lambda_{\max}(P)$, $\gamma = \frac{v_1+v_2}{2v_2}$, μ, $\delta_2 > 0$, $\tilde{V}(0)$ and matrix $P > 0$ are the same as Theorem 1. Constants $d, \eta > 0$, $0 < h < 1$, $\hat{M}_l > 1$ and $\bar{N} \in \mathbb{N}^+$ are later given in Corollary 1.

Corollary 1. *Under Assumptions 1–2, the coupled NNs (Eqs. 1 and 4) can achieve FTBS, if there are $P > 0$ and $\delta_2 > 0$ such that (Eq. 15) holds. Suppose that there exist $d > 0$, $\eta > 0$ and $0 < h < 1$ such that $d_k = d$, $t_{k+1} - t_k - d = \eta$, and $s_{i+1}^k - s_i^k = hd$. Moreover, constants $\hat{M}_k > 1$, $k \in \mathbb{N}$ satisfy*

$$\hat{M}_k - \mu^{1-\gamma} \le \hat{W}_{1k},\ t \in [s_1^k, s_2^k), \tag{25}$$

and

$$\hat{M}_k^{\bar{N}-1}(\hat{M}_k - \mu^{1-\gamma}) \le \hat{W}_k,\ t \in [s_{\bar{N}}^k, t_k + d), \tag{26}$$

where $\bar{N} = \ulcorner(h^{-1} - 1)\urcorner$, $\omega_0 = 0$, $\prod_{l=0}^{-1}\hat{M}_l^{\bar{N}} = 1$ and $k \le k_0$, $k_0 \in \mathbb{N}$ is a constant.

Moreover, the ST is

$$\hat{T}_1 \le \frac{1}{\delta_2(\gamma-1)}\left[\ln\prod_{l=0}^{k_0-1}\hat{M}_l^{\bar{N}} + \ln\tilde{V}(0)\right].$$

Proof. Firstly, by Theorem 1, one has

$$\begin{cases}\dot{V}(t) \le -\delta_1 V^{\gamma}(t) + \delta_2 V(t), \quad t \in [t_k, t_k + d),\\ V(s_i^k) = \mu V(s_i^{k-}), \quad t \ne s_i^k, \quad k \in \mathbb{N}, i \in \mathbb{N},\\ \dot{V}(t) \le \delta_2 V(t), \quad t \in [t_k + d, t_{k+1}).\end{cases}$$

Further, since the impulses occur periodically, satisfying $s_{i+1}^k - s_i^k = hd$, then, it holds that $\bar{N}$ impulses occur in each intermittently controlled interval, where $\bar{N} = \ulcorner(h^{-1} - 1)\urcorner$. Similarly, by mathematical induction, for $t \in [t_k, t_k + d)$, $t_k < s_1^k < \cdots < s_{\bar{N}}^k < t_k + d$, $k \in \mathbb{N}$, one has

$$\check{V}(t) \le \hat{M}_k^{\mathfrak{N}(t_k,t)} \prod_{l=0}^{k-1} \hat{M}_l^{\bar{N}} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}. \tag{27}$$

Suppose that there is $k_0 \in \mathbb{N}$ such that $V(t_{k_0}) > 0$ and $V(t_{k_0} + d) \le 0$. If $k_0 = 0$, then, the network (Eq. 1) does not need intermittent control, and can achieve FTBS under appropriate IC. If $k_0 > 0$, there is $T_1 \in (t_{k_0}, t_{k_0} + d)$, which is the ST. According to (Eq. 27), let

$$\check{V}(t) \le \hat{M}_{k_0}^{\mathfrak{N}(t_k,t)} \prod_{l=0}^{k_0-1} \hat{M}_l^{\bar{N}} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} = 0.$$

Then, the ST is $\hat{T}_1 \le \frac{1}{\delta_2(\gamma-1)}[\ln \prod_{l=0}^{k_0-1} \hat{M}_l^{\bar{N}} + \ln \tilde{V}(0)]$, which means that $V(t) \to 0$ when $t \to \hat{T}_1$. Therefore, the coupled NNs (Eqs. 1 and 4) can achieve FTBS under hybrid controllers (Eqs. 10 and 11). □

3.2. AIC and IC occur in different time periods

Next, we derive the criteria of FTBS of coupled NNs (Eq. 1) when the IC occurs in the uncontrolled intervals of AIC, and give the estimation of the corresponding ST. In other words, based on the design considerations outlined in the second part for the second form of the hybrid controller and the error network (Eq. 14), we can establish the sufficient conditions for achieving FTBS of the coupled NNs and provide an estimate of the ST, the details are as follows.

Theorem 2. *Under Assumptions 1–2, the coupled NNs (Eqs. 1 and 4) can achieve FTBS, if there are matrix $P > 0$ and constant $\delta_2 > 0$ such that (Eq. 15) holds. Moreover, constant $M > 1$ satisfies*

$$M^k - M^{k-1} \le \frac{\tilde{W}_k}{\tilde{V}(0)}, \ t \in [t_k, t_k + d_k), k \in \mathbb{N}_+, \tag{28}$$

where $\tilde{W}_k = \mu^{(\gamma-1)\sum_{l=0}^{k-1} \mathfrak{N}(t_l+d_l,t_{l+1})} e^{\delta_2(\gamma-1)t_k} - \mu^{(\gamma-1)\sum_{l=0}^{k-2} \mathfrak{N}(t_l+d_l,t_{l+1})} e^{\delta_2(\gamma-1)(t_{k-1}+d_{k-1})}$, $k \le k_1$, $k_1 \in \mathbb{N}$ *is a constant, and* $\sum_{l=0}^{-1} \mathfrak{N}(t_l + d_l, t_{l+1}) = 0$.

In addition, the ST is

$$T_2 \le \frac{1}{\delta_2(\gamma-1)}\left[\ln(M^{k_1} \mu^{(1-\gamma)\sum_{l=0}^{k_1-1} \mathfrak{N}(t_l+d_l,t_{l+1})}) + \ln \tilde{V}(0)\right].$$

Proof. Let $V(t) = \sum_{p=1}^N e_p^T(t) P e_p(t)$. Then, Combining Theorem 1 and (Eq. 14), one gets

$$\begin{cases}\dot{V}(t) \le -\delta_1 V^{\gamma}(t) + \delta_2 V(t), \quad t \in [t_k, t_k + d_k), k \in \mathbb{N},\\ \dot{V}(t) \le \delta_2 V(t), \quad t \in [t_k + d_k, t_{k+1}), t \ne \theta_j^k, j \in \mathbb{N}^+,\\ V(\theta_j^k) = \mu V(\theta_j^{k-}).\end{cases}$$

Further, for $t \in [0, d_0)$, by Lemma 3, it holds that

$$\check{V}(t) \le e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.$$

If $d_0 \ge T_0$, then, the coupled NNs (Eqs. 1 and 4) have achieved FTBS before d_0. If $d_0 < T_0$, then, for $t \in [d_0, t_1)$, we have $d_0 < \theta_1^0 < \cdots < \theta_{\mathfrak{N}(d_0,t_1)}^0 < t_1$. Specifically, for $t \in [d_0, \theta_1^0)$,

$$V(t) \le e^{\delta_2(t-d_0)} V(d_0).$$

That is,

$$\check{V}(t) \le e^{\delta_2(1-\gamma)(t-d_0)} \check{V}(d_0) \le e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-d_0)}.$$

For $t \in [\theta_1^0, \theta_2^0)$, one has

$$\check{V}(t) \le e^{\delta_2(1-\gamma)(t-\theta_1^0)} \mu^{1-\gamma} \check{V}(\theta_1^{0^-}) \le \mu^{1-\gamma}\left[e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-d_0)}\right].$$

By parity of reasoning, for $t \in [\theta_{\mathfrak{N}(d_0,t_1)-1}^0, \theta_{\mathfrak{N}(d_0,t_1)}^0)$,

$$\check{V}(t) \le \mu^{(\mathfrak{N}(d_0,t_1)-1)(1-\gamma)}[e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-d_0)}].$$

For $t \in [\theta_{\mathfrak{N}(d_0,t_1)}^0, t_1)$, we have

$$\check{V}(t) \le \mu^{\mathfrak{N}(d_0,t_1)(1-\gamma)}\left[e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-d_0)}\right],$$

and

$$\check{V}(t_1^-) = \check{V}(t_1) \le \mu^{\mathfrak{N}(d_0,t_1)(1-\gamma)}\left[e^{\delta_2(1-\gamma)t_1}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t_1-d_0)}\right]. \tag{29}$$

Similarly, for $t \in [t_1, t_1 + d_1)$,

$$\begin{aligned}\check{V}(t) &\le \mu^{\mathfrak{N}(d_0,t_1)(1-\gamma)} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \mu^{\mathfrak{N}(d_0,t_1)(1-\gamma)} \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-d_0)}\\ &\quad - \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-t_1)} + \frac{\delta_1}{\delta_2}\\ &\le M \mu^{\mathfrak{N}(d_0,t_1)(1-\gamma)} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}.\end{aligned}$$

For $t \in [t_1 + d_1, t_2)$, we have $t_1 + d_1 < \theta_1^1 < \cdots < \theta_{\mathfrak{N}(t_1+d_1,t_2)}^1 < t_2$. Specifically, for $t \in [t_1 + d_1, \theta_1^1)$,

$$V(t) \le e^{\delta_2(t-t_1-d_1)} V(t_1 + d_1).$$

That is,

$$\check{V}(t) \le e^{\delta_2(1-\gamma)(t-t_1-d_1)} \check{V}(t_1 + d_1) \le M \mu^{\mathfrak{N}(d_0,t_1)(1-\gamma)} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-t_1-d_1)}.$$

For $t \in [\theta_1^1, \theta_2^1)$, one has

$$\check{V}(t) \le e^{\delta_2(1-\gamma)(t-\theta_1^1)} \mu^{1-\gamma} \check{V}(\theta_1^{1^-}) \le M \mu^{(1-\gamma)[\mathfrak{N}(d_0,t_1)+1]} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \mu^{1-\gamma} \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-t_1-d_1)}.$$

By parity of reasoning, for $t \in [\theta_{\mathfrak{N}(t_1+d_1,t_2)}^1, t_2)$, one gets

$$\check{V}(t) \le M \mu^{(1-\gamma)[\mathfrak{N}(d_0,t_1)+\mathfrak{N}(t_1+d_1,t_2)]} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \mu^{(1-\gamma)\mathfrak{N}(t_1+d_1,t_2)} \frac{\delta_1}{\delta_2} e^{\delta_2(1-\gamma)(t-t_1-d_1)}.$$

By mathematical induction, for $t \in [t_k, t_k + d_k)$, $k \in \mathbb{N}$

$$\check{V}(t) \le M^k \mu^{(1-\gamma)\sum_{l=0}^{k-1} \mathfrak{N}(t_l+d_l,t_{l+1})} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2}. \tag{30}$$

Suppose that there is constant $k_1 \in \mathbb{N}$ such that $V(t_{k_1}) > 0$ and $V(t_{k_1} + d_{k_1}) \le 0$. If $k_1 = 0$, then, the network (Eq. 1) does not need IC, and can achieve FTBS before ST T_0. If $k_1 > 0$, there is $T_2 \in (t_{k_1}, t_{k_1} + d_{k_1})$, which is the ST. According to (Eq. 30), let

$$\check{V}(t) \le M^{k_1} \mu^{(1-\gamma)\sum_{l=0}^{k_1-1} \mathfrak{N}(t_l+d_l,t_{l+1})} e^{\delta_2(1-\gamma)t}\left(\check{V}(0) - \frac{\delta_1}{\delta_2}\right) + \frac{\delta_1}{\delta_2} = 0.$$

Then, we can obtain that the ST is $T_2 \le \frac{1}{\delta_2(\gamma-1)}[\ln(M^{k_1} \mu^{(1-\gamma)\sum_{l=0}^{k_1-1} \mathfrak{N}(t_l+d_l,t_{l+1})}) + \ln \tilde{V}(0)]$. Therefore, the coupled NNs (Eqs. 1 and 4) can achieve FTBS under hybrid controllers (Eqs. 10 and 12). □

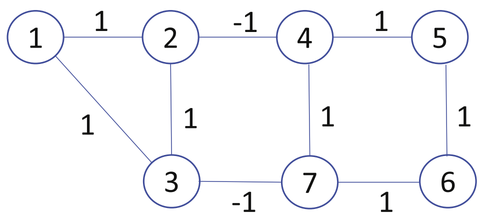


Fig. 1. The signed graph $\mathcal{G}^s$.

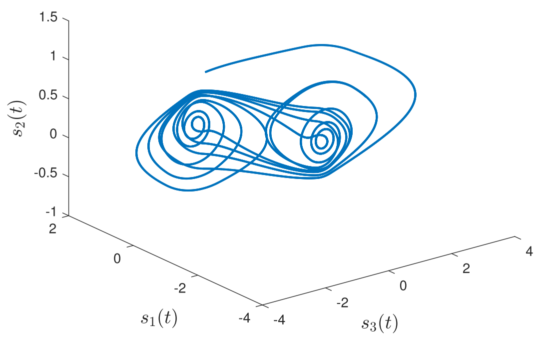


Fig. 2. The trajectory of leader network (4) in Example 1.

Remark 5. By designing AIC and IC in different intervals, we propose the criteria of FTBS of coupled NNs, which is easy to verify. In addition, Theorem 2 shows that by adding IC, we can reduce the control time of AIC in coupled NNs. In other words, the appropriate IC can effectively reduce the time of finite-time control (i.e., $-\varsigma e_p^{v_1/v_2}(t), t \geq 0$), which is different from the previous single intermittent control and hybrid IC (Jiang et al., 2019; Xi, Liu, & Li, 2024).

Further, if both intermittent control and IC occur periodically, the following theoretical results can be derived by Theorem 2.

Corollary 2. *Under Assumptions 1–2, the coupled NNs (Eqs. 1 and 4) can reach FTBS, if there are $P > 0$, and $\delta_2 > 0$ such that (Eq. 15) holds. Suppose that there exist $d > 0$, $\eta > 0$ and $0 < \tilde{h} < 1$ such that $d_k = d$, $t_{k+1} - t_k - d = \eta$, and $\theta_{j+1}^k - \theta_j^k = \tilde{h}\eta$. Moreover, constant $\check{M} > 1$ satisfies*

$$\check{M}^k - \check{M}^{k-1} \leq \frac{\check{W}_k}{\check{V}(0)}, \quad t \in [t_k, t_k + d), k \in \mathbb{N}_+, \tag{31}$$

where $\check{W}_k = \mu^{(\gamma-1)(k-1)\tilde{N}} e^{\delta_2 k(\gamma-1)(d+\eta)}[\mu^{(\gamma-1)\tilde{N}} - e^{\delta_2(1-\gamma)\eta}]$, $k \leq k_1$, $k_1 \in \mathbb{N}$, and $\tilde{N} = \ulcorner(\tilde{h}^{-1} - 1)\urcorner$.

Moreover, the ST is

$$\check{T}_2 \leq \frac{1}{\delta_2(\gamma - 1)}\Big[\ln(\check{M}^{k_1}\mu^{(1-\gamma)(k_1-1)\tilde{N}}) + \ln \check{V}(0)\Big].$$

Proof. According to the derivation of Theorem 2 and Corollary 1, we can easily reach the conclusion of Corollary 2, and the proof will be omitted here. □

4. Numerical simulations

In this section, the rationality of the proposed bipartite synchronization theory results and control methods for the coupled NNs will be verified through the Chua's circuit system model (Chen, Jia, Wang, Xie, & Qiu, 2024; Mao et al., 2022). Based on this, we can obtain the coupled NNs model as shown in network (1).

Example 1. For the coupled NNs (1), the signed graph $\mathcal{G}^s$ considered in this paper is consistent with that in reference Chen et al. (2024). Then, select other network parameters $N = 7$, $n = 3$, $c = 2.5$, $f(x_p(t)) = (0.5(|x_{p1}(t) + 1| - |x_{p1}(t) - 1|), 0.5(|x_{p2}(t) + 1| - |x_{p2}(t) - 1|), 0.5(|x_{p3}(t) + 1| - |x_{p3}(t) - 1|))^T$, $H = I_3$, and $K = \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.25 & 1 & 0.9 \\ 0 & 2.2 & 1.5 \end{bmatrix}$. The topology of the coupled NNs (1) is considered in Fig. 1, which is satisfies Assumption 2. Then, we have $\mathcal{V}_1 = \{1, 2, 3\}$ and $\mathcal{V}_2 = \{4, 5, 6, 7\}$. In addition, the leader network (4) with $s(0) = (0.9, 0.7, -0.8)^T$ is chaotic attraction (Lu, 2002), as shown in Fig. 2. Moreover, the state trajectories of the coupled NNs (1) with $x_1(0) = (1.1, 1.1, -1.5)^T$, $x_2(0) = (1.2, 1.0, -1.3)^T$, $x_3(0) = (1.0, 1.2, -1.1)^T$, $x_4(0) = (-1.6, -1.3, 1.1)^T$, $x_5(0) = (-1.1, -1.0, 1.3)^T$, $x_6(0) = (-1.2, -1.5, 1.5)^T$, $x_7(0) = (-1.5, -1.3, 1.1)^T$ and $u_p(t) = 0$ are presented in Fig. 3. The corresponding error evolution of networks (1) and (4) is presented in Fig. 4 (*a*). That is, FTBS cannot be achieved under the original NNs without control. Further, for the hybrid control $u_p(t)$, choose $\varsigma = 7.5$, $v_1 = 3$, $v_2 = 5$, $\nu = 0.6$, $t_1 = 0.5$, $t_2 = 1.1$, $d_0 = 0.2$, $d_1 = 0.3$, $d_2 = 0.5$, $s_1^0 = 0.1$, $s_2^0 = 0.15$, $s_1^1 = 0.55$, and $s_2^1 = 1.15$, then, it holds that $\delta_2 = 4.6$, $M_0 = 2.2$, $M_1 = 1.7$, and $P = \begin{bmatrix} 0.6254 & 0.0201 & 0.1522 \\ 0.0201 & 0.6491 & -0.1506 \\ 0.1522 & -0.1506 & 0.4757 \end{bmatrix}$. Based on Theorem 1 and (18), the coupled NNs (1) and (4) can achieve FTBS, and the ST $T_1 \leq 1.58$, as shown in Fig. 4 (*b*). However, the error trajectories of NNs (1) and (4)

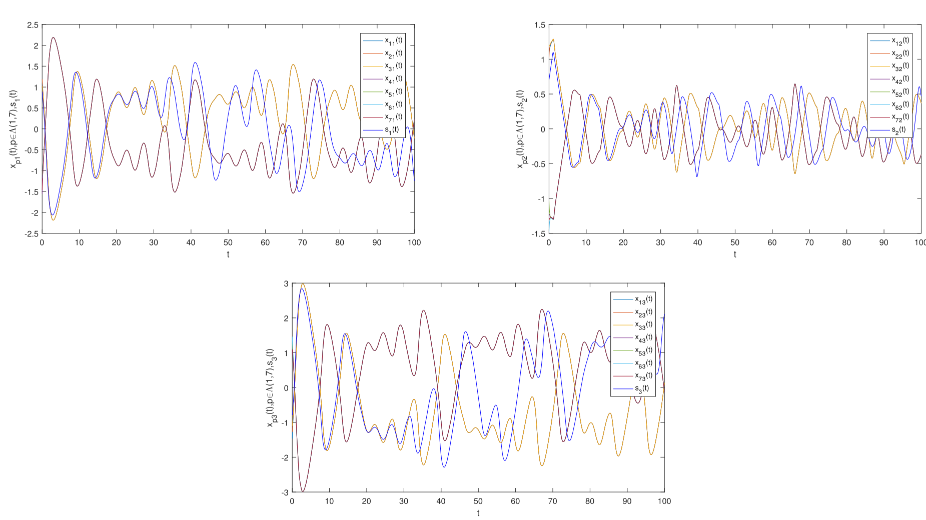
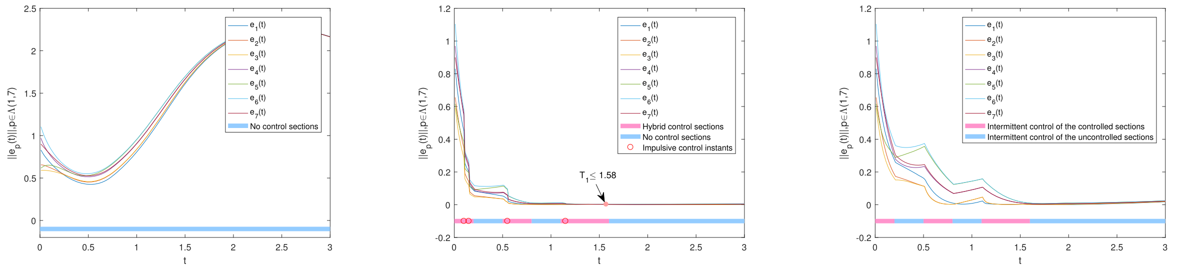
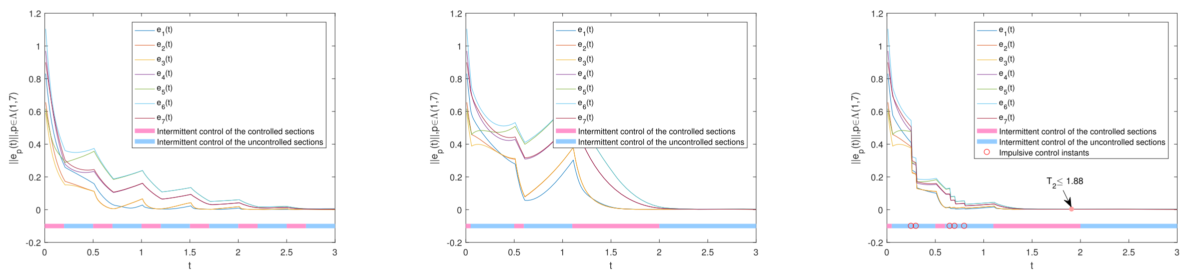


Fig. 3. The state evolutions of $x_p(t), p \in \Lambda(1, 7)$ and $s(t)$ without control in Example 1.



(a) Without control (b) With hybrid control (10) and (11) (c) With aperiodically intermittent control

Fig. 4. The error trajectories of NNs (1) and (4) in Example 1.



(a) With periodically intermittent control (b) With aperiodically intermittent control (c) With hybrid control (10) and (12)

Fig. 5. The error trajectories of NNs (1) and (4) under different control in Example 2.

without IC (11) are given in Fig. 4 (c), which show that the networks cannot be synchronized only under intermittent control. Therefore, compared to Fig. 4 (b), the efficacy of the hybrid controller designed in this paper is illustrated.

Example 2. The parameters of the coupled NNs (1) and (4) are the same as in Example 1. Firstly, select $\varsigma = 7.5$, $\upsilon_1 = 3$, $\upsilon_2 = 5$, and $d_k = 0.2$, that is, periodically intermittent control occurs. Fig. 5 (a) depicts the error evolution process of NNs (1) and (4). Distinct from reference Mao et al. (2022), which solely focuses on periodically intermittent control, our work extends the scope of exploration by designing an AIC mechanism. Then, choose $t_1 = 0.5$, $t_2 = 1.1$, $d_0 = 0.05$, $d_1 = 0.1$, $d_2 = 0.9$, that is, design a suitable AIC (10). Compared to Fig. 5 (a), Fig. 5 (b) presents that the NNs (1) and (4) are more easily synchronized with appropriate AIC. Further, for the IC $\check{u}_p(t)$, choose $\nu = 0.6$, $\theta_1^0 = 0.25$, $\theta_2^0 = 0.3$, $\theta_1^1 = 0.65$, $\theta_2^1 = 0.7$, and $\theta_3^1 = 0.8$, then, combined with the aforementioned AIC, it holds that $\delta_2 = 4.6$ and $M = 1.5$. Based on Theorem 2, the coupled NNs (1) and (4) can achieve FTBS, and the ST $T_2 \leq 1.88$, as presented in Fig. 5 (c). Moreover, compared with Fig. 5 (a), it can be seen from Fig. 5 (c) that the addition of IC can shorten the intervals of intermittent control and speed up the FTBS of NNs.

5. Conclusion

The FTBS problem for coupled NNs with signed graphs under AIC and IC has been investigated. It was revealed that the combination of these two forms of discontinuous control significantly reduces the network's control cost and the duration of continuous control input. Initially, the paper presented the theoretical frameworks and necessary conditions related to the signed graphs. Subsequently, specific forms of the two types of AIC and IC were illustrated separately. Theoretical results and numerical simulations shown that when the IC occurs within the controlled segments of AIC, it will expedite the synchronization of the coupled NNs. Furthermore, when the IC occurs in the uncontrolled segments of the AIC, it will diminish the controlled periods of the AIC. Consequently, based on the two hybrid controllers previously designed, the FTBS criteria for the coupled NNs have been established and the corresponding ST has been estimated. Future explorations may entail the deployment of an appropriate event-triggering mechanism to orchestrate the sequence of AIC and IC. Moreover, given the ubiquity of uncertainty, further investigation of the FTBS of stochastic NNs under AIC and IC warrants consideration.

CRediT authorship contribution statement

Qian Cui: Conceptualization, Investigation, Writing – original draft, Writing – review & editing; **Jinde Cao:** Writing – review & editing, Validation, Supervision; **Mahmoud Abdel-Aty:** Methodology, Supervision.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This work was jointly supported by the National Natural Science Foundation of China under Grant No. 12326311, the Guangxi Natural Science Foundation under Grant No. 2025GXNSFDA069040, and the SEU Innovation Capability Enhancement Plan for Doctoral Students under Grant No. CXJH_SEU 25118.

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