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Common fixed point theorems for weakly compatible self-mappings sustaining integral type contractions

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ABSTRACT

In this paper, for the self quadruple mappings (SQMs) $\mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4 : \mathcal{X} \to \mathcal{X}$, a common fixed point theorem (CFPT) is presented, where $(\mathcal{X}, \blacktriangledown_d)$ is a metric space and $(\mathfrak{F}_1, \mathfrak{F}_2)$ is a cyclic $(\alpha, \lambda)_{(\mathfrak{F}_3, \mathfrak{F}_4)}$ admissible pair.

Keywords: self quadruple mappings, fixed point theorems, admissible pair.

2000 Mathematics Subject Classification: 47H10.

1 Introduction

The fixed point theorems (FPTs) always attracted interest of researchers due to their useful applications in functional equations, differential equations, integral equations, existence and uniqueness of solutions for equations, and so forth; see, for instance, (Ege and Karaca, 2015), (Jleli and Samet, 2015), (Liu, Han, Kang and Ume, 2014), (Shatanawi, Samet and Abbas,

2012). Different techniques are followed for the new FPTs. The notion of integral type contractions in a metric space was introduced by Branciari (Branciari, 2002) who established several FPTs that are virtuous generalizations of the Banach's contraction principle. Liu et al. (Liu, Li and Kang, 2013) and Vetro (Vetro, 2010) extended the work of Branciari (Branciari, 2002) and obtained new FPTs by the help of integral type contractions for weakly compatible mappings (WCMs). Hussain et al. (Hussain, Isik and Abbas, 2016) introduced the notion of generalized almost rational contraction with respect to a pair of self mappings on a complete metric space, and they obtained several CFPTs for such mappings. Using the Closed Range Property (CLR property) of the involved pairs, some CFPTs for two pairs of WCMs satisfying contractive condition of integral type in complex valued metric spaces are reported in the paper by Zada et al. (Zada, Sarwar, Rahman and Imdad, 2016). Baleanu *et al.* (Baleanu, Agarwal, Khan, Khan and Jafari, 2015) gave an application of fixed point theorems.
Motivated by the above works, the aim of this paper is to generalize the CFPTs of Hussain et al. (Hussain et al., 2016) by the help of integral type contractions for the SQMs $\mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4 : \mathcal{X} \to \mathcal{X}$ on a metric space $(\mathcal{X}, \blacktriangledown_d)$. For this, we utilize integral type contractions with the assumptions that $(\mathfrak{F}_1, \mathfrak{F}_3)$ shares $CLR_{\mathfrak{F}_1}$ property, $(\mathfrak{F}_1, \mathfrak{F}_3)$, $(\mathfrak{F}_2, \mathfrak{F}_4)$ are WCMs, and $\mathfrak{F}_1\mathcal{X} \subseteq \mathfrak{F}_4\mathcal{X}$, $\mathfrak{F}_2\mathcal{X} \subseteq \mathfrak{F}_4\mathcal{X}$. In the proof of our results, we don't need to require the completeness of the metric space $(\mathcal{X}, \blacktriangledown_d)$.
In what follows, we provide some necessary and related definitions from the literature (Aydi, 2011; Aydi, Jellali and Karapinar, 2016) and references therein.

Definition 1.1. (see (Hussain et al., 2016)) Let $\mathfrak{F}_1, \mathfrak{F}_2 : \mathcal{X} \to \mathcal{X}$ be two self mappings on a complex valued metric space $(\mathcal{X}, d)$. Then $\mathfrak{F}_1$ and $\mathfrak{F}_2$ are said to satisfy the common limit range property with respect to $\mathfrak{F}_1$ (denoted by $CLR_{\mathfrak{F}_1}$) if there exists a sequence $\{x_n\}$ in $\mathcal{X}$ such that

$$\lim_{n\to\infty} \mathfrak{F}_1 x_n = \lim_{n\to\infty} \mathfrak{F}_2 x_n = \mathfrak{F}_1 x$$

for some $x \in \mathcal{X}$.

Definition 1.2. Two self mappings $\mathfrak{F}_1, \mathfrak{F}_2 : \mathcal{X} \to \mathcal{X}$ are said to be weakly compatible if they commute on their coincident point, that is, if there exists a point $x \in \mathcal{X}$ such that $\mathfrak{F}_1 x = \mathfrak{F}_2 x$, then $\mathfrak{F}_1\mathfrak{F}_2 x = \mathfrak{F}_2\mathfrak{F}_1 x$.

Definition 1.3. (see (Hussain et al., 2016)) A mapping $\tau : \mathbb{R}^+ \to \mathbb{R}^+$ is termed a generalized altering distance function if

(1) τ is nondecreasing;

(2) $\tau(x) = 0$ if and only if $x = 0$.

We also adopt the following notation for a compact presentation of our results.
$F = \{\tau : \mathbb{R}^+ \to \mathbb{R}^+ :\ \tau$ satisfies (1) and $(2)\}$.
$\Phi = \{\phi : \mathbb{R}^+ \to \mathbb{R}^+ :\ \phi$ is right upper semi-continuous, nondecreasing, and for all $x > 0, \tau(x) > \phi(x)$ and $\tau \in F\}$.
$\Psi_1 = \{\psi_1 : {\mathbb{R}^+}^6 \to \mathbb{R}^+ :\ \psi_1$ satisfies $(P_1) - (P_3)\}$, where
(P_1) ψ_1 is continuous and nondecreasing in each coordinate;

(P_2) $\psi_1(x,x,x,x,x,x) \leq x$ for all $x \geq 0$;
(P_3) $\psi_1 = 0$ if and only if all the components of ψ_1 are zero;
$\Psi_2 = \{\psi_2 : \mathbb{R}^{+^4} \to \mathbb{R}^+ :$ ψ_2 is continuous and $\psi_2 = 0$ if any component of ψ_2 is zero$\}$.

Recently, Hussain et al. (Hussain et al., 2016) introduced the notion of cyclic (α, λ)-admissible pair of maps $(\mathfrak{F}_1, \mathfrak{F}_2)$ on a metric space $(\mathcal{X}, \blacktriangledown_d)$ and defined generalized almost $(\mathfrak{F}_3, \mathfrak{F}_4)$-rational contraction pair $(\mathfrak{F}_1, \mathfrak{F}_2)$ as below:

Definition 1.4. (Hussain et al., 2016) Let $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$ be SQMs of a nonempty set X and $\alpha,\, \lambda : \mathcal{X} \to \mathbb{R}^+$. Then, the pair $(\mathfrak{F}_1, \mathfrak{F}_2)$ is called cyclic $(\alpha, \lambda)_{(\mathfrak{F}_3, \mathfrak{F}_4)}$-admissible if
(i) $\alpha(\mathfrak{F}_3\sigma^*) \geq 1$ for some $\sigma^* \in \mathcal{X}$ implies $\lambda(\mathfrak{F}_1\sigma^*) \geq 1$;
(ii) $\lambda(\mathfrak{F}_4\sigma^*) \geq 1$ for some $\sigma^* \in \mathcal{X}$ implies $\alpha(\mathfrak{F}_2\sigma^*) \geq 1$.

2 Main results

Here, we prove our main theorem for the SQMs $\mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4$ by the help of an integral-type inequality:

Theorem 2.1. *Let $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$ be SQMs of a metric space $(\mathcal{X}, \blacktriangledown_d)$ and $(\mathfrak{F}_1, \mathfrak{F}_2)$ be cyclic $(\alpha, \lambda)_{(\mathfrak{F}_3, \mathfrak{F}_4)}$-admissible. Suppose that the following conditions hold:*
(a) there exists a $\mu_0^* \in \mathcal{X}$ such that $\alpha(\mathfrak{F}_3\mu_0^*) \geq 1$ and $\lambda(\mathfrak{F}_4\mu_0^*) \geq 1$;
(b) if $\{\mu_n^*\}$ is a sequence in X such that $\alpha(\mu_n^*) \geq 1,\ \lambda(\mu_n^*) \geq 1$ for all $n \in \mathbb{N}_0$ and $\mu_n^* \to \mu^*$ as $n \to \infty$, then $\alpha(\mu^*) \geq 1$ and $\lambda(\mu^*) \geq 1$;
(c) for $\tau,\ \phi,\ \psi_1,\ \psi_2$ as defined in the Definition 1.3, and for some $\mu^* \in \mathcal{X}$, such that $\alpha(\mathfrak{F}_3\mu^*)\lambda(\mathfrak{F}_4\mu^*) \geq 1$, implies:

$$\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1\mu^*, \mathfrak{F}_2\sigma^*)} \Gamma(t)dt\Big) \leq \phi\Big(\int_0^{\psi_1(M(\mu^*,\sigma^*)) + L\psi_2(N(\mu^*,\sigma^*))} \Gamma(t)dt\Big), \tag{2.1}$$

for all $\mu^*,\ \sigma^* \in \mathcal{X}$, where $\Gamma(t)$ is a Lebesgue integrable function with finite integral such that $\int_0^\delta \Gamma(t)dt > 0$ for all $\delta > 0$, $L \geq 0$, and

$$\begin{aligned} \psi_1(M(\mu^*,\sigma^*)) \;=\; \max\Big\{ & \blacktriangledown_d(\mathfrak{F}_2(\sigma^*), \mathfrak{F}_4(\sigma^*)), \\ & \blacktriangledown_d(\mathfrak{F}_1(\mu^*), \mathcal{F}(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_4(\sigma^*), \mathfrak{F}_3(\mu^*)), \\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*), \mathfrak{F}_4(\sigma^*)) + \blacktriangledown_d(\mathfrak{F}_1(\mu^*), \mathfrak{F}_3(\mu^*))}{2}, \\ & \frac{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*), \mathfrak{F}_3(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*), \mathfrak{F}_2(\sigma^*))}{1 + \blacktriangledown_d(\mathfrak{F}_2(\sigma^*), \mathfrak{F}_1(\mu^*))}, \\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*), \mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*), \mathfrak{F}_4(\sigma^*))}{1 + \blacktriangledown_d(\mathfrak{F}_2(\sigma^*), \mathfrak{F}_3(\mu^*))}\Big\}, \end{aligned} \tag{2.2}$$

$$\begin{aligned} \psi_2(N(\mu^*,\sigma^*)) \;=\; \psi_2\Big(& \blacktriangledown_d(\mathfrak{F}_1(\mu^*), \mathfrak{F}_4(\sigma^*)), \\ & \blacktriangledown_d(\mathfrak{F}_2(\sigma^*), \mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_4(\sigma^*), \mathfrak{F}_3(\mu^*)), \\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*), \mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*), \mathfrak{F}_4(\sigma^*))}{1 + \blacktriangledown_d(\mathfrak{F}_2(\sigma^*), \mathfrak{F}_3(\mu^*))}\Big); \end{aligned} \tag{2.3}$$

(d) the pair $(\mathfrak{F}_1, \mathfrak{F}_3)$ shares $CLR_{\mathfrak{F}_1}$ property;
(e) $(\mathfrak{F}_1, \mathfrak{F}_3)$ and $(\mathfrak{F}_2, \mathfrak{F}_4)$ are WCMs.
Suppose that $\mathfrak{F}_1(\mathcal{X}) \subseteq \mathfrak{F}_4(\mathcal{X})$ and $\mathfrak{F}_2(\mathcal{X}) \subseteq \mathfrak{F}_3(\mathcal{X})$. Then the SQMs $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$ have a unique CFP in $(\mathcal{X}, \blacktriangledown_d)$.

Proof. By our assumption of the $CLR_{\mathfrak{F}_1}$ property of the pair $(\mathfrak{F}_1, \mathfrak{F}_2)$, there exists a sequence $\{\mu_n^*\}$ in the metric space $(\mathcal{X}, \blacktriangledown_d)$, such that

$$\lim_{n\to\infty} \mathfrak{F}_1(\mu_n^*) = \lim_{n\to\infty} \mathfrak{F}_3(\mu_n^*) = \mathfrak{F}_1\mu^* \text{ for some } \mu^* \in \mathcal{X}. \tag{2.4}$$

By the help of (a) we have $\alpha(\mathfrak{F}_3(\mu_0^*)) \geq 1$ and $\lambda(\mathfrak{F}_4(\mu_0^*)) \geq 1$, where the pair $(\mathfrak{F}_1, \mathfrak{F}_2)$ is cyclic $(\alpha, \lambda)_{(\mathfrak{F}_3,\mathfrak{F}_4)}$-admissible. Therefore, $\alpha(\mathfrak{F}_3(\mu_0^*)) \geq 1$ implies that $\lambda(\mathfrak{F}_1\mu_1^*) \geq 1$ and since $\mathfrak{F}_1(\mathcal{X}) \subseteq \mathfrak{F}_4(\mathcal{X})$, this further implies $\lambda(\mathfrak{F}_1(\mu_0^*)) = \lambda(\mathfrak{F}_4\mu_1^*) \geq 1$ for some $\mu_1^* \in \mathcal{X}$. From $(\mathfrak{F}_1, \mathfrak{F}_2)$ being a cyclic $(\alpha, \lambda)_{(\mathfrak{F}_3,\mathfrak{F}_4)}$-admissible pair and $\lambda(\mathfrak{F}_4\mu_1^*) \geq 1$, we have $\alpha(\mathfrak{F}_2(\mu_1^*)) \geq 1$. But $\mathfrak{F}_2(\mathcal{X}) \subseteq \mathfrak{F}_3(\mathcal{X})$, we can have $\alpha(\mathfrak{F}_2(\mu_1^*)) = \alpha(\mathfrak{F}_3(\mu_2^*)) \geq 1$ for some $\mu_2^* \in \mathcal{X}$. Continuing this process upto n times, we are having $\alpha(\mathfrak{F}_3(\mu_{2n}^*)) \geq 1$, $\lambda(\mathfrak{F}_4(\mu_{2n+1}^*)) \geq 1$, for all $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, which provide us

$$\alpha(\mathfrak{F}_3(\mu_n^*)) \geq 1\ ,\ \lambda(\mathfrak{F}_4(\mu_n^*)) \geq 1, \text{ for all } n \in \mathbb{N}_0. \tag{2.5}$$

By (2.5) and (b), we have $\alpha(\mathfrak{F}_3(\mu^*))\lambda(\mathfrak{F}_4(\mu^*)) \geq 1$. Since, $\mathfrak{F}_1(\mathcal{X}) \subseteq \mathfrak{F}_4(\mathcal{X})$, so there exists some $u \in \mathcal{X}$, such that $\mathfrak{F}_1\mu^* = \mathfrak{F}_4 u$. We prove that $\mathfrak{F}_2 u = \mathfrak{F}_4 u$. For this, we assume the contrary, that is, $\mathfrak{F}_2 u \neq \mathfrak{F}_4 u$. Therefore, from (2.1) we have

$$\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1 u, \mathfrak{F}_2\sigma_n^*)} \Gamma(t)dt\Big) \leq \phi\Big(\int_0^{\psi_1(M(u,\sigma_n^*))+L\psi_2(N(u,\sigma_n^*))} \Gamma(t)dt\Big), \tag{2.6}$$

For $\mu^* = \mu_n^*$ and $\sigma^* = u$ in the inequality (2.6), where

$$\begin{aligned}
\psi_1(M(\mu_n^*, u)) \quad = \quad \max\Big\{ & \blacktriangledown_d(\mathfrak{F}_1(\mu_n^*), \mathfrak{F}_4(u)),\ \blacktriangledown_d(\mathfrak{F}_2(u), \mathfrak{F}_4(u)), \blacktriangledown_d(\mathfrak{F}_4(u), \mathfrak{F}_3(\mu_n^*)),\\
& \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*), \mathfrak{F}_4(u)) + \blacktriangledown_d(\mathfrak{F}_1(\mu_n^*), \mathfrak{F}_3(\mu_n^*))}{2},\\
& \frac{\blacktriangledown_d(\mathfrak{F}_2(u), \mathfrak{F}_3(\mu_n^*))\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*), \mathfrak{F}_2(u))}{1 + \blacktriangledown_d(\mathfrak{F}_2(u), \mathfrak{F}_1(\mu_n^*))},\\
& \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*), \mathfrak{F}_4(u))\blacktriangledown_d(\mathfrak{F}_3(\mu_n^*), \mathfrak{F}_4(u))}{1 + \blacktriangledown_d(\mathfrak{F}_2(\mu_n^*), \mathfrak{F}_3(\mu_n^*))}\Big\},
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
\psi_2(N(\mu_n^*, u)) = \psi_2\Big(& \blacktriangledown_d(\mathfrak{F}_1(\mu_n^*), \mathfrak{F}_4(u)),\ \blacktriangledown_d(\mathfrak{F}_2(u), \mathfrak{F}_4(u)),\ \blacktriangledown_d(\mathfrak{F}_4(u), \mathfrak{F}_3(\mu_n^*)),\\
& \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*), \mathfrak{F}_4(u))\blacktriangledown_d(\mathfrak{F}_3(\mu_n^*), \mathfrak{F}_4(u))}{1 + \blacktriangledown_d(\mathfrak{F}_2(u), \mathfrak{F}_3(\mu_n^*))}\Big).
\end{aligned} \tag{2.8}$$

Taking the $\lim_{n\to\infty}$ in (2.6)–(2.8), we get

$$\begin{aligned}
\lim_{n\to\infty}\psi_1(M(\mu_n^*,u)) &= \lim_{n\to\infty}\max\Big\{\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*),\mathfrak{F}_4(u)),\ \blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_4(u)),\\
&\quad \blacktriangledown_d(\mathfrak{F}_4(u),\mathfrak{F}_3(\mu_n^*)),\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*),\mathfrak{F}_3(\mu_n^*))+\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*),\mathfrak{F}_4(u))}{2},\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_3(\mu_n^*))\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*),\mathfrak{F}_2(u))}{1+\blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_1(\mu_n^*))},\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*),\mathfrak{F}_4(u))\blacktriangledown_d(\mathfrak{F}_3(\mu_n^*),\mathfrak{F}_4(u))}{1+\blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_3(\mu_n^*))}\Big\} \qquad (2.9)\\
&= \max\Big\{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_1(\mu^*)),\ \blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_1(\mu^*)),\\
&\quad \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_1(\mu^*)),
\end{aligned}$$

$$\begin{aligned}
&\frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_1(\mu^*))+\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_1(\mu^*))}{2},\\
&\frac{\blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_1(\mu^*))\blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_1(\mu^*))}{1+\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(u))},\\
&\frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_1(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_1(\mu^*))}{1+\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(u))}\Big\}\\
=&\ \max\{\blacktriangledown_d(0,\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(u)),0,0,\frac{\blacktriangledown_d^2(\mathfrak{F}_2(u),\mathfrak{F}_1(\mu^*))}{1+\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(u))},0\}\\
=&\ \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(u)),
\end{aligned}$$

$$\begin{aligned}
\lim_{n\to\infty}\psi_2(N(\mu_n^*,u)) &= \lim_{n\to\infty}\psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*),\mathfrak{F}_4(u)),\ \blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_4(u)),\\
&\quad \blacktriangledown_d(\mathfrak{F}_4(u),\mathfrak{F}_3(\mu_n^*)),\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu_n^*),\mathfrak{F}_4(u))\blacktriangledown_d(\mathfrak{F}_3(\mu_n^*),\mathfrak{F}_4(u))}{1+\blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_3(\mu_n^*))}\Big) \qquad (2.10)\\
&= \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_1(\mu^*)),\ \blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_1(\mu^*)),\\
&\quad \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\ \mathfrak{F}_1(\mu^*)),\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\ \mathfrak{F}_1(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\ \mathfrak{F}_1(\mu^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(u),\ \mathfrak{F}_1(\mu^*))}\Big)\\
&= \psi_2\Big(0,\blacktriangledown_d(\mathfrak{F}_2(u),\mathfrak{F}_1(\mu^*)),\ 0,\ 0\Big)=0,
\end{aligned}$$

$$\lim_{n\to\infty}\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1\mu_n^*,\mathfrak{F}_2u)}\Gamma(t)dt\Big)\leq\lim_{n\to\infty}\phi\Big(\int_0^{\psi_1(M(\mu_n^*,u))+L\psi_2(N(\mu_n^*,u))}\Gamma(t)dt\Big).$$

From (2.9)–(2.11), we get

$$\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1\mu^*,\mathfrak{F}_2u)}\Gamma(t)dt\Big)\ \leq\ \phi\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2u)}\Gamma(t)dt\Big). \qquad (2.11)$$

From Definition 1.3, we have $\tau(\mathcal{X})>\phi(\mathcal{X})$, for all $x>0$. Thus, (2.11) is a contradiction and is due to our supposition that $\mathfrak{F}_2(u)\neq\mathfrak{F}_4(u)$, and hence

$$\mathfrak{F}_2(u)=\mathfrak{F}_4(u)=\mathfrak{F}_1\mu^*. \qquad (2.12)$$

But from our assumption that $\mathfrak{F}_2(\mathcal{X}) \subseteq \mathfrak{F}_3(\mathcal{X})$, there exists some $z \in \mathcal{X}$ such that $\mathfrak{F}_2 u = \mathfrak{F}_3 z$. Now, we show that $\mathfrak{F}_3 z = \mathfrak{F}_1 z$. For this, we assume the contrary, that is, $\mathfrak{F}_3 z \neq \mathfrak{F}_1 z$. By putting $\mu^* = z$ and $\sigma^* = u$ in (2.1), we have

$$\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_2 u)} \Gamma(t)dt\Big) \leq \phi\Big(\int_0^{\psi_1(M(z,u))+L\psi_2(N(z,u))} \Gamma(t)dt\Big), \tag{2.13}$$

where

$$\begin{aligned} \psi_1(M(z,u)) &= \max\Big\{\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_4 u),\ \blacktriangledown_d(\mathfrak{F}_2(u), \mathfrak{F}_4(u)),\ \blacktriangledown_d(\mathfrak{F}_4(u), \mathfrak{F}_3 z), \\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_4(u)) + \blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_3 z}{2}, \frac{\blacktriangledown_d(\mathfrak{F}_2(u), \mathfrak{F}_3 z)\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_2 u)}{1+\blacktriangledown_d(\mathfrak{F}_2 u, \mathfrak{F}_1 z)}, \\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_4 u)\blacktriangledown_d(\mathfrak{F}_3 z, \mathfrak{F}_4 u)}{1+\blacktriangledown_d(\mathfrak{F}_2 u, \mathfrak{F}_3 z)}\Big\} \\ &= \max\Big\{0, \blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_2 u), 0, \blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_2 u), 0, 0\Big\} = \blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_2 u), \end{aligned} \tag{2.14}$$

$$\begin{aligned} \psi_2(N(z,u)) &= \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_4 u),\ \blacktriangledown_d(\mathfrak{F}_2 u, \mathfrak{F}_4 u),\ \blacktriangledown_d(\mathfrak{F}_4 u, \mathfrak{F}_3 z), \\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_4 u)\blacktriangledown_d(\mathfrak{F}_3 z, \mathfrak{F}_4 u)}{1+\blacktriangledown_d(\mathfrak{F}_2 u, \mathfrak{F}_3 z)}\Big) \\ &= \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_4 u),\ 0,\ 0,\ 0\Big) = 0. \end{aligned} \tag{2.15}$$

From (2.13)–(2.15), we get

$$\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_2 u)} \Gamma(t)dt\Big) \leq \phi\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1 z, \mathfrak{F}_2 u)} \Gamma(t)dt\Big). \tag{2.16}$$

From Definition 1.3, we have $\tau(x) > \phi(x)$, for all $x > 0$. Thus, (2.16) is a contradiction and is due to our supposition that $\mathfrak{F}_3 z \neq \mathfrak{F}_1 z$, and hence:

$$\mathfrak{F}_3 z = \mathfrak{F}_1 z. \tag{2.17}$$

Thus, by virtue of (2.12) and (2.17), we have

$$\mathfrak{F}_1 z = \mathfrak{F}_3 z = \mathfrak{F}_2 u = \mathfrak{F}_4 u = x \text{ (say)}. \tag{2.18}$$

Note that the pairs $(\mathfrak{F}_1, \mathfrak{F}_3)$, $(\mathfrak{F}_2, \mathfrak{F}_4)$ are weakly compatible. Therefore, we have

$$\begin{aligned} &\mathfrak{F}_1 z = \mathfrak{F}_3 z \implies \mathfrak{F}_3\mathfrak{F}_1 z = \mathfrak{F}_1\mathfrak{F}_3 z \implies \mathfrak{F}_1 x = \mathfrak{F}_3 x, \\ &\mathfrak{F}_2 u = \mathfrak{F}_4 u \implies \mathfrak{F}_4\mathfrak{F}_2 u = \mathfrak{F}_2\mathfrak{F}_4 u \implies \mathfrak{F}_4 x = \mathfrak{F}_2 x. \end{aligned} \tag{2.19}$$

From (2.19), we conclude that $x \in \mathcal{X}$ is the coincident point of the pairs $(\mathfrak{F}_1, \mathfrak{F}_3)$, $(\mathfrak{F}_2, \mathfrak{F}_4)$. Next, we show that $x \in \mathcal{X}$ is a CFP of the mappings $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$. For this, putting $\mu^* = x$ and $\sigma^* = u$ in (2.6), and by the help of Definition 1.3, we obtain

$$\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1(\mathcal{X}), \mathfrak{F}_2(u))} \Gamma(t)dt\Big) \leq \phi\Big(\int_0^{\psi_1(M(x,u))+L\psi_2(N(x,u))} \Gamma(t)dt\Big), \tag{2.20}$$

where

$$\begin{aligned}
\psi_1(M(x,u)) &= \max\Big\{\blacktriangledown_d(\mathfrak{F}_1(\mathcal{X}),\mathfrak{F}_4u),\ \blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_4u),\ \blacktriangledown_d(\mathfrak{F}_4u,\mathfrak{F}_3x),\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_4u)+\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_3u)}{2},\ \frac{\blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_3x)\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_2u)}{1+\blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_1x)},\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_4u)\blacktriangledown_d(\mathfrak{F}_3x,\mathfrak{F}_4u)}{1+\blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_3x)}\Big\}\\
&= \max\Big\{\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_1x),\ \blacktriangledown_d(x,x),\ \blacktriangledown_d(x,\mathfrak{F}_1x),\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1x,x)+\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_1x)}{2},\\
&\quad \frac{\blacktriangledown_d(x,\mathfrak{F}_1x)\blacktriangledown_d(\mathfrak{F}_1x,x)}{1+\blacktriangledown_d(x,\mathfrak{F}_1x)},\frac{\blacktriangledown_d(\mathfrak{F}_1x,x)\blacktriangledown_d(\mathfrak{F}_1x,x)}{1+\blacktriangledown_d(x,\mathfrak{F}_1x)}\Big\}\\
&= \max\Big\{\blacktriangledown_d(\mathfrak{F}_1x,x),0,\blacktriangledown_d(x,\mathfrak{F}_1x),\frac{\blacktriangledown_d^2(\mathfrak{F}_1x,x)}{1+\mathfrak{F}_1x},\frac{\blacktriangledown_d^2(\mathfrak{F}_1x,x)}{1+\mathfrak{F}_1x}\Big\}\\
&= \blacktriangledown_d(\mathfrak{F}_1x,x),
\end{aligned}\tag{2.21}$$

$$\begin{aligned}
\psi_2(N(x,u)) &= \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_4u),\ \blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_4u),\ \blacktriangledown_d(\mathfrak{F}_4u,\mathfrak{F}_3x),\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_4u)\blacktriangledown_d(\mathfrak{F}_3x,\mathfrak{F}_4u)}{1+\blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_3x)}\Big)\\
&= \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1x,x),\ \blacktriangledown_d(x,x),\ \blacktriangledown_d(x,\mathfrak{F}_1x),\frac{\blacktriangledown_d(\mathfrak{F}_1x,x)\blacktriangledown_d(\mathfrak{F}_1x,x)}{1+\blacktriangledown_d(x,\mathfrak{F}_1x)}\Big)\\
&= \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1x,x),\ 0,\ \blacktriangledown_d(x,\mathfrak{F}_1x),\frac{\blacktriangledown_d^2(\mathfrak{F}_1x,x)}{1+\blacktriangledown_d(x,\mathfrak{F}_1x)}\Big)=0.
\end{aligned}\tag{2.22}$$

From (2.20)–(2.22), we get

$$\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1x,x)}\Gamma(t)dt\Big) \ \leq\ \phi\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1x,x)}\Gamma(t)dt\Big).\tag{2.23}$$

This contradiction implies that $\blacktriangledown_d(\mathfrak{F}_1x,x)=0$, or $\mathfrak{F}_1x=x$, that is, x is a CFP of $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4$. Finally, we prove that the CFP of the SQMs $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4$ is unique. For this, we again presume a contrary path, that is, let there exist two different CFP $x,\ u\in\mathcal{X}$ for the SQMs $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4$ such that $\mathfrak{F}_1x=\mathfrak{F}_3x=x,\ \mathfrak{F}_2u=\mathfrak{F}_4u=u,\ x\neq u$. Putting $\mu^*=x$ and $\sigma^*=u$ in (2.6), and using Definition 1.3, we have:

$$\tau\Big(\int_0^{\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_2(u))}\Gamma(t)dt\Big)=\tau\Big(\int_0^{\blacktriangledown_d(x,u)}\Gamma(t)dt\Big)\leq\phi\Big(\int_0^{\psi_1(M(x,u))+L\psi_2(N(x,u))}\Gamma(t)dt\Big),\tag{2.24}$$

where

$$\begin{aligned}
\psi_1(M(x,u)) &= \max\Big\{\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_3x),\ \blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_4u),\ \blacktriangledown_d(\mathfrak{F}_4u,\mathfrak{F}_3x),\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_4u)+\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_3x)}{2},\ \frac{\blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_3x)\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_2u)}{1+\blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_1x)},\\
&\quad \frac{\blacktriangledown_d(\mathfrak{F}_1x,\mathfrak{F}_4u)\blacktriangledown_d(\mathfrak{F}_3x,\mathfrak{F}_4u)}{1+\blacktriangledown_d(\mathfrak{F}_2u,\mathfrak{F}_3x)}\Big\}\\
&= \max\Big\{\blacktriangledown_d(x,x),\ \blacktriangledown_d(u,u),\ \blacktriangledown_d(u,x),\frac{\blacktriangledown_d(x,u)+\blacktriangledown_d(x,x)}{2},\\
&\quad \frac{\blacktriangledown_d(u,x)\blacktriangledown_d(x,u)}{1+\blacktriangledown_d(u,x)},\frac{\blacktriangledown_d(x,u)\blacktriangledown_d(x,u)}{1+\blacktriangledown_d(u,x)}\Big\}=\blacktriangledown_d(x,u)
\end{aligned}\tag{2.25}$$

and

$$\begin{aligned}\psi_2(N(x,u)) &= \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1 x,\mathfrak{F}_4 u),\ \blacktriangledown_d(\mathfrak{F}_2 u,\mathfrak{F}_4 u),\ \blacktriangledown_d(\mathfrak{F}_4 u,\mathfrak{F}_3 x),\\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_1 x,\mathfrak{F}_4 u)\blacktriangledown_d(\mathfrak{F}_3 x,\mathfrak{F}_4 u)}{1+\blacktriangledown_d(\mathfrak{F}_2 u,\mathfrak{F}_3 x)}\Big)\\ &= \psi_2\Big(\blacktriangledown_d(x,u),\ \blacktriangledown_d(u,u),\ \blacktriangledown_d(u,x),\ \frac{\blacktriangledown_d(x,u)\blacktriangledown_d(x,u)}{1+\blacktriangledown_d(u,x)}\Big) \qquad (2.26)\\ &= \psi_2\Big(\blacktriangledown_d(x,u),\ 0,\ \blacktriangledown_d(u,x),\ \frac{\blacktriangledown_d(x,u)\blacktriangledown_d(x,u)}{1+\blacktriangledown_d(u,x)}\Big)=0.\end{aligned}$$

Using (2.25) and (2.26) in (2.24), we have

$$\tau\Big(\int_0^{\blacktriangledown_d(x,u)}\Gamma(t)dt\Big) \ \le\ \phi\Big(\int_0^{\blacktriangledown_d(x,u)}\Gamma(t)dt\Big), \qquad (2.27)$$

which implies that, by Definition 1.3, we have $\tau(x) > \phi(x)$ for all $0 < x$. Thus, (2.27) is a contradiction and this is due to our assumption $x \neq u$ and therefore, the CFP of the SQMs $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$, is unique. □

Corollary 2.2. *Let $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$ be SQMs on a metric space $(\mathcal{X},\blacktriangledown_d)$ and the pair $(\mathfrak{F}_1,\mathfrak{F}_2)$ be cyclic $(\alpha,\lambda)_{(\mathfrak{F}_3,\mathfrak{F}_4)}$-admissible. Provided that, the following inequality is satisfied:*

$$\alpha(\mathfrak{F}_3\mu^*)\lambda(\mathfrak{F}_4\sigma^*)\tau\left(\int_0^{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}\Gamma(t)dt\right) \le \phi\Big(\int_0^{\psi_1(M(\mu^*,\sigma^*))+L\psi_2(N(\mu^*,\sigma^*))}\Gamma(t)dt\Big),$$

for all μ^*, $\sigma^* \in \mathcal{X}$, where $\Gamma(t)$ is a Lebesgue integrable function with finite integral such that $\int_0^\delta \Gamma(t)dt > 0$ for all $\delta > 0$ and τ, ϕ, ψ_1, ψ_2 are the mappings as defined in Definition 1.3, $L \ge 0$, and

$$\begin{aligned}\psi_1(M(\mu^*,\sigma^*)) &= \max\Big\{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathcal{F}(\sigma^*)),\\ &\quad \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)),\\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))+\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_3(\mu^*))}{2},\\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_1(\mu^*))},\\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big\},\end{aligned}$$

$$\begin{aligned}\psi_2(N(\mu^*,\sigma^*)) &= \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)),\\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big).\end{aligned}$$

Suppose that the conditions (a)-(e) of Theorem 2.1 are sustained and $\mathfrak{F}_1(\mathcal{X}) \subseteq \mathfrak{F}_4(\mathcal{X})$ and $\mathfrak{F}_2(\mathcal{X}) \subseteq \mathfrak{F}_3(\mathcal{X})$. Then the SQMs $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$ have a unique CFP in $(\mathcal{X},\blacktriangledown_d)$.

By assuming $\alpha(\mathfrak{F}_3\mu^*) = 1 = \lambda(\mathfrak{F}_4\sigma^*)$ and $\tau(t) = \phi(t) = t$ in Corollary 2.2, we have the following corollary.

Corollary 2.3. *Let $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4$ be SQMs of a metric space $(\mathcal{X}, \blacktriangledown_d)$ with $L \geq 0$, such that*

$$\int_0^{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))} \Gamma(t)dt \leq \int_0^{\psi_1(M(\mu^*,\sigma^*))+L\psi_2(N(\mu^*,\sigma^*))} \Gamma(t)dt,$$

for all $\mu^*,\ \sigma^* \in \mathcal{X}$, where $\Gamma(t)$ is a Lebesgue integrable function with finite integral such that $\int_0^\delta \Gamma(t)dt > 0$ for all $\delta > 0$ and ψ_1, ψ_2 are the mappings as defined in Definition 1.3, $L \geq 0$, and

$$\begin{aligned}\psi_1(M(\mu^*,\sigma^*)) \;=\;& \max\Big\{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathcal{F}(\sigma^*)),\\ & \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)),\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)) + \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_3(\mu^*))}{2},\\ & \frac{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_1(\mu^*))},\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big\},\end{aligned}$$

$$\begin{aligned}\psi_2(N(\mu^*,\sigma^*)) \;=\;& \psi_2\Big(\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)),\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big).\end{aligned}$$

Assume that the following conditions are satisfied:
(a) the pair $(\mathfrak{F}_1, \mathfrak{F}_3)$ shares $CLR_{\mathfrak{F}_1}$ property;
(b) $(\mathfrak{F}_1, \mathfrak{F}_3)$ and $(\mathfrak{F}_2, \mathfrak{F}_4)$ are WCMs, and $\mathfrak{F}_1(\mathcal{X}) \subseteq \mathfrak{F}_4(\mathcal{X})$, $\mathfrak{F}_2(\mathcal{X}) \subseteq \mathfrak{F}_3(\mathcal{X})$.
Then the SQMs $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4$ have a unique CFP in $(\mathcal{X}, \blacktriangledown_d)$.

If the value of $L = 0$, in Corollary 2.2, then we get following result.

Corollary 2.4. *Let $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4$ be SQMs of a metric space $(\mathcal{X}, \blacktriangledown_d)$ and the pair $(\mathfrak{F}_1, \mathfrak{F}_2)$ be cyclic $(\alpha,\lambda)_{(\mathfrak{F}_3,\mathfrak{F}_4)}$-admissible and the following inequality holds:*

$$\alpha(\mathfrak{F}_3\mu^*)\lambda(\mathfrak{F}_4\sigma^*)\tau\left(\int_0^{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))} \Gamma(t)dt\right) \leq \phi\left(\int_0^{\psi_1(M(\mu^*,\sigma^*))} \Gamma(t)dt\right),$$

for all $\mu^*,\ \sigma^* \in \mathcal{X}$. where $\Gamma(t)$ is a Lebesgue integrable function with finite integral such that $\int_0^\delta \Gamma(t)dt > 0$ for all $\delta > 0$ and $\tau,\ \phi, \psi_1$ are the mappings as defined in Definition 1.3, where

$$\begin{aligned}\psi_1(M(\mu^*,\sigma^*)) \;=\;& \max\Big\{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathcal{F}(\sigma^*)),\\ & \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)),\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)) + \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_3(\mu^*))}{2},\\ & \frac{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_1(\mu^*))},\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big\}.\end{aligned}$$

If the conditions (a)–(e) of Theorem 2.1 are sustained and $\mathfrak{F}_1(\mathcal{X}) \subseteq \mathfrak{F}_4(\mathcal{X})$, $\mathfrak{F}_2(\mathcal{X}) \subseteq \mathfrak{F}_3(\mathcal{X})$, then SQMs $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4$ have a unique CFP in $(\mathcal{X}, \blacktriangledown_d)$.

By assuming $\tau(t) = \phi(t)$ in Corollary 2.4, we get the following.

Corollary 2.5. *Let $\mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4$ be SQMs of a metric space $(\mathcal{X}, \blacktriangledown_d)$ and the pair $(\mathfrak{F}_1, \mathfrak{F}_2)$ be cyclic $(\alpha, \lambda)_{(\mathfrak{F}_3,\mathfrak{F}_4)}$-admissible and the following inequality holds:*

$$\alpha(\mathfrak{F}_3\mu^*)\lambda(\mathfrak{F}_4\sigma^*)\tau\left(\int_0^{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}\Gamma(t)dt\right) \leq \tau\left(\int_0^{\psi_1(M(\mu^*,\sigma^*))}\Gamma(t)dt\right),$$

for all μ^*, $\sigma^* \in \mathcal{X}$, where $\Gamma(t)$ is a Lebesgue integrable function with finite integral such that $\int_0^\delta \Gamma(t)dt > 0$ for all $\delta > 0$ and τ, ψ_1 are the mappings as defined in the Definition 1.3, where

$$\begin{aligned}\psi_1(M(\mu^*,\sigma^*)) \;=\; & \max\Big\{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathcal{F}(\sigma^*)),\\ & \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)),\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)) + \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_3(\mu^*))}{2},\\ & \frac{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_1(\mu^*))},\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big\}.\end{aligned}$$

If the conditions (a)–(e) of the Theorem 2.1 are sustained. Then, the SQMs $\mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4$, have a unique CFP in $(\mathcal{X}, \blacktriangledown_d)$.

If $\alpha(\mathfrak{F}_3\mu^*) = \lambda(\mathfrak{F}_4\sigma^*) = 1$ in Corollary 2.5, then we have the following result.

Corollary 2.6. *Let $\mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4$ be SQMs of a metric space $(\mathcal{X}, \blacktriangledown_d)$ and the pair $(\mathfrak{F}_1, \mathfrak{F}_2)$ be cyclic $(\alpha, \lambda)_{(\mathfrak{F}_3,\mathfrak{F}_4)}$-admissible such that*

$$\tau\left(\int_0^{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}\Gamma(t)dt\right) \leq \tau\left(\int_0^{\psi_1(M(\mu^*,\sigma^*))}\Gamma(t)dt\right),$$

for all μ^*, $\sigma^* \in \mathcal{X}$, where $\Gamma(t)$ is a Lebesgue integrable function with finite integral such that $\int_0^\delta \Gamma(t)dt > 0$ for all $\delta > 0$ and τ, ψ_1 are the mappings as defined in Definition 1.3, where

$$\begin{aligned}\psi_1(M(\mu^*,\sigma^*)) \;=\; & \max\Big\{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathcal{F}(\sigma^*)),\\ & \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)),\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)) + \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_3(\mu^*))}{2},\\ & \frac{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_1(\mu^*))},\\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big\}.\end{aligned}$$

Assume that the following conditions are sustained:
(a) the pair $(\mathfrak{F}_1, \mathfrak{F}_2)$ shares $CLR_{\mathfrak{F}_1}$ property,
(b) $(\mathfrak{F}_1, \mathfrak{F}_3)$ and $(\mathfrak{F}_2, \mathfrak{F}_4)$ are WCMs, and $\mathfrak{F}_1(\mathcal{X}) \subseteq \mathfrak{F}_4(\mathcal{X})$, $\mathfrak{F}_2(\mathcal{X}) \subseteq \mathfrak{F}_3(\mathcal{X})$.
Then SQMs $\mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4$, have a unique CFP in $(\mathcal{X}, \blacktriangledown_d)$.

3 Applications

Example 3.1. Let $(\mathcal{X} = [0,1], \blacktriangledown_d)$ be a metric space with $\blacktriangledown_d(x,y) = |x-y|$, for $x, y \in \mathcal{X}$. Define $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$ and $\mathfrak{F}_4$ as follows:

$$\mathfrak{F}_1(\mathcal{X}) = \begin{cases} 0.4 \text{ if } x \in [0,0.5], \\ \frac{1}{6} \text{ if } x \in (0.5,1], \end{cases} \qquad \mathfrak{F}_3(\mathcal{X}) = \begin{cases} 0.4 \text{ if } x \in [0,0.5], \\ \frac{1}{7} \text{ if } x \in (0.5,1], \end{cases} \tag{3.1}$$

$$\mathfrak{F}_2(\mathcal{X}) = \begin{cases} 0.4 \text{ if } x \in [0,0.5], \\ \frac{1}{8} \text{ if } x \in (0.5,1], \end{cases} \qquad \mathfrak{F}_4(\mathcal{X}) = \begin{cases} 0.4 \text{ if } x \in [0,0.5], \\ \frac{1}{9} \text{ if } x \in (0.5,1]. \end{cases} \tag{3.2}$$

Let $\alpha(x) = 0.9 + x,\ \lambda(x) = 0.95 + x$ for all $x \in \mathcal{X}$. Then $\alpha(\mathfrak{F}_3 x) \geq 1$ for all $x \in \mathcal{X} = [0,1]$, and $\lambda(\mathfrak{F}_1 x) = 0.95 + \mathfrak{F}_1 x > 1$, for all $x \in \mathcal{X} = [0,1]$. Also, $\lambda(\mathfrak{F}_1 x) \geq 1$ for all $x \in \mathcal{X}$, implies that $\alpha(\mathfrak{F}_2 x) = 0.9 + \mathfrak{F}_2 x > 1$, for all $x \in \mathcal{X} = [0,1]$. Thus, the pair $(\mathfrak{F}_1, \mathfrak{F}_2)$ is a cyclic $(\alpha,\lambda)_{(\mathfrak{F}_3,\mathfrak{F}_4)}$-admissible. Next, we show that the pair $(\mathfrak{F}_1, \mathfrak{F}_3)$ shares $CLR_{\mathfrak{F}_1}$ property. For this, let us consider the sequence

$$\{x_n\} = \left\{0.4 - \frac{1.5}{3n^2+0.1}\right\}. \tag{3.3}$$

By the help of (3.1), (3.2) and (3.3), we have:

$$\lim_{n\to\infty} \mathfrak{F}_1(x_n) = \lim_{n\to\infty} \mathfrak{F}_1\Big(0.4 - \frac{1.5}{3n^2+0.1}\Big) = 0.4, \tag{3.4}$$

$$\lim_{n\to\infty} \mathfrak{F}_3(x_n) = \lim_{n\to\infty} \mathfrak{F}_3\Big(0.4 - \frac{1.5}{3n^2+0.1}\Big) = 0.4. \tag{3.5}$$

From (3.4) and (3.5), for the sequence x_n in $\mathcal{X}$, we have $\lim_{n\to\infty} \mathfrak{F}_1 x_n = \lim_{n\to\infty} \mathfrak{F}_3 x_n = 0.4 = \mathfrak{F}_1 x$ for all $x \in [0,0.5]$. So that, $\lim_{n\to\infty} \mathfrak{F}_1 x_n = \lim_{n\to\infty} \mathfrak{F}_2 x_n = 0.4 = \mathfrak{F}_1 x$ for some $x \in [0,0.5] \subset \mathcal{X}$. Therefore, $(\mathfrak{F}_1, \mathfrak{F}_2)$ sustains $CLR_{\mathfrak{F}_1}$ property.

Case I For $x \in [0,\, 0.5]$, we have $\mathfrak{F}_1(\mathcal{X}) = \mathfrak{F}_2(\mathcal{X}) = \mathfrak{F}_3(\mathcal{X}) = \mathfrak{F}_4(\mathcal{X}) = 0.4$, which implies $\blacktriangledown_d(\mathfrak{F}_1,\, \mathfrak{F}_2) = 0$, $\psi_1(M) = 0$ and $\psi_2(N) = 0$. Therefore, the equality is trivially satisfied.

Case II For $\sigma^*, \mu^* \in (0.5,\, 1]$, we have $\mathfrak{F}_1\mu^* = 1/6$, $\mathfrak{F}_3\mu^* = 1/7$, $\mathfrak{F}_2\sigma^* = 1/8$, $\mathfrak{F}_4\sigma^* = 1/9$, and

$$\begin{aligned} \psi_1(M(\mu^*,\sigma^*)) \quad = \quad & \max\Big\{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)), \\ & \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)), \\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)) + \blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_3(\mu^*))}{2}, \\ & \frac{\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_2(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_1(\mu^*))}, \\ & \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big\} \\ = \quad & \max\Big\{\blacktriangledown_d(\frac{1}{8},\frac{1}{9}),\ \blacktriangledown_d(\frac{1}{6},\frac{1}{9}),\ \blacktriangledown_d(\frac{1}{9},\frac{1}{7}), \\ & \frac{\blacktriangledown_d(\frac{1}{8},\frac{1}{9}) + \blacktriangledown_d(\frac{1}{6},\frac{1}{9})}{2},\ \frac{\blacktriangledown_d(\frac{1}{8},\frac{1}{7})\blacktriangledown_d(\frac{1}{6},\frac{1}{7})}{1+\blacktriangledown_d(\frac{1}{6},\frac{1}{8})}, \\ & \frac{\blacktriangledown_d(\frac{1}{6},\frac{1}{9})\blacktriangledown_d(\frac{1}{7},\frac{1}{9})}{1+\blacktriangledown_d(\frac{1}{8},\frac{1}{7})}\Big\} = \frac{3}{54}, \end{aligned} \tag{3.6}$$

$$\begin{aligned}\psi_2(N(\mu^*,\sigma^*)) &= \min\Big\{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*)),\ \blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_4(\sigma^*)),\\ &\quad \blacktriangledown_d(\mathfrak{F}_4(\sigma^*),\mathfrak{F}_3(\mu^*)),\\ &\quad \frac{\blacktriangledown_d(\mathfrak{F}_1(\mu^*),\mathfrak{F}_4(\sigma^*))\blacktriangledown_d(\mathfrak{F}_3(\mu^*),\mathfrak{F}_4(\sigma^*))}{1+\blacktriangledown_d(\mathfrak{F}_2(\sigma^*),\mathfrak{F}_3(\mu^*))}\Big\} \\ &= \min\Big\{\blacktriangledown_d(\frac{1}{6},\frac{1}{9}),\ \blacktriangledown_d(\frac{1}{8},\frac{1}{9}),\ \blacktriangledown_d(\frac{1}{9},\frac{1}{7}),\ \frac{\blacktriangledown_d(\frac{1}{6},\frac{1}{9})\blacktriangledown_d(\frac{1}{7},\frac{1}{9})}{1+\blacktriangledown_d(\frac{1}{8},\frac{1}{7})}\Big\}\\ &= 0.00173.\end{aligned} \tag{3.7}$$

Since $\alpha(x) \geq 1,\ \lambda(x) \geq 1$ for all $x \in \mathcal{X}$, by the virtue of (2.6), (3.6) and (3.7), $L = 0.2,\ \tau(t) = 0.9t,\ \psi(t) = 0.85t,\ \phi(t) = 0.8t,\ \Gamma(t) = 2t$, we have

$$\begin{aligned} 0.001475 &= \tau\Big(\int_0^{\blacktriangledown_d(\frac{1}{6},\frac{1}{8})} 2tdt\Big)\\ &\leq \phi\Big(\int_0^{\psi_1(M(\mu^*,\sigma^*))+L\psi_2(N(\mu^*,\sigma^*))}\Gamma(t)dt\Big)\\ &= \phi\Big(\int_0^{\frac{3}{54}+0.000346} 2tdt\Big) = 0.0026253.\end{aligned} \tag{3.8}$$

Thus, the inequality (2.6) is also satisfied. Consequently, all the conditions of Theorem 2.1 are sustained and therefore the SQMs $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4$ have a unique CFP 0.4.

4 Conclusions

In this paper, we have presented new FPTs for the SQMs, $\mathfrak{F}_1,\ \mathfrak{F}_2,\ \mathfrak{F}_3,\ \mathfrak{F}_4 : \mathcal{X} \to \mathcal{X}$ on a metric space $(\mathcal{X}, \blacktriangledown_d)$, sustaining certain integral type inequalities. For the new FPTs, we assumed that $(\mathfrak{F}_1,\ \mathfrak{F}_3)$ shares $CLR_{\mathfrak{F}_1}$ property, $\mathfrak{F}_1,\ \mathfrak{F}_3)$, $(\mathfrak{F}_2,\ \mathfrak{F}_4)$ are WCMs, and $\mathfrak{F}_1\mathcal{X} \subseteq \mathfrak{F}_4\mathcal{X},\ \mathfrak{F}_2\mathcal{X} \subseteq \mathfrak{F}_4\mathcal{X}$. In the proof of our results, we don't need to require the completeness of the metric space $(\mathcal{X}, \blacktriangledown_d)$. Our results generalize many FPTs in the available literature. An expressive examples is given for the application of our Theorem 2.1.

Competing interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Author's contributions

All the authors have equal contributions in this article.

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