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Quantum effects in two-qubit systems interacting with two-mode fields: Dissipation and dipole-dipole interplay effects

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ABSTRACT

We explore two coupled dipole qubits interacting with two-mode cavity fields through non-degenerate two-photon processes and under the influence of intrinsic noise. Our investigation is based on the influence of the coupling constants between different constituents of the system on the quantum coherence and entropy squeezing. We investigate the phase space information, entanglement, and purity via the Husimi function. It is found that the robustness of the positivity and the negativity of the superposition of generalized coherent state is highly sensitive to the coupling constants. The phase space quantum information, the matter-light entanglement, and the stationary mixture state can be controlled by the two-qubit coupling as well as the coupling to the environment. The effect of the intrinsic dissipation and the two-qubit coupling on the generated squeezing via the qubit-system variable squeezing or the entropy squeezing is investigated. They lead to revoke the squeezing phenomenon.

1. Introduction

Qubit-fields interactions present several useful proprieties for quantum information, such as quantum coherence and quantum correlations [1–3]. The simple theoretical manipulation of these interactions was introduced by Jaynes-Cummings model (JC model) [4]. It leads to the prediction of a wide range of experimentally interesting phenomena and applications [5]. More quantum effects and quantum coherence were investigated by JC model and its generalizations including: the interaction between qubits and a one-mode field, the interaction between multi-level atom and multi fields [6], and its extension with multi-photon transitions, Kerr-like medium [7] and the nonlinear regime [8].

Quantum coherence, quantum correlations and non-classical effects played an important role in quantum processing. Quasi-probability Husimi function (HF) is always a positive distribution [9]. The HF, Wehrl density and Wehrl entropy are good measures to quantify the phase space information and the quantum coherence [10].

The quantum information entropies [von Neumann entropy [11], linear entropy [12]] are used to measure the entanglement in closed systems [3] and the coherence loss in open systems [13]. Wehrl entropy gives quantitative and qualitative phase space information on the purity and the entanglement. The temporal evolution of the Wehrl entropy is similar to that of the von Neumann entropy. The applications of the HF were limited to the cavity fields, the 2-level [14] and 3-level systems [15].

The squeezing phenomenon is one of the quantum effects that attracted more attentions due to its potential application in different directions as: optical communications [16], weak signals [17], and the atomic fountain clock [18]. Especially, the atomic squeezing, that based on the quantum information entropy and the variance of the system observable according to Heisenberg uncertainty relation [19], which has several applications in quantum optic [20] and quantum information [21,22]. Quantum spin squeezing characterizes the sensitivity of a state with respect to Su(2) rotations. It is significant for detecting quantum chaos, quantum phase transitions, and both entanglement detection and high-precision metrology [23]. The atomic squeezing has been intensively investigated [24–27].

The coherent states of the Su(2) and the Su(1, 1) algebras [28] and generalized coherent states for Su(n) were analyzed [29]. One of the important definitions of Su(1, 1) coherent states was introduced by Barut and Girardello [30]. It is analogue to the harmonic oscillator

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coherent state, being an eigenstate of the annihilation operator with complex eigenvalues. It is worth to mention that Barut-Girardello states are calculated by using spherical harmonics and can describe the entanglement [31]. For certain quantum models, Barut-Girardello states appear when using second-order differential operators as realization of Su(1, 1)algebra by associated Laguerre functions [32]. Recently, more constructions for the Barut-Girardello coherent states with various methods are realized [33–35].

In a real open system, the interaction with the environment causes decoherence. This decoherence shall deteriorate the quantum coherence and quantum effects [36]. Therefore, the decoherence process is a hot topic in quantum processing. The intrinsic decoherence (ID) of Milburn equation describes the degradation of the quantum coherence as the system evolves [37–39]. It is found that ID does not completely destroy the quantum coherence and the quantum effects [40,41].

It is well known that the analytical study of open quantum systems presents several mathematical difficulties. The previous investigations of quantum coherence and atomic squeezing in open quantum system, via the quasi-probability Husimi function, were limited to cavity-field, one-qubit and one-qutrit systems. Therefore, the study for multi-qubit/multi-qutrit systems need more attentions.

Motivated by this fact, we use the Husimi distribution, the Wehrl entropy and the information entropy squeezing to analytically investigate the quantum coherence and atomic squeezing of a two-qubit system. The considered system, here, is formed by two coupled dipole qubits interacting with two-mode cavity fields. These fields are initially in Barut-Girardello coherent states. The influence of the coupling constants as well as the interaction with the environment is explored. The results open the door for applying the used technique for analyzing similar systems in quantum information.

The organization of the paper is: In Section 2, we review the physical model. In Section 3, Husimi distribution and the Wehrl entropy are used to analyze the phase space information and the quantum coherence (entanglement/mixture). While the squeezing phenomenon will be investigated In Section 4. The conclusion is presented in Section 5.

2. The physical model

Here, we consider a non-degenerate two-photon JC model of two coupled two-level atoms (two qubits) via dipole-dipole interplay. Each two-level atom interacts with two mode fields with different frequencies $\omega_i (i = A, B)$. The Hamiltonian of the system is described by

$$\begin{aligned}\widehat{H}=&\omega_A\left(\widehat{\psi}_A^{\dagger}\widehat{\psi}_A+\frac{1}{2}\right)+\omega_B\left(\widehat{\psi}_B^{\dagger}\widehat{\psi}_B+\frac{1}{2}\right)+\frac{\omega}{2}\left(\widehat{\sigma}_z^A+\widehat{\sigma}_z^B\right)\\&+\Omega\left(\sigma_+^A\sigma_-^B+\sigma_-^A\sigma_+^B\right)+\sum_{j=A,B}\lambda\left(\widehat{\psi}_A\widehat{\psi}_B\sigma_+^j+\widehat{\psi}_A^{\dagger}\widehat{\psi}_B^{\dagger}\sigma_-^j\right),\end{aligned}\tag{1}$$

where $\widehat{\psi}_i^{\dagger}(\widehat{\psi}_i)$ and ω_i $(i = 1, 2)$ are the creation and annihilation operators of the two mode fields. ω is the qubit frequency. The qubit operators $\widehat{\sigma}_{\pm}^i$ and $\widehat{\sigma}_z^i$ design the Pauli operators with up $|1_i\rangle$ and down $|0_i\rangle$ states $(i = A, B)$. The λ is the interaction coupling between the qubits and the cavity fields and Ω represents the coupling constant of the interaction between the two qubits. If we focus on the case $\omega_A + \omega_B = 2\omega$, then the Hamiltonian can be written as function of the Su(1,1) and Su (2) operators. The Su(1, 1) generators, $\widehat{K}_{\pm}$ and $\widehat{K}_0$, defined by:

$$\begin{aligned}&\widehat{K}_-=\widehat{\psi}_1\widehat{\psi}_2=\widehat{K}^{\dagger}{}_+,\\&\widehat{K}_0=\tfrac{1}{2}\left(\widehat{\psi}_1^{\dagger}\widehat{\psi}_1+\widehat{\psi}_2^{\dagger}\widehat{\psi}_2+1\right).\end{aligned}\tag{2}$$

where $[\widehat{K}_0, \widehat{K}_{\pm}] = \pm\widehat{K}_{\pm}$ and $[\widehat{K}_-, \widehat{K}_+] = 2\widehat{K}_0$. From these relations we have: $\widehat{K}_+\widehat{K}_- = \widehat{K}_0^2 - (\widehat{K}_0 + \widehat{K}^2)$. In term of the Bargmann number k, the Casimier operator is defined as: $\widehat{K}^2=\widehat{K}_0^2-\frac{1}{2}(\widehat{K}_+\widehat{K}_-+\widehat{K}_-\widehat{K}_+)=k(k-1)\widehat{I}$.

The Hamiltonian can be rewritten as

$$\widehat{H}=\omega K_o+\sum_{i=A,B}\frac{\omega}{2}\widehat{\sigma}_i^z+\lambda\left(K_-\sigma_+^i+K_+\sigma_-^i\right)+\Omega\left(\sigma_+^A\sigma_-^B+\sigma_-^A\sigma_+^B\right),\tag{3}$$

this represents the interaction between Su(2) and Su(1, 1) systems. $\widehat{K}_{\pm}$ and $\widehat{K}_0$ verify:

$$\begin{aligned}&\widehat{K}_-|n,k\rangle=S_{k,n}|n-1,k\rangle,\ \widehat{K}_+|n,k\rangle=S_{k,n+1}|n+1,k\rangle,\widehat{K}_0|n,k\rangle=(n+k)|n,k\rangle,\ \widehat{K}^2|n,k\rangle\\&=k(k-1)|n,k\rangle,\end{aligned}\tag{4}$$

where $S_{k,n}=\sqrt{n(n+2k-1)}$.

To study the effect of intrinsic noise [37] on the quantum effects, we adopt the intrinsic decoherence model (ID), in which the dynamics of the density operator is described by the following Milburn equation:

$$\frac{d}{dt}\rho(t)=-i\left[\widehat{H},\rho\right]-\frac{\gamma}{2}\left[\widehat{H},\left[\widehat{H},\rho\right]\right],\tag{5}$$

where $\gamma$is the ID damping factor.

To calculate the time-dependent system state $\widehat{\rho}(t)$ of Eq. 5, we assume that the initial state of the Su(2) systems is $\widehat{\rho}^{AB}(0)=|1_A1_B\rangle\langle 1_A1_B|$, while the initial state of the quantum field of the Su(1, 1)-system is a Barut-Girardello coherent state [30], that is given by

$$\left|\alpha,k\right\rangle=\sum_{n=0}^{\infty}\alpha^n\sqrt{\frac{|\alpha|^{2k-1}}{n!I_{2k-1}(2|\alpha|)\Gamma(2k+n)}}\left|n,k\right\rangle=\sum_{n=0}^{\infty}\Lambda_n\left|n,k\right\rangle,\tag{6}$$

where, $I_\nu(x)$ is the modified Bessel function. Here, $k=\frac{1}{2}$, therefore, $|\alpha,\frac{1}{2}\rangle$ describes a nonlinear coherent state which may appear as a stationary state of the trapped ions center-of-mass motion [42]. For the two modes represented by Eq. 2, the Barut-Girardello coherent state is a pair of coherent states [43].

The solution of Eq. 5 is given by

$$\widehat{\rho}(t)=\sum_{m,n=0}\sum_{l=1,3,4}\Lambda_m\Lambda_n^*\left\{a^{l1}\left|\Psi_l^m\right\rangle\left\langle\Psi_1^n\right|+a^{l3}\left|\Psi_l^m\right\rangle\left\langle\Psi_3^n\right|+a^{l4}\left|\Psi_l^m\right\rangle\left\langle\Psi_4^n\right|\right\},\tag{7}$$

with

$$\begin{aligned}&a^{11}=c_{11}^mc_{11}^nY_{11},\quad a^{31}=c_{31}^mc_{11}^nY_{31},\quad a^{41}=c_{41}^mc_{11}^n\,Y_{41},\\&a^{13}=c_{11}^mc_{31}^nY_{13},\quad a^{33}=c_{31}^mc_{31}^nY_{33},\quad a^{34}=c_{31}^mc_{41}^n\,Y_{34},\\&a^{14}=c_{11}^mc_{41}^nY_{14},\quad a^{43}=c_{41}^mc_{31}^nY_{43},\quad a^{44}=c_{41}^mc_{41}^n\,Y_{44},\\&Y_{r,s}=e^{-i\lambda(E_r^m-E_s^n)t}d_{r,s},\end{aligned}$$

where $d_{r,s}=e^{-\frac{\gamma}{2}(E_r^m-E_s^n)^2t}$ is ID term. In the space states $\{|1\rangle=|1_A1_B,n\rangle,|2\rangle=|1_A0_B,n+1\rangle,|3\rangle=|0_A1_B,n+1\rangle,|4\rangle=|0_A0_B,n+2\rangle\}$, the eigenstates $|\Psi_l^n\rangle$are given by

$$\left|\Psi_l^n\right\rangle=\sum_{j=1}^{4}c_{lj}\left|j\right\rangle,\left(l=1,2,3,4\right)\tag{8}$$

where, the c_{lj} can be obtained from: $\widehat{H}|\Psi_l^n\rangle=E_l^n|\Psi_l^n\rangle$. The eigenvalues, E_l^n, of the Hamiltonian (3) are

$$E_1^n=\omega(n+k+1)\qquad E_2^n=\omega(n+k+1)-J,\tag{9}$$

$$\begin{aligned}&E_3^n=\omega(n+k+1)+\tfrac{1}{2}J-\tfrac{1}{2}\sqrt{J^2+8\lambda^2(S_{k,n+1}^2+S_{k,n+2}^2)},\\&E_4^n=\omega(n+k+1)+\tfrac{1}{2}J+\tfrac{1}{2}\sqrt{J^2+8\lambda^2(S_{k,n+1}^2+S_{k,n+2}^2)}.\end{aligned}$$

The quantum effects of the two Su(2)-system $\widehat{\rho}^{AB}(t)$, will be studied via the Husimi distributions, the Wehrl entropy and the information entropy squeezing. The reduced density matrix of the Su(2)-system is given by

$$\widehat{\rho}^{AB}(t)=\mathrm{Tr}_{\mathrm{field}}\left\{\rho(t)\right\}=\sum_{n=0}^{\infty}\left\langle n,k\left|\widehat{\rho}(t)\right|n,k\right\rangle,\tag{10}$$

that is used for calculating the reduced density matrices of the k-qubit $(k=A,B)$ by $\rho^{A(B)}(t)=\mathrm{Tr}_{B(A)}\{\rho^{AB}(t)\}$. In the system, we observe that the two Su(2)-system are identical and therefore we can study the quantum effects for only one atom of Su(2)-subsystems, say A, and the reduced density matrix, $\hat{\rho}^A(t) = Tr_B\{\hat{\rho}^{AB}(t)\}$ is given by

$$\hat{\rho}^A(t) = \begin{pmatrix} \rho_{11}^{AB} + \rho_{22}^{AB} & \rho_{13}^{AB} + \rho_{24}^{AB} \\ \rho_{31}^{AB} + \rho_{42}^{AB} & \rho_{33}^{AB} + \rho_{44}^{AB} \end{pmatrix}. \tag{11}$$

3. Husimi distribution

The phase space quasi-probability distributions is used to quantify the non-classicality of a quantum state. It is known that the phase space quasi-probability distributions depend on the density matrix elements of a quantum system. Therefore, the phase space information about all its operators can be given by the Husimi distributions.

3.1. Husimi function

The Husimi distribution and the Wehrl entropy represented [44] in the family of spin coherent states $|\theta, \phi\rangle$, are localized at points (θ, ϕ) of the phase space. These states also called Su(2) vector coherent states [45]. The spin coherent state representation is the most adapted description due to the algebraic characteristics of the angular momentum operator J. The Su(2) coherent state, for the defined angular momentum j, is given by [44]

$$\left|\theta, \phi\right\rangle = \sum_{n=-j}^{n=j} \frac{e^{i(j-n)\phi}}{2^j} \sqrt{\frac{(2j)!}{(j-n)!(j+n)!}} \sin^j\theta \cot^n\frac{\theta}{2} \left|j, n\right\rangle. \tag{12}$$

For spin-$\frac{1}{2}$ ($j = \frac{1}{2}$), the Bloch coherent quantum state of n-th Su(2)-system, $|\mu_n\rangle$, is given by

$$\left|\mu_n\right\rangle = \cos\frac{\theta_n}{2}\left|1_n\right\rangle + \sin\frac{\theta_n}{2}e^{i\phi_l}\left|0_n\right\rangle, \; n = A, B. \tag{13}$$

With the n-th Su(2)-system phase space parameters θ_n and ϕ_n and $d\mu_n = \sin\theta_n d\theta_n d\phi_n$. Therefore, the H-function (HF) of the n-th Su(2)-system is given by [9]

$$H^n\left(\mu_n, t\right) = \frac{1}{2\pi}\left\langle \mu_n\left|\rho^n(t)\right|\mu_n\right\rangle. \tag{14}$$

In the Su(2)-system, $\hat{\rho}^{AB}(t)$, phase space represented by its basis states $\{|\varpi_i\rangle\}$, the Bloch coherent of the two Su(2)-system states, $|\Phi\rangle$ is defined as:

$$\left|\Phi\right\rangle = \cos\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\left|\varpi_1\right\rangle + e^{i\phi_B}\cos\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\left|\varpi_2\right\rangle + e^{i\phi_A}\sin\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\left|\varpi_3\right\rangle + e^{i(\phi_A+\phi_B)}\sin\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\left|\varpi_4\right\rangle. \tag{15}$$

Therefore, the HF of the two Su(2)-systems $\rho^{AB}(t)$ is given by

$$H^{AB}\left(\Phi, t\right) = \frac{1}{4\pi^2}\left\langle \Phi\left|\rho^{AB}(t)\right|\Phi\right\rangle, \tag{16}$$

and its partial HFs of the n-th Su(2)-system (for example $n = A$) is

$$H^A\left(\mu_A, t\right) = \int_0^\pi \int_0^{2\pi} H^{AB}\left(\Phi, t\right) d\mu_B. \tag{17}$$

The effects of the rates of the ID and the coupling between the two Su(2)-system on the phase space information of the partial H-function, $H^A(\mu_A, t) = H^A(\theta, \phi, t)$, and its counter are shown in Fig. (1). Where, $H^A(\theta, \phi, t)$ is plotted in the phase space $\theta \in [0, 3\pi]$ and $\phi \in [0, 2\pi]$ when the Su(1, 1)-system is initially in a generalized Barut-Girardello coherent state, $|\alpha, k\rangle$, $\alpha = 3$ and $\lambda t = 2.2\pi$ for different cases $(\Omega, \gamma) = (0, 0)$, $(\Omega, \gamma) = (20\lambda, 0)$, $(\Omega, \gamma) = (0, 0.01\lambda)$.

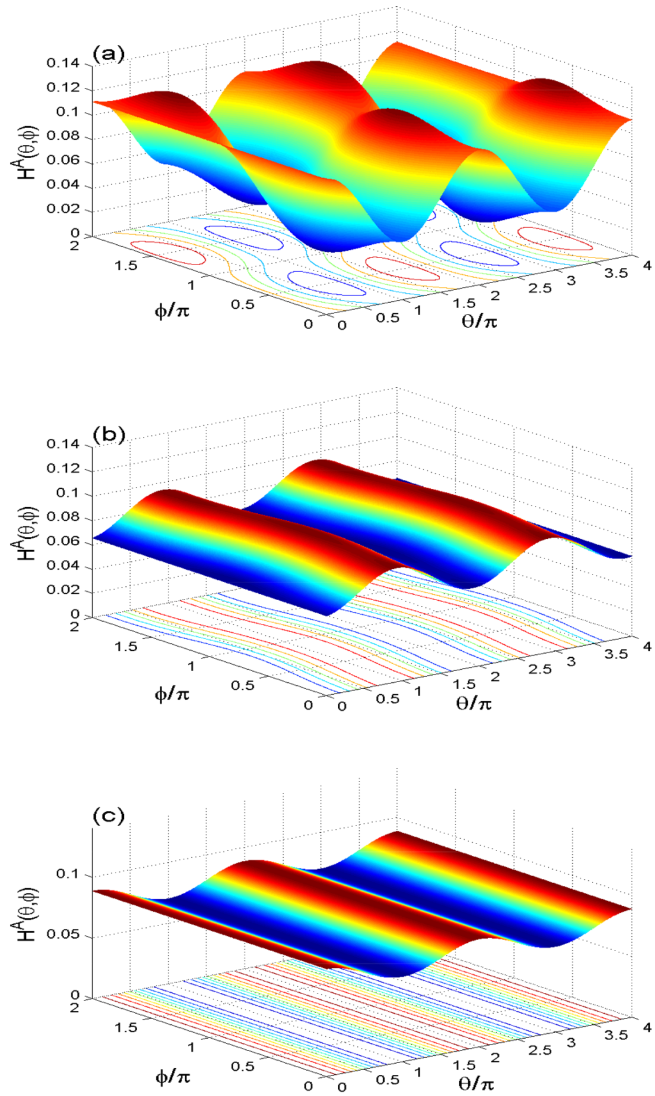

Fig. 1. $H^A(\theta, \phi, t)$ at $\lambda t = 2.2\pi$ with $\alpha = 3$ with the cases $(\Omega, \gamma) = (0, 0)$ in (a), $(\Omega, \gamma) = (20\lambda, 0)$ in (b), $(\Omega, \gamma) = (0, 0.01\lambda)$ in (c) when $\alpha = 3$ and $k = \frac{1}{2}$.

From Fig. (1) a, the $H^A(\theta, \phi)$ of A-th Su(2)-system has distributed regularity in its phase space (θ, ϕ) with 2π period in respect to both the θ and ϕ axes. The oscillations, along the θ-axis, of $H^A(\theta, \phi)$ and their amplitudes are more and larger than that along the ϕ-axis. The counter curves of $H^A(\theta, \phi)$ confirm that phase space HF distribution dependence on the angular variables θ and ϕ.

From Fig. (1) b, the dependence of the distribution of $H^A(\theta, \phi)$ on the rate of the coupling between the two Su(2)-systems, Ω/λ, is shown. The amplitudes of the oscillations along the θ-axis decreases whereas the oscillations along the ϕ-axis disappear completely. Approximatively, there is no dependence on ϕ-variable, i. e., the partial HF distribution depends only on the θ-angle.

The effect of the large value of the intrinsic decoherence rate, $\gamma = \lambda$, on the $H^A(\theta, \phi)$, is shown in Fig. (1) c, We note only regular oscillations along the θ-axis with weaker amplitudes compared to the case of $\gamma = 0$. The generated maxima and minima of HF distribution, see Fig. (1) a, have notable changes due to the non-zero value of $\gamma = \lambda$. For large value of ID rate, the counter curves of $H^A(\theta, \phi)$ become parallel straight lines with respect to ϕ-axis, i.e. the HF of A-th Su(2)-system appears as a constant value along the ϕ-axis.

3.2. Wehrl entropy

One of HF applications is Wehrl entropy [46], it is useful measure to the coherence loss in open systems and the entanglement in closed systems [47]. The partial Wehrl entropies are given by

$$E^n(t) = \int_0^{2\pi} \int_0^{\pi} D^n\left(\mu_n, t\right) d\mu_n, \; n = A, B \tag{18}$$

where the partial Wehrl densities, $D^n(\mu_n, t)$, of the n-qubit are

$$D^n(\mu_n, t) = -H^n(\mu_n, t)\ln[H^n(\mu_n, t)]. \tag{19}$$

Here, the phase space information of a system means that the HF distribution formed by the angular variables [15]. The function $D^n(\mu_n, t)$ is used to measure the information loss of the n-th Su(2)-system. For the initial sate of the two qubits $|1_A 1_B\rangle$, the initial value of the Wehrl density is given by

$$\begin{aligned} E^A(0) &= \frac{-1}{2}\int_0^{\pi/2} (2\sin 2\theta + \sin 4\theta)\ln[\cos^2(\theta)/2\pi]\, d\theta \\ &= 2.3379. \end{aligned} \tag{20}$$

Therefore, it grows in time as [48],

$$E^A(0) \leqslant E^A(t) \leqslant \ln(4\pi). \tag{21}$$

Where, in the considered system, there are two quantum coherence sources in the Su(2)-Su(1, 1) system: one of them is due to its unitary Su(2)-Su(1, 1) interaction, it is the entanglement between the Su(2) and Su(1, 1) systems and "coherence loss". The second due the interaction between the Su(2)-Su(1, 1) system and the environment, the system becomes open system, it causes the coherence loss (the purity loos of the quantum state of any one of them) and the Wehrl entropy [46] is used to measure this coherence loss.

In the absence of the ID and for the Su(1, 1)-system initially in a generalized coherent state, the partial Wehrl entropy, $E^A(t)$, of A-th Su(2)-system is plotted in Fig. (2) a, for $\alpha = 3$ and for different cases $\Omega = 0$ (solid curves), $\Omega = 20$ (dash curves). In this case $\gamma = 0$, the $E^A(t)$ is used to quantify the entanglement between the Su(2) and Su(1, 1) systems. We note that the Su(2)-Su(1, 1) entanglement grows with quasi-regular oscillatory behavior.

Solid curve of Fig. (2)a shows the effect of the coupling to the environment, Ω/λ, on the quasi-regular oscillatory behavior of the $E^A(t)$ with $J = 20\lambda$. The minima of the partial Wehrl entropy is shifted upward to its limiting value $\ln(4\pi)$.

In Fig. (2)b, we explore the effect of the ID rate with non-zero value $\gamma = 0.01\lambda$. In this case $E^A(t)$ is used to measure the coherence loss between the states of the A-th Su(2)-system. It has damped oscillatory behavior, and function $E^A(t)$ reaches its stationary value $E^A(\infty) \simeq \ln(4\pi)$, i.e., the A-th Su(2)-system reaches mixed state and it loses completely its purity. The coupling between the two Su(2)-systems may be weakened due to the effect of the ID rate and leads to delaying the state of the Su(2)-system to reaching its stationary mixture state.

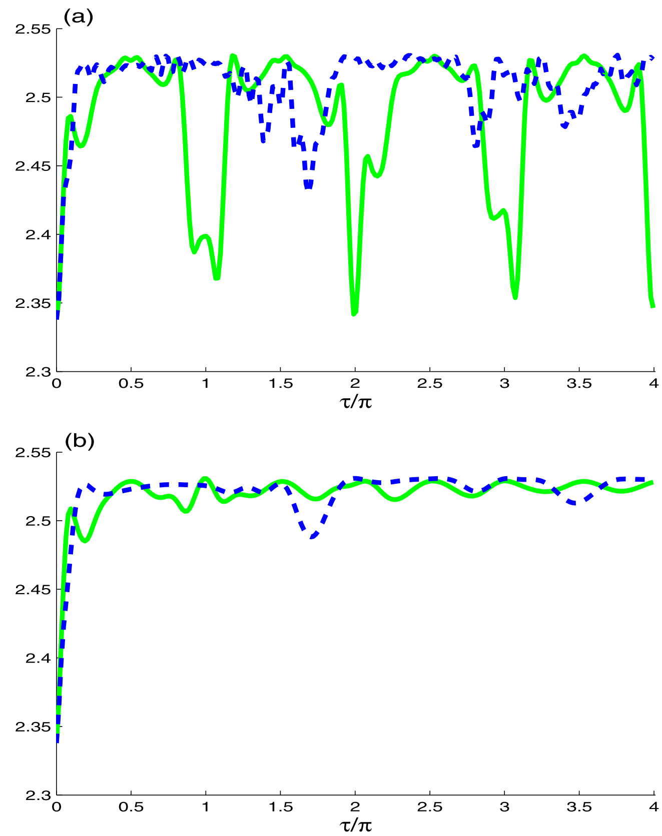

Fig. 2. Wehrl entropy $E^A(t)$ for $\alpha = 3$ and $k = \frac{1}{2}$ with different cases $\Omega = 0$ (solid plots), $\Omega = 20$ (dashed plots) for with different values of $\gamma = 0$ in (a) $\gamma = 0.01\lambda$ in (b).

4. Squeezing phenomenon

The squeezing phenomenon is one of the interesting quantum phenomena, it has different kinds as: normal squeezing, variance and entropy squeezing. Here, we deal with just two types of squeezing phenomena;

4.1. Su(2)-system variable squeezing

As is well known that the Pauli operators of the j-th Su(2)-system, $\hat{\sigma}_x^j$, $\hat{\sigma}_y^j$ and $\hat{\sigma}_z^j$, determine its real and the imaginary parts of the dipole moment and the energy. The $\hat{\sigma}_s^j(s = x, y, z)$ operators satisfy the following commutation rule

$$[\hat{\sigma}_x^j, \hat{\sigma}_y^j] = 2i\hat{\sigma}_z^j, \tag{22}$$

with the Heisenberg uncertainty relation

$$\langle(\Delta\hat{\sigma}_x^j)^2\rangle\langle(\Delta\hat{\sigma}_y^j)^2\rangle \geqslant |\langle\hat{\sigma}_z^j\rangle|^2, \tag{23}$$

where $\langle(\Delta\hat{\sigma}_s^j)^2\rangle = \langle(\hat{\sigma}_s^j)^2\rangle - \langle\hat{\sigma}_s^j\rangle^2$. The qubit variance squeezing is based on the Hisenberg uncertainty relation of the $\hat{\sigma}_s^j$ operators. The fluctuations in the component $\hat{\sigma}_s^j$ are defined as

$$V_r^j(t) = \frac{1}{2}\left[\sqrt{1 - \langle\hat{\sigma}_r^j\rangle^2} - \sqrt{|\langle\hat{\sigma}_z^j\rangle|}\right]; \quad r = x, y, \tag{24}$$

and if $V_l^j(t) < 0$, the squeezing occurs.

In the absence of the ID and when the Su(1, 1)-system is initially in a generalized coherent state, the partial Wehrl entropy, $V_y^A(t)$, of A-th Su(2)-system is plotted in Fig. (2)a, for $\alpha = 8$ and for different cases $\Omega = 0$ (solid curves), $\Omega = 20$ (dash curves). It is found that the phenomenon of squeezing does not occurs with the fluctuations in the component $\hat{\sigma}_x^j$, but it occurs only in the second quadrature $V_y^A(t)$ several times during a short period, see Fig. 3a. The coupling between the two Su(2)-systems leads to disappearing the phenomenon of squeezing. The phenomenon of squeezing are very sensitive ID damping rate γ/λ. It disappears with weak ID rate $\gamma = 0.001\lambda$, see Fig. 3b. Therefore, both the ID damping rate and the coupling between the two Su(2)-systems tend to deteriorate the qubit variable squeezing.

4.2. Entropy squeezing

For an arbitrary Su(2)-system, the squeezing of information entropy can be described by the probability distributions for two possible components measurement outcomes of the operator σ_s^j,

$$P_j(\sigma_r^j) = \frac{1}{2}\left[1 - \zeta\langle\hat{\sigma}_r^j\rangle\right], \qquad r = x, y, z. \tag{25}$$

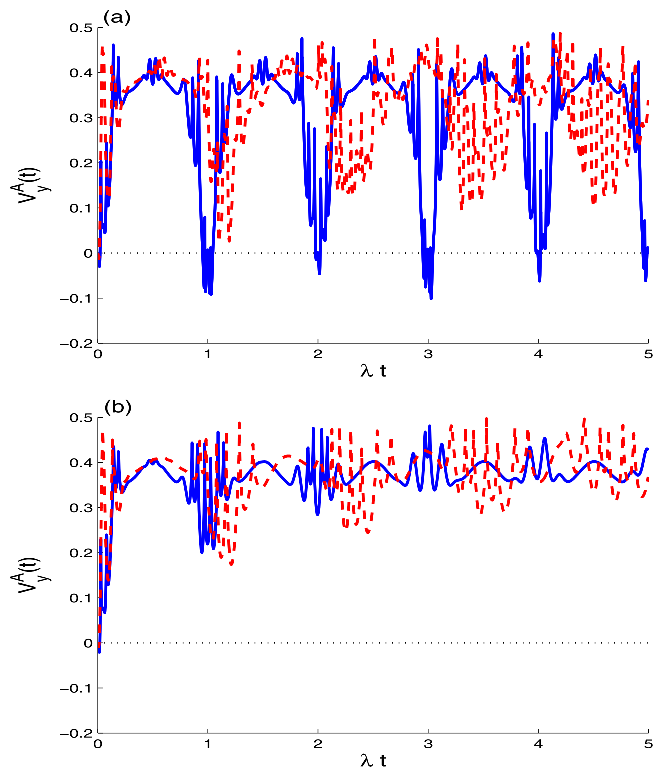

Fig. 3. The variance squeezing $V_y^A(t)$ with $k = \frac{1}{2}$ and $\alpha = 8$ for $\Omega = 0$ (solid plots) and $\Omega = 20$ (dashed plots) with $\gamma = 0$ in (a) and $\gamma = 0.001$ in (b).

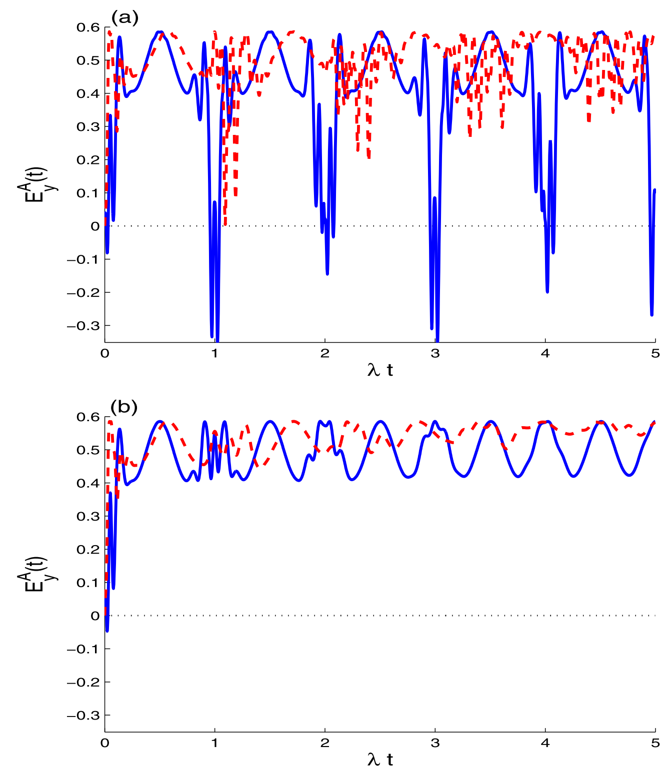

Fig. 4. Entropy squeezing $E_y^A(t)$ with with $k = \frac{1}{2}$ and $\alpha = 8$ for $\Omega = 0$ (solid plot) and $\Omega = 20$ (dashed plot) with $\gamma = 0$ in (a) and $\gamma = 0.001$ in (b).

for the Su(2)-system, ζ takes the values $\pm$ 1 while $\langle \hat{\sigma}_r^j \rangle$is the expectation value of the operators $\hat{\sigma}_r^j$. The corresponding information entropy of the Pauli operators $\hat{\sigma}_r^j$ for a j-th Su(2)-system is given by [49,50]

$$S_r^j = -\frac{1}{2} \sum_{i=1,2} [1 + (-1)^i \langle \hat{\sigma}_r^j \rangle] \ln \frac{1}{2} [1 + (-1)^i \langle \hat{\sigma}_r^j \rangle]. \tag{26}$$

Thus the uncertainty relation of the information entropy can be used as a general criterion for the squeezing in the entropy. Therefore, we have $0 \leqslant S_r^j \leqslant \ln 2$, and provided we take $\delta S_r^j \equiv \exp[S_r^j]$, the entropy uncertainty relation becomes

$$\delta S_x^j \delta S_y^j \delta S_z^j \geqslant 4, \tag{27}$$

consequently the information entropies of these operators satisfy the entropy uncertainty relation [51]

$$S_x^j + S_y^j + S_z^j \geqslant 2 \ln 2. \tag{28}$$

For the A-th Su(2)-system, its information entropies are

$$\begin{aligned} S_x^A =& - \left[\frac{1}{2} + \mathrm{Re}(\rho_{13}^{AB} + \rho_{24}^{AB})\right] \ln \left[\frac{1}{2} + \mathrm{Re}(\rho_{13}^{AB} + \rho_{24}^{AB})\right] \\ & - \left[\frac{1}{2} - \mathrm{Re}(\rho_{13}^{AB} + \rho_{24}^{AB})\right] \ln \left[\frac{1}{2} - \mathrm{Re}(\rho_{13}^{AB} + \rho_{24}^{AB})\right]. \\ S_y^A =& - \left[\frac{1}{2} + \mathrm{Im}(\rho_{13}^{AB} + \rho_{24}^{AB})\right] \ln \left[\frac{1}{2} + \mathrm{Im}(\rho_{13}^{AB} + \rho_{24}^{AB})\right] \\ & - \left[\frac{1}{2} - \mathrm{Im}(\rho_{13}^{AB} + \rho_{24}^{AB})\right] \ln \left[\frac{1}{2} - \mathrm{Im}(\rho_{13}^{AB} + \rho_{24}^{AB})\right]. \\ S_z^A =& - [\rho_{11}^{AB} + \rho_{22}^{AB}] \ln [\rho_{11}^{AB} + \rho_{22}^{AB}] \\ & - [\rho_{33}^{AB} + \rho_{44}^{AB}] \ln [\rho_{33}^{AB} + \rho_{44}^{AB}]. \end{aligned} \tag{29}$$

The entropy squeezing of the $\hat{\sigma}_r^A$ operator of the A-th Su(2)-system is given by

$$E_r^A(t) = \delta S_r^A - \frac{2}{\sqrt{\delta S_z^A}}, \qquad r \equiv x \text{ or } y. \tag{30}$$

The fluctuations in the component $\hat{\sigma}_r^A$ has “squeezed in its entropy squeezing” if the condition $E_r^A(t) < 0$ is fulfilled. It is found that $\hat{\sigma}_x^A$ does not squeezed, i. e., $E_x^A(t) > 0$. Therefore, we study only the dynamical behavior of the entropy squeezing of $E_y^A(t)$.

In Fig. (4), the effects of the rates of the ID and the coupling between the two Su(2)-systems on the dynamical behavior of $E_y^A(t)$ is shown with the same data as in Fig. (3). Solid curve of Fig. (4)a shows that the entropy squeezing presents regularity during several intervals. While the dashed curve shows that the coupling between the two Su(2)-systems enhances the minima of the entropy function $E_y^A(t)$ with more irregular fluctuations, the generated squeezing intervals are disappeared for $\Omega = 20$. When the ID rate takes place there is an increase in the minima of the entropy function $E_y^A(t)$, we observe irregular damped fluctuations without squeezing in the entropy, see Fig. (4)b. Thus the rates of the ID and the coupling between the two Su(2)-systems lead to revoke the squeezing. We can deduce that, there is no entropy squeezing in x-component, and it occurs only in y-component during some finite time intervals.

5. Conclusion

We have explored an open system formed by two qubits interacting with two mode cavities. The dynamical control of the information loss and the quantum coherence (entanglement/purity) via the Husimi distributions are investigated. We have analyzed the influence of the different interaction between the Su(2) and Su(1, 1) and between the two Su(2)-systems as well as the interaction with the environment. We have shown that the couplings control the phase space information of the Husimi distribution and its partial distributions. The growth of Su(2)-Su(1, 1) entanglement can be enhanced by the coupling between the two Su(2)-systems. The purity loss of the Wehrl entropy for the k-th Su(2)-system ($k = A, B$) and its stationary mixture state depend on both the coupling to the environment and to the two Su(2)-systems. Furthermore, we have investigated the effects of the intrinsic decoherence and the two-qubit coupling on the squeezing entropy.

Conflict of interest

The authors declare no conflicts of interest.

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