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Managing yield by lot splitting in a serial production line with learning, rework and scrap

Mohamad Y. Jaber *, Mehmood Khan

Department of Mechanical and Industrial Engineering, Ryerson University, Toronto, ON, Canada M5B 2K3

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ABSTRACT

Wright's (1936) learning curve (WLC) depicts that the time required to accomplish a repetitive operation decreases with each subsequent repetition. The WLC model assumes that the components produced in an operation are all of perfect quality. On the other hand, in many production operations, some of the components require rework. Some components are even scrapped if they cannot be reworked. We employ the WLC model by considering the learning process in the time to produce and the time to rework a given lot. This composite learning curve was developed by Jaber and Guiffrida (2004). The impact of splitting a production lot into batches of equal size is studied through this composite learning curve. The objective of the study is to maximize a combination of performance of average processing time and process yield with respect to the number of batches. The effect of varying the learning curve parameters in production and in rework is studied for the developed model.

1. Introduction

There has been a lot more emphasis on quality control in the recent years. But it is very difficult to ensure that a manufacturing process is defect free. There would always be some items that demand reworking (Buzacott, 1999). Rework is required when a product or a service does not meet the internal or external quality requirements, and could be defined as doing something at least one extra time due to non-conformance to these requirements (e.g., Love et al., 1999, Terwiesch and Bohn, 2001). For a single stage production system, rework can take place instantaneously, i.e. right after a defective is detected or it could be delayed until the end of a cycle (e.g., Jamal et al., 2004). In a multistage production system, rework can take place within each cycle or after a number of production cycles (e.g., Sarker et al., 2008) where not all defective items are repairable. Some of the reworked items may confirm to lesser quality requirements, or may need further repair, or could be scrapped (e.g., Terwiesch and Bohn, 2001; Flapper et al., 2002). Wein (1992) modeled a random yield at each stage of a multistage production system that undergoes rework and scrap at each stage. They used dynamic programming approach to solve a Markov process model for a semiconductor wafer production system. Grosfeld-Ner et al. (2006) developed heuristics to determine inventory policies in a two-echelon supply chain where the production yield at different stages in the first echelon is random. They tested their models for binomially distributed yields. Ben-Zvi and Grosfeld-Ner (2007) also proposed a heuristic for three different types of random yields in a serial production system. Keren (2009) discussed a two level supply chain in a single period inventory problem where yield risks are uniformly distributed. He suggested tightened supply chain integration for the improvement in the profit. Scrap rate and yield rate have a complimentary relationship, i.e., yield rate= 1 − scrap rate (Dolinsky et al., 1990).

In recent years, a technique known as lot splitting has received attention as a scheduling tool to improve quality (Kim and Ha, 2003). Eynan and Li (1997) developed an algorithm for solving a lot-splitting model where the unit production time follows a learning curve where their objective is to maximize the net present value of the total revenue collected at the delivery of the batches. A batch is the portion of a lot transported to the next stage (Szendrovits and Drezner, 1980). Serin and Kayaligil (2003) extended the work of Eynan and Li (1997) by first addressing the issue of extending lot splitting decisions to satisfy a revenue requirement, and second by incorporating the capability of studying several separate shipments to different customers. Grosfeld-Nir and Gerchak (2004) reviewed the literature on multiple lot-sizing with realistic complexities. They discussed the trade-off between producing large lots with overproduction and smaller lots with multiple production and setups. Bukchin and Masin (2004) studied the impact of lot splitting on the performance of a single product m-machine flow-shop. The objective function was a tradeoff between makespan and the mean flow time. Readers may refer to Chang and Chiu (2005) for a comprehensive survey on lot splitting. Another similar, but 'different' concept is the interruption of a production process to

* Corresponding author. Fax: +1 416 979 5265.
E-mail addresses: mjaber@ryerson.ca, myjaber@gmail.com (M.Y. Jaber).

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restore its quality (Khouja, 2005). This practice is common in some production environments where workers are authorized to stop the production process if a quality or production problem arises, e.g., the production process going out-of-control (e.g., Inman and Brandon, 1992). Khouja (2005) derived closed-form expressions for the optimal lot sizes and number of minor set-ups within each cycle. Jaber (2006a) extended Khouja's work by assuming that the setup cost reduces because of learning effects, and that the rate of generating defects reduces because the production process benefits from any changes for eliminating the defects, and thus reduces with every minor setup. None of the above surveyed works assumed learning in production and reworks, scrap, and lot splitting in multistage production.

Learning captured by a learning curve is a combination of employees, organizational systems, and outside factors (Argote, 1993). The power form of the classical learning curve (WLC; Wright, 1936) states that total time per unit decreases as the cumulative number of units produced increases. The WLC has been found to fit empirical data quite well (e.g., Jaber, 2006b). Yield improvement has been linked to learning within the manufacturing organization (Bohn and Terwiesch, 1999; Terwiesch and Bohn, 2001), and the importance of learning curves in the semiconductor industry are noted (e.g., Irwin and Klenow, 1994; Dance and Jarvis, 1992). Many researchers have investigated whether a quality learning curve does exist (Badiru, 1995; Li and Rajagopalan, 1997; Lapré et al., 2000; Terwiesch and Bohn, 2001; Jaber and Bonney, 2003; Allwood and Lee, 2004). However, the only analytical model available that has strong theoretical foundation, with its assumptions empirically validated (Jaber and Bonney, 2003), is that of Jaber and Guiffrida (2004).

Jaber and Guiffrida (2004) extended the WLC by assuming imperfect production. They proposed a composite learning curve, which is the sum of two learning curves, i.e. learning in production and reworks. However, their quality learning curve has limitations. Firstly, the model does not apply to cases where defective items are discarded or scrapped. Secondly, the model assumes the rate of generating defective items is constant, which means that the production process does not benefit from any changes for eliminating the defective items. Thirdly, the model is developed for a single stage and a single cycle production facility. Jaber and Guiffrida (2008) worked on the second assumption of their earlier model (Jaber and Guiffrida, 2004) through interrupting the production process to restore its quality (Khouja, 2005). They concluded that introducing these interruptions into the learning process might improve the performance when the percentage of the production time that represents this restoration is smaller than the production-learning rate. They also found that the plateau value decreases as the number of restoration interruption increases allowing for additional improvements in performance.

Although, and as aforementioned, much has been done distinctly in the areas of learning, rework, scrap and lot splitting, these dimensions have never been investigated altogether for a serial (multistage) production. That is, many researchers have investigated learning in production and few have also discussed learning with lot splitting. This paper is the first attempt to study a serial (multistage) production line with lot splitting with learning in production and rework processes when some defective items are scrapped at each stage. The objective is to optimize the performance of the average processing time and the process yield together. The time to produce and rework are assumed to reduce because of learning (Jaber and Guiffrida, 2004). The impact of splitting the production lot (into equal batches) is studied on the combined performance measure and an optimal number of batches is obtained for a given batch size and learning parameters. Assuming equal batches has been shown to be quite effective and convenient in practice (e.g. Şen et al., 1998). The batches are assumed to flow from one stage to another without any buffer, with this issue left for future research. Furthermore, there is no backlog or lost sale in this production facility. A sensitivity analysis is carried out to study the effects of the learning parameters and the times in production and reworks, on the overall performance of the facility.

The objectives defined above are related to two limitations of the model of Jaber and Guiffrida (2004) i.e. accounting for scrap (first limitation) and assuming a multistage production line (third limitation). Splitting a lot into smaller batches addresses another limitation of the works of Jaber and Guiffrida (2004) that is multiple production cycles. The second limitation (i.e. probability of the process going out-of-control reduces because of learning) is not addressed in this paper due to lack of data that support such an argument (e.g., Chand, 1989; Jaber and Bonney, 2003). Addressing this limitation is left for future work.

The remainder of this paper is organized as follows. Section 2 gives a brief description of Jaber and Guiffrida (2004) model. Section 3 is for mathematical modeling of the objective function. Section 4 provides numerical examples, sensitivity analysis and discusses results. Section 5 is for summary and conclusions.

2. Jaber and Guiffrida (2004) model

They adopted Wright's learning curve (WLC; Wright, 1936) to come up with a combined learning curve for the production and rework time in a manufacturing facility. The total time, $T(x)$, to produce a lot of size x is the sum of the production time, $Y(x)$, and the rework time, $R(x)$, and is given as

$$T(x)=Y(x)+R(x)=\frac{y_1}{1-b}x^{1-b}+\frac{r_1}{1-\varepsilon}\left(\frac{\rho}{2}\right)^{1-\varepsilon}x^{2-2\varepsilon} \tag{1}$$

where y_1 and r_1 are the times to produce the first unit and rework the first defective unit respectively; b and ε are the learning rates of the two processes respectively; whereas ρ is the probability that the process goes out of control. They followed Porteus (1986) assumption that for very small values of ρ (Khouja, 2005), the expected number of defective items in a lot of size x could be approximated as

$$d(x)=\frac{\rho x^2}{2} \tag{1a}$$

The marginal time function, or the learning curve, for the combined process, is given from (1) as

$$t(x)=y(x)+r(x)=y_1x^{-b}+2r_1\left(\frac{\rho}{2}\right)^{1-\varepsilon}x^{1-2\varepsilon} \tag{2}$$

They discussed three behavioral patterns: convex ($0\le\varepsilon<1/2$) with an optimal lot size of $x^*=\{by_1/[2r_1(1-2\varepsilon)\ \ (\rho/2)^{1-\varepsilon}]\}^{1/(1+b-2\varepsilon)}$ that minimizes (2), plateau (ε=1/2), or standard time, and monotonically decreasing ($1/2<\varepsilon<1$), like the WLC, for the composite learning model in (2). These patterns provided valuable managerial insights. A convex behavior may caution managers to not speed up production without as well improving the quality of the process. The plateau behavior may provide managers with an indication to when an additional investment in training or new technology might be required to break the plateau barrier. This suggests that plateauing usually observed in learning curve data might be attributed to problems in quality. Finally, a monotonically decreasing behavior of the quality learning curve could be because the time to rework a defective item becomes insignificant.

In this paper, two limitations of the composite learning curve model in (2) will be discussed. These limitations are: (i) the model in (2) does not apply to cases where defective items are discarded or scrapped, and (ii) the quality learning curve model in (2) was developed for a single stage production facility.

3. Mathematical modeling

This section extends upon the quality learning curve of Jaber and Guiffrida (2004) as follows. First, it is assumed that some reworked units do not meet the quality requirements and thus are scrapped. That is, similar to the production process, the rework process may go out of control generating scrap with probability λ. Second, it is assumed that the production line consists of N serial stages. It should be noted that when processing a number of batches, a machine may be idle before processing a batch in serial production, which depends on the processing times of different stages. This fact will be explained through a simple numerical example in the next section. After making the modifications to the quality learning curve of Jaber and Guiffrida (2004), a mathematical cost function (Z) that combines the performance measures of average processing time (Z_1) and the yield rate (Z_2), is developed. The next section defines the symbolic notations for the model. We then illustrate the process yield and average processing time with lot splitting to derive the modified learning curve and the objective function of the paper.

3.1. Notations

i	a subscript identifying the stage number along the production line, where $i=1,2,\ldots,N$
j	a subscript identifying the number of batches, $j=1,2,\ldots,n$
n	number of batches (a decision variable)
N	number of production stages
LS	lot size quantity
b_i	production learning exponent for stage i in a multistage production line
y_{1i}	time to produce the first unit at stage i
$y_i(x_i)$	time to process the x_ith unit at stage i
ρ_i	probability of the production process going out of control at stage i
$Y_i(x_i)$	time to produce x_i units (good and defective items) at stage i
ε_i	rework learning exponent for stage i in a multistage production line
r_{1i}	time to rework the first defective item in stage i
$r_i(x_i)$	time to rework the x_ith unit in stage i
λ_i	probability of the rework process going out of control at stage i
$R_i(x_i)$	time to rework x_i units at stage i
$T_i(x_i)$	time to produce x_i units at stage i
$t(x)$	marginal time function
$t_i(x_i)$	marginal time function for stage i
$T_{ji}(x_i)$	cumulative time to produce jth batch (equally sized) of x_i units at stage i (i.e., $x_i=x=LS/n$)
w_{ji}	cumulative idle time at stage i before processing batch j
L	production lead time, where $L=\sum_{i=1}^{N} T_i(x_i)$
$d_i(x_i)$	number of defective items in a lot of size x_i at stage i
$s_i(x_i)$	number of scrapped units in a lot of size x_i at stage i
π_i	yield rate at stage i
π_p	process yield rate for all stages

3.2. Process yield

A batch of size x (or x_1), flows sequentially from stage 1 to stage N, where x_{N+1} is the final batch size that exits stage N. Fig. 1 illustrates the flow along the production line.

In this paper, it is assumed that all units are inspected at each stage, and defective items are reworked before the lot is pushed forward to the next stage (Zargar, 1995; Buzacott, 1999). Like the production process, the rework process may go out-of-control at stage $i=1, 2,\ldots,N$ with probability λ_i, producing defective items, where these defective items (twice defective) are scrapped for quality or cost reasons (e.g., Bohn and Terwiesch, 1999; Flapper et al., 2002). Thus, as discussed in Section 2, the number of scrapped units in a lot of size x_i at stage i, are

$$s_i(x_i)=\frac{\lambda_i}{2}d(x_i)^2=\frac{\lambda_i}{2}\left(\frac{\rho_i}{2}x_i^2\right)^2 \tag{3}$$

where $x_i=x_1-\sum_{j=1}^{i} s_j(x_j)$ The process yield at station i is given from (3) as

$$\pi_i=1-\frac{s_i(x_i)}{x_i}=1-\frac{\lambda_i}{2x_i}\left(\frac{\rho_i}{2}x_i^2\right)^2 \tag{4}$$

The process yield of all stages is computed as

$$\pi_p=1-\sum_{i=1}^{N} s_i(x_i)/x_1=1-\frac{1}{x_1}\sum_{i=1}^{N}\frac{\lambda_i}{2}\left(\frac{\rho_i}{2}x_i^2\right)^2 \tag{5}$$

It should be noted here that to calculate the process yield with n splits, one should use x_1/n in place of x_i to calculate x_i and π_p in Eq. (5), where $x_2=x_1/n-s_1(x_1/n)$, $x_3=x_2-s_2(x_2)$, etc.

3.3. Lot splitting

Lot splitting refers to breaking a large lot of parts into smaller transfer batches that can move through the job's routing independently. The batches can be in process simultaneously at different operations on the job's routing, allowing the operations to overlap. To formulate the time to produce a split, we use the fact that for batches of equal size, the time to produce a split would decrease by increasing the number of batches. The reader may refer to Szendrovits (1975) that treats cycle time as a function of the lot size in a multi-stage production. Large savings could be obtained by using this model instead of the conventional EPQ model. For simplicity of computations, this paper assumes no transfer of learning between batches and that machines require no time to be restored before processing the next batch. We can write

Time to finish nth batch at stage i = Time to finish the first batch at stage i + "time to finish" the remaining batches at stage i + "idle time" at stage i

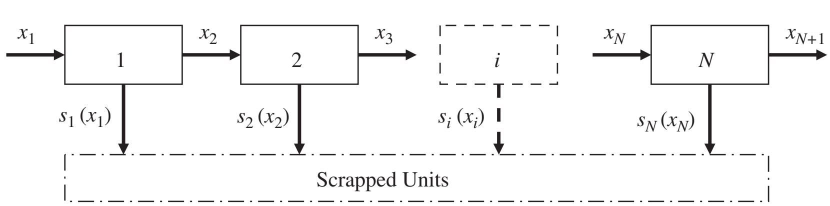

Fig. 1. An illustrative sketch of the production line of interest.

The cumulative idle time of a machine at stage i, before processing batch j,

$$w_{ji} = \begin{cases} (j-1)[T_{j,i-1}(x_i) - T_{j-1,i}(x_i)] & \text{if } T_{j,i-1} \geq T_{j-1,i} \\ 0 & \text{otherwise} \end{cases} \tag{6a}$$

Thus, the cumulative time for batch j at stage i from the start of production (i.e., time zero) is written as

$$T_{ji}(x_i) = T_{1i}(x_i) + (j-1)T_i(x_i) + w_{ji} \tag{6b}$$

To comprehend the above formula, take an example of a production facility with five stages that have to serve three equal-sized batches of a lot. The service times of the machines are 5, 15, 10, 25 and 20 units respectively. The processing time for the first batch, $T_{1i}(x_i)$ at the five stages would be 5, 20, 30, 55 and 75 units respectively, while at the first stage, the two other batches finish at T_{21} = 10 and T_{31} = 15 units of time. To compute the idle time for the second batch at the second stage, i.e. w_{22}, we need to compute the difference (T_{21}–T_{12}) using (6a), i.e. (10–20). Thus, from (6a), the negative idle time would be ignored (w_{22} = 0 since $T_{21} < T_{12}$). So, using Eq. (6b), the time T_{22} is [20+(2–1) × 15+0], i.e. 35 units. Similarly, the idle time w_{35} is computed from (6a) as [(3–1) × (105–100)], i.e. 10 units. Thus, the time T_{35} is [75+(3–1) × 20+10], i.e. 125 units. To further elaborate the example, the time to finish three batches at stage four and five is shown through Gantt charts in Fig. 2.

3.4. The modified quality learning curve

The total time to produce x_i units, rework $d_i(x_i)$ units, and scrap $s_i(x_i)$ at stage i is given from (1) as

$$T_i(x_i) = Y_i(x_i) + R_i(x_i) = \frac{y_{1i}}{1-b_i} x_i^{1-b_i} + \frac{r_{1i}}{1-\varepsilon_i}\left(\frac{\rho_i}{2}\right)^{1-\varepsilon_i} x_i^{2-2\varepsilon_i} \tag{7}$$

As discussed earlier, $s_i(x_i)$ units are scrapped in a lot of size x_i, with x_{i+1} being the number of good units that enters stage $i+1$ from stage i, where $x_{i+1}=x_i-s_i(x_i)$. The average time to process x_{i+1} units at stage i is given as (dividing (7) by x_{i+1})

$$t_i(x_{i+1}) = y_i(x_i) + r_i(x_i) = \frac{y_{1i}}{(1-b_i)x_{i+1}} x_i^{1-b_i} + \frac{r_{1i}}{(1-\varepsilon_i)x_{i+1}}\left(\frac{\rho_i}{2}\right)^{1-\varepsilon_i} x_i^{2-2\varepsilon_i} \tag{8}$$

where (8) is the average learning curve for stage i. The average time to process x_{N+1} with n batches in N stages would be given by

$$P(x_{N+1}) = \frac{T_{nN}(x_{N+1})}{n x_{N+1}} \tag{9}$$

It should be noted that to calculate the average time in (9), one has to (i) calculate the individual process times T_i, for each stage using (7) and then (ii) use the results of (i) in (6b) to calculate the time to finish the nth batch at stage N. Furthermore, note that x_{i+1} is a function of $x_1 = x$ for all i=1,2,…,N, and $P(x_N)$ could be written as $P(x)$.

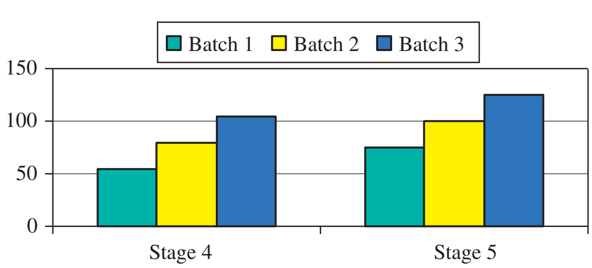

Fig. 2. Time to finish the three batches at stages four and five.

3.5. The objective function

We take our first performance measure regarding the processing time as

$$Z_1 = 1 - \frac{P(x)}{P_0} \tag{9a}$$

where P_0 is the processing time with no learning in production and reworks and with no lot splitting. The worst average performance can be attained when $P(x)$ approaches P_0, while the best performance would be attained when $P(x_{i+1}) \to 0$. Similarly, we take our second performance measure, regarding the process yield, given in (5), to be

$$Z_2 = \pi_p \tag{9b}$$

where the best process yield occurs when π_p = 1, corresponding to $\sum_{i=1}^{N} s_i(x_i) = 0$, and the worst yield occurs when $\pi_p \to 0$, corresponding to $\sum_{i=1}^{N} s_i(x_i) \to 1$. It should be noticed that Z_1 and Z_2 are conflicting functions as shown in Appendix. Thus, we make a tradeoff by defining Z_1 as in Eq. (9a) and combining it with Z_2 (Eq. (9b)) to optimize the number of splits for a given batch size. So, our objective function would be

$$\text{Maximize} \quad Z = Z_1 + Z_2 = 2 - \frac{P(x_{i+1})}{P_0} - \frac{1}{x_1}\sum_{i=1}^{N} s_i(x_i) \tag{10a}$$

$$\text{Subject to}: \quad x_i > 0, \quad i = 1, 2, \ldots, N \tag{10b}$$

It is obvious that the number of defective items at a stage cannot exceed the number of components that enter the stage, i.e.

$$d(x_i) = \frac{\rho_i}{2}(x_i)^2 < x_i, \quad \text{or} \quad x_i < 2/\rho_i \tag{10c}$$

In a similar fashion, it can be said, from (3), that the number of the scrapped units at stage i cannot exceed the number of defective items at that stage, i.e., $s(x_i) < d(x_i)$, then

$$\frac{\lambda_i}{2}\left[\frac{\rho_i}{2}(x_i)^2\right]^2 < \frac{\rho_i}{2}(x_i)^2, \quad \text{or} \quad x_i < \sqrt{4/\lambda_i \rho_i} \tag{10d}$$

We can combine the constraints (10c,d) as

$$x_i < \min[2/\rho_i, \sqrt{4/\lambda_i \rho_i}], \quad i = 1, 2, \ldots, n \tag{10e}$$

4. Numerical examples

Eqs. (5,9b) indicates that the process yield remains independent of b and ε and the process times, but it does depend on the number of batches of a production lot. On the other hand, Eq. (9a) indicates that the process time performance measure depends on both the learning exponents as well as the number of batches.

Before we analyze the effects of learning and lot splitting, let us take a simple example to elaborate the use of the model given in this paper. Consider a lot of 200 units that is processed (and reworked) at two stages in series. We will compute the base performance P_0 for this lot, ignoring learning and without dividing it into batches. We will then compute the two performance measures, i.e. Z_1 and Z_2, for splitting the lot into four equal-sized batches of 50, with a learning exponent of 0.25, both in production and rework in the two stages. Assume that the probability of the process (production and rework) going out-of-control is 0.005. Now we compute the base performance. Using Eq. (3), the number of defectives at the production and rework stations of the first stage is 100 and 25 units respectively. The defectives from the rework station, i.e., 25 units, are scrapped. Therefore, 175 units would enter the second stage. Assuming that per unit time of production and rework is 10 and 5 units respectively, at the two stages, the total processing time at the first stage 1, using Eq. (7), would be 2500 units of time (with no learning). Similarly, the second stage will produce 77 and 15 defective units in production and rework, respectively, with the 15 units scrapped. The sum of production and rework time at stage 2 is 2133 units of time (with no learning), which makes the total processing time of the line 4633 units of time. So, there would be 160 (200–25–15) items coming out of the line. This, by the use of Eqs. (5) and (9), results in a base performance yield as 0.8 and base average processing time as 28.96. Now, we split the lot into four equal-sized batches with learning in production and rework. In a similar fashion, the number of defectives at production and rework at the first stage would be 6 and approximately zero (no scrap). Again, 6 and approximately zero units are found defective at the production and rework at the second stage and 50 units will come out. The first batch would finish at the second stage at 554 units while the last batch, using Eqs. (6a), (6b), would finish at 1385 units of time. There would be an idle time of about one unit. The yield and the average processing time would be 0.996 and 6.95 respectively. Using, Eqs. (9a), (9b), the values of Z_1 and Z_2 would be 0.760 and 0.996. This results in Z=1.755. Following the same method for splitting the lot into 5, 6, 7 and 8 batches would result in the Z values as 1.758, 1.757, 1.755 and 1.753 and respectively. Thus, the optimal number of batches for the given set of data is 5 (i.e., $1.755 < Z = 1.758 > 1.757$).

Let us now analyze the effect of varying the learning exponents and the process times on the overall performance. Consider now a production line with three stages, i=1, 2, and 3, that has the following parameters at each stage (Jaber and Guiffrida, 2004). In addition to these, we take a lot size of 400 and then analyze the effect of dividing it into batches of equal size, on the cumulative performance measure defined in (9a).

Description of parameter	*Symbol*	*Numerical value*
Production learning exponent	b_i	0.25, 0.5, and 0.75
Rework learning exponent	ε_i	0.25, 0.5, and 0.75
Time to produce the first unit	y_1	10, 20, 30 units
Time to rework the first unit	r_1	5 units
Probability of the production to go out-of-control	ρ	0.001
Probability of the rework process to go out-of-control	λ	0.001

For simplicity, and without loss of generality, we assumed $y_{1i}=y_1$, $r_{1i}=r_1$, $\rho_i=\rho$, and $\lambda_i=\lambda$, where i=1, 2, 3. The overall performance for varying values of b, ε and y/r is shown in Table 1 and Figs. 3 and 4. Overall performance Z is the sum of the performance Z_1 and the yield Z_2. Table 1 reports the optimal values for the number of batches, Z_1, Z_2 and Z while the figures depict the behavior of overall performance Z only, with the number of batches.

The first part of Table 1 and Fig. 3 depicts the variation of overall performance with respect to ε and the number of batches, n at fixed values of b and y/r. It should be noted that at fixed values of b, the increase in ε decreases the number of batches and the overall performance. The rationale for this is that with an increase in ε, the learning curve component for reworks in (8), $r_i(x_i)$, gets closer to the x-axis intersecting the learning curve component for production in (8), $y_i(x_i)$, at higher values of x. An increase in the batch size decreases the number of batches. This increase in ε, on the other hand also decreases the performance. The reason is that

Table 1
Variation of the performance with N, b, ε and y/r.

b	ε	y/r	n	Z_1	Z_2	Z
0.25	0.25	2.00	12	0.8811	0.9977	1.8789
	0.50		11	0.8805	0.9972	1.8778
	0.75		9	0.8782	0.9955	1.8738
0.50	0.25		9	0.9270	0.9955	1.9225
	0.50		8	0.9289	0.9940	1.9229
	0.75		7	0.9287	0.9916	1.9203
0.75	0.25		8	0.9433	0.9940	1.9373
	0.50		7	0.9466	0.9916	1.9382
	0.75		7	0.9445	0.9916	1.9362
0.25	0.25	2.00	12	0.8811	0.9977	1.8789
		3.00	11	0.8269	0.9972	1.8241
		4.00	10	0.7733	0.9965	1.7698
0.50		2.00	9	0.9270	0.9955	1.9225
		3.00	8	0.8980	0.9940	1.8921
		4.00	7	0.8710	0.9916	1.8626
0.75		2.00	8	0.9433	0.9940	1.9373
		3.00	7	0.9240	0.9916	1.9156
		4.00	6	0.9074	0.9877	1.8951
0.25	0.50	2.00	11	0.8805	0.9972	1.8778
		3.00	10	0.8270	0.9965	1.8235
		4.00	9	0.7740	0.9955	1.7696
	0.75	2.00	9	0.8782	0.9955	1.8738
		3.00	9	0.8243	0.9955	1.8199
		4.00	8	0.7721	0.9940	1.7662

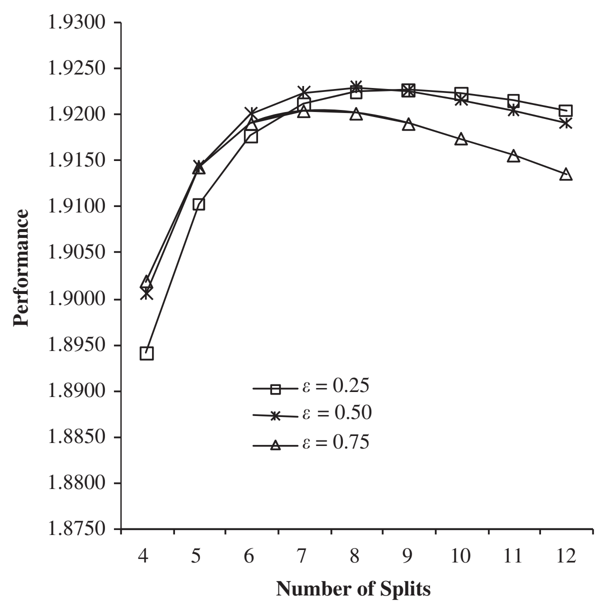

Fig. 3. Overall Performance Z with respect to n and ε (b=0.50, y/r=2).

although the second term of Eq. (8) is found to be decreasing with ε, the first term increases as the lot size increases while subsequently x_{n+1} decrease because of higher scrap rate.

On the other hand, for a fixed value of learning in reworks in these curves, it should be noted that the performance improves with the increase in the learning in production. The reason is that with a higher rate of learning in production the learning curve component for production in (8), $y_i(x_i)$, intersects the learning curve component for reworks in (8), $r_i(x_i)$, at lower values of x. An important observation in these curves is that the overall performance has a decreasing trend when the fixed value of b is 0.25 which it does not have for the cases where b=0.50 and 0.75. The rationale for this is that Eq. (9) decreases when b goes from 0.25 to 0.5 while it increases when b goes from 0.50 to 0.75.

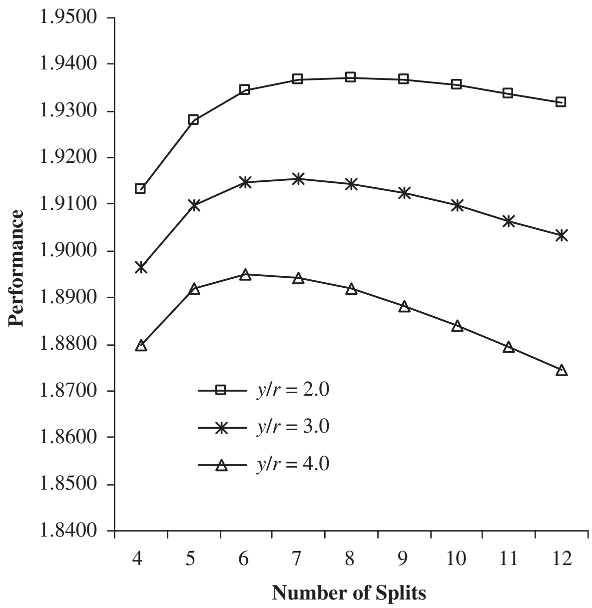

Fig. 4. Overall Performance Z with respect to N and y/r (b=0.75, ε=25).

This suggests that to improve the overall performance it is recommended to increase the number of batches when learning in reworks is slow.

The second part of Table 1 and Fig. 4 depict the variation of the overall performance with respect to y/r and the number of batches n, at fixed values of b and ε. It can be seen that increasing y decreases the number of batches and the performance. The rationale for this is that with an increased y the production learning curve intersects the rework learning curve at a higher level of time which increases the lot size thus decreasing the number of batches and the performance (9a). The effect of learning becomes more obvious when the parameters of the production learning curve, i.e. y and b, increase at the same time, where we note a further decrease in the number of batches accompanied by an improved performance.

To enhance the results of this research, an experiment was designed for 5000 examples. For each example, a random set of input parameters was generated, and the output in terms of individual performance Z_1, production yield Z_2 and the overall performance Z was optimized and recorded. To generate the random input parameters, the following ranges of the data set were employed.

Lot size (LS)	200–2200
Learning rates in production/rework	0.25–0.75
Time to produce/rework the first unit	0.25–1.00

The results from the analysis of variance for the overall performance Z are shown in Table 2.

The subscripts 1, 2 and 3 represent the three stages of production, respectively. The degree of significance of each factor is represented in Table 2 by its p-value (less than 0.05). The table shows that the learning rate in production in all three stages has a significant effect on the overall performance while the learning rate in reworks becomes insignificant as the number of stages, N increase. This implies that, as workers learn more in production, there is lesser and lesser number of defective items going to be reworked in the successive stages. This effect is pronounced when workers learn in reworks, too. Therefore, in the third stage of reworks, the learning rate has no significance. This therefore would suggest that the workers with higher learning rates should be appointed for earlier stations of rework, than for later stations.

Table 2
Significance of the input parameters for Z.

	Coefficients	St. error	t-Stat	P-Value
Intercept	1.8426806	0.0010823	1702.5685	0
b_1	0.0945904	0.0007226	130.89936	0
b_2	0.0173941	0.0007297	23.836387	5.06E−119
b_3	0.0176643	0.0007291	24.225996	1.13E−122
ε_1	0.0019479	0.0007206	2.703166	0.0068915
ε_2	0.0016532	0.0007277	2.2718573	0.0231375
ε_3	0.0007281	0.000723	1.0070728	0.3139487
y_1/r_1	3.134E−05	0.000526	0.0595751	0.9524964
y_2/r_2	0.0009402	0.0005239	1.7946103	0.0727763
y_3/r_3	0.0005573	0.0005262	1.0590032	0.2896496
LS	1.631E−05	2.338E−07	69.789625	0

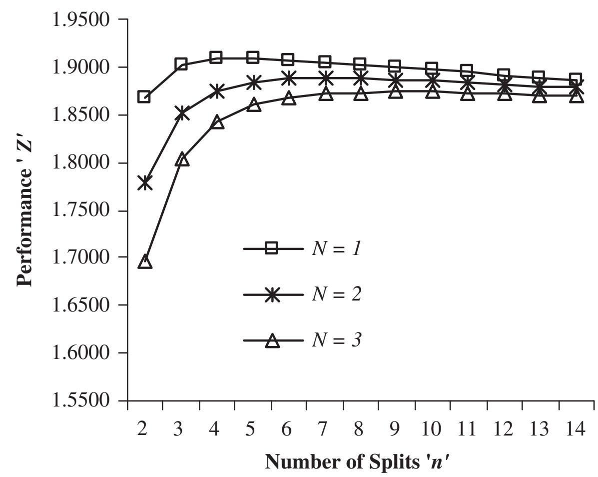

Fig. 5. Performance Z with respect to the number of Stages N and the number of batches n (b=ε=0.25).

Table 2 also indicates that the ratio of the times a part spends in production and in reworks is insignificant for the performance. The rationale is that the time spent in production or reworks affects the performance but not their ratio. For example, the ratio between these times remains the same whether they are (1000, 100), (100,10) or (50,5) while the amount of learning in these situations is totally different. In other words, the span of time one spends in a process (production or rework), affects his learning and thus the number of defective items in that process.

Lot size, on the other hand, has a significant effect on the performance. The rationale for this is that the more is the lot size, the more is the room for learning. Once a worker learns more, the number of defective items would decrease and the overall performance would improve.

The above examples leave us wondering if the number of production/rework stages has a role in the overall performance. Redoing one of the examples in Fig. 3, for different number of stages, shows that the performance has a decreasing trend with the number of stages. The results are shown in Fig. 5. This explains the fact that the total processing time and total scrap, both go on increasing with an increased number of stages, which decreases both performance measures. The optimal number of batches therefore has to take a higher value with each stage, in an effort to improve the performance.

5. Summary, conclusions and future research

The composite learning curve developed by Jaber and Guiffrida (2004) is extended in this paper to optimize a combination of two performance measures. The first performance measure studies the average processing time a batch spends in a production facility which comprises of production and rework; the second performance measure studies the overall process yield. Production and rework, both are assumed to follow learning. The defective items from the production station are sent to the rework station where they are classified as to be reworkable or scrap. This is better than simply classifying the items to be good or defective. The impact of splitting a production lot into equal-sized batches, on the overall performance is studied.

In contrast to the available literature, Jaber and Guiffrida (2004) model can take three different scenarios of learning; convex, plateau and decreasing. Most of the earlier work has studied only the last scenario, i.e. decreasing processing time. Therefore, this work has a wider scope of application where one has to study the performance of a production facility through three variables, i.e. learning in production, learning in reworks and number of batches.

The effect of the varying the learning rates of production and reworks, on the individual and overall performance is studied through graphical analysis. An experimental study with 5000 examples is also carried out for the performance of the system with respect to varying learning rates and the times for production and reworks.

The study indicated that the optimal performance improves by increasing the learning exponent in production and deteriorates with an increase in the learning exponent in reworks. The rationale was the effect of these variations on the batch size. It was also noted that increasing the production time decreases the number of batches. Besides, the time spent in production or reworks affects the performance but their ratio remains insignificant. Furthermore, the lot size has a significant effect on the performance. Lastly, the study suggested that the performance of the system deteriorates by increasing the number of stages in the serial production and that the workers with higher learning rates should be appointed for earlier stations of rework than for the later stations.

The work presented herein could be extended to consider the case of dual resource constrained (DRC) system where the number of machines on a production line exceeds the number of workers (e.g., Jaber et al., 2003), which requires the workforce to be flexible, whereby workers are cross-trained to operate machines in functionally different departments. Another potential extension to the work is considering stochastic percentage of defectives that result in random yields in the different stages of the serial line. This would add to the practicality of the work presented herein.

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Appendix

The number of defectives for a batch of size x (Porteus, 1986) is $d(x)=\rho x^2/2$, with the percentage defectives being $\rho x/2$. When there are n splits, at a single stage system, the number of defectives and the percentage of defectives respectively become $d_n(x)=\rho(n/2)(x/n)^2=(\rho/2n)x^2$ and $(\rho/2n)x$. Similarly the number of scrap, $s(x)=\lambda/2[\rho x^2/2]^2$. And for n splits at a single stage system, $s_n(x)=(\lambda/2)n[\rho/2(x/n)^2]^2=(\lambda\rho^2/8n^3)x^4$. For simplicity, let $\theta=(\lambda\rho^2/8n^2)x^3$. So, $s_n(x)=\theta x/n$ and the percentage scrap becomes θ/n, where $0<\theta<1$. Note that this simplistic relationship provides the same behavioral trend for increasing or decreasing values of x and n; i.e. $(\lambda\rho^2/8n^3)x^4(\theta x/n)$ increases (or decreases) for increasing values of x (or n). Now, for a single stage case, for n splits, the average processing time is

$$\begin{aligned} P(x) &= \frac{y_1 n}{(1-b)}\frac{(x/n)^{1-b}}{(x-x\theta/n)}+\frac{r_1 n}{1-\varepsilon}\left(\frac{\rho}{2}\right)^{1-\varepsilon}\frac{(x/n)^{2-2\varepsilon}}{x-x\theta/n} \\ &= \frac{y_1}{(1-b)}\frac{(x/n)^{-b}}{(1-\theta/n)}+\frac{r_1}{1-\varepsilon}\left(\frac{\rho}{2}\right)^{1-\varepsilon}\frac{(x/n)^{1-2\varepsilon}}{1-\theta/n} \\ &= \frac{y_1 n}{(1-b)}\frac{(x/n)^{-b}}{(n-\theta)}+\frac{r_1 n}{1-\varepsilon}\left(\frac{\rho}{2}\right)^{1-\varepsilon}\frac{(x/n)^{1-2\varepsilon}}{n-\theta} \end{aligned}$$

Now, the base performance would be: $P_0(x)=(y_1/(1-\theta))+r_1(\rho/2)(x/(1-\theta))$. To have $P(x)<P_0(x)$, we must have

$$\text{(a)}\ \frac{y_1 n}{1-b}\frac{(x/n)^{-b}}{n-\theta}<\frac{y_1}{1-\theta}\Rightarrow\frac{n(x/n)^{-b}}{(1-b)(n-\theta)}<\frac{1}{1-\theta}$$
$$\Rightarrow\frac{(x/n)^{-b}}{1-b}<\frac{n-\theta}{(1-\theta)n}$$
$$\Rightarrow\frac{(x/n)^{1-b}}{1-b}<\frac{n-\theta}{(1-\theta)n^2}x$$

Let $v=x/n\ge 1$, then

$$\frac{v^{1-b}}{1-b}<\frac{n-\theta}{(1-\theta)n}v\Rightarrow\frac{d}{dv}\left(\frac{v^{1-b}}{1-b}\right)<\frac{d}{dv}\left(\frac{n-\theta}{(1-\theta)n}v\right)$$
$$\Rightarrow v^{-b}<\frac{n-\theta}{(1-\theta)n}$$

Since $n\ge 1$, $0<b<1$, $0<\theta<1$, and $v^{-b}<1$, then

$$\frac{y_1 n}{1-b}\frac{(x/n)^{-b}}{n-\theta}<\frac{y_1}{1-\theta}$$

is "True".

$$\text{(b)}\ \frac{r_1 n}{1-\varepsilon}\left(\frac{\rho}{2}\right)^{1-\varepsilon}\frac{(x/n)^{1-2\varepsilon}}{n-\theta}<r_1\left(\frac{\rho}{2}\right)\frac{x}{1-\theta}$$
$$\Rightarrow\frac{1}{1-\varepsilon}\left(\frac{\rho}{2}\right)^{1-\varepsilon}\frac{(x/n)^{2-2\varepsilon}}{n-\theta}<\left(\frac{\rho}{2}\right)\frac{(x/n)^2}{1-\theta}\Rightarrow\frac{1}{1-\varepsilon}$$

$$\frac{1}{1-\varepsilon}\left(\frac{\rho}{2}\times\left(\frac{x}{n}\right)^2\right)^{1-\varepsilon}<\frac{n-\theta}{1-\theta}\times\frac{\rho}{2}\times\left(\frac{x}{n}\right)^2.$$

Let $u=\rho/2(x/n)\ge 1$, then

$$\frac{1}{1-\varepsilon}u^{1-\varepsilon}<\frac{n-\theta}{1-\theta}u\Rightarrow\frac{d}{du}\left(\frac{1}{1-\varepsilon}\right)u^{1-\varepsilon}<\frac{d}{du}\left(\frac{n-\theta}{1-\theta}u\right)$$
$$\Rightarrow u^{1-\varepsilon}<\frac{n-\theta}{(1-\theta)}$$

Since $n\ge 1$, $0<\varepsilon<1$, and $u^{-\varepsilon}<1$, then

$$\frac{r_1 n}{1-\varepsilon}\left(\frac{\rho}{2}\right)^{1-\varepsilon}\frac{(x/n)^{1-2\varepsilon}}{n-\theta}<r_1\left(\frac{\rho}{2}\right)\frac{x}{1-\theta}$$

is "True".

Therefore, for a fixed value of n, $P(x)<P_0(x)$ is True and $Z_1=1-P(x)/P_0(x)$ is an increasing function in x. In can be shown in a similar manner to the above analysis, that for a fixed value of x, the average processing time, $P(x)$, increases as the number of splits increases causing Z_1 to decrease.

Similarly, Z_2's best and worst performance values are 1 and 0 respectively. It can be easily deduced by looking at Z_2,

$$Z_2=1-\frac{s_n(x)}{x}=1-\frac{\lambda\rho^2}{8n^3}x^3$$

that it has a direct relation with n and an inverse relation with x. For example for a fixed value of n, increasing the batch size

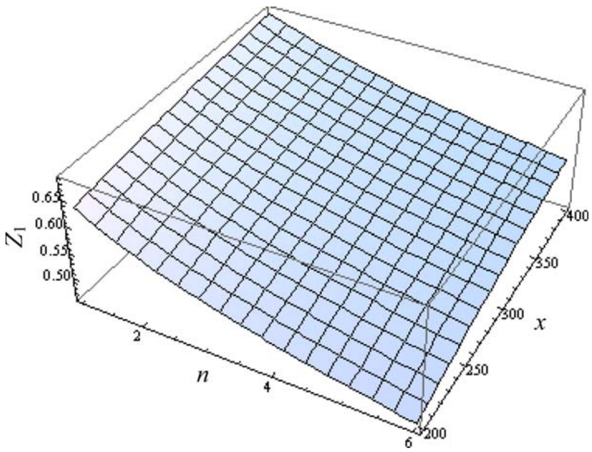

Fig. A1. Sensitivity of Z_1 to the number of splits and the batch size.

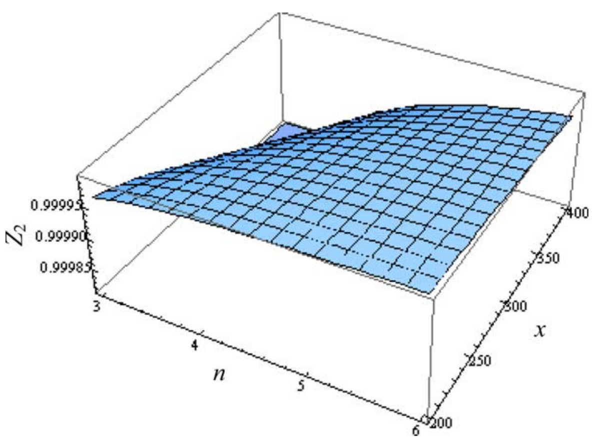

Fig. A2. Sensitivity of Z_2 to the number of splits and the batch size.

increases the scrap and thus reduces the process yield. The results from the above analysis are summarized below.

	n	x	Z_1	Z_2
x	–	↑ (↓)	↑ (↓)	↓ (↑)
n	↑ (↓)	–	↓ (↑)	↑ (↓)

This behavior of Z_1 and Z_2 is now illustrated numerically. Assume $\rho=\lambda=0.001$, $y_1=10$, $r_1=5$ and $b=\varepsilon=0.25$. Figs. A1 and A2 show, respectively, how Z_1 and Z_2 behave on varying n and x.

It is clear from Fig. A1 that Z_1 has a direct relationship with the batch size while it has an inverse one with the number of splits. On the other hand, Fig. A2 shows that Z_2 has a direct relationship with the number of splits while it has an inverse relationship with the batch size.

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