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Review

# A review of the extensions of a modified EOQ model for imperfect quality items

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ABSTRACT

Imperfect items in the raw material and production stages of a supply chain directly impact the coordination of the product flow within a supply chain. In response to this concern, production and inventory lot sizing models, which incorporate imperfect items into their formulation have become an important and growing area of research. The contribution of Salameh and Jaber (2000) is one of the fundamental models on lot sizing when procured items are of imperfect quality. Over the past decade, there has been a noticeable amount of interest in the EOQ model for imperfect items that was set forth in Salameh and Jaber (2000). Several researchers have published adaptations and extensions of this original model that address supply chain coordination, quality improvement and yield management, and the impact of human error on production and inventory systems. In this paper, we summarize the current body of research that has extended the Salameh and Jaber (2000) EOQ model for imperfect items. Some possible future research directions are identified at the end of the paper.


**Contents**



## 1. Introduction

The economic order quantity (EOQ) model of Harris (1913) is the foundation of modern-day inventory models. Within the inventory management literature, a great amount of effort has been devoted to developing lot sizing models that overcome the restrictive assumptions of the EOQ model. For example, the EOQ assumption of perfect quality is unrealistic in most industrial applications and if adopted, can lead to errors in determining the order size and related inventory control parameters, such as the inventory cycle time, backorder size.

This assumption has led many researchers to come up with a number of practical and realistic lot sizing models when quality is imperfect and hence supply is unreliable. Karlin (1958) was one of the first to address this issue. He studied the assumptions underlying the structure of the inventory cost components and presented three single stage newsvendor models to characterize the optimal ordering policy under random supply. Porteus (1986) studied the effect of defective items on the basic EOQ model in a case where it was assumed that a process goes out-of-control with a fixed probability. Rosenblatt and Lee (1986) suggested producing in smaller lots when the process is not perfect. They assumed that the time between the in-control and the out-of-control state of a

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process follows an exponential distribution and that the defective items are reworked instantaneously. In a later paper, Lee and Rosenblatt (1987) studied a joint lot sizing and inspection policy for an EOQ model with a fixed fraction of defective products. Urban (1998) modeled the defect rate of a process as a function of the run length and derived closed-form solutions for the economic production quantity. This model accounted for either positive or negative learning effects in production processes.

Researchers have also investigated lot sizing with imperfect items under conditions of stochastic production yield and demand rates. Gerchak et al. (1988) analyzed a single period production problem where the production process was characterized by uncertain demand and variable yield. The single period model that was developed for this problem was then extended to an *n*-period model. Yano and Lee (1995) presented a review of the literature on lot sizing models with stochastic production or procurement yields. Grosfeld-Nir and Gerchak (2004) also reviewed the literature on single stage and multistage imperfect production systems that involve random yield and inspection. Inderfurth (2004) determined an optimal production policy for a uniformly distributed demand and yield rate. For a detailed review of the articles dealing with the relationship between quality and inventory the reader should consult Wright and Mehrez (1998).

The above surveyed works assumed imperfect production processes that generate defects that are either reworked or scrapped. Unlike these works, Salameh and Jaber (2000) developed an extended EOQ model where imperfect quality items are salvaged at a discounted price. Incoming lots of raw material containing items of imperfect quality that occur as a random fraction with a known probability distribution, undergo a screening in which defective items in the lot are removed by the end of the screening period and sold at a discounted price. Recently, this paper has received a high level of attention in the literature with several researchers extending the model. Table 1 summarizes some important extensions of the model in Salameh and Jaber (2000).

As outlined in Table 1, the extensions of the model in Salameh and Jaber (2000) have considered single stage and multistage production systems, supply chains, shortages and backorders, and fuzzy logic. The broad scope and application of these extensions suggests that a comprehensive review of these models with imperfect quality would be helpful to future researchers.

In this paper we review the literature that extends the model in Salameh and Jaber (2000). The review presented is organized around the following six themes: (*i*) EOQ/EPQ models for imperfect items, (*ii*) shortages and backordering, (*iii*) quality, (*iv*) supply

**Table 1**
Works that modified/extended the work of Salameh and Jaber (2000).

| Category | Article | Remarks |
|---|---|---|
| **EOQ/EPQ models for imperfect items** | Goyal and Cárdenas-Barrón (2002) | A simplification of Salameh and Jaber (2000) |
| | Konstantaras et al. (2007) | Acceptable lot is shipped in batches; reworked the defective items |
| | Jaber et al. (2008) | Fraction of defectives follows learning |
| | Maddah and Jaber (2008) | Introduced renewal reward theorem; imperfect items carried for several cycles |
| | Yoo et al. (2009) | Manufacturer commits errors in screening; reworked and salvaged the screened and returned items |
| | El-Kassar (2009) | Imperfect and perfect items have continuous demand |
| | Wahab and Jaber (2010) | Different unit holding costs for imperfect and perfect items; fraction of defectives follows learning |
| | Maddah et al. (2010) | Order-overlapping to avoid shortages |
| | Khan et al. (in press-a) | Buyer commits errors in screening |
| | Khan et al. (2010b) | Screening rate follows learning; stockouts result in lost sales and backorders |
| | Chang and Ho (2010b) | Used renewal reward theorem for the first model in Konstantaras et al. (2007) |
| **Shortages backordering** | Rezaei (2005) | Shortages fully backordered |
| | Yu et al. (2005) | Partial backordering and lost sales |
| | Papachristos and Konstantaras (2006) | Conditions in Salameh and Jaber (2000) are not enough to avoid shortages |
| | Wee et al. (2006) | On-going deterioration with partial backordering |
| | Wee et al. (2007) | Shortages fully backordered |
| | Eroglu and Ozdemir (2007) | Shortages fully backordered |
| | Chang and Ho (2010a) | Renewal reward theorem in Wee et al. (2007) |
| | Roy et al. (in press) | Different causes of shortages |
| **Quality** | Chan et al. (2003) | Various scenarios to dispose defective items |
| | Tsou (2007) | Quality follows normal distribution |
| | Tsou et al.(2009) | Continuous quality characteristic and rework |
| **Supply chains** | Huang (2002) | Vendor–buyer supply chain |
| | Goyal et al. (2003) | Vendor–buyer supply chain following Hill (1997) and Goyal and Cárdenas-Barrón (2002) |
| | Huang (2004) | Vendor–buyer supply chain following Hill (1997) and Goyal and Salameh and Jaber (2000) |
| | Khan and Jaber (2009) | Three level supply chain with a window for percentage of defective items |
| | Khan and Jaber (2011) | Supplier-vendor coordination for the first two mechanisms in Khouja (2003) |
| | Khan et al. (in press-b) | Several human factors in Khan and Jaber (2011) |
| | Khan et al. (2010a) | Screening errors and learning in production in Huang (2002) |
| | Khan et al. (2010c) | Vendor-buyer supply chain with defective items and consignment stocking policy |
| | Sana (2011) | Three level supply chain with unequal-sized shipments |
| **Fuzzy** | Chang (2004) | Use of triangular fuzzy numbers |
| | Ouyang et al. (2006) | Triangular fuzzy numbers in Huang (2002) |
| **Others** | Chung and Huang (2006) | Permissible delay in payments |
| | Chung et al.(2009) | Two warehouses with limited storage |
| | Hsu and Yu (2009) | Price discount to motivate buyers |
| | Hsu and Yu (2011) | Price increase at different times to make the buyer order in larger quantities |

chains, ($v$) fuzzy set theory, and ($vi$) other extensions of the model in Salameh and Jaber (2000).

The remainder of this paper is organized as follows. Section 2 overviews the mathematical structure of the model of Salameh and Jaber (2000) thereby providing the reader with a foundation for future extensions of the model. Section 3 examines the methodologies used in the extensions of this model and overviews the key mathematical aspects of the extensions. Section 4 summarizes and concludes the paper.

## 2. EOQ model for imperfect items (Salameh and Jaber, 2000)

The work of Salameh and Jaber (2000) was the first model, to our knowledge, that provided an economic order quantity for a buyer who receives imperfect lots. They extended the traditional EOQ model by accounting for imperfect quality items under the following set of assumptions: ($i$) the demand is deterministic, ($ii$) the orders are replenished instantaneously, ($iii$) the lot contains a fixed fraction $\gamma$ of defectives with known probability density function, ($iv$) a 100% screening is performed to separate these defective items, and ($v$) the defective items are sold as a single batch at a discounted price. The inventory profile for this model is illustrated in Fig. 1.

Where $Q$ is the lot size, $\gamma$ is the fraction of imperfect quality items, $D$ is the demand rate, $\tau$ is the screening time, and $T$ is the cycle time. To prevent shortages, Salameh and Jaber (2000) adopted the condition (E is for expectation)

$$\mathrm{E}[Q,\gamma] \geq Dt \tag{1}$$

Under this condition the inventory cycle time becomes

$$T = \frac{(1-\gamma)Q}{D} \tag{2}$$

The total cost in an inventory cycle is

$$TC(Q) = A_b + pQ + dQ + h_b\left\{\frac{Q(1-\gamma)T}{2} + \frac{\gamma Q^2}{x}\right\} \tag{3}$$

where $A_b$ is the (buyer's) order cost, $p$ is the unit purchase price, $d$ is the unit screening cost, $h_b$ is the (buyer's) unit holding cost, and $x$ is the screening rate ($x > D$).

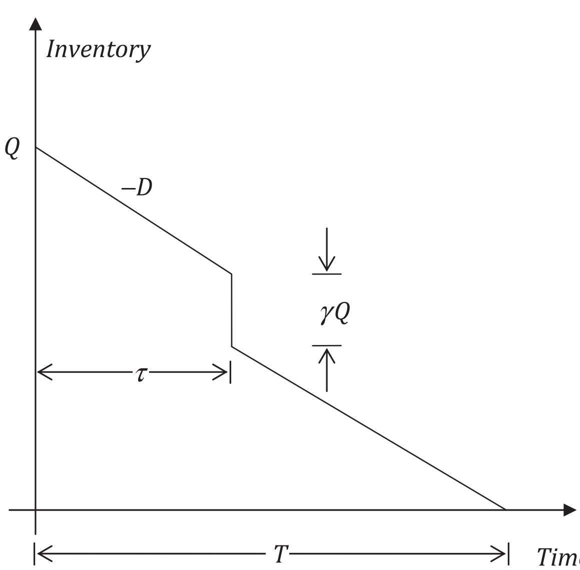


**Fig. 1.** Behavior of the inventory with time for the model of Salameh and Jaber (2000).

The total profit per cycle was given as

$$TP(Q) = sQ(1-\gamma) + vQ\gamma - \left[A_b + pQ + dQ + h_b\left\{\frac{Q(1-\gamma)T}{2} + \frac{\gamma Q^2}{x}\right\}\right] \tag{4}$$

where $s$ and $v$ are the selling and salvage prices, respectively. The expected per unit time profit function is computed as

$$\mathrm{E}[TPU(Q)] = \mathrm{E}\left[\frac{TP(Q)}{T}\right] \tag{5}$$

or

$$\mathrm{E}[TPU(Q)] = D\left(s - v + \frac{h_b Q}{x}\right) + D\left(v - \frac{h_b Q}{x} - p - d - \frac{A_b}{Q}\right)\mathrm{E}\left[\frac{1}{1-\gamma}\right] - \frac{h_b Q(1-\mathrm{E}[\gamma])}{2} \tag{6}$$

The optimal order quantity, $Q^*$, that maximizes (6) was given as

$$Q^* = \sqrt{\frac{2A_b D\mathrm{E}\,[1/(1-\gamma)]}{h_b[1-\mathrm{E}[\gamma]-2D(1-\mathrm{E}\,[1/(1-\gamma)])/x]}} \tag{7}$$

The number "2" in the denominator of the above equation was omitted in Salameh and Jaber (2000), but was corrected by Cárdenas-Barrón (2000). The model defined by (7) is hence referred to as the modified EOQ model for imperfect items.

## 3. Extensions of the model in Salameh and Jaber (2000)

The purpose of this section is to introduce the reader to the underlying mathematics used to describe models with imperfect production and/or supply. The model of Salameh and Jaber (2000) has been extended for a number of scenarios in production and supply chains. In this section, a brief review of these extensions is presented. The following notations (some of them were defined earlier) will be utilized:

| | |
|---|---|
| $p$ | purchasing price of unit product |
| $d$ | screening cost per unit |
| $\tau$ | screening time in a cycle |
| $b$ | learning exponent |
| $c_{rj}$ | cost of rejecting an item |
| $c_{rw}$ | cost of reworking an item |
| $X$ | actual value of the quality characteristic |
| $Y(X)$ | quality distribution function (normal distribution function) |
| $L(X)$ | loss of poor quality per unit products |
| $G$ | Taguchi loss parameter |
| $h_b$ | buyer's holding cost per unit per year |
| $h_{bo}$ | buyer's holding cost per unit per year for his own warehouse |
| $h_{br}$ | buyer's holding cost per unit per year for the rented warehouse |
| $h_{bg}$ | buyer's holding cost per unit per year for non-defective (good) items |
| $h_{bd}$ | buyer's holding cost per unit per year for defective items |
| $h_v$ | vendor's holding cost per unit per year |
| $h'_v$ | financial component of vendor's unit holding cost |
| $A_v$ | vendor's setup cost per batch |
| $A_b$ | buyer's ordering cost per cycle |
| $A_s$ | supplier's setup cost per cycle |
| $LSL$ | lower specification limit |
| $USL$ | upper specification limit |
| $LSL_1$ | lower specification limit for perfect items |
| $USL_1$ | upper specification limit for perfect items |
| $LSL_2$ | lower specification limit for imperfect items |
| $USL_2$ | upper specification limit for imperfect items |
| $U$ | allowed deviation of a quality characteristic |

$\gamma$ or $\gamma_1$ fraction of defective items
$\gamma_2$ fraction of items that can be reworked
$\gamma_3$ fraction of items to be rejected/scrapped
$\gamma\ (n)$ fraction of defectives in $n$th cycle of supplier's learning
$\varphi$ complement of the fraction of defectives $(1-\gamma\ )$
$\Delta_1$, $\Delta_2$ upper and lower limits of $\varphi$
$\gamma_s$ fraction of defective parts supplied by supplier $s$
$\gamma_{max}$ the highest fraction of defectives in suppliers' items
$\pi_s$ the fraction used to accommodate the leftovers before ordering parts of type $s$, in a cycle $(1-(\gamma_{max}-\gamma_s))$
$\gamma_{s0}$ fraction of defective parts acceptable to the vendor
$\gamma_{v0}$ fraction of defective products acceptable to the buyer
$D$ demand rate ($D_p$ for perfect items and $D_i$ for imperfect items)
$P$ production rate
$M$ production and inspection rate together
$M_R$ reworking rate
$M_D$ permissible delay from the buyer in settling accounts
$I_e$ interest which can be earned per \$ in a year
$I_k$ interest charges per \$ investment in inventory per year
$I_s$ cost per unit idle time for a supplier
$I_b$ cost per unit idle time for a vendor
$B$ maximum level of backorder allowed
$F$ vendor's transportation cost per batch
$R$ vendor's total production quantity
$\theta$ deterioration rate of an item
$P_p$ production rate of good items ($P_p=1-\gamma_1-\gamma_3$)
$\beta$ a ratio ($\beta=E[\gamma_1]/E[P_p]$)
$s$ unit selling price of items within the specification limits (non-defectives)
$Q$ lot size per cycle
$k$ decrease in range of purchasing cost per unit in case of discount
$Q_p$ order size in case of discount at purchasing cost $c$ per unit
$Q_s$ order size in case of discount with a decrease of $k$ per unit
$q$ stock level when the discount is offered
$x$ screening rate
$\sigma$ population standard deviation of quality
$\mu$ target quality characteristic
$\mu_s$ number of parts from supplier $s$ required for a product
$\rho$ fraction of scrap in the defective items
$r$ fraction of rework in the defective items
$T_b$ buyer's cycle time
$T_v$ vendor's cycle time
$n$ number of batches delivered in a cycle
$v$ unit selling price of defective items, $v<p$
$a$ cost of sending a batch of acceptable items to the working inventory warehouse
$d$ unit screening cost
$W$ vendor's warranty cost for defective items
$w$ unit rework cost
$cc$ production cost per unit for a vendor
$c$ production cost per year for a vendor
$z$ a random variable representing the number of non-defective items
$c_a$ cost of accepting a defective item.
$c_r$ cost of rejecting a non-defective item.
$c_S$ cost of disposing a scrap item
$c_L$ cost of lost sales per unit
$c_B$ cost of backorders per unit for a buyer
$c_{Ba}$ cost of backorders per unit for a buyer per unit time
$c_{Dv}$ deterioration cost per unit for a vendor
$c_{Db}$ deterioration cost per unit for a buyer
$t$ unit return cost in sales return
$t_s$ time for possible shortages/backordering
$c_g$ goodwill cost for items misclassified
$\Psi$ capacity of a buyer's own warehouse

### 3.1. EOQ/EPQ models for imperfect items

Goyal and Cárdenas-Barrón (2002) presented a simpler approach to the model of Salameh and Jaber (2000). They suggested computing the expected total annual revenue and the expected total annual cost as

$$E[TRU(Q)]=\{s(1-E[\gamma])+vE[\gamma]\}D \tag{8}$$

and,

$$E[TCU(Q)]=\frac{A_bD}{Q}+\frac{h_bQ}{2E\,[1/(1-\gamma)]} \tag{9}$$

This modifies the expected total annual profit in (6) as

$$E[TPU(Q)]=\{s(1-E[\gamma])+vE[\gamma]\}D-\frac{A_bD}{Q}-\frac{h_bQ}{2E\,[1/(1-\gamma)]} \tag{10}$$

The above expression can be used to compute the optimal order size as

$$Q^*=\sqrt{\frac{2A_bD}{h_b}E\left[\frac{1}{(1-\gamma)}\right]}=EOQ_{trad}\sqrt{E\left[\frac{1}{(1-\gamma)}\right]} \tag{11}$$

Examining (11) we note that that this lot size calculation is simpler and easier to implement than the lot size defined in the original model of Salameh and Jaber (2000). The robust nature of the simplified lot sizing model was demonstrated by direct comparison using data sets illustrating varying levels of the fraction of defectives. Results of the model comparisons indicated near zero differences in the annual profits of the modified and original models.

Konstantaras et al. (2007) extended the model in Salameh and Jaber (2000) along two dimensions. First, they assumed that the acceptable items are entered into work-in-process inventory in batches and not on unit-by-unit basis. Second, items that were defect free after completing rework were used to meet the demand.

In the first case, defectives are kept in stock until the end of inspection and are then sold as a single batch. The expected annual profit of a buyer in this case is given by

$$E[TPU(Q,n)]=D\left\{s-v+(v-d-p)E\left[\frac{1}{1-\gamma}\right]\right\}-\frac{1}{Q}\left\{(A_bD+naD)E\left[\frac{1}{1-\gamma}\right]\right\}$$
$$-\frac{Q}{2}\left\{h_b\left(1-E[\gamma]-\frac{2D}{x}\left(1-E\left[\frac{1}{1-\gamma}\right]\right)\right)+\frac{2D}{nx}\right\} \tag{12}$$

In the second case, imperfect quality items are kept in stock until the end of the inspection process, reworked and then enter the working inventory in a single batch. The expected annual profit of a buyer in this case is given by

$$E[TPU(Q,n)]=\{Ds-(d+p)D-E[\gamma]wD\}$$
$$-\left\{\frac{A_bD}{Q}+\frac{(n+1)aD}{Q}+\frac{DQh_b}{x}+\frac{Qh_b}{2}-\frac{(n-1)DQh_b}{nx}\right\} \tag{13}$$

Using data collected from the automotive industry, Jaber et al. (2008) extended the model in Salameh and Jaber (2000) and showed, using empirical data that the fraction of defective items in a supplier's shipment reduces in conformance with a learning curve. A logistic learning curve was used to capture the reduction in the number of defective units, the shipment size, and cost as a result of learning. The logistic curve was

$$\gamma(n)=\frac{a_1}{g+e^{b_1n}} \tag{14}$$

where $a_1$, $b_1$ and $g$ are the model fit parameters given as $a_1=70.067$, $b_1=0.7932$ and $g=819.76$. The model indicates that the number of defective units, the shipment size and the total cost of the buyer reduce as learning increases. The term $\gamma$ was replaced with $\gamma(n)$ and $Q$ with $Q_n$ in Eqs. (2) and (4), where $\gamma(1)>\gamma(2)>\ldots>\gamma(n)$ and $Q_1>Q_2>\ldots>Q_n$. Jaber et al. (2008) provided a closed form solution.

Maddah and Jaber (2008) mentioned that the expected annual profit in Salameh and Jaber (2000) is not accurate and that the exact annual profit could be calculated using the renewal–reward theorem. This implies that (5) be rewritten as

$$\mathrm{E}[TPU(Q)]=\frac{\mathrm{E}[TP(Q)]}{\mathrm{E}[T]} \tag{15}$$

and that the annual profit as defined in Salameh and Jaber (2000) be modified to

$$\mathrm{E}[TPU(Q)]=\frac{\{s(1-\mathrm{E}[\gamma])+v\mathrm{E}[\gamma]-p-d\}D-\frac{A_bD}{Q}-h_bQ\left\{\frac{(\mathrm{E}[(1-\gamma)^2])}{2}+\frac{\mathrm{E}[\gamma]D}{x}\right\}}{1-\mathrm{E}[\gamma]} \tag{16}$$

The above two equations are simpler and require less computational effort than those found in Salameh and Jaber (2000). In addition, Maddah and Jaber (2008) suggested consolidating the imperfect lots for $n_0$ cycles before disposing or salvaging them. Under this consolidation a transportation and holding cost for the imperfect items is now included and the expected annual cost becomes

$$\mathrm{E}[TCU(Q,n_0)]=\frac{\{(A_b+F_d/n_0)\,(D/Q)+h_bQ(\mathrm{E}[(1-\gamma)^2])/2-(2(n_0-1)/n_0)\mathrm{var}[\gamma]+(n_0-1)\mathrm{E}[\gamma](1-\mathrm{E}[\gamma])+(2\mathrm{E}[\gamma]D/x)\}}{1-\mathrm{E}[\gamma]} \tag{17}$$

where $F_d$ represents the transportation cost of defective items. Yoo et al. (2009) incorporated two types of inspection error and a sales return option for customers who receive defective products as a result of inspection errors committed by the vendor. Following the methodology of Salameh and Jaber (2000), annual profit in this model was defined as

$$\begin{aligned}\mathrm{E}[TPU(Q)]&=sD+h_v(cc+d)(D/M)Q-\frac{h_v(cc+d+t)}{2}Q\mathrm{E}[\gamma m_2]\\&-\left[(cc+d)+\frac{A_b}{Q}+\frac{h_v(cc+d)Q}{2M}\right]D\mathrm{E}\left[\frac{1}{\alpha}\right]-(s+t+c_g)D\mathrm{E}\left[\frac{\gamma m_2}{\alpha}\right]+vD\mathrm{E}\left[\frac{\delta}{\alpha}\right]\\&-(v+w)D\mathrm{E}\left[\frac{\delta r}{\alpha}\right]-\frac{h_vtQ}{2}\mathrm{E}\left[\frac{\delta\gamma m_2r^2}{\alpha}\right]-\frac{h_v(cc+d)Q}{2}\mathrm{E}\left[\frac{\varphi(\varphi+2\delta r)}{\alpha}\right]\\&-\frac{h_v}{2}\left[cc+d+w\left(1-\frac{D}{M_R}\right)\right]Q\mathrm{E}\left[\frac{\delta^2r^2}{\alpha}\right]\end{aligned} \tag{18}$$

where

$$\alpha=\varphi+\delta r,\quad \varphi=\gamma m_2+(1-\gamma)(1-m_1),\quad\text{and}\quad \delta=\gamma+(1-\gamma)m_1$$

The authors demonstrated that increasing the fraction of defectives and the Type I error rate may lead to an increase in the annual profit and concluded that managers need to make more careful decisions to control the production/inspection process.

El-Kassar (2009) generalized Salameh and Jaber (2000) by allowing the demand for both perfect and imperfect quality items to be continuous throughout the inventory cycle as opposed to only allowing imperfect quality in the incoming lots of the buyer. Their annual profit function was of the form

$$TP(Q)=sD_p+\frac{v(1-\gamma)D_p}{\gamma}-\frac{pD_p}{\gamma}-\frac{dD_p}{\gamma}-\frac{A_bD_p}{\gamma Q}$$

$$-\frac{h_bQ\{(1-\gamma)D_pD_i+(1-\gamma)(\gamma D-D_p)D_p+(\gamma D-D_p)^2\}}{2\gamma D_i^2} \tag{19}$$

Wahab and Jaber (2010) extended the model in Salameh and Jaber (2000) by introducing different values for the annual holding costs of the buyer for good items and for defective items. Two cases of learning in the supplier's quality level were studied under the different holding costs.

Case 1: No learning in supplier's quality

The optimal lot size in this case was given by

$$Q_1^*=\sqrt{\frac{2DA_b\mathrm{E}\,[1/(1-\gamma)]}{h_{bg}(1-\mathrm{E}[\gamma])+\frac{D}{x}(h_{bg}+h_{bd})\mathrm{E}\,[1/(1-\gamma)]-\frac{D}{x}(h_{bg}+h_{bd})}} \tag{20}$$

Case 2: With learning in supplier's quality

The optimal lot size in this case was given by

$$Q_n^*=\sqrt{\frac{2DA_b\mathrm{E}\,[1/(1-\gamma)]}{h_{bg}\{1-\gamma(n)\}+(D/x)\,(h_{bg}+h_{bd})\,\{1/1-\gamma(\mathrm{n})\}-(D/x)\,(h_{bg}+h_{bd})}} \tag{21}$$

The analyses presented showed that the lot size under different holding costs for non-defective and defective items increases with the fraction of defectives. In the case of learning in supplier's quality, incorporating different holding costs for perfect and imperfect items into the lot sizing decision led to larger lot sizes when the number of shipments was small. When the number of shipments was large, no difference in the lot size was observed thus suggesting that for a high volume of shipments, maintaining separate inventory holding costs terms for perfect and imperfect items is not necessary.

Khan et al. (in press-a) noted that the assumption of an error-free screening process in Salameh and Jaber (2000) is limiting and extended this model to include Type I and Type II errors in the inspection process. Hence, the number of defective items will be those perceived by the screening process. Under this extended model, the condition to avoid shortages as originally defined in Salameh and Jaber (2000) would now become

$$Q(1-\gamma)(1-m_1)\geq DT \tag{22}$$

where $m_1$ and $m_2$ represent the probabilities of Type I and Type II errors, respectively. So, for the limiting case, the cycle length can be written as

$$T=\frac{Q(1-\gamma)(1-m_1)}{D} \tag{23}$$

The expected annual profit in their model is given by

$$\begin{aligned}\mathrm{E}[TPU(Q)]&=sD+\frac{sD\mathrm{E}[\gamma]\mathrm{E}[m_2]}{(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])}+\frac{vD\mathrm{E}[m_1]}{(1-\mathrm{E}[m_1])}+\frac{vD\mathrm{E}[\gamma]}{(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])}\\&-\frac{D\left[\frac{A_b}{Q}+p+d+c_r(1-\mathrm{E}[\gamma])\mathrm{E}[m_1]+c_a\mathrm{E}[\gamma]\mathrm{E}[m_2]+\frac{h}{2}\left\{\left(\frac{2}{x}-\frac{D}{x^2}+\frac{\mathrm{E}[V^2]}{D}\right)Q\right\}\right]}{(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])}\\&-\frac{hQ\mathrm{E}[\gamma]\mathrm{E}[m_2]}{2}\end{aligned} \tag{24}$$

where

$$V=1-\frac{D}{x}-(m_1-\gamma)+\gamma(m_1+m_2)$$

Khan et al. (2010b) further extended the model in Salameh and Jaber (2000) in two directions. First, they assumed that the rate of screening for defectives follows a learning curve and that stockouts (which are treated as both lost sales and backorders) may occur when the screening rate is slower than the demand rate. Secondly, they accounted for transfer of knowledge in learning when the screening moves from one cycle to another in three possible scenarios: (*i*) no transfer of learning, (*ii*) complete transfer of learning, and (*iii*) partial transfer of learning. The total profit for the lost sales case in the *i*th cycle of partial transfer of learning was given by

$$\begin{aligned}\mathrm{E}[TP_{iL}] &= \{s(1-\mathrm{E}[\gamma])+v\mathrm{E}[\gamma]-p\}Q_i-A_b-c_LZ_i-\frac{h_b}{2D}\left\{Q_i^2\mathrm{E}\left[(1-\gamma)^2\right]\right.\\ &\quad \left.+2Q_iZ_i(1-\mathrm{E}[\gamma])+Z_i^2\right\}+\frac{h_bD}{2}t_{si}^2\\ &\quad -\frac{(h_bQ_i\mathrm{E}[\gamma]+d)\{(Q_i+u_i)^{1-b}-u_i{}^{1-b}\}}{x_1(1-b)}\\ &\quad -h_bQ_{si}t_{si}+h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)}t_{si}{}^{2-b/1-b}\end{aligned} \tag{25}$$

where $Z_i{=}b/1-bQ_{si}, t_s{=}t_{si}{=}Q_{si}/D(1-b)$, and $\tau=\tau_i=Q_i^{1-b}/(1-b)x_1>0$, where $Q_{si}{=}(D/x_1)^{1/b}$. The subscript '1' in the screening rate represents the first unit. For the backorder case, this total profit was

$$\begin{aligned}\mathrm{E}[TP_{iB}] &= \{s(1-\mathrm{E}[\gamma])+v\mathrm{E}[\gamma]-p\}Q_i-A_b-c_B\left(t_X-\frac{t_{si}}{2}\right)B\\ &\quad -\frac{h_b}{2D}\left\{Q_i^2\mathrm{E}\left[(1-\gamma)^2\right]\right\}+\frac{h_bD}{2}t_{si}^2-\frac{\{hQ_i\mathrm{E}[\gamma]+d\}\{(Q_i+u_i)^{1-b}-u_i{}^{1-b}\}}{x_1(1-b)}\\ &\quad -h_bQ_{si}t_{si}-h_bt_XB+h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)}t_{si}{}^{2-b/(1-b)}\end{aligned} \tag{26}$$

They showed that the total transfer of learning gives better results for both the lost sales and backorder cases. It was also found that an increase in the unit screening cost reduces the annual profit to a great extent at the slower rates of learning.

Chang and Ho (2010b) applied the renewal reward theorem to simplify the profit function of the first case discussed by Konstantaras et al. (2007) to present a simpler profit function, which is given by

$$f(n|Q^*)=2\sqrt{\frac{Dh_b(A_b+nv)}{\mathrm{E}[1-\gamma]}\left\{\frac{D}{x}\left[\frac{1}{\mathrm{E}[1-\gamma]}-\frac{n-1}{n}\right]+\frac{\mathrm{E}[(1-\gamma)^2]}{2\mathrm{E}[1-\gamma]}\right\}} \tag{27}$$

where

$$Q^*=\sqrt{\frac{2D(A_b+nv)}{h_b((2D/x)\,[\mathrm{E}[\gamma]+\mathrm{E}[1-\gamma]/n]+\mathrm{E}[(1-\gamma)^2])}} \tag{28}$$

and the optimal number of acceptable batches was found by

$$f((n^*-1)|Q^*)\geq f((n^*)|Q^*)\leq f(|(n^*+1)|Q^*)$$

The above presented works investigated lot sizing decisions in a supply chain from the perspective of a buyer.

The extensions of the model in Salameh and Jaber (2000) that were surveyed in this section assumed shortages (backordering or lost sales) not to be allowed. The following section examines extensions to the model of Salameh and Jaber (2000) where shortages are allowed.

### 3.2. Shortages backordering

Rezaei (2005) extended the model in Salameh and Jaber (2000) by assuming that shortages in a cycle resulting from defective items are completely backordered in the beginning of each cycle. Their model determined an optimal size of the order and the backorder, i.e. $Q$ and $B$, respectively. The annual profit of the buyer in a cycle for their model is given by

$$\begin{aligned}\mathrm{E}[TPU(Q,B)] &= D\left(s-v+\frac{h_bQ}{x}\right)+D\left(v-\frac{h_bQ}{x}-p-d-\frac{A_b}{Q}\right)\mathrm{E}\left[\frac{1}{1-\gamma}\right]\\ &\quad -\left[h_b\left\{\frac{Q(1-\mathrm{E}[\gamma])}{2}-B\right\}+\frac{B^2(c_B+h_b)}{2Q(1-\mathrm{E}[\gamma])}\right]\end{aligned} \tag{29}$$

Yu et al. (2005) extended the model in Salameh and Jaber (2000) to include product deterioration and partial backordering in the production process. They considered a ratio of the backorder and lost sales costs in their model and used an iterative methodology to determine the lower bound of this ratio in support of a feasible profit function. The annual cost of a buyer in their model is

$$\begin{aligned}TCU(T,B) &= \frac{A_b}{T}+\left(p+d+c_{Db}+\frac{h_b}{\theta}\right)\\ &\quad\times\left[BD\gamma+D+\left\{\frac{\gamma\theta D^2}{x}+\frac{\theta D}{2}(1-\gamma)^2\right\}T+\left(\frac{\gamma\theta^2D^3}{2x^2}\right)T^2\right]\\ &\quad-\left(c_{Db}+\frac{h_b}{\theta}\right)(BD\gamma+D)+\frac{c_BDB\gamma^2T}{2}+(1-B)c_LD\gamma\end{aligned} \tag{30}$$

Papachristos and Konstantaras (2006) revisited the work of Salameh and Jaber (2000) and Chan et al. (2003) and showed that the conditions given in these models are not enough to ensure non-shortages. They identified an inconsistency in (2) whereby a random variable was equated with a deterministic number and suggested a more accurate representation of the form

$$\mathrm{E}[\gamma]\leq 1-\frac{D}{x} \tag{31}$$

which is similar to the condition used by Chan et al. (2003). Despite this correction there is no assurance that shortages could be eliminated. Therefore, they proposed a better condition given by

$$z\geq D \tag{32}$$

Papachristos and Konstantaras (2006) further argued that Salameh and Jaber (2000) assumes that defective items are sold at the end of a cycle while their model represents a situation where the defective lot is sold at the end of screening time. Thus, they proposed adding the component $\gamma Q(T-t)$ to the holding cost of the defective lot in Salameh and Jaber (2000). They noticed that with these modifications, the change in the lot size and the expected annual profit is quite minimal because the ordering cost in Salameh and Jaber (2000) is quite higher than the holding cost. Although Salameh and Jaber (2000) were not explicit about this point, the Figure 1 (p. 61) in their article, that depicts the behavior of inventory over a cycle, shows that imperfect quality items are withdrawn by the end of the screening period and "not" by the end of the cycle.

Wee et al. (2006) extended the model in Salameh and Jaber (2000) by allowing product deterioration to exist in a joint buyer–vendor integrated lot sizing model. The product in the model was characterized by an *on-going* deterioration with partial backordering and imperfect quality. Shortages due to imperfect items were completely backordered. The vendor's annual cost in their model was

$$TC_v=\frac{A_v}{T_v}+\frac{nF}{T_v}+\left(\frac{R-nQ}{T_v}\right)\left(\frac{h_v}{\theta}+c_{D_v}\right) \tag{33}$$

And the buyer's annual cost was

$$TC_b=\frac{A_b}{T_b}+\left(p+d+c_{Db}+\frac{h_b}{\theta}\right)\frac{Q}{T_b}-\left(c_{Db}+\frac{h_b}{\theta}\right)(D\gamma+D)+\frac{c_BD\gamma^2T_b}{2} \tag{34}$$

Imperfect quality, deterioration and complete backordering were shown to significantly affect the total annual cost resulting from the optimization of the joint vendor–buyer cost functions. They also showed that the buyer can use the model to select vendors (suppliers) based on their fraction of defectives and the deterioration rate of the products supplied.

Wee et al. (2007) also extended the model in Salameh and Jaber (2000) for the case where shortages are backordered in each cycle. The resulting cost function, which is similar to (29), is

$$E[TPU(Q,B)] = D\left(s-v+\frac{h_bQ}{x}\right)+\left(B-\frac{Q}{2}\right)h_b+\frac{h_bQ}{x}\mathrm{E}[\gamma]$$
$$+\left[D\left(v-p-d-\frac{h_bQ}{x}-\frac{A_b}{Q}\right)-\frac{(c_B+h_b)B^2}{2Q}\right]\mathrm{E}\left[\frac{1}{1-\gamma}\right] \tag{35}$$

They also showed that the rate of change in their annual profit, as compared to that in Salameh and Jaber (2000), decreases with an increasing value of the unit backordering cost.

Eroglu and Ozdemir (2007) extended the model in Salameh and Jaber (2000) by allowing shortages to be backordered. They suggest that part of the good quality items in each cycle is used to eliminate backorders. The annual profit in their model is given by

$$\mathrm{E}[TPU(Q)] = sD+\frac{vD(1-\rho)\mathrm{E}[\gamma]}{\mathrm{E}_1}-\frac{D(p+d+c_S\rho\mathrm{E}[\gamma])}{E_1}-\frac{A_bD}{Q\mathrm{E}_1}-\frac{h_b\mathrm{E}_4Q}{2\mathrm{E}_1}$$
$$+hB-\frac{(h_b+c_{Ba})\mathrm{E}_2B^2}{2Q\mathrm{E}_1} \tag{36}$$

where

$$\mathrm{E}_1 = 1-E[\gamma],\ \mathrm{E}_2=\mathrm{E}\left[\frac{1-\gamma}{1-\gamma-D/x}\right],\ \mathrm{E}_3=\mathrm{E}\left[(1-\gamma-D/x)^2\right],$$
$$\mathrm{E}_4=\frac{D(2-D/x)}{x}+\mathrm{E}_3$$

The optimal order and the backorder quantity are,

$$Q^* = \sqrt{\frac{2A_bD}{h_b(\mathrm{E}_4-h_b\mathrm{E}_1^2/((h_b+c_{Ba})\mathrm{E}_2))}} \tag{37}$$

$$B^* = \frac{h_b\mathrm{E}_1Q^*}{(h_b+c_{Ba})\mathrm{E}_2} \tag{38}$$

They also examined how the effects of different levels of the fraction of defective items impacts the lot size and expected total profit and showed that the optimal total profit per unit time decreases when defective and scrap rates increase individually.

Chang and Ho (2010a) revisited the work of Wee et al. (2007) and adopted the suggestion of Maddah and Jaber (2008) to use renewal reward theorem to derive the expected profit per unit time for their model. Exact closed-form solutions were derived for the optimal lot size, backordering quantity and maximum expected profit. The annual profit function in their simplified model was given by

$$f(B,Q) = \frac{(h_b+c_B)\mathrm{E}_1}{2Q}\left[B-\frac{h_bQ}{(h_b+c_B)\mathrm{E}_1}\right]^2$$
$$+\frac{h_bQ}{2\mathrm{E}_1}\left[\mathrm{E}_1\mathrm{E}_2+\frac{2D\mathrm{E}_1\mathrm{E}_3}{x}-\frac{h_b}{(h_b+c_B)}\right]+\frac{DA_b\mathrm{E}_1}{Q} \tag{39}$$

with

$$B=\frac{h_bQ}{(h_b+c_B)\mathrm{E}_1}\quad\text{and}\quad Q=\sqrt{\frac{2DA_b}{h_b\{\mathrm{E}_2/\mathrm{E}_1+2D\mathrm{E}_3/(x\mathrm{E}_1)-h_b/((h_b+c_B)\mathrm{E}_1^2)\}}}$$

where

$$\mathrm{E}_1=\frac{1}{\mathrm{E}[1-\gamma]},\ \mathrm{E}_2=\frac{\mathrm{E}[(1-\gamma)^2]}{\mathrm{E}[1-\gamma]},\ \mathrm{E}_3=\frac{\mathrm{E}[\gamma]}{\mathrm{E}[1-\gamma]}$$

Roy et al. (in press) revisited the model in Salameh and Jaber (2000) for the case where a buyer's cycle starts with shortages that may have occurred due to lead time or labor problems. The fraction of the demand in this stock-out period that varies with time was back ordered while the time invariant demand was left as lost sale.

$$\mathrm{E}[TPU(Q)] = \frac{1}{(\mathrm{E}_1Q-B)/D+t_s}\left[\left(s-v+\frac{h_bB}{D}\right)\mathrm{E}_1Q+(v-p-d)Q-\frac{h_b\mathrm{E}_2Q^2}{2D}\right.$$
$$\left.-\frac{1}{2D}\left\{2KD+h_bB^2+\left(2D^2/\delta^2\right)((c_B-c_L\delta)(1-e^{-\delta t_s})-\delta t_s(c_Be^{-\delta t_s}-c_L\delta))\right\}\right] \tag{40}$$

with

$$\mathrm{E}_1 = 1-\mathrm{E}[\gamma]\quad\text{and}\quad \mathrm{E}_2=\mathrm{E}[(1-\gamma)^2]$$

Their analysis showed that the buyer does not face any financial or goodwill loss if the shortage period is very small.

Maddah et al. (2010) made a simple modification to the model of Salameh and Jaber (2000) to avoid shortages. They assumed that an order is placed when the inventory level is just enough to meet the demand during the screening period. This was named as order-overlapping. The demand during the screening period is met from the items from the previous order. The expected annual profit in this case, was given by

$$\mathrm{E}[TPU(Q)] = \frac{\{s(1-\mathrm{E}[\gamma])+v\mathrm{E}[\gamma]-p-d\}D-\frac{A_bD}{Q}-\left(\frac{h_bQ}{2}\right)\left(\mathrm{E}\left[(1-\gamma)^2\right]+\frac{2D}{x}\right)}{1-\mathrm{E}[\gamma]} \tag{41}$$

They showed that their model leads to a negligible loss of profitability as compared to that in Salameh and Jaber (2000) in exchange for an improved service level to avoid shortages.

There are some works that generalized the model of Salameh and Jaber (2000) to allow for other options besides just salvaging them at a price. They considered that a fraction of these defective items are scrapped at a cost, while others are reworked and kept in inventory. Other studies modified the work of Salameh and Jaber (2000) by calculating the fraction of defectives as percentage of items that will fall outside the quality acceptance limits. These works are discussed in the next section.

### 3.3. Quality

Chan et al. (2003) integrated lower pricing, rework and reject situations into a single economic production quantity (EPQ) model. It was assumed that rework items can be kept in stock at a cost. Similar to Salameh and Jaber (2000), defective items that are disposed are sold at a discounted price under the following three cases: (*i*) when the defective item is identified, (*ii*) at the end of production and (*iii*) just before the start of the next cycle. The EPQ that maximized expected total profit for each of the scenarios was determined. To ensure no shortages they assumed that

$$\mathrm{E}[1-\gamma_1-\gamma_3] \geq \frac{D}{P} \tag{42}$$

Case 1: Imperfect items sold as identified and not kept in stock

The expected total profit in this case was given by

$$\mathrm{E}[TP_1(Q)]$$
$$= Ds+Dv\beta-\left\{\frac{h_bQ[\mathrm{E}[P_P]-D/P]}{2}+\left[\frac{A_b}{Q}+c_{rj}\mathrm{E}[\gamma_3]+c_{rw}\mathrm{E}[\gamma_2]+d+cc\right]\left[\frac{D}{\mathrm{E}[P_P]}\right]\right\} \tag{43}$$

which gives an optimal production size as

$$Q_1^* = \sqrt{\frac{2DA_b}{h_b\left(\mathrm{E}[P_P]-D/P\right)\mathrm{E}[P_P]}} \tag{44}$$

Case 2: Imperfect items sold at the end of production period

The expected total profit in this case was given by

$$\mathrm{E}[TP_2(Q)] = Ds+Dv\beta-\left\{\frac{h_bQ[\mathrm{E}[P_P]-(D/P)\ (1-\beta)]}{2}\right.$$

$$+\left[\frac{A_b}{Q}+c_{rj}\mathrm{E}[\gamma_3]+c_{rw}\mathrm{E}[\gamma_2]+d+cc\right]\left[\frac{D}{\mathrm{E}[P_P]}\right]\Bigg\} \tag{45}$$

which gives an optimal production size as

$$Q_2^*=\sqrt{\frac{2DA_b}{h_b(\mathrm{E}[P_P]-(D/P)\,(1-\beta))\mathrm{E}[P_P]}} \tag{46}$$

Case 3: Imperfect items sold at the end of cycle

The expected total profit in this case was given by

$$\mathrm{E}[TP_3(Q)]=Ds+Dv\beta-\Bigg\{\frac{h_bQ[\mathrm{E}[P_P]+2\mathrm{E}[\gamma_1]-(D/P)\,(1+\beta)]}{2}$$
$$+\left[\frac{A_b}{Q}+c_{rj}\mathrm{E}[\gamma_3]+c_{rw}\mathrm{E}[\gamma_2]+d+cc\right]\left[\frac{D}{\mathrm{E}[P_P]}\right]\Bigg\} \tag{47}$$

which gives an optimal production size as

$$Q_3^*=\sqrt{\frac{2DA_b}{h_b(\mathrm{E}[P_P]+2\mathrm{E}[\gamma_1]-(D/P)\,(1+\beta))\mathrm{E}[P_P]}} \tag{48}$$

They noticed that the lot size tends to decrease if the defective items are kept in stock for long.

Tsou (2007) extended the model in Salameh and Jaber (2000) by assuming that the quality of a lot follows a normal distribution function. Items failing to meet specifications were scrapped while those items within the specification limits were sold with a discount based on a cost of poor quality as determined by Taguchi's loss function. The quality distribution function was defined as

$$Y(X)=\frac{1}{\sigma\sqrt{2\pi}}e^{(X-\mu)^2/2\sigma^2},\ \int_{-\infty}^{\infty}Y(X)dX=1 \tag{49}$$

The cost of poor quality with Taguchi's loss function was given by

$$L(X)=G(X-\mu)^2;\ LSL\le X\le USL \tag{50}$$

where

$LSL=p;\ X\le LSL,\ X\ge USL$

$G=p/U^2;\ U=(USL-\mu)=(\mu-LSL)$

The total revenue per cycle was computed using Taguchi's loss concept such as

$$TR(Q)=Q\int_{LSL}^{USL}Y(X)(s-L(X))dX=Qs\int_{LSL}^{USL}Y(X)dX-Q\int_{LSL}^{USL}Y(X)L(X)dX \tag{51}$$

which can be written as

$$TR(Q)=Qs(1-\gamma)-Q\int_{LSL}^{USL}\frac{1}{\sigma\sqrt{2\pi}}e^{(X-\mu)^2/2\sigma^2}\mathrm{x}\cdot G(X-\mu)^2dX \tag{52}$$

The total cost per cycle from Salameh and Jaber (2000) was used to calculate the total annual profit. The optimal order quantity from this concave function was given as

$$Q^*=\sqrt{\frac{DA_b}{h_b(1-\gamma)^2/2+\gamma h_bD/x}} \tag{53}$$

Tsou et al. (2009) follow the study of Salameh and Jaber (2000) and Chan et al. (2003) to develop an EPQ model with *continuous* quality characteristic and rework. That is

$$\int_{-\infty}^{\infty}f(x)=1 \tag{54}$$

and

$$\gamma_1=\int_{LSL_2}^{LSL_1}f(x)dx+\int_{USL_1}^{USL_2}f(x)dx,\qquad \gamma_2=\int_{USL_2}^{\infty}f(x)dx$$
$$\gamma_3=\int_{-\infty}^{LSL_2}f(x)dx \tag{55}$$

with this assumption, they revisited the model of Chan et al. (2003) for the case where imperfect items are sold as identified and are not kept in stock.

### 3.4. Supply chains

Huang (2002) extended the model in Salameh and Jaber (2000) to present an integrated inventory policy for a vendor and a buyer where an order is replenished by a vendor in $n$ equal sized shipments. A transportation cost $F$ was introduced on the part of buyer while the vendor incurs a warranty cost $v$ for the defective items withdrawn by the buyer. The total annual cost that was minimized to find $Q$, the size of one shipment, is

$$TC(Q,n)=\left\{\frac{(A_v+A_b)D}{nQ}+\frac{FD}{Q}+(d+v)D+\frac{(h_b-h_v)DQ}{x}\right\}\mathrm{E}\left[\frac{1}{1-\gamma}\right]-vD$$
$$+\left\{\frac{DQ}{P}+\frac{nQ}{2}\left(1-\frac{D}{P}\right)\right\}h_v-\frac{(h_b-h_v)DQ}{x}+\frac{Q(1-\mathrm{E}[\gamma])}{2}(h_b-h_v) \tag{56}$$

The optimal lot size quantity that minimize Eq. (56) as a function of $n$ was given as

$$Q^*=\sqrt{\frac{2D\{(A_v+A_b)/n+F\}\mathrm{E}\,[1/(1-\gamma)]}{\{2D(h_b-h_v)/x\}\{\mathrm{E}\,[1/(1-\gamma)]-1\}+(\frac{2D}{P}+n(1-D/P))h_v+(1-\mathrm{E}[\gamma])(h_b-h_v)}} \tag{57}$$

Hill (1997) had studied a single vendor single buyer system. He presented a generalized inventory policy where each successive shipment within a production batch increases by a fixed factor. An equal-sized shipment would be a special case of this policy. Goyal et al. (2003) used the approach in Hill (1997) and Goyal and Cárdenas-Barrón (2002) to extend the model in Salameh and Jaber (2000) for developing a model for a vendor–buyer supply chain. Their total expected annual cost was

$$ETC(Q,n)=\frac{(A_v+A_b+nF)D}{Q}+\frac{Q}{2n}\left\{[1+(n-2)(1-D/P)]h_v+\frac{h_b}{\mathrm{E}[1/(1-\gamma)]}\right\} \tag{58}$$

They followed the same methodology as in Huang (2002) to find the optimal number of shipments per cycle. Their optimal shipment size was

$$Q^*=\sqrt{\frac{2n(A_v+A_b+nF)D}{\{[1+(n-2)(1-D/P)]h_v+h_b/E[1/(1-\gamma)]\}}} \tag{59}$$

They also carried out a sensitivity analysis of their model for the fraction of defectives, the transportation cost, the demand size and the holding costs.

Huang (2004) reformulated the model in Goyal et al. (2003) but they used the work of Salameh and Jaber (2000) to model the annual cost of the buyer. The expected annual cost of the supply chain in their model was

$$ETC(Q,n)=\left\{\frac{(A_v+A_b)D}{nQ}+\frac{FD}{Q}+(d+v)D-\frac{(n-2)DQh_v}{2P}+\frac{DQh_b}{x}\right\}\mathrm{E}\left[\frac{1}{1-\gamma}\right]-vD$$
$$+\frac{(n-1)Q}{2}h_v-\frac{DQ}{x}h_b+\frac{Q(1-\mathrm{E}[\gamma])}{2}h_b \tag{60}$$

and the shipment size in their model was

$$Q(n)=\sqrt{\frac{2D\{(A_v+A_b)/n+F\}\mathrm{E}[1/(1-\gamma)]}{\{2Dh_b/x-(n-2)Dh_v/P\}\mathrm{E}[1/(1-\gamma)]+(n-1)h_v-\frac{2D}{x}h_b+(1-\mathrm{E}[\gamma])h_b}} \tag{61}$$

Khan et al. (in press-b) extended the model in Salameh and Jaber (2000) for a two level multi-supplier, single-vendor supply chain where a vendor is supposed to ask for a number of components from different suppliers, which are needed to make a single product. Suppliers are believed to be providing a certain fixed fraction of defectives in their supplies. They studied two coordination mechanisms as in Khouja (2003), i.e. equal cycle time and integer multiplier cycle time. The total cost for the equal cycle time mechanism was given by

$$TC = \frac{D(A_v + \sum_{s=1}^{m} A_s)}{Q} + \frac{cD}{P} + \frac{h_{v2}Q}{2}\left(1-\frac{D}{P}\right) + D\sum_{s=1}^{m}\left[(a_{vs}+d_s)\mu_s\pi_s + \frac{h_{v1s}Q}{2}\left\{\frac{u_s^2\pi_s^2(1+\gamma_s)}{x} + \left(\frac{\mu_s\pi_s}{P}\right)\left(1-\gamma_s-\frac{D}{x}\right)\right\}\right] + \sum_{s=1}^{m} h_{v1s}Q\mu_s(1-\pi_s)(1-D/P) \tag{62}$$

and for integer multiplier mechanism, this cost was

$$TC = \frac{D(A_v + \sum_{s=1}^{m} A_s/K_s)}{Q} + \frac{cD}{P} + \frac{Q}{2}\left[h_{v2}\left(1-\frac{D}{P}\right) + \sum_{s=1}^{m}(K_s-1)h_s\mu_s\right] + D\sum_{s=1}^{m}\left[(a_{vs}+d_s)\mu_s\pi_s + \frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\pi_s^2(1+\gamma_s)}{x} + \left(\frac{\mu_s\pi_s}{P}\right) \times \left(1-\gamma_s-\frac{D}{x}\right)\right\}\right] + \sum_{s=1}^{m} h_{v1s}Q\mu_s(1-\pi_s)(1-D/P) \tag{63}$$

They showed that integer multiplier mechanism performs better as it is a relaxed and practical approach of coordination in a supply chain.

They further extended this model from a human factors perspective that involves: (*i*) errors in screening, (*ii*) learning in vendor's production process, and (*iii*) learning in supplier's quality. The total cost of the supply chain for the case of learning during production, was given as

$$TC = \frac{A_v D}{Q} + \sum_{s=1}^{m}\left(\sqrt{2A_s D h_s \mu_s} - \frac{Qh_s\mu_s}{2}\right) + h_{v2}\left[\frac{Q}{2} - \frac{T_1 D Q^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right] + D\sum_{s=1}^{m}\left[\frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\pi_s^2(1+\gamma_s)}{x} + \frac{Q^{-b_i}\mu_s\pi_s T_1\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)} \times \left(1-\gamma_s-\frac{D}{x}\right)\right\} + (a_{vs}+d_s)\mu_s\pi_s\right] + \sum_{s=1}^{m} h_{v1s}Q\mu_s(1-\pi_s)(1-D/P) + \frac{cT_1 D Q^{-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)} \tag{64}$$

They showed that screening errors tend to reduce the carrying cost of the vendor and thus the overall annual cost of the supply chain. The annual cost of the supply chain was found to decrease as a result of learning in the production component of the supply chain. As a result of learning, improvement in the supplier's fraction of defectives improved the system to a state where there are no defectives from the suppliers. Khan and Jaber (2011) explored the model in Khan et al. (in press-b) to optimize cycle time for three coordination mechanisms.

Khan et al. (2010a) studied a vendor–buyer supply chain and extended the model in Huang (2002) to include errors in buyer's screening process and learning in vendor's production process. The expected total cost for this supply chain with screening errors is given by

$$\mathrm{E}[TCU(Q,n)] = \frac{D}{(1-\mathrm{E}[\gamma_e])Q}\left\{\frac{A_v}{n} + \frac{A_b}{n}\right\} + \frac{D}{(1-E[\gamma_e])}\left(d+\frac{c}{P}\right) + \frac{Q}{2}\left[\frac{h_v}{(1-E[\gamma_e])}\left\{(n-1)-(n-2)\frac{D}{P}\right\} + h_b\{(1-\mathrm{E}[\gamma_e])+\}\frac{2D\mathrm{E}[\gamma_e]}{x(1-\mathrm{E}[\gamma_e])}\right] \tag{65}$$

when learning is incorporated into the vendor's production process, this annual cost is

$$\mathrm{E}[TCU_i] = \frac{D}{(1-\mathrm{E}[\gamma])Q}\left\{\frac{A_v}{n} + A_r\right\} + \frac{dD}{(1-\mathrm{E}[\gamma])} + \frac{h_v Q^{1-b}}{P(1-\mathrm{E}[\gamma])(1-b)}\left[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b} - \frac{n^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{(2-b)}\right] + \frac{h_v(n-1)Q}{2(1-E[\gamma])} + \frac{c(nQ)^{-b}\{i^{1-b}-(i-1)^{1-b}\}}{P(1-\mathrm{E}[\gamma])(1-b)} + \frac{h_r Q(1-\mathrm{E}[\gamma])}{2} + \frac{h_r\mathrm{E}[\gamma]QD}{x(1-\mathrm{E}[\gamma])} \tag{66}$$

The optimal values of batch size and the number of shipments per cycle were determined through an iterative procedure. They noticed that the more non-defective items are misclassified the higher is the order size and so is the annual cost of the supply chain. Besides, learning in production process brought a substantial saving for the supply chain.

Khan et al. (2010c) again discussed a vendor–buyer supply chain but for a relationship that uses consignment stocking policy. Their annual cost was

$$\mathrm{E}[TC] = \frac{DM_{2e}}{Q}\left(\frac{A_v}{n} + A_b\right) + dDM_2 + \frac{QM_{2e}}{2}\left[\frac{h_v D}{P} + (h_b + h_v')\left\{(1-M_{1e})\left(n+\frac{D}{P}\right) + D\left(\frac{2M_1}{x} - \frac{n}{P}\right)\right\}\right] \tag{67}$$

where

$$M_1 = \mathrm{E}[\gamma], \quad M_{2e} = \mathrm{E}\left[\frac{1}{1-\gamma}\right]$$

Again, an iterative procedure was used to determine the optimal batch size and the number of shipments. They showed that the fraction of defectives has a sharp impact on the annual cost of the supply chain. In comparison to the results of Khan et al. (2010a), it was observed that the vendor's holding cost is the major consideration affecting model performance in terms of annual cost.

Khan and Jaber (2009) extended the model in Salameh and Jaber (2000) for a three level supply chain where at each stage of the chain the supplier and the vendor use a 100% inspection process to screen out defective items in their lots. The three stakeholders were assumed to follow an agreement that they will bear a penalty cost per part/product once the fraction of defectives they send, goes beyond a set value. The annual cost of the supply chain in their model was given by

$$\mathrm{E}[TC_i] = \frac{D}{Q(1-\mathrm{E}[\gamma_v])}\left(\frac{A_s}{n_s n_v} + A_b + \frac{A_v}{n_v}\right) + \frac{Q}{(1-\mathrm{E}[\gamma_v])}\left(\frac{n_v(1-\mathrm{E}[\gamma_v])(n_s-1)}{2}\sum_{s=1}^{m} h_s\mu_s + h_b\left\{\frac{(1-\mathrm{E}[\gamma_v])^2}{2} + \frac{\mathrm{E}[\gamma_v]D}{x_b}\right\} + n_v D\sum_{s=1}^{m} h_{v1s}\left\{\frac{\mu_s(1-\mathrm{E}[\gamma_s])}{2P} + \frac{\mathrm{E}[\gamma_s]\mu_s^2}{x_s}\right\} + \frac{h_{v2}}{2}\left[(n_v-1) - \frac{(n_v-2)D}{P}\right]\right) + \frac{D}{Q(1-\mathrm{E}[\gamma_v])}\left\{\alpha_1\sum_{s=1}^{m}\mu_s(\mathrm{E}[\gamma_s]-\mathrm{E}[\gamma_{s0}]) + d_b + (a+d_v)\sum_{s=1}^{m}\mu_s + \beta_1\sum_{s=1}^{m}(\mathrm{E}[\gamma_v]-\mathrm{E}[\gamma_{v0}])\right\} \tag{68}$$

where $\alpha_1$ and $\beta_1$ are the penalty costs per item from the supplier and the vendor, respectively. Sana (2011) used the methodology in Salameh and Jaber (2000) in their model of a three level supply chain to screen out the defective items/products from the supplier and vendor when the vendor's production rate is not fixed.

An equal cycle time was assumed for all the stages of the supply chain and inventory lots were sent in unequal shipments to the downstream stage as soon as they are available or are produced. The annual profit of the supplier, vendor and buyer in their model was given by

$$APS=\frac{1}{T}\left[s_s(1-\gamma_s)Q_s+v_s\gamma_sQ_s-A_s-h_s\left\{\frac{(1-\gamma_s)^2{Q_s}^2}{2P}+\frac{\gamma_s{Q_s}^2}{x_s}\right\}-(d_s+p_s)Q_s -I_s\left(T-\frac{(1-\gamma_s)Q_s}{P}\right)\right] \quad (69)$$

$$APV=\frac{1}{T}\left[\left\{s_v+v_v\left(\frac{\gamma_v}{1-\gamma_v}\right)\right\}D_bT_p-A_v-d_v\frac{D_bT_p}{1-\gamma_v}-\frac{h_v}{2}\left\{1-\frac{D_b}{(1-\gamma_v)P}\right\}D_b{T_p}^2 -h_v\left\{\frac{D_bT_p\gamma_v}{2P(1-\gamma_v)^2}+\frac{P\gamma_v}{x_v(1-\gamma_v)}\right\}D_bT_p-\frac{cD_bT_p}{1-\gamma_v}-I_v(T-T_s)\right] \quad (70)$$

$$APB=\frac{1}{T}\left[s_bDT-A_b-s_vDT-\frac{1}{2}h_bD\left(1-\frac{D}{D_b}\right)T^2\right] \quad (71)$$

respectively. The subscripts $s$, $v$ and $b$ represent the parameters for the supplier, the vendor and the buyer, respectively.

Some researchers suggested using fuzzy set theory (Zadeh, 1965) when imprecise information is available or there is ambiguity about setting the values of a model's parameters (e.g., demand, lead-time, inventory-related costs). This theory has been widely used and applied in inventory management. Several studies have fuzzified the model of Salameh and Jaber (2000) and its extensions. These models are discussed in the next section. Interested readers may refer to Guiffrida and Nagi (1998) and Guiffrida (2009) for reviews on the application of fuzzy set theory in production and inventory management.

### 3.5. *Fuzzy set theory*

When historical data on the defective rate is unavailable to parameterize $f(\gamma)$, fuzzy set theory can be used to represent the defective rate in terms of linguistic descriptions of the defect rate. Chang (2004) reformulated the EOQ model of Salameh and Jaber (2000) to capture the uncertainty in the defective rate using fuzzy set theory. In this extension, the complement of the defect rate, $\varphi=1-\gamma$, is represented as a triangular fuzzy number $\tilde{\varphi}$ of the form $\tilde{\varphi}=(\varphi-\Delta_1,\varphi,\varphi+\Delta_2)$. The parameters $\Delta_i(i=1,2)$ are determined by the decision maker's linguistic interpretations of the magnitude of the complement of the defective rate and are constrained as $0<\Delta_1<\varphi$ and $0<\Delta_2<1-\varphi$. The resulting fuzzy total profit per unit time was defined as

$$TP(Q)=D\left(s-v+\frac{h_bQ}{x}\right)-\left[\frac{D}{2}\left(\frac{h_bQ}{x}+\frac{A_b}{Q}+p+d-v\right) \left\{\frac{1}{\Delta_1}\ln\left(\frac{\varphi}{\varphi-\Delta_1}\right)-\frac{1}{\Delta_2}\ln\left(\frac{\varphi}{\varphi+\Delta_2}\right)\right\}+\frac{h_bQ}{2}\left(\varphi+\frac{\Delta_2-\Delta_1}{4}\right)\right] \quad (72)$$

which leads to the optimal lot size in the fuzzy sense as

$$Q^*=\sqrt{\frac{DA_b\left\{(1/\Delta_1)\ln(\varphi/(\varphi-\Delta_1))-\frac{1}{\Delta_2}\ln(\varphi/(\varphi+\Delta_2))\right\}}{h_b[(\varphi+(\Delta_2-\Delta_1)/4)+(D/x)\{(1/\Delta_1)\ln(\varphi/(\varphi-\Delta_1))-(1/\Delta_2)\ln(\varphi/(\varphi+\Delta_2))-2\}]}} \quad (73)$$

The fuzzy lot size defined by (74) reduces to the crisp lot size of Salameh and Jaber (2000) when $\varphi$ and $\Delta_i$ reach their limiting values

Ouyang et al. (2006) further extended the integrated vendor–buyer lot sizing model of Huang (2002) with imperfect quality by treating the defective rate as the triangular fuzzy number $\tilde{\gamma}=(\gamma-\Delta_1,\gamma,\gamma+\Delta_2)$ where $0<\Delta_1,\ <\gamma$ and $0<\Delta_2,\ <1-\gamma$. The resulting fuzzy joint total expected cost per unit time is

$$TP(Q,n)=Q[K(n)+Z(n)-R(n)]-DW-QK(n)\left(\gamma+\frac{\Delta_2-\Delta_1}{4}\right) +\left\{\frac{H(n)}{Q}+QR(n)+Dd+DW\right\}\left\{\frac{1}{2\Delta_1}\ln\left(\frac{1-\gamma+\Delta_1}{1-\gamma}\right) -\frac{1}{2\Delta_2}\ln\left(\frac{1-\gamma-\Delta_2}{1-\gamma}\right)\right\} \quad (74)$$

where

$$H(n)=\frac{D(A_v+A_b+Fn)}{\{1+(n-1)P/D\}}$$

$$K(n)=\frac{(h_{bg}-h_v)\{1+(n-1)(P/D)^2\}}{2\{1+(n-1)P/D\}}$$

$$R(n)=\frac{D(h_{bd}-h_v)\{1+(n-1)(P/D)^2\}}{x\{1+(n-1)P/D\}}$$

$$Z(n)=h_v\left\{\frac{D}{P}+\frac{1+(n-1)P/D}{2P}(P-D)\right\}$$

and from Eq. (74) it can be written as

$$Q(n)=\sqrt{\frac{H(n)\left\{\frac{1}{2\Delta_1}\ln\left(\frac{1-\gamma+\Delta_1}{1-\gamma}\right)-\frac{1}{2\Delta_2}\ln\left(\frac{1-\gamma-\Delta_2}{1-\gamma}\right)\right\}}{K(n)\left(1-\gamma-\frac{\Delta_2-\Delta_1}{4}\right)+Z(n)-R(n)\left\{1-\frac{1}{2\Delta_1}\ln\left(\frac{1-\gamma+\Delta_1}{1-\gamma}\right)+\frac{1}{2\Delta_2}\ln\left(\frac{1-\gamma-\Delta_2}{1-\gamma}\right)\right\}}} \quad (75)$$

The optimal solutions for $Q$ and $n$ can be determined using an iterative algorithm.

There are other studies that have extended the model in Salameh and Jaber (2000) for a specific inventory situation. Because these studies are few in number and highly specialized, we group them under the general heading of "other extensions" and address them collectively in the next section.

### 3.6. *Other extensions of the model in Salameh and Jaber (2000)*

Chung and Huang (2006) extended the model in Salameh and Jaber (2000) and relaxed the assumption that the buyer must pay for the items as they are received. They developed two theorems for determining a buyer's ordering quantity when permissible delay in payments can occur and interest charges are levied on inventory costs of a buyer:

Case 1: Permissible delay $\leq$ cycle time

The expected annual profit in this case was given by

$$E[TPU_1(T)]=\left[(sD+pI_kDM_D)-\frac{pDT}{2}(h_b+I_k)-\frac{A_b}{T}-\frac{DM_D^2}{2T}(pI_k-sI_e)\right] +vD(1+I_eM_D)E\left[\frac{\gamma}{1-\gamma}\right]-D(p+d)E\left[\frac{1}{1-\gamma}\right]-D^2T(h_b+vI_e)\frac{E[\gamma/(1-\gamma)^2]}{x} \quad (76)$$

Case 2: Permissible delay $\geq$ cycle time

The expected annual profit in this case was given by

$$E[TPU_1(T)]=\left[(sD+sI_eDM)-\frac{DT}{2}(h_b+sI_e)-\frac{A_b}{T}\right] +vD(1+I_eM_D)E\left[\frac{\gamma}{1-\gamma}\right]-D(p+d)E\left[\frac{1}{1-\gamma}\right]-D^2T(h_b+vI_e)\frac{E[\gamma/(1-\gamma)^2]}{x} \quad (77)$$

Chung et al. (2009) generalized the work of Salameh and Jaber (2000) to develop an inventory model where the warehousing of items is subject to a constraint on the available item storage capacity. To avert the warehousing space shortage, the model allows for capacity to be introduced from a rented warehouse (RW) as an alternative to the existing owned warehouse (OW).

Their annual profit under this warehousing configuration was

$$\mathrm{E}[TPU(Q)] = \begin{cases} \mathrm{E}[TPU_1(Q)] & \text{if } Q \le \psi \\ \mathrm{E}[TPU_2(Q)] & \text{if } Q \ge \psi \end{cases} \tag{78}$$

where

$$\mathrm{E}[TPU_1(Q)] = D\left\{s - v + \frac{h_{bo}Q}{x}\right\} + D\left\{v - \frac{h_{bo}Q}{x} - p - d - \frac{A_b}{Q}\right\}\mathrm{E}\left[\frac{1}{1-\gamma}\right] - \frac{h_{bo}Q\{1-\mathrm{E}[\gamma]\}}{2} \tag{79}$$

and

$$\begin{aligned}\mathrm{E}[TPU_2(Q)] = {} & D\left\{s - v + \frac{h_{br}(Q-\psi)^2}{xQ} + \frac{h_{bo}\psi^2}{xQ}\right\} \\ & + D\left\{v - \frac{A_b}{Q} - p - d - \frac{h_{br}(Q-\psi)^2}{xQ} - \frac{h_{bo}\psi^2}{xQ}\right\}\mathrm{E}\left[\frac{1}{1-\gamma}\right] \\ & - \frac{h_{br}(Q-\psi)^2\{1-\mathrm{E}[\gamma]\}}{2Q} - h_{bo}\left(\psi\{1-\mathrm{E}[\gamma]\} - \frac{\psi^2\{1-\mathrm{E}[\gamma]\}}{2Q}\right)\end{aligned} \tag{80}$$

An interesting feature of their model is the use of a piecewise concave function for determining the expected total annual profit.

Hsu and Yu (2009) extended the model in Salameh and Jaber (2000) for the case where a supplier offers a special price discount to motivate buyers to buy in larger than normal order quantities. This is a situation where a supplier faces either a surplus in inventory, or a change in the production run of a product. Thus, the buyers would generally reduce their total purchasing costs by placing larger lot sizes of order when the special price discount takes effect. They discussed three cases for when to offer this discount, with respect to the last EOQ cycle associated with the unrevised price:

Case 1: At the terminal time of the last EOQ cycle

The optimal order size in this case was given by

$$Q_s^* = \frac{cQ_p}{2(p-k)} + \frac{(k + A_b/Q_p)}{2(p-k)h_b\{(1-\gamma)^2/(2D) + \gamma/x\}} \tag{81}$$

Case 2: Before the end of screening in the last EOQ cycle

The optimal order size in this case was given by

$$Q_s^* = \frac{cQ_p}{2(p-k)} + \frac{(k + A_b/Q_p) - (p-k)h_b\{(2q - q\gamma - 3Q_p\gamma + 2Q_p\gamma^2)/(2D) + 2Q_p\gamma/x\}}{2(p-k)h_b\{(1-\gamma)^2/(2D) + \gamma/x\}} \tag{82}$$

and

Case 3: After the end of screening in the last EOQ cycle

The optimal order size in this case was given by

$$Q_s^* = \frac{cQ_p}{2(p-k)} + \frac{(k + A_b/Q_p) - (p-k)h_b\{q(1-\gamma)/D\}}{2(p-k)h_b\{(1-\gamma)^2/(2D) + \gamma/x\}} \tag{83}$$

They showed that a special order is profitable for the buyer in case 1 above, while it is not always the case in cases 2 and 3 unless the price decrease exceeds a critical value.

Hsu and Yu (2011) extended the model in Salameh and Jaber (2000) for the case where a price increase is introduced at any time during the infinite planning horizon of a product. This policy enforces the buyer to buy in larger than normal order quantities. They described three situations for the possible time ($t$) at which the price-increase is announced. That is, ($i$) $t$ is at the end of an EOQ cycle, ($ii$) $t$ is after the screening finishes in the last EOQ cycle, and ($iii$) $t$ is before the screening finishes in the last EOQ cycle. Assuming $Q_s$ is the special order size at the time that a price increase is effective, the procurement cost for the first case would be

$$PC_1 = A_b + pQ_s \tag{84}$$

Now, if the price increase takes place when the remaining stock level in a cycle is $q$, the procurement cost for the second and the third situation described above would be:

$$PC_2 = A_b + p(q + Q_s) \tag{85}$$

## 4. Concluding remarks

Inventory models with imperfect items have received a good deal of attention in the literature over the past decade. This paper has reviewed the advancement of this literature by focusing on studies that have extended the EOQ model for imperfect items set forth by Salameh and Jaber (2000). The review of these studies has been organized along the following six categories: ($i$) EOQ/EPQ models for imperfect items, ($ii$) shortages and backordering, ($iii$) quality, ($iv$) supply chains, ($v$) fuzzy set theory, and ($vi$) other extensions of the model in Salameh and Jaber (2000).

The review presented herein provides a useful resource for researchers currently engaged in the work on inventory systems with imperfect items, and hopefully will provide new ideas to further stimulate this field of research. As evident by the literature reviewed in this paper, inventory models for items of poor quality have been studied by several researchers. However opportunities for additional research on the topic are abound. For example to our knowledge, the effect of imperfect items on the performance measures of an internal supply chain where learning is involved in production (and demand), has not been investigated yet. This modeling opportunity could be found in an environment such as the production assembly line of a manufacturing facility. The few models that have touched upon this important area have not considered the transfer of learning from cycle to cycle. In short, the study of a supply chain would remain incomplete until the researchers consider a number of practical scenarios. These scenarios can be, but are not limited to: imperfect items from suppliers, imperfect items in a production stage, transfer of knowledge from cycle to cycle and different strategies of dispatch of products to the buyers. Future research could also concentrate on examining the effect that quality improvement has on future demand rates which could be stochastic. Research that addresses these issues may lead to modeling insights that could strengthen coordination between all members of a supply chain.

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