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Effect of relativistic motion on superconducting quantum bits under decoherence

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ABSTRACT

In the present manuscript, we consider a quantum system of two super-conducting (SC) qubits that move at relativistic speeds and each qubit coupled to a resonator mode (RM). Here, the relativistic motion of the SC-qubits results from the modulation of the time-varying coupling between the SC-qubits and the resonator for which the system resembles the rotating approximation and anti-Jaynes-Cummings (JC) dynamics. We study the time variation of some interesting current quantumness measures with respect to the time-varying coupling (TVC) considering von Neumann entropy, negativity and Mandel parameter. We illustrate the dynamical behavior of the SC-qubits–RMs entanglement, SC-qubit–SC-qubit entanglement and photon statistics of the RMs at relativistic motion in the absence and presence of dissipative environment.

Introduction

Circuit quantum electrodynamics provides a suitable area for studying relativistic effects, either directly or through simulation [1–3]. The dynamical Casimir (DC) effect, which is caused by the modification of the electromagnetic field's boundary conditions at relativistic speeds, was seen in SC devices [4,5]. DC radiation has been shown to hold several types of correlations [6–9] which may be transferred to SC-qubits [10,11]. The Unruh effect is a noteworthy scientific phenomena in which thermal radiation is measured in the vacuum using an accelerated detector [12,13]. It has recently been demonstrated that modulating the coupling strength of a SC-qubit may be used to mimic the generation of acceleration radiation [14,15]. The ability to simulate both the DC effect and acceleration radiation in a SC design is dependent on the ability to execute an ultrafast change of the magnetic flux via a SC quantum interferometric device [16,17].

In light of Josephson junctions, SC circuits are promising options for implementing qubits, studying the physical idea of charge qubits, and controlling two-level systems in different domains of quantum information processing (QIP) [18–31]. A small SC island with a tunnel junction couples n excess Cooper-pair charges with capacitor and Josephson coupling energy to a SC electrode and an electron charging energy in the most efficient Josephson junction qubit. Specifically, when accounting for a qubit, only two charge-states with $n = 0$ and 1 will contribute, while all other states with much higher energy can be ignored. A SC-qubit then reacts as a two-charge state system [32], also known as an artificial two-level atom, in a Cooper box, where the gate charge is utilized to alter the difference in energies between the two states. The possibility of realizing various QIP tasks in the context of circuit QED, as described in Ref. [33], is particularly fascinating. Some quantum characteristics of such a quantum system have already been seen experimentally [34–40]. Recently, a critical step toward attaining the quantum regime has been made. Recent work has examined and handled the coupling among photons and solid-state circuits in the context of cavity QED, and it has been proven that a SC-qubit may be interacted with microwave photons [41]. (see Refs. [42,43] and references therein).

The maximal knowledge of a composite system may be specified in quantum mechanics theory, and the state of each subsystem can be determined by its relationship to other subsystems, resulting in entangled states [44,45]. Quantum entanglement, which postulated nonlocal correlations between distinct physical systems [46–48], is one of the most exciting phenomena in quantum technology. In this

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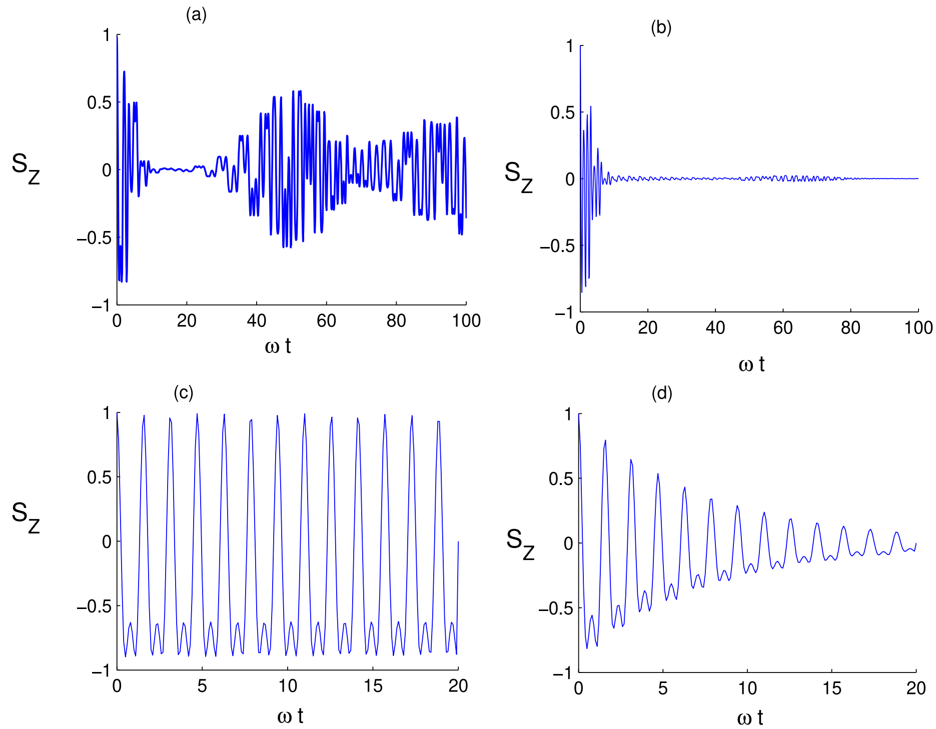

Fig. 1. Dynamics of the population inversion S_Z versus the time ωt according to the system responses with $\alpha = \sqrt{5}$ and $\theta = 0$. Panels (a) and (b) correspond to $g(t) = g_0 \cos\left(\pi + \pi \cos \omega_d t\right)$ and panels (c) and (d) correspond to $g(t) = g_0 \cos\left(\frac{\pi}{2} + \frac{\pi}{2} \cos \omega_d t\right)$. Panels (a) and (b) are for $\gamma = 0$ and $\gamma = 0.01$, respectively. Panels (c) and (d) are for $\gamma = 0$ and $\gamma = 0.01$, respectively.

way, studying entanglement as the most common kind of quantum correlation and the results of quantum measurements led to a better understanding and solution of many physical issues [49,50]. More recently, the advancement of QIP has resulted in the accumulation of a significant amount of information and grown the literature around the phenomenon of entanglement, which leads to the optimal performance of QIP and quantum metrology [51–56]. The importance of entanglement in a variety of applications has prompted researchers to look into and analyze physical systems in high-dimensional spaces, revealing a new feature for this type of correlation in many-particle systems [57]. Furthermore, while studying nonclassicality phenomena in quantum mechanics, the production of this type of correlation is considered as a goal in quantum experimental implementation.

At the core of quantum technology is the study of interactions between SC-qubits and coherent fields. SC-qubits that are coupled to a single-mode field in an optical cavity using the rotating wave approximation [58,59] , are one of the simplest examples. This model has been extended in several ways [60,61]. Currently, time-dependent coupling can be considered critical for various physical processes in quantum information and optics, as established in previous research, and it is also utilized to maximize different aspects of quantum systems. Circuit quantum electrodynamics has quickly become a leading quantum technology for studying quantum mechanics principles and implementing many quantum information theory applications. The ability to control parameters describing the interaction between quantized fields and SC-qubits quickly allows for the experimental realization of optical systems in unexplored regimes, permitting for the exploration of a wide range of new quantum systems [62,63]. In this paper, we introduce a system of two SC-qubits that move at relativistic motion speeds, where each SC-qubit is coupled to a RM. We study the time evolution of some interesting current quantumness measures when the quantum system resembles the rotating approximation and anti-JC dynamics. We illustrate the dynamical behavior of the SC-qubits–RMs entanglement, SC-qubit–SC-qubit entanglement and the statistical properties of the RMs at relativistic motion in the absence and presence of the environment.

The following is the paper's structure. We discuss the quantum model and its solution in Section "Physical system and quantumness measures", as well as the various quantumness measures. Section "Discussion and numerical results" is devoted to provide numerical results and discuss the different physical phenomena predicted during relativistic motion in the presence and absence of the environment. Finally, in Section "Conclusions", some conclusions are presented.

Physical system and quantumness measures

Recently, the non-Hermitian Hamiltonian has been widely considered in the literature [64–68]. This Hamiltonian is used to study the influence of the dissipative of environment considering the damping effect in the JC model [69–71].

We consider a quantum system that consists of two SC-qubits of frequency gaps ω_{sc_i}, where each SC-qubit is coupled to a RM with frequency ω_i. Accordingly, we can formulate the non-Hermitian that describes this model as

$$H = \hbar \sum_{i=1}^{2} \omega_i a_i^{\dagger} a_i + \sum_{i=1}^{2} \left[\frac{\hbar \Omega_i}{2} \sigma_z^i + g_i \cos\left(k x_{sc_i}\right) \sigma_x^i \left(\hat{a}_i^{\dagger} + \hat{a}_i\right) + \mathrm{i}\frac{\gamma_i}{2} |+_i\rangle\langle +_i| \right], \quad (1)$$

where the function g_i describes the coupling strength, x_{sc_i} represents the SC-qubit position [72–74], σ_{z_i} and σ_{x_i} denoting the Pauli matrices of each SC-qubit, and a_i ($a_i^{\dagger}$) represents the annihilation (creation) operator for the RMs. The dynamics of the quantum system is realized as an external flux f_i that oscillates with the amplitude Δf_i and the frequency ω_{d_i} around the offset f_{0_i}.

$$k x_{sc_i}(t) = f_{0_i} + \Delta f_i \cos\left(\omega_{d_i} t\right), \quad (2)$$

Here, the tunable coupling periodically depends on the external flux function, $g_i = g_{0_i} \cos(f_i)$, and that the coupling modulation takes off the SC-qubit motion $x_{sc_i}(t)$ inside the RM. The relativistic motion of the SC-qubits and fields is due to the effective motion.

Let us suppose that the RMs are initially described in the uncorrelated two-mode coherent states, $|\alpha_1, \alpha_2\rangle$, with $|\alpha_i\rangle = \exp\left(-|\alpha_i|^2/2\right)$

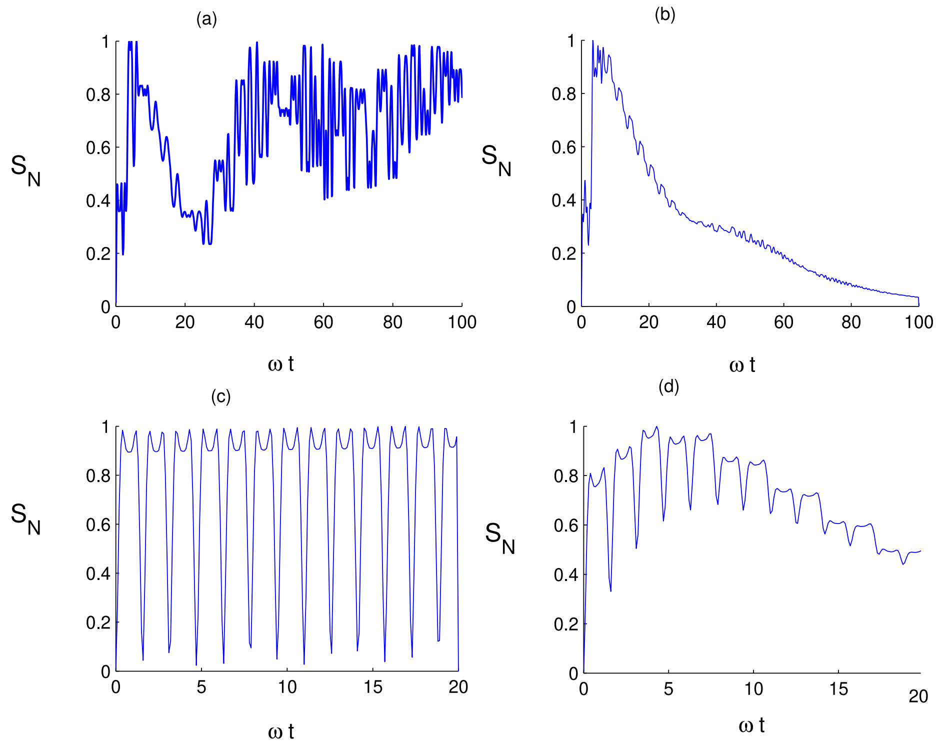

Fig. 2. Dynamics of the von Neumann entropy S_N versus the time ωt according to the system responses with $\alpha = \sqrt{5}$ and $\theta = 0$. Panels (a) and (b) correspond to $g(t) = g_0 \cos\left(\pi + \pi \cos \omega_d t\right)$ and panels (c) and (d) correspond to $g(t) = g_0 \cos\left(\frac{\pi}{2} + \frac{\pi}{2} \cos \omega_d t\right)$. Panels (a) and (b) are for $\gamma = 0$ and $\gamma = 0.01$, respectively. Panels (c) and (d) are for $\gamma = 0$ and $\gamma = 0.01$, respectively.

$$\Lambda = \begin{bmatrix} \lambda_1 - \frac{i}{2}\left(\gamma_1 + \gamma_2\right) & g_2 \cos\left(k x_{sc_2}\right)\sqrt{n_2+1} & g_1 \cos\left(k x_{sc_1}\right)\sqrt{n_1+1} & 0 \\ g_2 \cos\left(k x_{sc_2}\right)\sqrt{n_2+1} & (n_1+1)\omega_1 + n_2\omega_2 - i\frac{\gamma_1}{2} & 0 & g_1 \cos\left(k x_{sc_1}\right)\sqrt{n_1+1} \\ g_1 \cos\left(k x_{sc_1}\right)\sqrt{n_1+1} & 0 & n_1\omega_1 + (n_2+1)\omega_2 - i\frac{\gamma_2}{2} & g_2 \cos\left(k x_{sc_2}\right)\sqrt{n_2+1} \\ 0 & g_1 \cos\left(k x_{sc_1}\right)\sqrt{n_1+1} & g_2 \cos\left(k x_{sc_2}\right)\sqrt{n_2+1} & \lambda_2 \end{bmatrix} \quad (6)$$

Box I.

$\sum_{n_i=0}^{\infty}\left(\alpha_i^{n_i}\right)/\sqrt{n_i!}|n_i\rangle$, while the SC-qubits are initially described in the superposition of the upper states, $|+_1,+_2\rangle$, and lower state $|-_1,-_2\rangle$. The initial state of the combined quantum system is given by

$$|\Psi(0)\rangle = \left(\cos\theta|+_1,+_2\rangle + \sin\theta|-_1,-_2\rangle\right) \otimes |\alpha_1,\alpha_2\rangle. \quad (3)$$

The general solution $|\Psi(t)\rangle$ for $t > 0$ takes the form

$$|\Psi(t)\rangle = \sum_{n_1,n_2=0}^{\infty} (X_1(t)|+_1,+_2\rangle|n_1,n_1\rangle + X_2(t)|+_1,-_2\rangle|n_1,n_2+1\rangle$$
$$+ X_3(t)|-_1,+_2\rangle|n_1+1,n_2\rangle + X_4(t)|-_1,-_2\rangle|n_1+1,n_2+1\rangle). \quad (4)$$

By using the Schrödinger equation,

$$i\hbar\frac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle, \quad (5)$$

the coefficients $X_j(t)$ $(j = 1,2,3,4)$ in the previous equation achieve the following differential equation,

$$\mathrm{i}\frac{\mathrm{d}X_j}{\mathrm{d}t} = \Lambda X_j,$$

with Λ given in Box I and

$$\lambda_\ell = (n_1 + \ell - 1)\omega_1 + (n_2 + \ell - 1)\omega_2 - \frac{(-1)^\ell}{2}\left[\Omega_1 + \Omega_2\right], \quad l = 1,2. \quad (7)$$

According to the dynamics of the SC-qubits, the relative probabilities are significant when the qubits reach their excited states. The population inversion, $\langle S_Z^i\rangle$, is defined as the difference of probabilities when the qubit is in its excited state and ground state:

$$\langle S_Z^i\rangle = -\mathrm{Tr}\left[\sigma_z^i \rho_{sc_i}\right]. \quad (8)$$

To quantify the entanglement of the qubits–RMs state, we employ the von Neumann entropy, which is also used to detect the purity and disorder in the quantum states. It is defined as

$$S(\rho) = -\mathrm{Tr}\left(\rho_{qubits}\ln\rho_{qubits}\right)$$
$$= \sum_i r_i \ln r_i, \quad (9)$$

where ρ_{qubits} is the state of the qubits with the eigenvalues r_i. Here, we examine the degree of the purity for the state ρ_{qubits}, that can be evaluated from the whole quantum state, $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, by performing a trace over the RMs. If we initially consider the qubits–RMs state as a pure state, then the state ρ_{qubits} provide the same entropy value during the dynamics, i.e., $S_N = S_{\text{qubits}}(t) = S_{\text{RMs}}(t)$. We just find the von Neumann entropy of the state of SC-qubits to evaluate the degree of entanglement for the qubits–RMs state. The function S_N

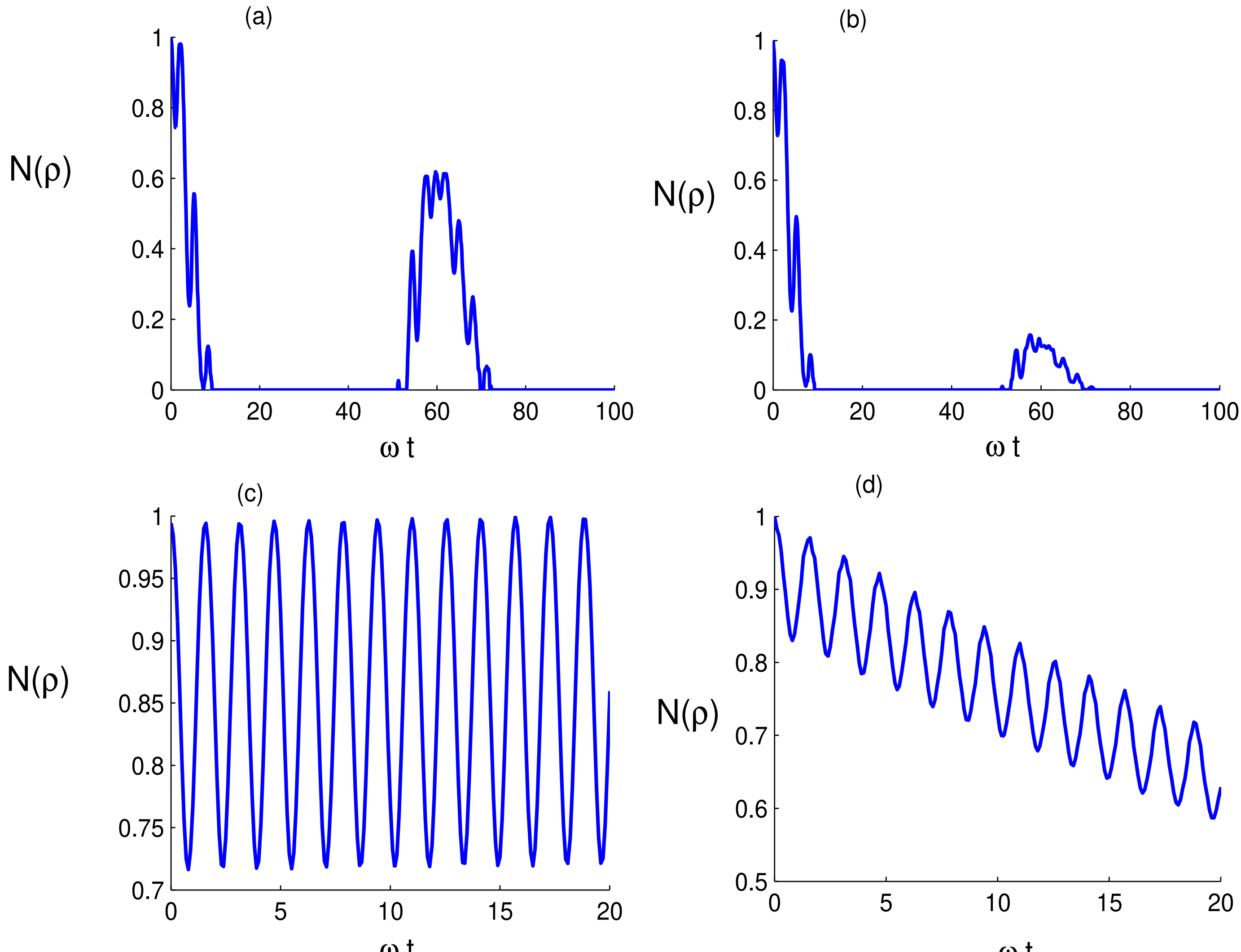

Fig. 3. Dynamics of the negativity N versus the time ωt according to the system responses with $\alpha = \sqrt{5}$ and $\theta = \pi/4$. Panels (a) and (b) correspond to $g(t) = g_0 \cos\left(\pi + \pi \cos \omega_d t\right)$ and panels (c) and (d) correspond to $g(t) = g_0 \cos\left(\frac{\pi}{2} + \frac{\pi}{2} \cos \omega_d t\right)$. Panels (a) and (b) are for $\gamma = 0$ and $\gamma = 0.01$, respectively. Panels (c) and (d) are for $\gamma = 0$ and $\gamma = 0.01$, respectively.

varies from 0 for a separable pure state to 1 for a maximally entangled state.

In order to analyze the SC-qubit–SC-qubit entanglement, we introduce the negativity measure defined by [75]

$$N(\rho) = \frac{\left\| \rho_{qubits}^{T_1} \right\| - 1}{2}, \tag{10}$$

in which $\rho_{qubits}^{T_1}$ is the partial transpose of ρ_{qubits} with respect to the first SC-qubit. The function $N(\rho)$ varies from 0 for a separable pure state to 1 for a maximally entangled state.

The Mandel's parameter is used to detect the non-classical properties of the fields, which is defined by [76]

$$M_Q = \frac{\langle (\Delta N)^2 \rangle - \langle N \rangle}{\langle N \rangle}. \tag{11}$$

Here, the quantity $\langle N \rangle$ represents the average of photon number, $N = a^\dagger a$, of the RM and ΔN represents the dispersion. The M_Q parameter can differentiate whether the distribution of the photons of the RM is sub-Poissonian ($M_Q < 0$) that indicates the non-classical case, Poissonian distribution ($M_Q = 0$) for the case of a semi-classical state and super-Poissonian distribution ($M_Q > 0$). This parameter is used to determine and show the effects of the main parameters of the model on the variation of the non-classical properties of the RMs.

Discussion and numerical results

In the present section, we explore the effect of the frustration parameter f and magnetic flux ϕ_0 on the dynamical behavior of the population inversion, von Neumann entropy, negativity and Mandel parameter in the absence and presence of dissipative environment. In the dissipative regime, we have considered the damping JC model to describe the SC-qubit interaction with the environment. We suppose that $f_{0_i} = \pi/2$, $\Delta f_i = \pi/2$ and $\omega_{d_i} = 2\omega_i$ in order that the system behaves as the anti-JC model. Moreover, we take $f_{0_i} = \pi$, $\Delta f_i = \pi$ and $\omega_{d_i} = \omega_i$ in which the counter-rotating photon transitions conduct as the rotating ones [72]. For the sake of simplicity, we assume $g_1 = g_2 = g$, $f_1 = f_2 = f$, $\Delta f_1 = \Delta f_2 = \Delta f$, $\omega_{d_1} = \omega_{d_2} = \omega_d$, $\alpha_1 = \alpha_2 = \alpha$ and $\gamma_1 = \gamma_2 = \gamma$.

Fig. 1 displays the dynamics of the SC-qubit population inversion versus the time in the absence and presence of dissipative effect with respect to the system responses. In the absence of the decay term, the population inversion represents randomly oscillations in the case of counter rotating photon transitions with collapse and revival phenomena. On the other side, the case of anti-JC model shows that the function S_Z experiences a periodic behavior during the time evolution. In case which decay rate is considered, the amplitude of the function S_Z is reduced with the time and it tends to attain small values as the time becomes significantly large.

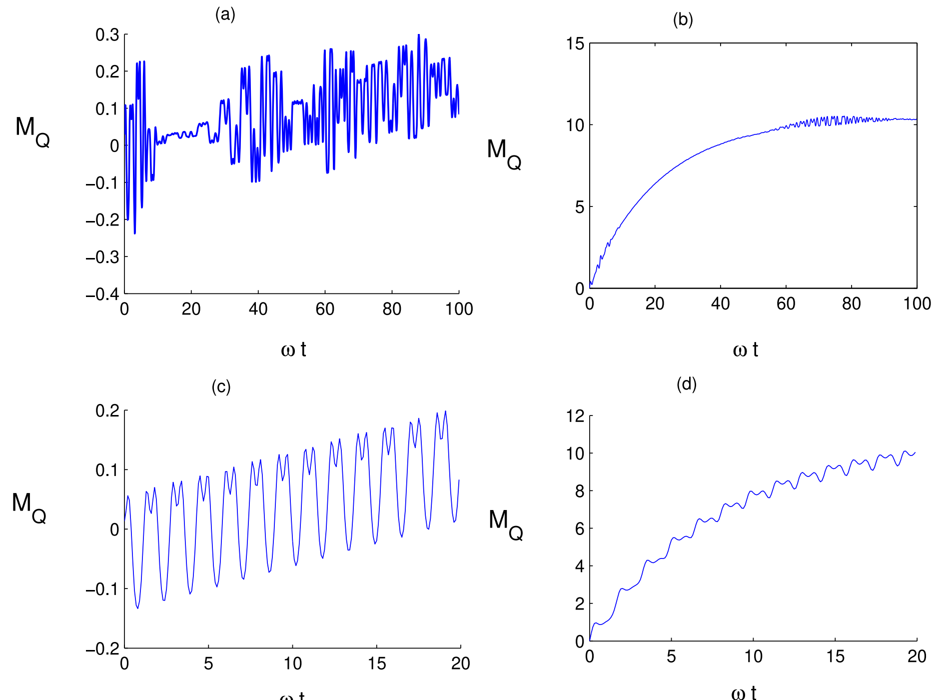

Fig. 4. Dynamics of the parameter M_Q versus the time ωt according to the system responses with $\alpha = \sqrt{5}$ and $\theta = 0$. Panels (a) and (b) correspond to $g(t) = g_0 \cos\left(\pi + \pi \cos \omega_d t\right)$ and panels (c) and (d) correspond to $g(t) = g_0 \cos\left(\frac{\pi}{2} + \frac{\pi}{2} \cos \omega_d t\right)$. Panels (a) and (b) are for $\gamma = 0$ and $\gamma = 0.01$, respectively. Panels (c) and (d) are for $\gamma = 0$ and $\gamma = 0.01$, respectively.

In Fig. 2, we show the time variation of the SC-qubit–RMs entanglement according to the choice of the TVC without and with dissipation effect. In case which counter rotating photon transitions and no decay rate is considered, the amount of the entanglement between the qubits and RMs increases at the beginning of the interaction to attain its maximal value and then the function S_N provides rapidly oscillations. For this propose, the stabilization and preservation of the entanglement can achieved through the regime in which the system is prepared. When the system resembles the anti-JC model, the function S_N experiences periodic oscillations during the dynamics. The presence of the decay rate leads to decrease the amount of the entanglement where the loss rate of the entanglement is dependent on the system responses.

In Fig. 3 we display the time variation of the entanglement of the SC-qubit–SC-qubit state in the absence and presence of the dissipative effect with respect to the system responses. In general the behavior of the negativity is very sensitive to the TVC and dissipative effect. In the absence of the decay rate term, we can observe that the negativity exhibits randomly and periodic oscillations in the rotating approximation and anti-JC model, respectively. When the decay rate term is considered, the negativity function decreases with the time and the loss rate of the negativity is dependent on the system responses.

Let us now illustrate the effects of the relativistic motion and decay rate on the statistical properties of the RMs. Fig. 4 shows the behavior of the Mandel parameter as a function of time in the absence and presence of the dissipative effect for both regimes when the RMs are initially defined in the canonical coherent states. In case which no decay rate is considered, the Mandel parameter has a negative value at the beginning of the interaction, exhibiting sub-Poissonian distribution of photons, and it tends to get positive values, displaying the super-Poissonian distribution of photons. On the other side, the Mandel's parameter takes positive values in the dissipative regime, showing the super-Poissonian distribution of photons. From the obtained results, we can infer that the control and protection of the bipartite entanglement and statistical properties can be realized by considerable choice of the physical parameters of the studied model.

Conclusions

In summary, we have considered a system of two SC-qubits that move at relativistic speeds and each SC-qubit coupled to a resonator mode (RM). We have considered that the relativistic motion of the SC-qubits resulting from the modulation of the TVC between the SC-qubits and the RM for which when the quantum system resembles the rotating approximation and anti-JC dynamics. We have studied the time variation of some interesting current quantumness measures considering von Neumann entropy, negativity and Mandel parameter. We have shown the dynamical behavior of the SC-qubits–RMs entanglement, SC-qubit–SC-qubit entanglement and statistics of the RMs at relativistic motion. Finally, we have examined the behavior of the quantumness measures at the relativistic motion in the presence of dissipative environment. In compared to other recent research, our phenomenological findings may be more helpful in the description of the behavior of nonclassicality phenomena in relativistic quantum systems considering interactions between SC-qubits and coherent fields and exploiting these phenomena to describe realistic quantum experimental implementation with optimal conditions.

CRediT authorship contribution statement

M.S. Al-Ghamdi: Visualization, Supervision, Project administration. **K. Berrada:** Conceptualization, Methodology, Writing Original Draft, Writing – reviewing and editing. **S. Abdel-Khalek:** Conceptualization, Methodology, Writing – original draft, Writing – reviewing and editing. **H. Eleuch:** Validation, Funding acquisition, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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