A Fast Directional Sigma Filter for Noise Reduction in Digital TV Signals

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Abstract — *This paper proposes a structure-oriented multi-directional Sigma filter for additive white Gaussian noise in digital TV signals. Filtering is restricted to homogeneous directions to reduce blurring by analyzing local structure using directional second derivatives. The proposed filter improves the Sigma estimate of denoised pixels by imposing a homogeneity constraint on the noise-adaptive selection of estimation pixels by the Sigma filter. It achieves noise-reduction gains of up to 4.8 dB Peak-Signal-to-Noise Ratio (PSNR) in real-time. The block size, shape and coefficients of the filter are adapted to both structure and noise level. The goal is to optimize the filter with regard to noise-reduction gain and structure preservation. A possible hardware-oriented design of the proposed filter is also presented. To show the effectiveness of the proposed method, comparisons between the proposed Sigma filter and referenced Sigma filters in terms of the PSNR gain and the modulation transfer function (MTF) are shown. Results show that the proposed method achieves a higher PSNR gain and contrast transfer ratio than referenced Sigma filters.*

Index Terms — Real-time noise filters, Sigma filter, spatial multi-directional filters, structure preservation, digital TV noise

I. INTRODUCTION

Video processing systems continuously increase in volume and complexity. Since noise is an undesirable yet inevitable phenomenon, noise reduction filters have become integral components added to these systems to improve the overall performance. Different system applications bring forth different requirements for noise reduction filters. For example, with the emergence of image and video enabled products and services, such as in portable TV devices, mobile phones, digital cameras, and iPods, rises the need for real-time noise reduction filters with high algorithmic speed, low memory consumption, and ability to handle variable noise levels. In general, noise filters work better if they are adaptive to both signal content, e.g., using directional filters, and to the noise level, e.g., using the Sigma filter. The problem is how to extract signal content and noise level information and combine them in real-time to achieve a balance between the amount of noise-reduction gain and the loss due to blurring.

This work was supported, in part, by the *Fonds de la recherche sur la nature et les technologies du Québec* (NATEQ) under grant numbers F00365 and F00361.

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The noise-adaptive methods in [1–3] are capable of achieving high noise-reduction gains in real-time, while preserving the high frequency content of TV signals. The method in [1] introduces a spatial filter that is based on the Sigma probability of the Gaussian distribution. Recent real-time noise-adaptive filters widely compare to [1], which is known as the Sigma filter. The Sigma filter uses the noise level to select which pixels to use or discard in estimating the value of the denoised pixel. The drawback of the Sigma filter is in not imposing any spatial constraints on pixels which pass the Sigma test. Thus, any random neighboring pixel whose value is within two standard deviations of noise from the value of the processed pixel is used in filtering, even if it belongs to a different population. The Sigma test can be further impaired when measuring the difference between pixels in the presence of high noise levels. The work presented in [2] and [3] defines a recursive Sigma filter which improves the Sigma filter by changing the shape of the block, making the filtering recursive, and using a multi-level weighting function. However, the recursive Sigma filter in [2] and [3] is not adaptive to content.

Content-adaptive denoising filters [4–8] preserve edges and structures by adapting to signal content. Directional filters [4, 5] achieve such adaptation through filtering along one selected direction in the block. The filter in [4] replaces the center pixel by the average of pixels in the direction with the minimum variance. The problem with [4] is the variance not being a reliable measure of homogeneity in the presence of noise [9]. The homogeneity-based directional filter in [5] applies filtering to the intensity most homogeneous direction as decided by examining directional derivatives. However, it performs filtering in only one direction and does not adapt to noise. Rational filters [6–8] adapt to signal content through modulating the coefficients of a linear low-pass filter to limit its actions in the presence of image details. They can be applied directionally for increased preservation of frame content. For example, the filter in [7] is applied to the 0^o, 45^o, 90^o and 135^o directions, but it does not adapt to more complex structures such as corners. The work in [7] is extended in [8] with the rational filter applied temporally, hence requiring multiple frame delays. The filter parameters in [7, 8] are set manually to tune the filter's response.

Filters in [10–17] are computationally expensive to be used in digital TV sets. A classical approach to spatial noise reduction is the spatial Wiener filter in [10] which introduces residual blurring near edges [18]. The work in [11] builds a variant of the Sigma filter with an enlarged block size and an

Contributed Paper
Manuscript received November 20, 2007 0098 3063/07/$20.00 © 2007 IEEE

improved multi-level weighting function. This variant of the Sigma filter gives higher noise-reduction gain at the expense of reduced performance in less noisy signals and slower execution. The filter in [12] is another filter to trade speed with higher gains by optimizing the filter's parameters leading to a significantly slow performance. The method in [13] uses Gauss curvature driven diffusion and is computationally expensive. Recent wavelet based methods [14–17] are also computationally expensive as they require transforming frames to the wavelet domain.

In this paper, a multi-directional Sigma filter, which adapts the size and shape of the filter block to signal and noise characteristics, is proposed. The proposed filter is adapted to signal content by filtering along detected homogeneous directions and is adapted to noise by utilizing the Sigma probability of the Gaussian distribution. The aim is to introduce a method by which multiple directions can be used safely in filtering to increase the noise-reduction gain while reducing the blurring side effect. The advantages of the proposed approach include: 1) noise-robust measure of local structure; 2) structure preservation through directional application of the Sigma test; 3) high noise-reduction gains; and 4) real-time performance.

Noise, in this paper, is an unwanted stochastic signal and not deterministic distortions such as shading or lack of focus. Generally, noise can be added to a signal (i.e., additive noise) or multiplied by a signal (i.e., multiplicative noise). It can also be signal dependent or independent, white or colored. The choice of the noise model for this paper is motivated by the several types of noise affecting charge-coupled device (CCD) cameras such as photon-shot and read-out noise [18]. The random arrival of photons at the sensor, which is governed by Poisson distribution, causes the photon-shot noise. Read-out noise is Gaussian and results from combining other sources of noise such as output amplifier noise, camera noise and clock noise. The high counting effect of photon arrivals leads to an aggregate noise effect which can be well approximated by a Gaussian distribution [19] according to the central limit theorem. Therefore, in this paper, an additive white Gaussian noise (AWGN) model is assumed. The choice here is also motivated by AWGN being the most common noise model for terrestrial TV [20]. AWGN samples are assumed to be independent and identically distributed (iid) Gaussian random variables. Accordingly, the noisy signal F_η is modeled as

$$F_\eta = F + \eta\,, \tag{1}$$

where F is the noise-free signal and η is the added noise. The problem now is to obtain $\hat{F}$, the estimate of F, given F_η only. Define $F(\mathbf{n})$ to be the intensity value at spatial position $\mathbf{n} = (n_1,\ n_2)$ in the sampling lattice Ω, where n_1 is the horizontal coordinate and n_2 is the vertical coordinate. Similarly, $F_\eta(\mathbf{n})$ and $\eta(\mathbf{n})$ are the intensity value and noise component at position $\mathbf{n}$ of F_η and η, respectively.

The remainder of the paper is organized as follows. In section II, the proposed multi-directional Sigma filter is presented. In section III, the modulation transfer function (MTF) is used to show that the proposed filter is more structure preserving than referenced filters. A possible hardware-oriented design of the proposed filter is also introduced in section III. Results are presented and discussed in section IV. Finally, section V concludes the paper.

II. Proposed Sigma Filter

A. Structure Measurement to Adapt to Signal Content

In this section, a method for performing an analysis of the structure around each pixel in F_η is proposed. Structure around a given noisy pixel $F_\eta(\mathbf{n})$ is measured using the magnitude of the directional second derivatives $|\nabla^2 F_\eta^d(\mathbf{n})|$ calculated using

$$\left|\nabla^2 F_\eta^d(\mathbf{n})\right| = (W-1)F_\eta(\mathbf{n}) - \sum_{\mathbf{m}\in S_d} F_\eta(\mathbf{n}+\mathbf{m})\,, \tag{2}$$

where W is the filter's block size and S_d is a set of vectors from $\mathbf{n}$ to the neighboring pixels in the direction d. Eight directions are used. Thus, S_d, $d \in \{1, 2, ..., 8\}$ when $W = 3$ is (see Fig. 1)

$$\begin{aligned} S_1 &= \{(1,0),(-1,0)\} & S_5 &= \{(1,0),(0,1)\} \\ S_2 &= \{(0,1),(0,-1)\} & S_6 &= \{(-1,0),(0,1)\} \\ S_3 &= \{(-1,-1),(1,1)\} & S_7 &= \{(0,1),(1,0)\} \\ S_4 &= \{(1,1),(-1,-1)\} & S_8 &= \{(0,-1),(1,0)\}. \end{aligned}$$

It is important that the noise susceptibility of (2) is reduced to improve structure measurement. Let S be the set of vectors from $\mathbf{n}$ to every pixel in the $W{\times}W$ neighborhood. S can be decomposed into the set R of vectors to neighboring pixels that are already noise reduced and the set N of vectors to neighboring pixels yet to be noise reduced (see Fig. 2(a)).

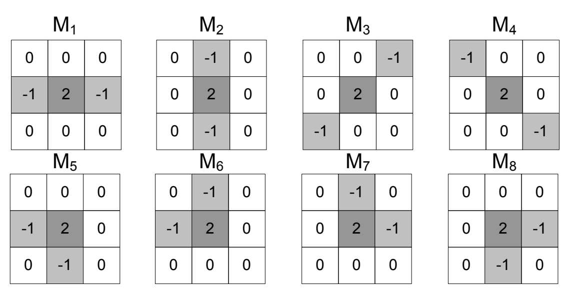


Fig. 1. Directions [9] along which activity is measured represented as the directional masks M_d.

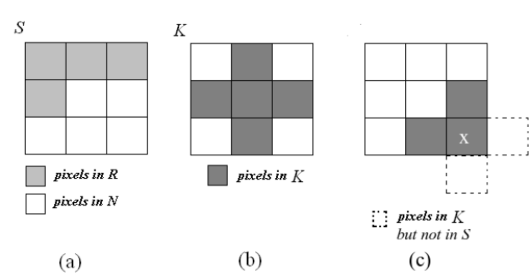


Fig. 2. (a) The set *S* separated into recursive pixels in *R* and non-recursive pixels in *N*, (b) set *K* of 4 immediate neighbors of the center pixel, and (c) set *K* of 4 immediate neighbors of X-marked pixel, two of which are in *S*.

Instead of using noisy pixels from F_η in (2), pixels from R and smoothed pixels from N are used instead as

$$\left|\nabla^2 F_\eta^d(\mathbf{n})\right| = (W-1)\frac{1}{|K|}\sum_{\mathbf{k}\in K} F_\eta(\mathbf{n}+\mathbf{k}) - \sum_{\mathbf{m}\in S_d\cap R}\hat{F}(\mathbf{n}+\mathbf{m}) - \frac{1}{|K|}\sum_{\mathbf{m}\in S_d\cap N}\sum_{\mathbf{k}\in K} F_\eta(\mathbf{n}+\mathbf{m}+\mathbf{k}) \quad (3)$$

where K is the set of four immediate neighbors as depicted in Fig. 2(b) for the center pixel. If not all immediate neighbors of a pixel in $S_d \cap N$ are in S (i.e., $\mathbf{m}\in S_d\cap N$ but $\mathbf{m}+\mathbf{k} \notin S$), only available ones are used. For example, only two of the immediate neighbors of the bottom right pixel in Fig. 2(c) (marked with a white X) are in S. Therefore, only these two neighbors along with the pixel itself are averaged to reduce noise. Next, the directions d's are ordered based on their measured activities $|\nabla^2 F_\eta^d(\mathbf{n})|$ to build the set of ordered directions ζ. The first element in ζ, $\zeta(1)$, is the direction d with the least amount of structure (the smallest absolute value of the directional second derivative) or

$$\zeta(1) = \arg\min_d \left|\nabla^2 F_\eta^d(\mathbf{n})\right|. \quad (4)$$

$\zeta(2)$ is the direction with the second least structure and $\zeta(8)$ is the direction d with the most structure. The first D directions in ζ are referred to as the homogeneous directions which are used in filtering, and the rest are abandoned to avoid blurring. ζ represents the structure analysis around $F_\eta(\mathbf{n})$.

B. Parameter Selection to Adapt to the Noise Level

A method is proposed in this section to find a scheme that assigns a W (block size) or a D (maximum number of homogeneous directions) for every noise level. The noise level is measured in Peak-Signal-to-Noise-Ratio (PSNR) and is denoted PSNR_η. The proposed method is trained on a set of representative data (different levels of structure and noise) to build such a scheme. The proposed procedure is comparable to quantization and can be explained as reducing noise from different noisy image and video signals and obtaining the noise-reduction gain *G[dB]*, then collecting the values of W and D which achieve the highest noise-reduction gain for every noise level PSNR_η. After the scheme is built, it is hardcoded into the filter as a set of rules (e.g., for $28 < \mathrm{PSNR}_\eta < 40$, use W=3). This procedure is run only once offline.

The noise reduction gain *G[dB]* is calculated using,

$$G(\mathrm{PS\hat{N}R}, \mathrm{PSNR}_\eta) = \mathrm{PS\hat{N}R} - \mathrm{PSNR}_\eta, \quad (5)$$

where $\mathrm{PS\hat{N}R}$ denotes the PSNR of the noise-reduced signal $\hat{F}$ and PSNR_η is the PSNR of the noisy signal F_η. $\mathrm{PS\hat{N}R}$ is either measured (from F and $\hat{F}$) if F is available or estimated in its absence (e.g., using noise estimation methods such as [9]). Finding the optimal W and D can be treated as a quantization problem. For both W and D, the region of support B of PSNR_η, $B = [\mathrm{PSNR}_\eta^{\min}, \mathrm{PSNR}_\eta^{\max}]$ (e.g., B = [20dB, 55dB]), is divided into intervals. For the block size W, these intervals are $B_l^W = [b_{l-1}^W, b_l^W)$ where $l \in \{1, 2, ..., L\}$ and L is the number of possible block sizes. For the number of directions D, the intervals are $B_i^D = [b_{i-1}^D, b_i^D)$ where $i \in \{1, 2, ..., I\}$ and I is the number of possible directions. Then, a level w_l is assigned from possible values of W (e.g., $w_l \in \{3, 5, 7, 9\}$) and a level h_i from possible values of D (e.g., $h_i \in \{1, 2, ..., 8\}$). The optimal values of w_l and their corresponding intervals B_l^W [21] which completely define the adaptation of W to the noise level can be found by solving the two equations

$$B_l^W = \{\mathrm{PSNR}_\eta : G_{w_l}(\mathrm{PS\hat{N}R}, \mathrm{PSNR}_\eta) \geq G_{\hat{w}_l}(\mathrm{PS\hat{N}R}, \mathrm{PSNR}_\eta)\}, \quad (6)$$

$$w_l = \arg\max_{w_l} E\{G_{w_l}(\mathrm{PS\hat{N}R}, \mathrm{PSNR}_\eta) \mid \mathrm{PSNR}_\eta \in B_l^W\}, \quad (7)$$

where G_{w_l} and $G_{\hat{w}_l}$ denote the gain in PSNR for using $W = w_l$ and $W = \hat{w}_l$, respectively, and $E\{.\}$ is the conditional mean. Similarly, the optimal values of h_i and their corresponding intervals B_i^D which define the adaptation of D to the noise level are found through solving

$$B_i^D = \{\mathrm{PSNR}_\eta : G_{h_i}(\mathrm{PS\hat{N}R}, \mathrm{PSNR}_\eta) \geq G_{\hat{h}_i}(\mathrm{PS\hat{N}R}, \mathrm{PSNR}_\eta)\}, \quad (8)$$

$$h_i = \arg\max_{h_i} E\{G_{h_i}(\mathrm{PS\hat{N}R}, \mathrm{PSNR}_\eta) \mid \mathrm{PSNR}_\eta \in B_i^D\}, \quad (9)$$

where G_{h_i} and $G_{\hat{h}_i}$ denote the gain in PSNR for using $D = h_i$ and $D = \hat{h}_i$, respectively. Equations (6) and (8) are known in quantization theory as the *generalized nearest neighbor conditions* and (7) and (9) as the *generalized centroid conditions* [21]. With absence of information about the probability density function of the PSNR_η, the Llyod algorithm for optimal scalar quantization [22] can be used on a representative set of training data to find the optimal noise adaptation schemes.

C. Proposed Multi-Directional Sigma Filter

The proposed structure analysis from section II.A and the noise level adaptation in section II.B are integrated to propose the multi-directional Sigma filter. The Sigma filter [1] is based on the Sigma probability of the Gaussian distribution. It smoothes noise by averaging neighborhood pixels which have intensity values within a fixed Sigma range of the center pixel.

Multi-directional filtering can be described using

$$\hat{F}(\mathbf{n}) = CF_\eta(\mathbf{n}) + \sum_{d=1}^{D}\sum_{\mathbf{m}\in S_{\zeta(d)}} \frac{(1-C)}{D(W-1)} F_\eta(\mathbf{n}+\mathbf{m}), \quad (10)$$

where C defines the filter's coefficients. Note that (10) generalizes directional filtering since it allows filtering by defining the structure analysis scheme ζ, the directions of interest S_d's (as sets of vectors from the current pixel to the neighboring pixels in the direction), and filter's block shape and size using W, D and C. Next, by combining the D most homogeneous directions and the Sigma probability, a block is created which best fits the image structure and noise level.

Specifically, the filter:

- Step 1: Populates the ordered set of mask indexes ζ based on the homogeneity $|\nabla^2 F_\eta^d(\mathbf{n})|$ of the direction d using (3) and adapts D to the noise level σ_η^2 using

$$D = \begin{cases} h_1 & : b_0^D \le \sigma_\eta^2 < b_1^D \\ h_2 & : b_1^D \le \sigma_\eta^2 < b_2^D \\ h_3 & : b_2^D \le \sigma_\eta^2 < b_3^D \end{cases}. \qquad (11)$$

- Step 2: Adapts the block size W to σ_η^2 as

$$W = \begin{cases} w_1 & : b_0^W \le \sigma_\eta^2 < b_1^W \\ w_1 & : b_1^W \le \sigma_\eta^2 < b_2^W \end{cases}. \qquad (12)$$

- Step 3: Changes $F_\eta(\mathbf{n}+\mathbf{m})$ in (10) to $f(F_\eta(\mathbf{n}+\mathbf{m}), \sigma_\eta^2)$ based on the Sigma probability to make pixel selection noise level adaptive as follows

$$f(F_\eta(\mathbf{n}+\mathbf{m}), \sigma_\eta^2) = \begin{cases} F_\eta(\mathbf{n}+\mathbf{m}) : F_\eta(\mathbf{n}+\mathbf{m}) - \gamma\sigma_\eta \le F_\eta(\mathbf{n}) \le F_\eta(\mathbf{n}+\mathbf{m}) + \gamma\sigma_\eta \\ 0 \quad : \text{otherwise} \end{cases}, \qquad (13)$$

  where γ is a parameter controlling sensitivity to noise.

- Step 4: Adapts the center pixel weight C as follows

$$C = \frac{\mathrm{PSNR}_\eta}{\mathrm{PSNR}_\eta^{\max}}, \quad \mathrm{PSNR}_\eta = 10\log_{10}\left(\frac{255^2}{\sigma_\eta^2}\right), \qquad (14)$$
$$\mathrm{PSNR}_\eta^{\max} = 55\,\mathrm{dB}.$$

  Non-center pixel weights C' can be calculated using $(1-C)/(D(W-1)-\mu)$, where μ is the number of zeroed coefficients by the Sigma test in (13).

III. Analysis of The Proposed Sigma Filter

A. Structure Preservation

In this section, the structure preservation capabilities of the proposed multi-directional Sigma filter are analyzed using the MTF [24] to illustrate the relative degradation of contrast due to blurring by the proposed multi-directional Sigma filter, the conventional Sigma probability filter [1], and the directional filter [5]. To approximate the MTF from a local region Z in a single image F, let $H(x)$ be the normalized local histogram calculated from pixels in Z as

$$H(x) = \frac{T_x}{T}, \; 0 \le x \le 255, \qquad (15)$$

where T_x is the number of pixels with gray level $x \in Z$ and T is the total number of pixels in Z. $H(x)$ approximates the probability mass function $p(x)$ of region Z in F. By finding the cumulative sum of $H(x)$ with

$$P(x) = \sum_{u=0}^{x} H(u), \; 0 \le x \le 255, \qquad (16)$$

an estimate of the cumulative distribution function (CDF), $P(x)$, of region Z is obtained [23]. $P(x)$ is related to the Edge-Spread Function (ESF), defined as the response of the system to an ideal edge [24], by $P(x) = x(ESF)$, when it is known that the locality of region Z on which the histogram was calculated represents an edge. The MTF measures the degradation of contrast due to blurring at different spatial frequencies. It can be calculated as the Fourier transform, $\mathcal{F}\{.\}$, of the first derivative of the ESF, that is,

$$MTF(\omega) = \mathcal{F}\left\{\frac{\partial ESF(x)}{\partial x}\right\}. \qquad (17)$$

Fig. 3 shows the MTF of the local region Z in Fig. 4 (*Saturn* image) for the proposed multi-directional Sigma filter and the filters in [1] and [5]. The proposed Sigma filter achieves a higher contrast transfer ratio than [1] and [5] due to using (11)-(13).

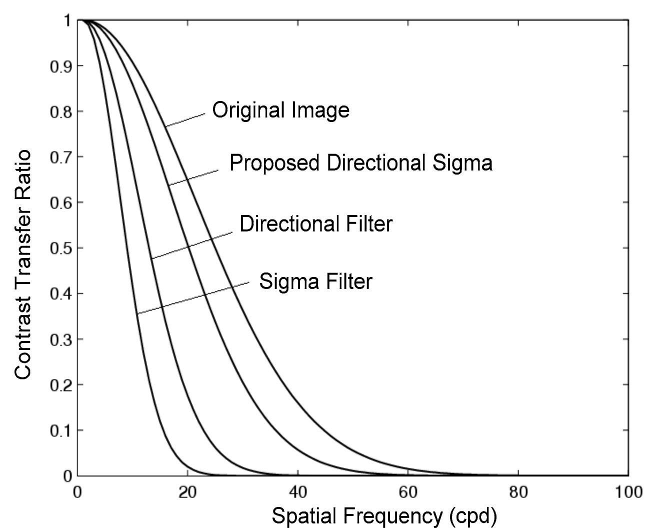


Fig. 3. Estimation of the MTF of region *Z* in Fig. 4 for the proposed muli-directional Sigma, the conventional Sigma filter [1] and the directional filter [5].

Local histograms shown in Fig. 4 are used to explain the improved structure-preservation capabilities of the proposed Sigma filter. The local histogram at area Z shows two distinct sample populations. The rectangles superimposed on the *Saturn* image are magnified blocks. If filtering takes place over pixels in region Z, the assumption being that they belong to the same population, blurring occurs. The multi-directional application of the Sigma probability acts as a low-pass filter which isolates one population so that less cross-population averaging takes place, hence, effectively reducing the blurring side effect. This is shown in Fig. 5.

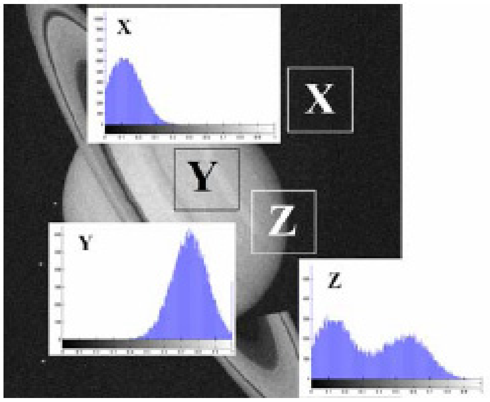


Fig. 4. Local histograms at structured and unstructured areas of the noisy *Saturn* (see Fig. 5).

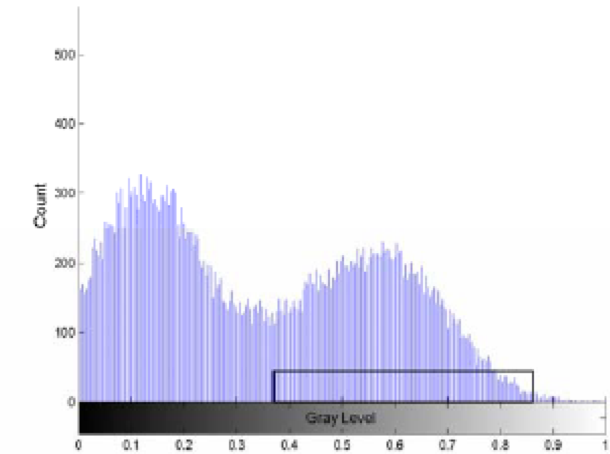


Fig. 5. The multi-directional Sigma probability isolates a sample population to prevent blurring. Assuming the block on region *Z* in Fig. 4 is centered on a high-intensity pixel.

Moreover, the improved structure-preservation capabilities of the proposed multi-directional Sigma filtering over the Sigma filter [1] is due to the fact that two criteria are used in the proposed method as opposed to one in the Sigma filter. The proposed method imposes an extra constraint that pixels have to be spatially grouped in a direction rather than scattered as in the Sigma filter. The proposed method ensures exclusion of pixels which randomly pass the Sigma probability without being part of the same population as the center pixel.

Fig. 6 shows the change in the shape of the filter's block at different noise level ranges between the proposed Sigma filter and the directional filter [5]. In Fig. 6(a,b), all pixels in the homogeneous direction are assumed to belong to one population and are used in filtering. Fig 6(c,d) illustrates how the proposed filter adapts the block size, shape and weighting to the frame and noise characteristics.

B. Complexity Analysis

This section discusses the hardware implementability of the proposed filter and suggests a hardware-oriented design for it. Fig. 7 shows such possible hardware-oriented design. The filter control parameters W, D and C come from an external noise adaptation unit which compares the noise level σ_η^2 to preset registers initialized to the boundary values b_l^W and b_i^D

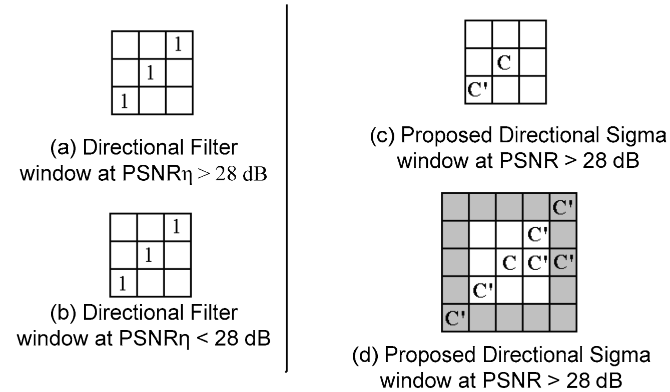


Fig. 6. Fixed directional filter [5] block size compared to the proposed variable block size, shape and weighting for the multi-directional Sigma filter at different noise levels. Pixels in gray belong to the $W = 5$ and not to the $W = 3$ block. The coefficients of these pixels are set to zero to reduce the filter's block size in hardware.

in (11) and (12), respectively. The noise adaptation unit outputs the corresponding levels w_l, h_i and the coefficients C and C' in (14).

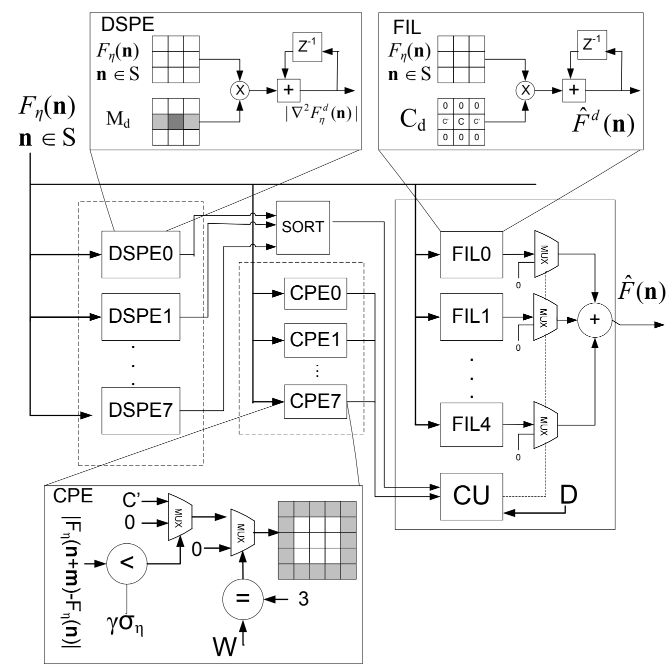


Fig. 7. A possible hardware design of the proposed multi-directional Sigma filter.

The design has three processing engines, namely, direction-selection processing engine (DSPE), coefficient-processing engine (CPE), and directional-filter engine (FIL). The input to the filter is a WxW two dimensional array extracted from a W (e.g., W=5) line delay buffer. At the first stage, the DSPEs compute the directional derivatives using the M_d masks (see Fig. 1) and feed them to a sorting unit. The sorting unit outputs the indices d of the D most homogeneous directions to the control unit (CU) of the filtering stage. Each one of the

FIL engines will output an estimate of $F(\mathbf{n})$ for pixels in the direction d. The CU will pass the multiplexers' outputs which correspond to the selected directions only to the adder. Each one of the filtering engines uses a coefficients mask denoted C_d (e.g., Fig. 6(d)) which is computed for each direction by the CPEs based on the Sigma test in (13), the noise sensitivity parameter γ, and the block size W. The design is based on the larger block size $W=w_1$. If the noise adaptation unit selects a smaller size block (e.g., $W=w_2$) instead, the outermost pixels are set to zero in the CPEs to reduce the mask size without much complexity due to the variable block size.

Table 1 shows the execution time and the time ratio (averaged over images in Fig. 8(a-h) for all noise levels) for the referenced Sigma filters compared to the execution time of the proposed multi-directional Sigma filter to process a 512×512 image when implemented using C++ under an Intel(R) Xeon(TM) CPU 2.40GHz machine running Linux. As shown in the table, the proposed multi-directional Sigma filter is faster than the conventional Sigma [1] and recursive Sigma filters [2]. It is more than three times faster than the modified Sigma filter [11] while producing higher gain.

TABLE I
TIME COMPLEXITY COMPARISON.

Algorithm	Time Ratio /512x512 Frame	Time (Seconds) /512x512 frame
Proposed Multi-Directional Sigma	1.00	0.1090
Conventional Sigma [1]	1.44	0.1567
Recursive Sigma [2]	1.67	0.1820
Modified Sigma Filter [11]	3.21	0.3489

IV. EXPERIMENTAL RESULTS

For related work comparison, the proposed Sigma filter was compared to other Sigma filters including the conventional Sigma filter [1], the recursive Sigma filter [2], and the modified Sigma filter [11].

To experimentally evaluate the proposed method, the PSNR gain at different noise levels is used. Eight images (Fig 8(a-h)) and five video sequences (Fig. 8(i-m)) are used in simulation. The images are selected to represent different levels of structure. The video sequences are selected to represent different types of video motion and levels of structure. Video sequences *Car* and *Prlcar* represent tracking camera motion. Video sequence *Train* represents translational object motion with fixed camera. Video sequence *Wheel* represents rotational object motion and video sequence *Kiel* represents zoom motion. The images are corrupted with noise levels ranging from 20-40 db PSNR in steps of 5 dB and the video sequences with levels 20-40 dB in steps of 10 dB.

To adapt the proposed method to the noise level as in section II. B, the training set: images *Fingerprint*, *Boat*, *Zelda*, and *Peppers* and video sequences *Flowergarden*, *Hall*, and *Coastguard,* is used.

A. Implementation Parameters

To maintain the fast algorithmic speed of the filter during execution, the block size is adapted to the noise level using (12). As shown in Fig. 9, B can be divided into two B_l^W intervals and two corresponding block sizes w_l. The first range $B_1^W = [b_0^W, b_1^W)$ = [20, 28) and the corresponding block size is $w_1 = 5$ (note in Fig. 9 the higher gain of the W=5 over the W=3 in this interval) and the second range $B_2^W = [b_1^W, b_2^W)$ = [28,∞) and the corresponding block size is $w_2 = 3$. Similarly, the maximum number of directions D is adapted as in (11) using $B_1^D = [b_0^D, b_1^D)$ = [20, 22) and $h_1 = 5$, $B_2^D = [b_1^D, b_2^D)$ = [22, 32) and $h_2 = 3$, and $B_3^D = [b_2^D, b_3^D)$ = [32,∞) and $h_3 = 1$. γ in (13) can range from 1.5 to 2.7 and it controls the sensitivity to noise. γ is set to 2 because it provides a compromise between noise reduction and structure preservation.

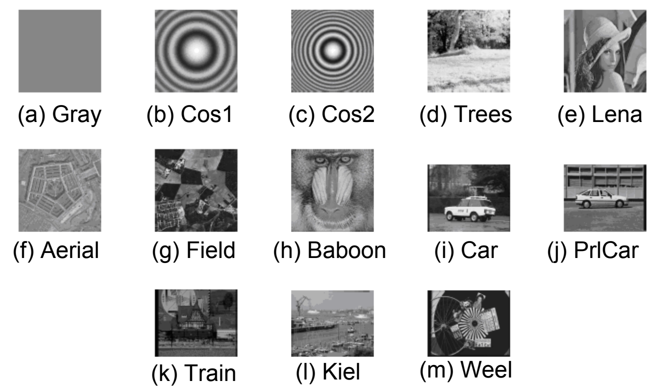


Fig. 8. Images and video sequences used in simulation.

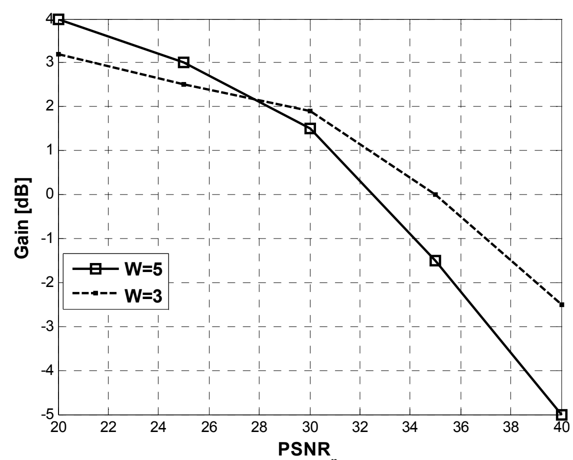


Fig. 9. Average PSNR gain at different noise levels for a W = 3 and a W = 5 block sizes. This figure represents an example of the training results used to obtain the intervals B_l^W for block size adaptation to noise.

B. PSNR Gain

The average gain (in PSNR dB) over time for the proposed multi-directional Sigma and referenced Sigma filters [1,2,11] is shown in Fig. 10 averaged for the first 50 frames of the video sequences in Fig. 8(i-m) corrupted with 25[dB] noise. It can be seen that the proposed filter outperforms the referenced methods. Fig. 11 shows a comparison of the gain achieved by the proposed multi-directional Sigma filter and referenced Sigma filter at different noise levels for the test images and

Fig. 12 shows the comparison for test video sequences. The proposed multi-directional Sigma filter outperforms the Sigma filter [1], the recursive sigma filter [2], and the modified Sigma filter [11] in terms of PSNR gain at different noise levels.

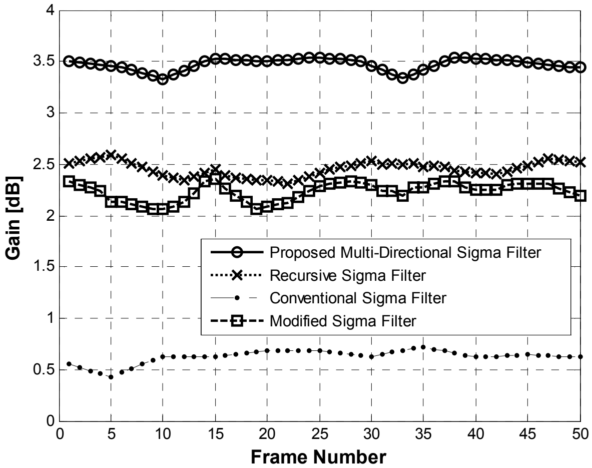


Fig. 10. Gain (in [dB]) over time achieved from applying proposed and referenced methods to 25[dB] noisy video sequences in Fig. 8(i-m).

C. Visual Comparison of the Proposed Sigma Filter

Fig. 13 shows how the proposed multi-directional Sigma filter has improved structure-preservation capabilities compared to the conventional Sigma filter [1] for a 25 dB noisy *Baboon* image. With the conventional Sigma filter, the mustache hair of the Baboon in Fig. 13(a) is merged together due to blurring. The hair lines are more visible in the output of the proposed multi-directional Sigma filter in Fig. 13(b). Fig. 14 shows the subjective improvement in an enlarged highly structured part of the noise-reduced *Lena* image from 20 dB noisy input. It can be seen again that with the proposed multi-directional application of the Sigma filter, the lines separating the hair are more visible in Fig. 14(b). The reason why the proposed filter introduces less blurring than the conventional Sigma filter is the proposed integration of directionality and the Sigma test.

V. CONCLUSION

This paper proposed a multi-directional adaptive spatial Sigma filtering of AWGN in digital TV signals. The adaptation to signal content is achieved through combining most homogeneous directions to tailor a block that best fits image structure. The adaptation to noise level is achieved through controlling the coefficients of the filter using the Sigma probability. The proposed filter is a low cost noise filter suitable for fast noise reduction. It achieves higher gains (up to 4.8 dB PSNR) and is faster than referenced Sigma filters. The proposed filter is analyzed with regard to its hardware-orientation and it was concluded that the proposed filter can be easily implemented in hardware due to its regular block-based parallel design. The MTF was used to measure the relative structure preservation capabilities of the proposed and referenced methods. It was shown that the proposed Sigma filter has a higher contrast transfer ratio than referenced filters indicating improved structure preservation. This is attributed to the multi-directional application of the Sigma test. Moreover, the proposed filter reduces the short tailed noise artifacts caused by the conventional Sigma filter.

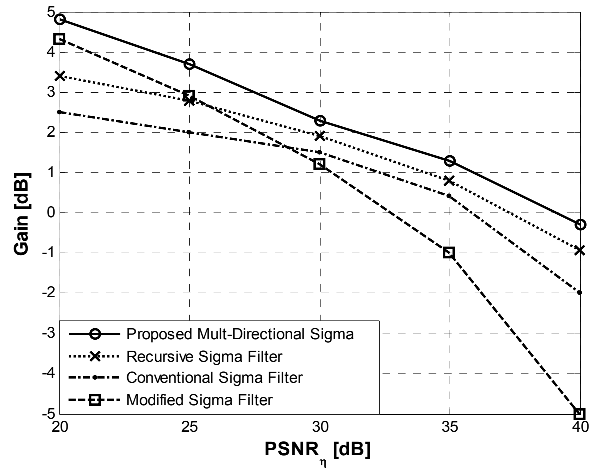


Fig. 11. Applied to images in Fig. 8(a-h), the average gain achieved by proposed multi-directional Sigma, conventional Sigma filter [1], recursive Sigma filter [2], and modified Sigma filter [11].

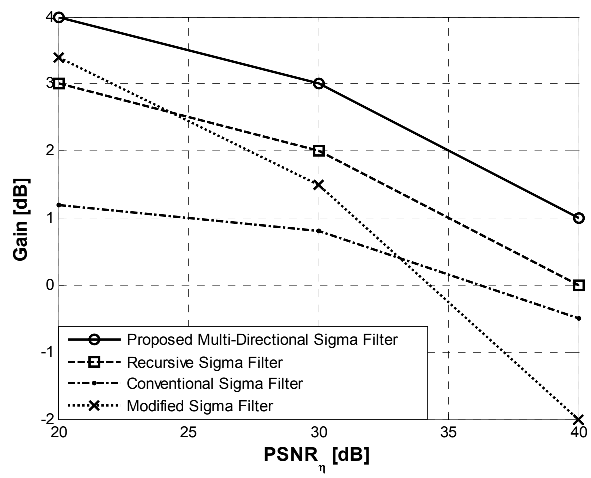
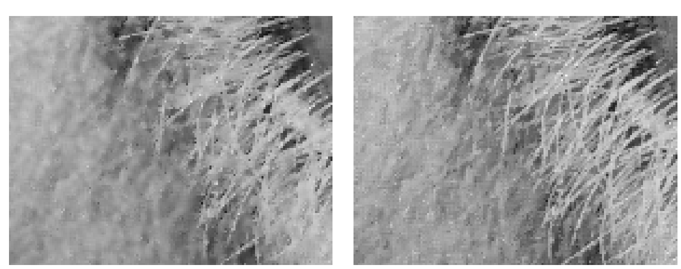
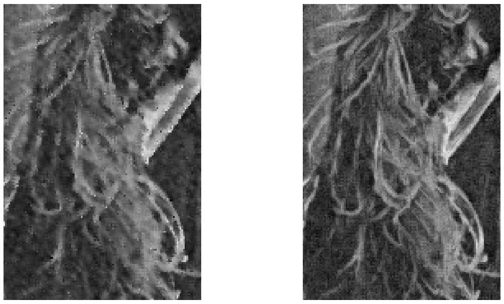


Fig. 12. Applied to video sequences in Fig. 8(i-m), the average gain achieved by proposed multi-directional Sigma, conventional Sigma filter [1], recursive Sigma filter [2], and modified Sigma filter [11].



(a) Sigma filter in [1] **(b) Proposed Sigma**

Fig. 13. Improved structure-preservation in the proposed multi-directional Sigma filter over the conventional Sigma filter for an enlarged part of the noise-reduced *Baboon* image from a 25 dB noisy input.



(a) Sigma filter [1] (b) Proposed Multi-Directional Sigma

Fig. 14. Improved structure-preservation in the multi-directional Sigma filter over the conventional Sigma filter for an enlarged part of the noise-reduced *Lena* image from a 20 dB noisy input.

ACKNOWLEDGMENT

The authors would like to thank Mr. Kumara Ratnayake for his help in the hardware-oriented design section III.B.

REFERENCES

[1] J. S. Lee, "Digital image smoothing and the sigma filter," *Computer Vision, Graphics and Image Processing*, vol. 24, pp. 225–269, 1983.

[2] G. de Haan, T. G. Kwaaitall-Spassova, M. M. Larragy, O. A. Ojo, and R. J. Schutten, "Television noise reduction IC," *IEEE Transactions on Consumer Electronics*, vol. 44, no. 1, pp. 143–154, 1998.

[3] G. de Haan, T. G. Kwaaitaal-Spassova, and O. A. Ojo, "Automatic 2-D and 3-D noise filtering for high quality television receivers," *Proceedings of the 7th International Workshop on Signal Processing and HDTV,* Oct. 1994.

[4] M. Nagao and T. Matsuyama, "Edge preserving smoothing," *Computer Graphics Image Processing*, vol. 9, pp. 394–407, Apr. 1979.

[5] K. Jostschulte, A. Amer, M. Schu, and H. Schröder, "Perception-Adaptive Temporal TV-Noise Reduction Using Contour Preserving Prefilter Techniques", *IEEE Transactions on Consumer Electronics* (TCE), vol. 44, no. 3, pp. 1091-1096, Aug. 1998.

[6] R. Castagno, S. Marsi, and G. Ramponi, "A simple algorithm for the reduction of blocking artifacts in images and its implementation," *IEEE Transactions on Consumer Electronics,* vol. 44, no. 3, pp. 1062–1070, Aug. 1998.

[7] G. Ramponi, "The rational filter for image smoothing," *IEEE Signal Processing Letters*, vol. 3, no. 3, pp. 63–65, March 1996.

[8] F. Cocchia, S. Carrato, and G. Ramponi, "Design and real-time implementation of a 3-D rational filter for edge preserving smoothing," *IEEE Transactions on Consumer Electronics*, vol. 43, no. 4, pp. 1291–1300, Nov. 1997.

[9] A. Amer and E. Dubois, "Fast and reliable structure-oriented video noise estimation," *IEEE Transactions on Circuits and Systems for Video Technolog*y, vol. 15, pp. 113–118, Jan. 2005.

[10] J. S. Lee, "Digital image enhancement and noise filtering by use of local statistics," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. PAMI-2, pp. 165-168, 1980.

[11] Y. Huang and L. Hui, "Adaptive spatial filter for additive Gaussian and impulse noise reduction in video signals," *Int'l Conf. on information Communication and Signal Processing*, vol. 1, pp. 523–526, Dec. 2003.

[12] F. Russo, "A method for estimation and filtering of Gaussian noise in images," *IEEE Transactions on Instrumentation and Measurements*, vol. 52, no. 4, pp. 1148–1154, Aug. 2003.

[13] S.-H. Lee and J. K. Seo, "Noise removal with Gauss curvature-driven diffusion," *IEEE Transactions on Image Processing*, vol. 14, no. 7, pp. 904–909, Jul. 2005.

[14] D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol. 81, no. 3, pp. 425–455, Apr. 1994.

[15] L. Sendurand and I. W. Selesnick, "Bivariate shrinkage functions for wavelet based denoising exploiting interscale dependency," *IEEE Transactions on Signal Processing*, vol. 50, pp. 2744–2756, Oct. 2002.

[16] S. G. Chang, B. Yu, and M. Vetterli, "Adaptive wavelet thresholding for image denoising and compression," *IEEE Transactions on Image Processing*, vol. 9, pp. 1532–1546, 2000.

[17] D. Cho and T. D. Bui, "Multivariate statistical modeling for image denoising using wavelet transforms," *Signal Processing: Image Communication*, vol. 20, pp. 77–89, 2005.

[18] J.-L. Starck, F. D. Murtagh, and A. Bijaoui, *Image Processing and Data Analysis: The Multiscale Approach*, pp. 46–74, Cambridge University Press, 1998.

[19] A. Bovik, *Handbook of Image and Video Processing*, vol. 2, pp. 275–285, ELSEVIER Academic Press, 2005.

[20] G. de Haan, M. Larragy T. G. Kwaaitall-Spassova, and O. A. Ojo, "Memory integrated noise reduction IC for television," *IEEE Transactions on Consumer Electronics*, vol. 42, pp. 175–181, 1996.

[21] Y. Wang, J. Ostermann, Y.-Q. Zhang, Y. q. Zhang, and J. Ostermann, *Video Processing and Communications*, vol. 1, pp. 241–248, Prentice Hall, 2002.

[22] S. Lloyd, "Least squares quantization in PCM," *IEEE Transactions on Information Theory*, vol. 28, pp. 129–137, Mar. 1982.

[23] R. C. Gonzalez and R. E. Woods, *Digital Image Processing - Second Edition*, pp. 230–231, Prentice Hall, 2002.

[24] M. Luxen and W. F¨orstner, "Characterizing image quality: Blind estimation of the point spread function from a single image," *Proceedings of the Photogrammetric Computer Vision and Image Analysis Symposium*, p. A: 205, 2002.

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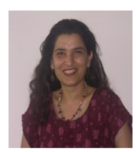

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